PHYSNET

MISN-0-492

VECTOR ANALYSIS

Classical Mechanics

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

VECTOR ANALYSIS

by

C. P. Frahm

Title: **Vector Analysis**

Author: C. P. Frahm, Physics Dept., Illinois State Univ

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Input Skills:

1. State the transformation properties of an arbitrary vector in matrix and/or component form (MISN-0-491).
2. State an expression for the total differential of a function of several variables in terms of the partial derivatives of the function.

Output Skills (Knowledge):

K1. Vocabulary: gradient, line integral, surface integral, volume integral.

K2. Define the derivative of a vector with respect to a scalar in terms of components, and give an alternative geometrical expression in terms of a limit. Write down the chain rule for the derivative of a vector product and a scalar product of two vectors.

K3. Define what is meant by the following: line integral, surface integral, volume integral.

K4. Derive general expressions for velocity and acceleration in cartesian, cylindrical and spherical coordinate systems, and also in terms of tangential and normal components.

Output Skills (Rule Application):

R1. Evaluate the gradient of a given scalar function.

Output Skills (Problem Solving):

S1. Given the components of two vectors calculate the angle between them.

S2. Evaluate simple line integrals where the line of integration is a straight line.

External Resources (Required):

1. J. Marion, *Classical Dynamics*, Academic Press (1988).
2. D. T. Greenwood, *Principles of Dynamics*, Prentice-Hall (1965).

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New authors, reviewers and field testers are welcome.

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1. Introduction

Many of the topics in this unit should be familiar to you from past courses in physics and mathematics. Hence this unit constitutes a review (with a slight extension) of the basic operations with vectors. The component (subscripted) notation will be extensively used since it lends itself most easily to discussions of transformations and can be extended to higher rank tensors. Vector algebra and some elements of vector calculus will be covered.

2. Procedures

1. Read Sections 1.10-1.12 of Marion. You may also find helpful some review reading from your General Physics text helpful.

   ▷ Work problems 1-7, 1-17 and 1-24 in Marion.

2. Read Sections 1.13-1.16 of Marion.

   Write down answers to all three parts of Output Skill K2.

   ▷ Work problem 1-27 in Marion.

3. Read Sections 1.14-1.15 of Marion. Marion does some of the analysis for motion in a plane. For motion in three dimensions it is convenient to extend the discussion of section 1.15 to the time derivative of a unit vector. A very good account of this is given on pages 34 - 39 of Greenwood. Greenwood also gives some examples on pp. 40 - 42.

   Read pages 34 - 42 of Greenwood.

   Sketch appropriate figures and derive the velocity and acceleration expressions for the four cases in Output Skill 3. Outlines of two of the derivations are appended to this outline. See your instructor if you need further assistance.

4. Read Section 1.16 of Marion.

   Write out definitions of the quantities in Output Skill K1.

   ▷ Work problems 1-30 and 1-31 in Marion.

5. Read Section 1.17 in Marion.

   Write out definitions of the three types of integrals in terms of limits of sums (not done in text).

   ▷ Work problems 1-32 and 1-33 in Marion. Note that these are special line integrals evaluated along the straight line "time axis."

Acknowledgments

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