PHYSNET

MISN-0-497

CALCULUS OF VARIATIONS

Classical Mechanics

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

CALCULUS OF VARIATIONS

by

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Title: **Calculus of Variations**

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Input Skills:

1. Calculate the area of the surface of revolution of a given curve.
2. Write down expressions for the potential and kinetic energies of an oscillator (MISN-0-495) and a linear pendulum (MISN-0-493).

Output Skills (Knowledge):

K1. State the basic problem of the calculus of variations and describe in general terms the method of solution.

K2. Derive Euler's equation for one dependent variable.

K3. Derive the "second form" of Euler's equation applicable when the integrand is not on explicit function of x (the independent variable).

K4. Derive Euler's equations for several dependent variables.

Output Skills (Problem Solving):

S1. Use the variational technique and Euler's equations to solve the brachistochrone problem, to find geodesics on simple surfaces, to find the function which has the minimum surface of revolution, and other such problems.

External Resources (Required):

1. J. Marion, *Classical Dynamics*, Academic Press (1988).

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1. Introduction

One of the most intriguing properties of nature is its apparent fondness for variational principles. Almost every class of physical phenomena from classical mechanics to general relativity can be formulated in terms of a variational principle. In classical mechanics the variational principle is known as Hamilton's Principle while in geometrical optics, it is known as Fermat's Principle. As well as providing a useful tool for the solution of certain kinds of problems, these variational principles provide an elegant new point-of-view from which one can gain new insights into the behavior of physical systems. This unit will cover the mathematical methods of variational calculus while subsequent units will treat Hamilton's principle and its application to mechanics.

2. Procedures

1. Read Sections 6.1 and 6.2 in Marion. Omit Example 6.1. The two examples given in the section 6.2 are a little misleading since they yield extrema only for a special class of trial functions. In general, one wishes to find extrema for arbitrary trial functions and, in fact, one usually does not specify the form of the trial functions.

2. Read section 6-3 in Marion and fill in any missing details. This is the fundamental variational technique and should be studied carefully.

3. Read sections 6.4 and 6.5 in Marion.

   Optional: Read Chapter 17 of Arfken. A number of worked-out examples are given in this reference.

4. Read Example 5.2 in Marion. In the brachistochrone problem, it is assumed that the particle slides on a frictionless surface under the influence of gravity. The problem is to find the shape of the surface that minimizes the travel time.

   Read example 6.4 on geodesics in Marion.

   ▷ Work problems 6-3, 6-4 in Marion.

   ▷ Work problem 6-5 in Marion, using eq. 6.18.

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