PHYSNET

MISN-0-530

ELECTRIC CURRENT DENSITY

Electricity and Magnetism

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

ELECTRIC CURRENT DENSITY

by

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Title: **Electric Current Density**

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Version: 10/18/2001 Evaluation: Stage B0

Length: 1 hr; 8 pages

Input Skills:

1. Vocabulary: thermal equilibrium (MISN-0-157), inelastic collision, thermal gradient, hydrodynamic motion, mass transport, valence electrons, steady state.
2. Given dielectric media in conjunction with conducting surfaces, use appropriate boundary conditions to determine the potential, electric field and displacement in the media (MISN-0-507).
3. Determine the capacitance per unit length of a pair of long, coaxial, cylindrical conducting shells (MISN-0-135).

Output Skills (Knowledge):

K1. Vocabulary: drift motion, thermal motion, conduction current, convection current, resistivity, conductivity, resistance.

K2. State the definition of the current density both in terms of macroscopic quantities only and microscopic quantities only.

K3. State the continuity equation that expresses the conservation of charge.

K4. State Ohm's law both in terms of current density and electric field and in terms of potential difference and current.

Output Skills (Problem Solving):

S1. Given a conducting medium bounded by known potential surfaces, use Ohm's law to calculate the current density, current per unit length, or current in the medium, and the resistance or resistance per unit length of the medium.

External Resources (Required):

1. J. Reitz, F. Milford and R. Christy, *Foundations of Electromagnetic Theory*, 4th Edition, Addison-Wesley (1993).

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1. Introduction

This unit is involved with electric current. After defining electric current and discussing the nature of electric current, the electric current density $\vec{J}(\vec{r})$ is defined. The electric current density $\vec{J}(\vec{r})$ so defined is another vector field which enters directly into the differential equations of electromagnetic theory, that is, Maxwell's equations. One of these differential equations, the equation of continuity, which is essentially a statement of the conservation of electric charge, relates the current density $\vec{J}(\vec{r},t)$ and the charge density $\rho(\vec{r},t)$.

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0\,.$$

Notice that an explicit time dependence is included in the current and charge density.

2. Objectives

You should be able to do each of the following without the aid of books or notes except where explicitly indicated otherwise:

1. Write down a definition or explanation in one or two sentences, using an explanatory equation where appropriate, for each of the following terms and concepts:

   Drift motion

   Thermal motion

   Resistivity, η

   Resistance, R

   Conduction current

   Convection current

   Conductivity, g($\vec{E}$)

2. Write down the definition of the current density $\vec{J}$ in terms of macroscopic quantities as in Eq. 7-5 and microscopic quantities as in Eq. 7-4. Write down the equation of continuity (Eq. 7-9) which expresses the principle of conservation of charge. Write down the empirical relation known as Ohm's law both in terms of the electric field $\vec{E}$ and current density $\vec{J}$ (Eq. 7-10) and in terms of potential difference ΔV and current I (Eq. 7-15).

3. Solve simple problems involving the above concepts and laws. The problems are listed in the Procedures below.

3. Procedures

1. Read Sections 7-1 through 7-3 in the text, including the introduction to Chapter 7. Write down or underline in the text the definitions or explanations of the following terms or concepts:

   Drift motion

   Thermal motion

   Conduction current

   Convection current

   These concepts are explained in the text. Also, write down or underline in the text the definitions or explanations of the following terms or concepts:

   Conductivity, $g(\vec{E})$

   Resistivity, η

   Resistance, R

2. Write down or mark in the text each of the following:

   Definition of current density $\vec{J}$ in terms of macroscopic quantities (Eq. 7-5)

   Definition of current density $\vec{J}$ in terms of microscopic quantities (Eq. 7-4)

   Equation of continuity (Eq. 7-9)

   Ohm's law in terms of $\vec{J}$ and $\vec{E}$ (Eq. 7-10)

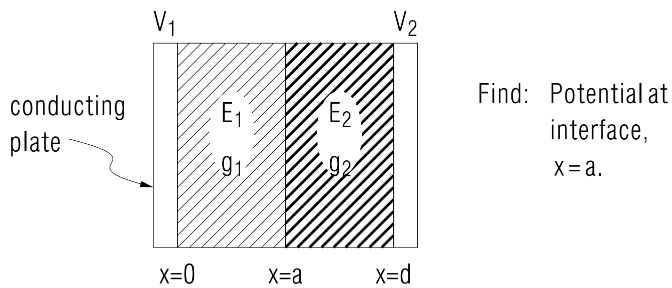
   Ohm's law in terms of ΔV and I (Eq. 7-15)

Be able to explain all quantities and symbols which appear in these definitions, etc.

3. Read the Supplementary Notes for an example of the types of problems you must solve in Procedure 4.
4. Solve the following problems: Problem 7-7, 7-8, 7-10

4. Supplementary Notes

4a. Problem 7-3.



At steady state, $\partial\rho/\partial t = 0 \Longrightarrow \vec{\nabla}\cdot\vec{J} = 0$. This condition on $\vec{J}$ can be used to show that the normal component of $\vec{J}$ is continuous across the interface. So,

$$J_{1n} = J_{2n} \,. \tag{1}$$

The proof is the same as outlined in Sec. 4-7 of the text for the displacement vector $\vec{D}$. From Ohm's law, $\vec{\nabla}\cdot\vec{J} = 0 \Longrightarrow \vec{\nabla}\cdot\vec{E} = 0$. At steady state, $\vec{E} = -\vec{\nabla}V$ so that $\vec{\nabla}\cdot\vec{E} = 0 \Longrightarrow \nabla^2 V = 0$. So V can be written as:

$$V_1(x) = \alpha_1 x + \beta_1 \,,$$
$$V_2(x) = \alpha_2 x + \beta_2 \,,$$

where

$$V_1(x=0) = V_1 \,, \qquad V_1(x=a) = V_2(x=a) = V_3 \,,$$
$$V_2(x=d) = V_2 \,.$$

Thus,

$$V_1 = \beta_1 \,,$$
$$V_3 = \alpha_1 a + V_1 = \alpha_2 a + \beta_2$$
$$V_2 = \alpha_2 d + \beta_2 \,.$$

These equations can be easily solved for α_1, α_2, β_1 and β_2 to get:

$$V_1(x) = \frac{V_3 - V_1}{a}x + V_1 \,,$$

$$V_2(x) = \frac{V_2 - V_3}{d-a}x + V_3 \,.$$

Thus,

$$E_1(x) = -\frac{V_3 - V_1}{a}; \qquad E_2(x) = -\frac{V_2 - V_3}{d-a} \,. \tag{2}$$

But, $J_{1n} = g_1 E_1$ and $J_{2n} = g_2 E_2$. So, Eqs. (1) and (2) give:

$$g_1 E_1 = g_2 E_2 \,,$$
$$g_1 \frac{V_3 - V_1}{a} = g_2 \frac{V_2 - V_3}{d-a} \,,$$
$$\Longrightarrow V_3 = \frac{V_1 g_1 (d-a) + V_2 g_2 a}{g_1(d-a) + g_2 a} \,.$$

You can calculate the free charge density on the interface from

$$D_{1n} - D_{2n} = \sigma \,.$$

4b. Answers to Assigned Problems. 7-8: $\eta = 2.7 \times 10^{-12}\Omega\,\mathrm{m}$

7-10: $\vec{J} = -\frac{g\phi_0}{R}\left(\hat{r}\cos\theta - \hat{\theta}\sin\theta\right)$, for a sphere of radius R.

Acknowledgments

The author would like to thank Illinois State University for support in the construction of this lesson. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.