PHYSNET

MISN-0-482

FOURIER ANALYSIS - SERIES: PART I

Math Physics

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

FOURIER ANALYSIS - SERIES: PART I

by

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Title: **Fourier Analysis - Series: Part I**

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Input Skills:

1. Vocabulary: periodic function, trigonometric function, sine, cosine, sound waves, electromagnetic waves, Taylor series.
2. Unknown: assume (MISN-0-481).

Output Skills (Knowledge):

K1. Write a definition or explanation of the following terms: periodic function, period, Fourier series or expansion, Fourier coefficients, Dirichet conditions, odd and even functions, half-range Fourier sine and cosine series.

Output Skills (Rule Application):

R1. Graph periodic functions when given the algebraic form of the function over one period and the period. Also, test whether a function is even or odd, given an algebraic or graphical statement of the function.

R2. Compute Fourier coefficients and write down the appropriate Fourier series when given the function.

R3. Test whether a function satisfies Dirichet conditions. Write the limit of the appropriate Fourier series at a point, given that the function satisfied Dirichet conditions.

External Resources (Required):

1. G. Arfken, Mathematical Methods for Physicist, Academic Press (1995).
2. Schaum's Outline: Murray Spiegel, *Theory and Problems of Advanced Mathematics for Scientists and Engineers*, McGraw-Hill Book Co. (1971).

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1. Introduction

Next to calculus for scalars and vectors, the most used mathematical tool in undergraduate physics is the theory of Fourier analysis. The Fourier series were discovered by Jean Baptiste Joseph Fourier (1768-1830), a French mathematician and physicist, during a study of heat conduction. These highly useful series enter into any analysis involving wave phenomena including sound waves, electromagnetic waves and matter waves in quantum mechanics as well as heat conduction problems. The list of applications could be extended even more. These series are useful in a theoretical sense and as a practical calculational technique when only the first few terms in the series are important. The numerical evaluation of series by means of Fourier analysis also led to new insights into number theory and real analysis and spurred dramatic advances in pure mathematics. Later in the course you will solve several practical physics problems using these series.

2. Procedures

1. Read pages 182 to 184 of Spiegel. Stop at the section entitled, "Complex Notation for Fourier Series." Read sections 14.1-14.3 of Arfken. You will not be tested on Laurent series and Sturm-Liouville theory.

2. Underline in the text or write out the definitions and explanations of the terms and concepts of Output Skill K1 using an explanatory equation where necessary. One or two sentences should be sufficient.

3. Memorize and write from memory Theorem 7-1. The conditions (1) to (3) in Theorem 7-1 are called Dirichlet conditions on the function.

4. Read through these Solved Problems in Spiegel:

7.1 (Graphing)	7.7 (Evaluation of series)
7.2 (Miscellaneous)	7.8 (Odd and even functions)
7.3 (Miscellaneous)	7.11 (Half range cosine series)
7.5 (Fourier series)	7.12 (Half range cosine and sine series)
7.6 (Fourier series)	

5. Solve these problems in the section on Supplementary Problems of Spiegel:

   7.26 (Graphing only)
   7.32 (Half range cosine and sine series)
   7.33 (Evaluation of series)

6. Solve these problems in Arfken:

   14.1.2 (Fourier series)
   14.2.2 (Half range series)
   14.3.1 (Fourier series)
   14.3.4 (Fourier series)*

* The correct answer to part (b) should be,

$$x = \frac{2L}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin\left(\frac{n\pi x}{L}\right).$$

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