PHYSNET

MISN-0-494

NEWTONIAN SYSTEM OF PARTICLES

Classical Mechanics

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

NEWTONIAN SYSTEM OF PARTICLES

by
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Title: **Newtonian System of Particles**

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Input Skills:

1. Vocabulary: momentum, angular momentum, potential energy (MISN-0-493).
2. Evaluate the gradient of a given function (MISN-0-492).
3. Evaluate simple line integrals (MISN-0-492).

Output Skills (Knowledge):

K1. Define center of mass.

K2. State Newton's law of universal gravitation. Define: gravitational field, gravitational potential, lines of force, equipotential surfaces.

K3. Show each of the following for a system of particles: the total momentum is equal to the momentum of the center of mass, the total angular momentum is equal to the angular momentum of the center of the center of mass plus the angular momentum about the center of mass, the total kinetic energy is equal to the kinetic energy of the center of mass plus the kinetic energy relative to the center of mass.

K4. Derive from Newton's laws each of the following for a system of particles: (a) the total external force acting on the system is equal to the time rate of change of the total momentum. (b) the total external torque is equal to the time rate of change of the total angular momentum. (c) the total work done on the system by non-conservative forces is equal to the change in total mechanical energy.

K5. Starting from Newton's laws derive the law governing the motion of a system with variable mass.

External Resources (Required):

1. J. Marion, *Classical Dynamics*, Academic Press (1988).
2. D. T. Greenwood, *Principles of Dynamics*, Prentice-Hall (1965).

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1. Introduction

Physical systems seldom contain only a single particle. Instead there are usually many particles making up larger objects whose size or internal structure cannot be ignored. The larger objects may be rotating, colliding with each other or flying apart. One can in principle apply Newton's laws as formulated in the preceding unit to each separate particle of each object and proceed to make calculations. Unfortunately for most systems of interest there are so many particles (molecules, for example) to deal with that this procedure is impractical. However, it is still possible to utilize Newton's mechanics in a very general and powerful way for such multi- particle systems. By introducing the concept of the center of mass of a system of particles, it is possible to deduce some very useful theorems regarding the motion of the system as a whole without having to deal with each separate particle. This unit is concerned with such analyses of multi-particle systems along with a review of some aspects of Newton's law of gravitation.

2. Procedures

1. a. Study Equations 9.2 to 9.3 of Marion.

   b. For the second, third, and fourth items in Output Skill K3 (momentum, angular momentum and kinetic energy), the total for the system is just the sum (scalar or vector, whichever is appropriate) of the corresponding quantities for the individual particles. In each case, the total can be expressed as the sum of two terms one associated with the motion of the center of mass (denoted by a subscript *com*) and another associated with the motion of the particles relative to the center of mass (denoted by a subscript *rel*). Thus for momentum, one has:

$$\vec{P} \equiv \sum_\alpha \vec{P}_\alpha = \sum_\alpha m_\alpha \dot{\vec{r}}_\alpha = \sum_\alpha m_\alpha (\dot{\vec{R}} + \dot{\vec{r}}_\alpha)$$

$$= \left(\sum_\alpha m_\alpha\right) \dot{\vec{R}} + \sum_\alpha m_\alpha \dot{\vec{r}}_\alpha = M\dot{\vec{R}} = M\vec{V} = \vec{P}_{com}\,.$$

The quantity M is the total mass of the system while V is the velocity of the center of mass. The expression $\sum_\alpha m_\alpha \vec{r}_\alpha$ is zero since:

$$\sum_\alpha m_\alpha \vec{r}_\alpha = \sum_\alpha m_\alpha (\vec{r}_\alpha - \vec{R}) = \sum_\alpha m_\alpha \vec{r}_\alpha - \sum_\alpha m_\alpha \vec{R}\,,$$

$$M\vec{R} - M\vec{R} = 0\,.$$

It should be noted that:

$$\vec{P}_{rel} = 0\,.$$

   c. Read Marion from eq. (9.13) to Example 9.2 and also Sect. 9.4 through Eq. 9.23 and note that:

$$\vec{L}_{com} = \vec{R} \times \vec{P}\,,$$

$$\vec{L}_{rel} = \sum_\alpha \vec{r}_\alpha \times \vec{p}_\alpha, \quad \vec{p}_\alpha \equiv m_\alpha \dot{\vec{r}}_\alpha\,.$$

   d. Read Marion Eq. 9.36 through Eq. 9.39 and note that:

$$T_{com} = \frac{1}{2} M V^2\,,$$

$$T_{rel} = \sum_\alpha \frac{1}{2} m_\alpha v_\alpha^2\,.$$

2. The simplest way to derive these laws is to make use of the results of the previous unit and sum over the particles of the system. (Marion seems to go through a lot of unnecessary gyrations). Recommended derivations are outlined below.

   a. From MISN-0-493 (Newton's second law for the α-th particle):

$$\vec{F}_\alpha = \frac{d\vec{P}_\alpha}{dt}\,.$$

Summing on all particles gives:

$$\sum_\alpha \vec{F}_\alpha = \sum_\alpha \frac{d\vec{P}_\alpha}{dt} = \frac{d}{dt}\sum_\alpha \vec{P}_\alpha = \frac{d\vec{P}}{dt}\,.$$

Now separate $\vec{F}_\alpha$ into an external part and an internal part (see Marion Section 9.3 through eq. (9.13):

$$\vec{F}_\alpha = \vec{F}_\alpha^{(\ell)} + \sum_\beta \vec{f}_{\alpha\beta}\,.$$

Then:

$$\sum_\alpha \vec{F}_\alpha = \sum_\alpha \vec{F}_\alpha^{(\ell)} + \sum_{\alpha,\beta} \vec{f}_{\alpha\beta} = \sum_\alpha \vec{F}_\alpha^{(\ell)} \equiv \vec{F}_\alpha^{(\ell)} .$$

Hence:

$$\vec{F}_\alpha^{(\ell)} = \frac{\vec{P}}{dt} .$$

b. From MISN-0-493 (Torque law for the α-th particle)

$$\vec{N}_\alpha = \frac{d\vec{L}_\alpha}{dt} .$$

Summing on all particles

$$\sum_\alpha \vec{N}_\alpha = \sum_\alpha \frac{d\vec{L}_\alpha}{dt} = \frac{d}{dt} \sum_\alpha \vec{L}_\alpha = \frac{d\vec{L}}{dt} .$$

As in part (a):

$$\sum_\alpha \vec{N}_\alpha = \sum_\alpha \vec{r}_\alpha \times \vec{F}_\alpha = \sum_\alpha \vec{r}_\alpha \times (\vec{F}_\alpha^{(\ell)} + \sum_\beta \vec{f}_{\alpha\beta})$$

$$= \sum_\alpha \vec{r}_\alpha \times \vec{F}_\alpha^{(\ell)} + \sum_{\alpha,\beta} \vec{r}_\alpha \times \vec{f}_{\alpha\beta} = \sum_\alpha \vec{r}_\alpha \times \vec{F}_\alpha^{(\ell)} = \sum_\alpha \vec{N}_\alpha^{(\ell)} \equiv \vec{N}^{(\ell)} .$$

Thus:

$$\vec{N}^{(\ell)} = \frac{d\vec{L}}{dt} .$$

c. From MISN-0-493 (Mechanical energy law the α-th particle):

$$W_{n,\alpha} = \Delta E_\alpha .$$

Summing over all particles:

$$\sum_\alpha W_{n,\alpha} = \sum_\alpha \Delta E_\alpha = \Delta \sum_\alpha E_\alpha ,$$

or

$$W_n = \Delta E ,$$

where

$$W_n = \sum_\alpha W_{n,\alpha} \text{ and } E = \sum_\alpha E_\alpha .$$

Note: W_n includes all work done by non-conservative forces whether internal or external. Similarly, E contains all potential energies whether internal or external.

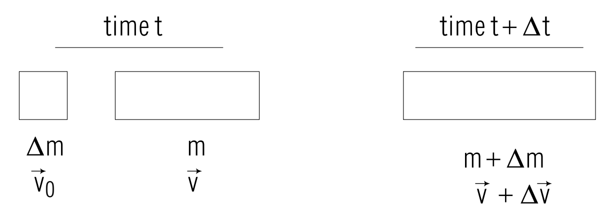
Note: Other laws such as the impulse law or the conservation laws can now be easily derived as in the single particle case.

3. Within the context of classical mechanics the mass of a particle is constant. Hence if one wishes to consider an object whose mass is variable, then that object should, strictly speaking, be thought of as a system of particles rather than as a single particle. However, if that one object is the dominant one under discussion, then it is often convenient to use a pseudo-particle formulation of Newtonian mechanics which treats the object as a single entity with a variable mass. Such a formulation can be derived from the impulse law for a system of particles. For a small time interval Δt this law takes the form:

$$\vec{F}^{(\ell)} \Delta t \approx \Delta \vec{P} .$$

Although the result is the same, it is convenient to consider two cases, one when the object gains mass and one where the object loses mass.

Case 1. Object gains mass (change in mass $= \Delta m > 0$):

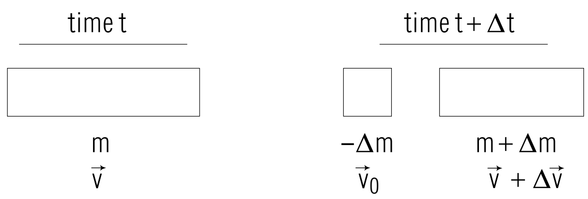


$$\vec{F}^{(\ell)} \Delta t \approx (m + \Delta m)(\vec{v} + \Delta \vec{v}) - (m\vec{v} + \Delta m \vec{v}_0) \approx m \Delta \vec{v} + (\vec{v} - \vec{v}_0) \Delta m .$$

Thus,

$$\vec{F}^{(\ell)} = m \frac{d\vec{v}}{dt} + (\vec{v} - \vec{v}_0) \frac{dm}{dt} .$$

Case 2. Object loses mass (change in mass $= \Delta m < 0$):



$$\vec{F}^{(\ell)}\Delta t \approx [(m+\Delta m)(\vec{v}+\Delta\vec{v}) - \Delta m\vec{v}_0] - m\vec{v} \approx m\Delta\vec{v} + (\vec{v}-\vec{v}_0)\Delta m\,.$$

Thus:

$$\vec{F}^{(\ell)} = m\frac{d\vec{v}}{dt} + (\vec{v}-\vec{v}_0)\frac{dm}{dt}\,.$$

Note: $\vec{v}$ is the velocity of the new mass *just before* it is added to the object in Case 1, and it is the velocity of the ejected mass *just after* it is ejected in Case 2.

4. If one wants to use Newton's second law in the predictor role to which we have assigned it, it is necessary to know in advance the force(s) acting on a particle. This knowledge can be obtained in one of two ways: first by measuring the force, that is, using the definition of force in terms of the standard mass; and second, by calculating the value of the force from another fundamental law (assuming such a law is known). Within the context of classical mechanics, there is only one type of force for which the force law is known (at least to a good approximation). This is, of course, gravitation and the force law is again due to Newton. There are three other known basic forces in nature: the electromagnetic force, the strong nuclear force and the weak nuclear force. Only for the first of these have we been successful in determining the force law (i.e. the Lorentz force).

   Read Marion, sections 5.1 - 5.3.

5. ▷ Work problems 2-50, 5-7 and 5-8 in Marion, see Example 5.3.

   Read section 4-7 of Greenwood.

   ▷ Work problems 4-1, 4-2, 4-7 and 4-9 in Greenwood.

   Bring all problem solutions to the exam.

Acknowledgments

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