PHYSNET

MISN-0-509

MAGNETOSTATICS; INDUCTION, VECTOR POTENTIAL

Electricity and Magnetism

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

MAGNETOSTATICS; INDUCTION, VECTOR POTENTIAL

by

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Title: **Magnetostatics; Induction, Vector Potential**

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Input Skills:

1. Vocabulary: conduction current, current density, ampere (MISN-0-530).
2. State Ohm's law in terms of current density and electric field (MISN-0-530).

Output Skills (Knowledge):

K1. Vocabulary: magnetic force, magnetic induction, Lorentz force, magnetic vector potential, magnetic moment of a circuit.

K2. State the expressions for the force and the torque on an infinitesimal element of current-carrying conductor, on a complete circuit, and on a magnetic dipole moment due to a magnetic induction.

K3. State the differential equations satisfied by the magnetic induction and the current density in the case of magnetostatics.

K4. State the expression for the magnetic vector potential and the magnetic induction due to a magnetic dipole moment.

Output Skills (Rule Application):

R1. Given a simple current distribution, determine the magnetic vector potential or use the Biot and Savart law to determine the magnetic induction.

Output Skills (Problem Solving):

S1. Given a current distribution of sufficiently high symmetry, use Ampere's law to determine the magnetic induction.

External Resources (Required):

1. J. Reitz, F. Milford and R. Christy, *Foundations of Electromagnetic Theory*, 4th Edition, Addison-Wesley (1993).

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1. Procedures

1. Read Chapter 8, Sec. 8-1 to 8-7.
2. Write down or underline in the text each of the equations called for in Output Skills K1-K4. Be sure that you can define or explain each of the symbols appearing in the various equations (except for standard mathematical notation like integral signs). The definition of the magnetic Lorentz force $\vec{F}_m$ is given to be: that part of the force exerted on a moving charge, which is neither electrostatic nor mechanical. The mathematical definition of the Lorentz force is given by equation 8-5.
3. Write down, or underline in the text, the equations for force and torque on a circuit with magnetic moment $\vec{m}$ due to a constant magnetic induction. Write down an expression for the vector potential $\vec{A}$ and magnetic induction $\vec{B}$ due to a magnetic dipole $\vec{m}$.
4. Re-read Sec. 8-4 very carefully for applications of the Biot- Savart Law to simple geometrics. These are proto-type calculations. Especially important is the approximation procedure involving the Taylor's series expansions through which equations 8-44 and 8-49 are obtained.
5. Re-read Sec. 8-5 very carefully for applications of Ampere's Law. You may wish to review earlier course work in this area.
6. Read the Supplementary Notes for additional model problems.

7. Solve these problems:

Problem Number	**Type**
8-3	Lorentz force and equation of motion
8-7, 8-8, 8-9, 8-11	Magnetic induction due to simple current distribution - Biot-Savart Law
8-15, 8-16	Magnetic induction due to simple current distribution - Ampere's Law
8-20	Vector potential due to simple current distribution

Hint: For 8-20 first calculate the vector potential due to a sing1e current-carrying wire of finite length. Add to that result, the expression for the current-carrying wire of the same length but oppositely-directed current. Then let the length of the two wires go to infinity together.

2. Supplementary Notes

Problem 8-1: Given the parameters shown in Fig. 1, find the motion of the particle.

Solution: Relevant theory is based on the Lorentz Force Law,

$$\vec{F} = q\vec{V} \times \vec{B}$$

and Newton's Second Law,

$$F = m\frac{d\vec{V}}{dt}\,.$$

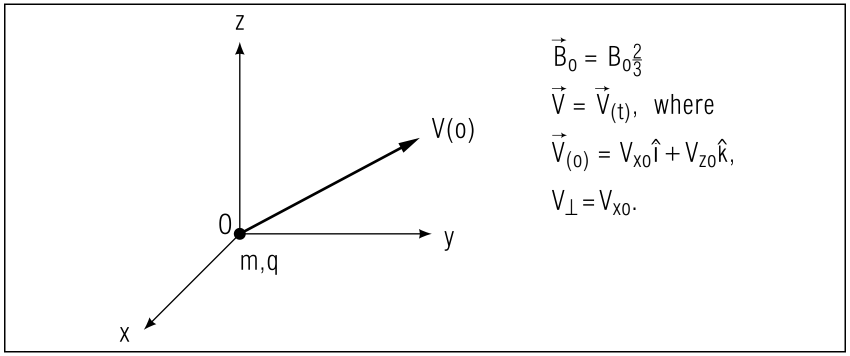

Figure 1. .

Thus, we get three equations from equating the right hand sides of the above two equations and carrying out the indicated cross products.

$$m\frac{dV_x}{dt} = +qV_yB_0 \tag{1}$$

$$m\frac{dV_y}{dt} = -qV_xB_0 \tag{2}$$

$$m\frac{dV_z}{dt} = 0 \tag{3}$$

Differentiating the first two of these three relations again gives

$$m\frac{d^2V_x}{dt^2} = qB_0\frac{dV_y}{dt} = -q^2B_0^2\frac{V_x}{m} \tag{4}$$

and

$$m\frac{d^2V_y}{dt^2} = -qB_0\frac{dV_x}{dt} = -q^2B_0^2\frac{V_y}{m} \tag{5}$$

These last two equations can be written as,

$$\ddot{V}_x + \omega^2V_x = 0 \tag{6}$$

$$\ddot{V}_y + \omega^2V_y = 0 \tag{7}$$

where $\omega = |qB_0/m|$. Solutions of these last two equations are

$$V_x = V_x(t) = A\sin\omega t + B\cos\omega t \tag{8}$$

$$V_y = V_y(t) = A'\sin\omega t + B'\cos\omega t \tag{9}$$

So,

$$V_x(0) = B = V_{x0} \text{ and } V_y(0) = B' = 0 \tag{10}$$

Thus,

$$V_x(t) = A\ sin\omega t + V_{x0}\cos\omega t \tag{11}$$

$$V_y(t) = A'\sin\omega t \tag{12}$$

From eq. (2),

$$-mA'\cos\omega t = qB_0[A\sin\omega t + V_{x0}\cos\omega t]$$

so that,

$$A' = -V_{x0} \text{ and } A = 0$$

Thus,

$$V_x(t) = V_{x0}\cos\omega t \tag{13}$$

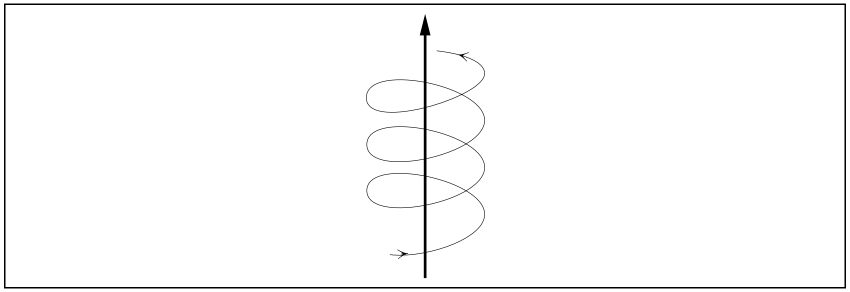

Figure 2. .

$$V_y(t) = -V_{x0}\sin\omega t \tag{14}$$

$$V_z(t) = V_{z0} \tag{15}$$

Eqs. (13), (14), and (15) can be solved to get

$$x(t) = \frac{V_{x0}}{\omega}\sin\omega t$$

$$y(t) = \frac{V_{x0}}{\omega}\cos\omega t$$

$$z(t) = V_{z0}t$$

These are the equations for a helix, the cross section of which is a circle (see Fig. 2.). The radius R is given by:

$$R = \sqrt{x(t)^2 + y(t)^2} = \frac{V_{x0}}{\omega} = \left|\frac{mV_{x0}}{qB}\right|.$$

Problem 8-13: Given the situation in Fig. 3.

Also,

$$B_r = B_r(r,z), B_z = B_z(r,z), B_\theta = 0$$

$$B_r(r,0) = 0, \frac{\partial B_z}{\partial r} < 0$$

And,

$$\vec{\nabla}\times\vec{B} = 0 \text{ in the gap}$$

Solution:

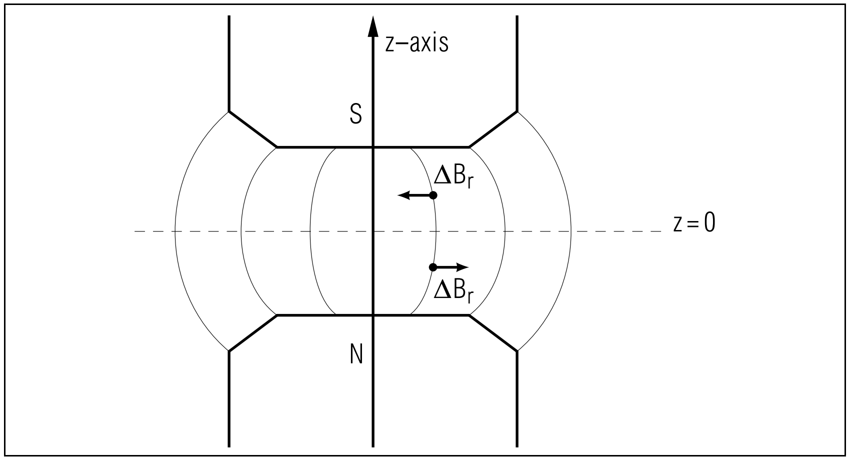

Figure 3. .

a. Now,

$$\vec{\nabla} \times \vec{B} = \hat{a}_\theta \left(\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r} \right) = 0$$

So,

$$\frac{\partial B_r}{\partial z} = \frac{\partial B_z}{\partial r} < 0$$

Then,

$$\Delta B_r = \frac{\partial B_r}{\partial z} \Delta z$$

If $\Delta z > 0 \Rightarrow \Delta B_r < 0$ while $\Delta z < 0 \Rightarrow B_r > 0$. Since $B_r = 0$ on the median plane, the lines tend to bow out as pictured.

b. Locally,

$$\vec{B} = B_r \hat{a}_r + B_z \hat{a}_z$$

and

$$\vec{V} = V_r \hat{a}_r + V_\theta \hat{a}_\theta + V_z \hat{a}_z$$

where $\vec{V}$ is the velocity of a charge drifting from the median plane. If the charge q is positive, one has counterclockwise motion of the charge so $V_\theta < 0$. If the charge is negative, one has clockwise motion so $V_\theta > 0$. So, $qV_\theta < 0$ for both cases. But,

$$\vec{F} = q\vec{V} \times \vec{B}$$

so that

$$F_z = -qV_\theta B_r < 0$$

because $B_r < 0$ above the median plane (from part a). Since, $\Delta z < 0 \Rightarrow$ *is a restoring force in the z-direction.* The same analysis goes through for drift below the median plane where $\Delta z < 0$ since $B_r > 0$ in that region.

Acknowledgments

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