

## REVIEW OF VECTOR ANALYSIS <br> by <br> R. D. Young

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## Title: Review of Vector Analysis

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## Input Skills:

1. Vocabulary: Cartesian coordinates, normal (direction).
2. Take partial derivatives of simple functions.
3. Perform multiple integration on simple functions.
4. Expand any given simple function in a Taylor series (MISN-0-4).

## Output Skills (Knowledge):

K1. Vocabulary: scalar, vector, scalar field, vector field, vector addition and subtraction, multiplication of a vector by a scalar, scalar product, vector product, unit vectors, rectangular unit vectors, spherical unit vectors.
K2. Write the mathematical expressions for these theorems: divergence, Stokes', Green's.

## Output Skills (Rule Application):

R1. Given a scalar field determine its gradient and Laplacian in cartesian and spherical coordinates.
R2. Given a vector field determine its divergence and curl in cartesian coordinates.

## Output Skills (Problem Solving):

S1. Given a table of vector identities involving differential operators, prove any given vector identity.
S2. Solve problems involving basic vector algebra, including vector addition and subtraction, cross products and dot products.

## External Resources (Required):

1. J. Reitz, F. Milford and R. Christy, Foundations of Electromagnetic Theory, 4th Edition, Addison-Wesley (1993).

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## 1. Introduction

You have already studied most of the mathematics needed for this course. However, there are probably some of the more formal vector identities (especially those involving the del operator, $\vec{\nabla}$ ) and vector theorems with which you are not too familiar. This unit will review vector analysis and refresh your memory regarding major vector theorems. The major vector theorems are summarized in Table 1-1. You should refer to this table whenever you meet a problem involving the differential operator, $\vec{\nabla}$.

## 1. Procedure

1. Read chapter 1 of the text (Reitz and Milford).
2. Underline in the text or write out the definitions and explanations of the terms in Output Skill K1.
3. Write down the various gradients, divergence, and curl called for in Output Skills R1 and R2. In particular,

- gradient in rectangular coordinates (eq. 1-20)
- gradient in spherical coordinates (eq. 1-23)
- divergence in rectangular coordinates (eq. 1-31)
- curl in rectangular coordinates (eq. 1-44)

4. Write down and memorize these theorems:

- Divergence Theorem (p. 13 and eq. 1-37)
- Stoke's Theorem (p. 16 and eq. 1-45)
- Green's Theorem (eq. 1-57)

5. Solve the following problems in Reitz and Milford using Table 1-1 and 1-2 where appropriate:

Problem 1-2, 1-16, 1-19, 1-20, 1-22, and 1-26.

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