PHYSNET

MISN-0-472

INTRODUCTION TO GENERAL RELATIVITY

Relativity

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

INTRODUCTION TO GENERAL RELATIVITY

by

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Title: **Introduction to General Relativity**

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Input Skills:

1. Vocabulary: invariant interval, metric, relativity principle.
2. Unknown: assume (MISN-0-471).

Output Skills (Knowledge):

K1. Define or explain each of the following: (a) primary motivation which led Einstein to the general theory of relativity, (b) inertial mass and gravitational mass, (c) active and passive gravitational mass, (d) weak, semi-strong and strong equivalence principles, (e) local inertial frame, and the dependence of a frame's extension upon the desired degree of accuracy (f) special relativity as a local theory, (g) bending of light and the gravitational Doppler shift as qualitative consequences of the equivalence principle.

K2. Define or explain: (a) Riemannian space and metric, (b) indefinite metric and signature, (c) geodesics, (d) geodesic separation (define and derive formula for), (e) curvature (for 2-dim and n-dim surfaces), (f) geodesic deviation, (g) isometric spaces.

K3. Outline the basic scheme of general relativity with special emphasis on the roles played by the equivalence principle, by geodesics and by masses.

K4. Derive expressions for (a) the gravitational Doppler shift (and time dilation), (b) the spacetime metric around a spherical mass.

External Resources (Required):

1. W. Rindler, *Essential Relativity*, van Nostrand (1977).

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1. Introduction

The general theory of relativity is generally conceded (at least by most physicists) to be the most elegant accomplishment of a single person in the history of mankind. It is mathematically difficult and conceptually challenging. Yet to the person who has the fortitude to struggle through the vast amount of mathematics involved to learn its subtle consequences, it can be a very self- satisfying endeavor. Besides all this there are several predictions of the theory which have been tested and even more which with modern technology, are becoming testable. It is even conceivable that there may ultimately be some practical applications of this marvelous theory. This unit will be concerned primarily with a conceptual introduction to the topic.

2. Procedures

1. Read sections 1.14 - 1.21 of Rindler and the last paragraph on p. 31 and section 49. Sections 1.15 - 1.17 are recommended as "food-for-thought" material. They should not be considered integral parts of general relativity. The material for Output Skill K1 is contained in sections 1.14, 1.18 - 1.21.

   Note: When there is no modifier present (weak or semi-strong) "the equivalence principle" refers to the strong equivalence principle which is stated at the top of p. 18. The second "all" in that statement is very important.

   Optional reading:

   (1) K. Ford, Basic Physics, Chapter 22

   (2) Taylor and Wheeler, Spacetime Physics, Chapter 3

   (3) P. Bergman, The Riddle of Gravitation

   (4) A. Einstein, On the Influence of Gravitation on the Propagation of Light; The Foundation of the General Theory of Relativity. (Two original papers reproduced in the Dover book The Principle of Relativity, pp. 99 and 111)

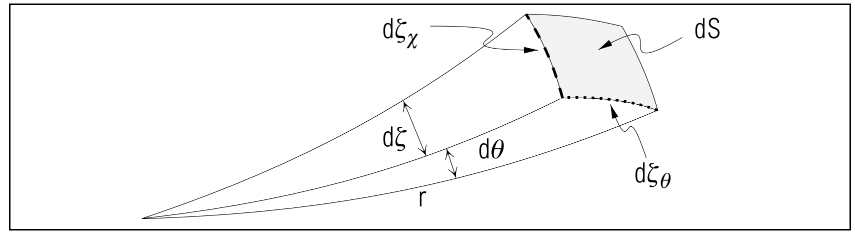

Figure 1.

   (5) A. Einstein, The Meaning of Relativity, p. 55.

   ▷ Exercise: Try to answer the questions posed in exercise 1.10 on p. 255 of Rindler.

   Organize (mentally or on paper) your responses to the items in Output Skill K1.

2. Read sections 7.1 - 7.3 of Rindler. Comments: A Geodesic has the following properties:

   a. it extremizes the "distance" between two points at least for a positive definite metric.

   b. it is uniquely determined either by two points or by a direction through a point.

   c. it satisfies a certain differential equation.

   d. it is represented locally by linear equations in local pseudo- Euclidian coordinates.

   ▷ Exercise: Fill in the details to arrive at equations 7.1, 7.2 and 7.3 of Rindler.

   ▷ Exercise: Justify the expressions used on pp.108 and 109 of Rindler.

$$A = \int C dr, V = \int S dr$$

   ▷ Exercise: Derive the expressions in equation 7.4 of Rindler.

   Hints:

   1) Consider a "plug" extracted from the sphere as shown in Fig. 1.
   then $dS = d\eta_\theta d\eta_x$
   with $d\eta_\theta = d\theta(r - 1/6kr^3 + \ldots)$, etc.

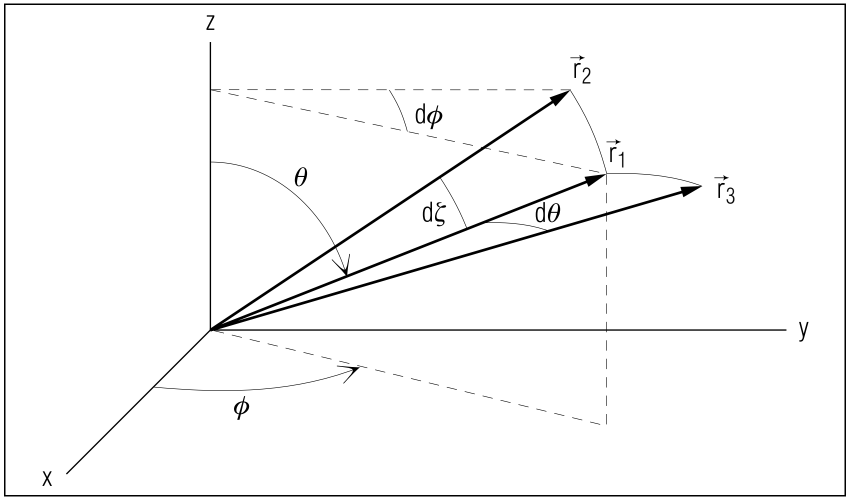

Figure 2. .

2) Consider a local region near 0 so that Euclidian coordinates may be used (see Fig. 2).

   $\vec{r}_1$, $\vec{r}_2$ and $\vec{r}_3$ are tangent at the origin to the three geodesies o the previous figure. Show that d is related to the differential of the usual spherical coordinate ϕ by

$$dx = d\phi \sin\theta$$

3) Combine results and integrate over the usual spherical coordinate θ and ϕ.

▷ Exercise: Derive equation 7.6 of Rindler.

3. Read section 7.4 of Rindler. Comments:

   1) The equivalence principle makes special relativity a local theory and it assumes therefore that space-time is locally pseudo- Eudlidean and hence globally Riemannian.

   2) The assumption that free-particles follow geodesics even in the presence of gravitating masses removes gravity from consideration as a force and replaces it by a curved space-time, i.e. a Riemannian geometry.

   3) The previous assumption implies that the distribution of masses determines the space-time metric.

4. Read sections 7.5 - 7.7 of Rindler.

   ▷ Exercise: Fill in the details leading to equation 7.17 of Rindler.
   ▷ Exercise: Work exercise 7-11 in Rindler.

   Questions:

   1) What is the interpretation of the radial coordinate in the metric of Rindler's equation 7.28?

   2) How are clock rates and settings synchronized in obtaining Rindler's equation 7.28?

   3) Under what conditions is Rindler's equation 7.25 expected to give the metric in the vicinity of a point-mass?

   ▷ Exercise: Work exercise 7.9 of Rindler. Note that the period a simple pendulum as obtained in beginning mechanics is $2\pi\sqrt{\ell/g}$, $\ell =$ length, $g =$ acceleration due to gravity).

Acknowledgments

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