PHYSNET

MISN-0-475

CLASSICAL TESTS OF GENERAL RELATIVITY

Relativity

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

CLASSICAL TESTS OF GENERAL RELATIVITY

by

C. P. Frahm

Title: **Classical Tests of General Relativity**

Author: C. P. Frahm, Dept. of Physics, Illinois State Univ.

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Input Skills:

1. Unknown: assume (MISN-0-474).

Output Skills (Knowledge):

K1. Derive the formulae for the three classical tests of general relativity using the Schwarzschild metric and summarize the observational evidence pertinent to each: (a) gravitational Doppler (or red) shift; (b) anomalous planetary perihelion advance; (c) gravitational deflection of light.

Output Skills (Problem Solving):

1. Solve problems of the type given in the Procedures.

External Resources (Required):

1. W. Rindler, *Essential Relativity*, van Nostrand (1977).

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1. Introduction

It is probably fair to say that general relativity is distinguished by its elegance, its mathematical complexity and its experimental truculence. Einstein himself proposed three tests - the perihelion advance of planets, the bending of light as it passes near the sun and the gravitational red-shift. These are known as the classical tests and were for a long time the only tests amenable to observation. In recent years there have been proposed a few new tests including radar echo delay and gyroscope precession. Unfortunately, however, most of these tests are not sufficiently precise with present technology to unambiguously verify general relativity and reject other similar theories that have been proposed in the last 50 years. However, it does appear that the results of these tests favor a geometric theory of gravitation (like general relativity) instead of the Newtonian theory. Furthermore, none of these tests give results inconsistent with general relativity. This unit will be devoted to the three classical tests.

2. Procedures

1. a. Read Rindler, pp. 143-144.

   ▷ Exercise - Fill in any missing details in the analysis leading to eq. 8.51 of Rindler.

   Hint: It is convenient to think of an oscillator or atomic radiator positioned at the point 1 and observed at point 2. If the oscillator emits N pulses in coordinate time Δt_1 then those N pulses must arrive at point 2 in the coordinate time Δt_2. Thus the oscillator's proper frequency (number of oscillations per unit proper time of the oscillator) is given by

$$\nu_1 = \frac{N}{\Delta S_1}$$

   where ΔS_1 is the proper (or standard clock) time interval at point 1 corresponding to the coordinate time interval Δt_1. On the other

hand the frequency observed at point 2 is

$$\nu_2 = \frac{N}{\Delta S_2}$$

where ΔS_2 is the proper (or standard clock) time interval at point 2 corresponding to the coordinate time interval Δt_2. The frequency ratio is then given by

$$D = \frac{\nu_1}{\nu_2} = \frac{\Delta S_2}{\Delta S_1}$$

It is now only necessary to express ΔS_1 and ΔS_2 in terms of Δt_1 and Δt_2 and use the result of eq.8.50 that $\Delta t_1 = \Delta t_2$.

Read section 38.5 in Misner, Thorne and Wheeler

b. Read Rindler pp. 143 - 146.

▷ Exercise - Start with the geodesic equations in MISN-0-474 (use $\lambda = s$) and show that

a) if a particle initially moves in the $\theta = \pi/2$ plane (i.e. $\theta = \pi/2$, $d\theta/dS = 0$ at the initial instant) then it remains in that plain (i.e. $d^n\theta/dS^n = 0$).

b) the geodesic equations for r and ϕ lead to Rindler's eqs. 8.52 and 8.53 when the motion is in the $\theta = \pi/2$ plane. Hints: 1) The definition of the metric itself requires the following condition for non-null geodesics

$$1 = \alpha\left(\frac{dt}{dS}\right)^2 - \frac{1}{\alpha}\left(\frac{dr}{dS}\right)^2 - r^2\left(\frac{d\theta}{dS}\right)^2 - r^2\sin^2\theta\left(\frac{d\phi}{dS}\right)^2$$

This relation can be used to considerably simplify the geodesic equation for the radial coordinate. 2) The result of the last exercise of procedure 3a in MISN-0-474 is also very useful.

Comment - By comparing eqs. 8.52 - 8.53 with eqs. 8.55 - 8.57 in Rindler one sees two differences. First there is the appearance of a second term on the right side of eq.8.53 and secondly the Newtonian absolute time increment is replaced by the particle's proper time. The latter difference points up the importance of properly identifying the coordinates and relating them to measured quantities. For planetary motion (in which the relative velocities are low and the gravitational field weak it is not unreasonable to expect $dS \approx dt$. Assuming that to be the case then the constant h in eq. 8.52 of

Rindler must be nearly the same as the Newtonian angular momentum (eq. 8.55). This then permits one to estimate the relative importance of the two terms on the right side of eq. 8.53.

$$\frac{3mu^2}{m/h} \approx 3\left(\frac{1}{r}\right)^2 \left(r^2\frac{d\phi}{dt}\right)^2 = 3\left(r\frac{d\phi}{dt}\right)^2 \approx 3v^2$$

where v is the orbital speed of the planet. Thus (remember $c = 1$) for planetary motion

$$\frac{3mu^2}{m/h} \ll 1$$

and the second term on the right side of eq. 8.53 can be treated as a small correction. The solution of eq. 8.53 then must not differ appreciable from the solution of eq. 8.57. In fact the solution of eq. 8.57 can be used to evaluate the small term on the right side of eq. 8.53. This is what Rindler has done to obtain his eq. 8.59.

▷ Exercise - Fill in the details in the analysis leading to eqs. 8.61 and 8.62 of Rindler.

Read pages 198-201 of Weinberg.

c. Read Rindler from the middle of p. 146 to the end of section 8.4.

Comment - The analysis should start with the geodesic equations using the parameter $\lambda \neq S$ since $dS^2 = 0$ for a light signal. However, the end result is the same as that obtained by setting $h = \infty$ in Rindler's eq. 8.53. Thus it is safe to begin with eq. 8.64 in Rindler.

(Optional) ▷ Exercise - Derive eq. 8.64 of Rindler directly from the geodesic equations using $dS^2 = 0$.

▷ Exercise - Fill in any missing details in the analysis leading from eq. 8.64 of Rindler to the result that the total deflection is $4m/R$.

Read pp. 191-194 of Weinberg.

2. Work problems 8.13, 8.14 and 8-15 of Rindler.

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