PHYSNET

MISN-0-488

CAUCHY INTEGRAL THEOREM, LAURENT SERIES AND RESIDUE

Math

Physics

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

CAUCHY INTEGRAL THEOREM,

LAURENT SERIES AND RESIDUE

by

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Title: **Cauchy Integral Theorem, Laurent Series and Residue**

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Version: 10/19/2000 Evaluation: Stage B0

Length: 2 hr; 9 pages

Input Skills:

1. Vocabulary: Taylor series, analyticity, function of complex variable, Cauchy-Riemann condition, partial differential equations, complex number, singularity, analyticity, complex function derivative, De Moiure's formula.
1. Unknown: assume (MISN-0-487).

Output Skills (Knowledge):

K1. Vocabulary: contour integral, Taylor series, Laurent senses, isolated singularity, pole of order n, simple pole, removable singularity, essential singularity, residue, double pole, simply connected region or domain, multiply connected region or domain.

K2. Write, and explain with a diagram, Cauchy's integral theorem for simply and multiply connected regions.

K3. Prove that $\oint z^{m-n-1}\,dz = 2\pi\,i\,\delta_{mn}$ when the contour encircles the origin and is counterclockwise.

Output Skills (Rule Application):

R1. Apply Cauchy's integral theorem and Cauchy's integral formulas to evaluate integrals in the complex plane.

R2. Determine Taylor's and Laurent's series for a given function: (a) using the Cauchy integral formulas; (b) using known expansions for common functions.

R3. Classify the singularities of a given function according to whether they are isolated singularities, poles (give orders n), branch points, removable singularities, or essential singularities. Calculate the residues at each pole.

External Resources (Required):

1. G. Arfken and M. Spiegel (see *ID Sheet* for MISN-0-487).

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New authors, reviewers and field testers are welcome.

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1. Introduction

In this unit, you will continue to explore the implications of analyticity of a function of a complex variable. In the previous unit, the main implication of analyticity appeared in the form of the Cauchy- Riemann conditions which are partial differential equations connecting the real and imaginary parts of an analytic formation. In this unit, you will see the Cauchy Integral Theorem and the Cauchy integral formulas which are integral relations satisfied by analytic functions. In this unit, you will also study Taylor and Laurent series for analytic functions as well as the concept of residue.

2. Procedures

1. Read these sections and pages in Arfken:

   Sec. 6.3, Cauchy's Integral Theorem.

   Sec. 6.4, Cauchy's Integral Formulas (This includes Morera's Theorem).

   Sec. 6.5, Taylor and Laurent Series.

   Sec. 7.1, Singularities.

   Sec. 7.2, Residues (to 9th line from top).

2. Read these sections and pages in Spiegel:

Section Title	Page
Integrals	287
Cauchy's Theorem*	287-288
Cauchy's Integral Formulas	288
Taylor's Series	288
Singular Points	288
Poles	288-289
Laurent's Series	289
Residues	289

* This should be called Cauchy's Integral Theorem

3. Write down or underline in the texts the definitions and concepts called for in Output Skill K1.

4. Write down Cauchy's Integral Theorem for simply connected and multiply-connect regions. Write down Morera's Theorem. Write down the Cauchy integral formulas. Be sure to include an explanatory diagram as well as any conditions regarding continuity and analyticity.

5. Read these Solved Problems in Spiegel:

   13.11 (Cauchy's Integral Theorem for simply connected regions)

   13.12 (Cauchy's Integral Theorem for multiply connected regions)

   13.13 (Proof of result in Output Skill K3)

   13.15 (Cauchy's Integral Formula)

   13.16 (Evaluating contour integrals using Cauchy's integral formulas)

   13.17 (Evaluating contour integrals using Cauchy's integral formulas)

   13.20 (Singularities)

   13.22 (Laurent series and singularities)

   13.23 (b only) (Residues)

   13.24 (Residues)

6. Solve these problems in Arfken:

   6.5.1 (Taylor series)

   6.5.2 (Taylor series. Use the Cauchy integral formulas)

   6.5.10 (Laurent series)

7. Solve these Supplementary Problems in Spiegel:

   13.70 (Application of Cauchy's Integral Theorem)

   13.72 (Application of Cauchy's Integral Theorem)

   13.73 (Application of Cauchy's Integral Formulas)

   13.79 (Singularities)

   13.80 (Laurent series)

   13.81 (Laurent series)

   13.83 (Residues)

13.84 (Residues)

* The binomial expansion is:

$$(1+z)^m = \sum_{p=0}^{m} \binom{m}{p} z^p$$

where

$$\binom{m}{p} \equiv \frac{m!}{p!(m-p)!}\,.$$

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