PHYSNET

MISN-0-495

LINEAR OSCILLATIONS

Classical Mechanics

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

LINEAR OSCILLATIONS

by

C. P. Frahm

Title: **Linear Oscillations**

Author: C. P. Frahm, Physics Dept., Illinois State Univ

Version: 2/1/2000 Evaluation: Stage B0

Length: 2 hr; 8 pages

Input Skills:

1. Apply Kirchhoff's circuit equation to find the potential drop around a circuit containing inductors and/or capacitors (MISN-0-119).
2. Define acceleration as a time derivative of the position coordinate for a single particle (MISN-0-493).
3. Write down a second order differential equation for the position coordinate of a single particle acted upon by several external forces (MISN-0-493.

Output Skills (Knowledge):

K1. Vocabulary: simple harmonic motion, damped oscillation.

Output Skills (Problem Solving):

S1. Set up and solve the differential equation of a simple harmonic oscillator with given initial conditions. Interpret the various symbols and quantities and represent the solutions on a phase diagram.

S2. Set up and solve the differential equation of a damped oscillator. Specify the results for the three cases of underdamped, critically damped and overdamped motion.

S3. Obtain the current as a function of time for an electrical oscillator without the use of the analogy between mechanical and electrical oscillations and/or with the use of such an analogy.

External Resources (Required):

1. J. Marion, *Classical Dynamics*, Academic Press (1988).

THIS IS A DEVELOPMENTAL-STAGE PUBLICATION
OF PROJECT PHYSNET

The goal of our project is to assist a network of educators and scientists in transferring physics from one person to another. We support manuscript processing and distribution, along with communication and information systems. We also work with employers to identify basic scientific skills as well as physics topics that are needed in science and technology. A number of our publications are aimed at assisting users in acquiring such skills.

Our publications are designed: (i) to be updated quickly in response to field tests and new scientific developments; (ii) to be used in both classroom and professional settings; (iii) to show the prerequisite dependencies existing among the various chunks of physics knowledge and skill, as a guide both to mental organization and to use of the materials; and (iv) to be adapted quickly to specific user needs ranging from single-skill instruction to complete custom textbooks.

New authors, reviewers and field testers are welcome.

LINEAR OSCILLATIONS

by

C. P. Frahm

1. Introduction

Among the simple systems encountered in mechanics, the harmonic oscillator and its modifications (for example, with damping and driving forces) are by far the most important. This follows from the fact that any system describable by a potential function and undergoing small amplitude oscillation near equilibrium behaves like a simple harmonic oscillator. Hence, a thorough investigation of harmonic oscillators and the effects of damping and driving forces on such oscillators constitutes an essential part of any study of mechanical systems. This unit reviews some (perhaps already familiar) material on simple harmonic oscillators and the effects of damping forces which are linear in the velocity.

2. Procedures

1. Read Appendix C of Marion, *Ordinary Differential Equations of Second Order*.

   Note that:

   1. The general solution of the homogeneous equation is always one of the forms given in equations C.10 and C.11 where the r's are given by the auxiliary equation.
   2. The general solution of the inhomogeneous equation is always the sum of the general solution of the homogeneous equation (the complementary function) and any solution of the inhomogeneous equation (a particular integral or solution).

   ▷ Work problems C-1 and C-2 at the end of Appendix C in Marion.

2. Read Sect. 3.1, 3.2, and 3.4 of Marion.

   ▷ Work problems 3-1, 3-3 and 3-6 in Marion.

   Optional: Read Sect. 3.3 of Marion.

3. Read section 3-5 of Marion.

   ▷ Work problems 3-2 and 3-11 in Marion.

4. Read section 3-8 of Marion.

   ▷ Work problems 3-27 and 3-29 in Marion.

5. ▷ Work problems 3-12 and 3-26 in Marion. (Hint for 3-26: See Wylie, pp. 79-80.)

▷ Exercise - A weight of 128 lb hangs from a spring of modulus 75 lb/in. The damping in the system is 28 percent of critical. Determine the motion of the weight if it is pulled downward 2 in. from its equilibrium position and suddenly released.

▷ Exercise - A 1-μf condenser with an initial charge $Q_0 = 10^{-5}$ coulomb is discharged through a resistance of 120 ohms and an inductance of 0.01 henry connected in series with it. Find the current as a function of time.

Acknowledgments

The author would like to thank Illinois State University for support in the construction of this lesson. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.