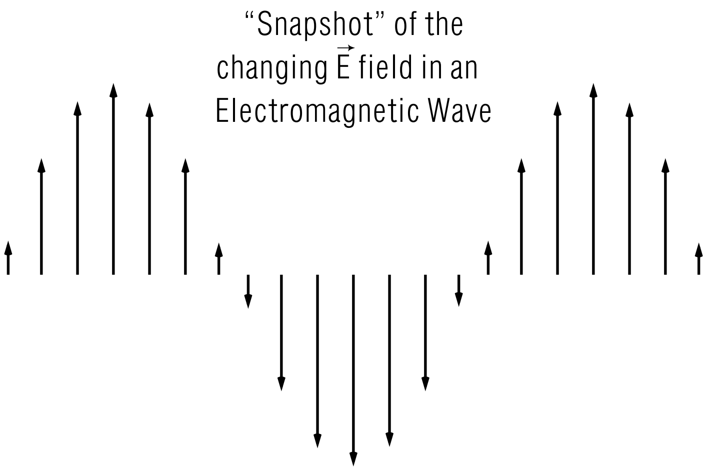
MISN-0-429

CHANGING FIELDS AND ELECTROMAGNETIC WAVES



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

CHANGING FIELDS AND ELECTROMAGNETIC WAVES

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Title: **Changing Fields and Electromagnetic Waves**

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Input Skills:

1. Calculate the magnitude and direction of the magnetic field produced by a moving charged particle (MISN-0-427).
2. State Faraday's law (MISN-0-428).

Output Skills (Knowledge):

K1. State how an induced magnetic field is produced and describe its magnitude and direction.

K2. Summarize the main points of Maxwell's electromagnetic theory.

K3. State how the speed of light is related to the electric and magnetic force constants.

K4. Describe an electromagnetic wave by drawing its electric and magnetic fields at various times.

Output Skills (Rule Application):

R1. Calculate the time required for an electromagnetic wave to travel a given distance.

Output Skills (Problem Solving):

S1. Given a charged particle in an electromagnetic field, determine the electromagnetic force on the particle.

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CHANGING FIELDS AND ELECTROMAGNETIC WAVES

Abstract:
The present unit presents a qualitative overview of all the electric and magnetic interactions between charged particles. All of these interactions, except one, have already been discussed in the preceding units. This remaining interaction involves the theoretical prediction that a changing electric field induces a magnetic field. Once this interaction is taken into account, one obtains a complete electromagnetic theory capable of predicting correctly *all* the electric and magnetic interactions between charged particles. This theory predicts, in particular, the existence of "waves" consisting of electric and magnetic fields traveling through a vacuum with a well-defined speed. Furthermore, the theory predicts that radio waves, light, and X-rays are merely special kinds of such electromagnetic waves.

SECT.

A INDUCED MAGNETIC FIELD

► *Incompleteness of theory*

In the 1860's, the Scottish physicist James Clerk Maxwell (1831-1879) set himself the task of summarizing in mathematical form all the known electric and magnetic interactions between charged particles. Maxwell soon realized that the known principles describing these interactions were incomplete. In particular, although these principles described properly the magnetic fields produced by steady currents, they did not describe in a consistent way what would happen in situations where charged particles accumulate in some region (e.g., in the situation of Fig. A-1 where a current I produces an accumulation of charged particles on the plates of a capacitor).

► *Existence of induced $\vec{B}$*

Maxwell then speculated that this deficiency of the theory could be overcome in a plausible way if there existed another kind of electromagnetic interaction which had not yet been discovered. In particular, as discussed in text section D of Unit 428, Faraday had discovered that a changing magnetic field $\vec{B}$ produces in its vicinity an induced electric field $\vec{E}_x$ (perpendicular to $\vec{B}$) such that $\vec{E}_x$ is proportional to the rate of change $d\vec{B}/dt$. Correspondingly, Maxwell assumed that the heretofore undiscovered interaction was just the converse of Faraday's interaction, i.e., Maxwell assumed the existence of this effect:

Induced magnetic field: A changing electric field $\vec{E}$ produces in its vicinity an induced magnetic field $\vec{B}_x$ (perpendicular to $\vec{E}$) such that $\vec{B}_x$ is proportional to the rate of change $d\vec{E}/dt$.	(A-1)

► *Example*

As a simple example, consider the situation illustrated in Fig. A-1 during the time where the current I flowing through the wire produces an accumulation of charges on the capacitor plates (i.e., before the current stops and the capacitor is fully charged). Then the moving charged particles responsible for the current in the wire produce a magnetic field in accordance with the principles discussed in text section A of Unit 427. But Maxwell's assumption implies that the changing electric field, pro-

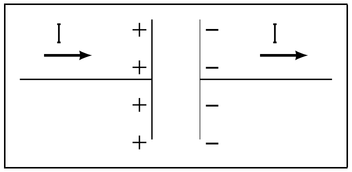

Fig. A-1: Current charging a capacitor.

duced between the capacitor plates as a result of the accumulation of charges on these plates, also produces a magnetic field.

► *Observable consequences*
The induced magnetic field, assumed to exist by Maxwell, is usually so small that it is very difficult to observe directly. However, as we shall see in Sec. C, the existence of this induced magnetic field has far-reaching implications which are confirmed by very striking experimental observations.

SECT.
B MAXWELL'S ELECTROMAGNETIC THEORY

We are now in a position to summarize the main features of Maxwell's complete theory of the electromagnetic interactions between charged particles.

► *Fields and forces*
The basic aim of the theory is to find the force $\vec{F}$ acting on a charged particle X because of its interaction with other charged particles X_1, X_2, ... To facilitate the description of this force, one introduces two auxiliary vectors $\vec{E}$ and $\vec{B}$, called the electric and magnetic fields. The problem of finding the force $\vec{F}$ can then be reduced to the solution of these two successive problems:

(1) *Production of fields*: One needs first to find the electric field $\vec{E}$ and magnetic field $\vec{B}$ produced by the charged particles X_1, X_2, X_3, ... at the position P of the particle X.

(2) *Production of force*: One needs then to find the force $\vec{F}$ exerted on the particle X by the fields $\vec{E}$ and $\vec{B}$.

PRODUCTION OF FIELDS

► *Electric field*
The electric field $\vec{E}$ produced at any point P by a charged particle consists of two parts and can be written as

$$\vec{E} = \vec{E}_c + \vec{E}_x \quad \text{(B-1)}$$

Here $\vec{E}_c$ is the electric field produced directly by the charged particle in accordance with Coulomb's law, statement (A-1) of Unit 419. The field $\vec{E}_x$ is the *induced* electric field produced by the changing magnetic field due to the particle, in accordance with Faraday's law, statement (D-1) of Unit 428. [In Maxwell's mathematical formulation, two equations describe quantitatively how the electric field $\vec{E}$ is produced by the charged particles and by the rate of change dB/dt of the magnetic field.]

► *Magnetic field*
The magnetic field $\vec{B}$ produced at any point P by a charged particle consists also of two parts and can be written as

$$\vec{B} = \vec{B}_c + \vec{B}_x \quad \text{(B-2)}$$

Here $\vec{B}_c$ is the magnetic field produced directly by the moving charged particles in accordance with the Relation (A-1) of Unit 427 and Relation (A-2) of Unit 427. The field $\vec{B}_x$ is the *induced* magnetic field produced by the changing electric field due to the particle, in accordance with Maxwell's assumption, Rule (A-1). [In Maxwell's mathematical formulation, two equations describe quantitatively how the magnetic field $\vec{B}$ is produced by the moving charged particle and by the rate of change dE/dt of the electric field.]

► *Superposition principle*

Suppose that a charged particle X_1 produces at a point P an electric field $\vec{E}_1$ and a magnetic field $\vec{B}_1$; that another charged particle X_2 produces at P an electric field $\vec{E}_2$ and a magnetic field $\vec{B}_2$; .... Then the electric field $\vec{E}$ and magnetic field $\vec{B}$ produced at P by all these particles present simultaneously is simply equal to the sum of the fields due to these individual particles. Thus

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots$$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \dots \qquad \text{(B-3)}$$

PRODUCTION OF FORCE

The electromagnetic force acting on a particle X with charge q consists of two parts and can be written as

$$\vec{F} = \vec{F}_e + \vec{F}_m \qquad \text{(B-4)}$$

Here $\vec{F}_e$ is the electric force which does *not* depend on the velocity of the particle, while $\vec{F}_m$ is the magnetic force which *does* depend on the velocity of the particle and is zero when the velocity of the particle is zero.

If $\vec{E}$ is the electric field at the position of the particle, the electric force on this particle is given by

$$\vec{F}_e = q\vec{E} \qquad \text{(B-5)}$$

If $\vec{B}$ is the magnetic field at the position of the particle moving with a velocity $\vec{v}$, the magnetic force on this particle is given by Relation (B-3) of Unit 426 so that

$$\vec{F}_m = |qv_\perp B| \text{ (direction given by right-hand rule)} \qquad \text{(B-6)}$$

The relations (B-1) through (B-6) summarize qualitatively all electromagnetic interactions between charged particles.

B-1 *Example: Superposition of fields:* A radio station sends out its broadcast signals by using two transmitting antennas. At a certain point P, equidistant from these antennas, the alternating electric fields produced by these antennas have always the same magnitude. However, the signals emitted by the antennas are such that, at any instant, the electric field produced at P by one of the antennas has always a direction opposite to that of the electric field produced at P by the other antenna. (a) Suppose that only one of the two antennas were operating. Would a transistor radio at P detect a radio signal? (b) Suppose that *both* of the antennas are simultaneously sending out signals in the manner described. What then would be the electric field produced at P by both antennas? Would a transistor radio at P detect any radio signal? *(Answer: 3)*

SECT.
C ELECTROMAGNETIC WAVES

► *Fields traveling in vacuum*

After Maxwell had postulated the existence of an induced magnetic field and had thus obtained a complete electromagnetic theory, he used this theory to make some very remarkable predictions. For example, suppose that a changing electric field exists at some point in a vacuum far from any charged particles. By Maxwell's assumption, this changing electric field then induces in its vicinity a changing magnetic field. By Faraday's law, this changing magnetic field then also induces in its vicinity a changing electric field. But this changing electric field again induces in its vicinity a changing magnetic field, which in turn induces in its vicinity a changing electric field, which in turn induces in its vicinity a changing magnetic field, ... Thus electric and magnetic fields can keep on mutually inducing each other at neighboring points, with the result that these fields travel through space. *

> * The situation is somewhat analogous to that of a row of dominoes falling so that each domino produces the fall of a neighboring one. The result is then a disturbance consisting of falling dominoes traveling along the entire row.

► *Existence of em waves*

In this way, a "disturbance" consisting of an electric field $\vec{E}$ and of a magnetic field $\vec{B}$ should be able to travel through space with some constant velocity $\vec{v}$. (Such a disturbance, illustrated in Fig. C-1, is called an "electromagnetic wave.") Indeed, as such a disturbance passes some point P, the electric and magnetic fields at P change at a rate depending on the velocity $\vec{v}$ of the disturbance. If this rate of change is of the right magnitude, the electric and magnetic fields induced at a neighboring point can then just have the same values $\vec{E}$ and $\vec{B}$ as those of the disturbance at P. We know from Rule (A-1) that the magnetic field induced by a changing electric field is perpendicular to this electric field. Similarly, we know from text section D of Unit 428 that the electric field induced by a changing magnetic field is perpendicular to this magnetic field. Hence the electric and magnetic fields in an electromagnetic wave are perpendicular to each other, as illustrated in Fig. C-1. The wave then travels through space with a velocity $\vec{v}$ perpendicular to both of these fields.

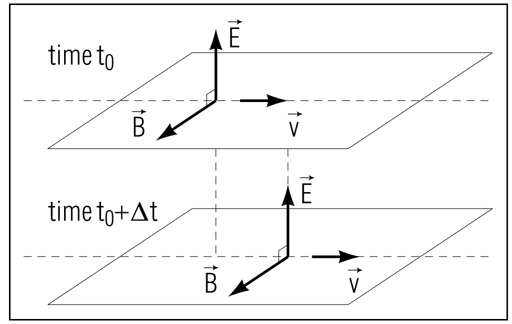

Fig. C-1: Electromagnetic wave traveling in space with a velocity $\vec{v}$. The electric and magnetic fields $\vec{E}$ and $\vec{B}$ are shown at two successive times.

Thus Maxwell used the equations of his theory to predict quantitatively the existence of electromagnetic waves which can travel through a vacuum and which have the basic properties illustrated in Fig. C-1. The existence of a special class of such electromagnetic waves, now known as "radio waves," was first demonstrated experimentally by the German physicist Heinrich Hertz (1857-1894) in 1887, long after Maxwell's death. Around 1900, the Italian engineer M.G. Marconi then first demonstrated the practical utility of such radio waves for long-distance communication. Needless to say, radio waves are nowadays used so commonly in all forms of communication (such as radio, television, radar, ...) that it seems strange that the mere existence of radio waves was just an unconfirmed theoretical prediction less than a century ago.

► *Speed of em waves*

On the basis of the quantitative equations of his theory, Maxwell predicted that the speed c of electromagnetic waves in vacuum should be a fundamental constant equal to

$$c = \sqrt{\frac{k_e}{k_m}} \qquad \text{(C-1)}$$

where k_e and k_m are the electric and magnetic force constants occurring in the theory. By using the known values of these force constants, Maxwell thus predicted that the speed of electromagnetic waves in vacuum should be

$$c = \sqrt{\frac{9\times10^{9}\,\mathrm{N\,m^2/C^2}}{10^{-7}\,\mathrm{N\,s^2/C^2}}} = \sqrt{9\times10^{16}\,\mathrm{m^2/s^2}}$$

or

$$c = 3\times10^{8}\,\mathrm{m/s} \qquad \text{(C-2)}$$

But this rather large speed is exactly the same as the experimentally measured speed of light in vacuum! (Indeed, if one uses the more accurately known values of k_e and k_m, the agreement of c with the measured speed of light remains true with greater precision.) Maxwell concluded that this agreement is not fortuitous, but indicates that light is a particular kind of electromagnetic wave. This conclusion has been very well borne out by all subsequent experimental observations. Thus it is now well established that radio waves, light, and X-rays are all merely different forms of electromagnetic waves.

Knowing About Electromagnetic Waves

C-1 *Light from the sun:* The distance of the earth from the sun is 1.5×10^8 km. (a) What is the time required for light from the sun to reach the earth? Express your answer in minutes. (b) What is the time required for X-rays emitted by the sun to reach the earth? *(Answer: 2) (Suggestion: [s-1])*

C-2 *Communication with the moon:* The distance of the moon from the earth is 3.8×10^5 km. (a) What is the time required for a radio signal sent from the earth to reach an astronaut on the moon? (b) Would this time be larger, smaller, or the same if one sent a radio signal of larger intensity (i.e., one with a larger magnitude of the electric field)? (c) Would this time be larger, smaller, or the same if one established communication with the astronaut by using light from a laser beam (instead of radio waves)? *(Answer: 4) (Suggestion: [s-1])*

C-3 *Forces exerted on a particle by an e.m. wave:* For an electromagnetic wave traveling in a vacuum, the ratio E/B of the magnitude E of the electric field compared to the magnitude B of the magnetic field is always equal to $E/B = c$, where c is the speed of electromagnetic waves in vacuum. Suppose that such an electromagnetic wave arrives at a point P where there is a charged particle. (a) What is the ratio F_m/F_e of the magnitude F_m of the magnetic force compared to the magnitude F_e of the electric force on this particle if the particle is at rest? (b) What is the ratio F_m/F_e if the particle moves with a velocity $\vec{v}$ parallel to the magnetic field? (c) What is the ratio F_m/F_e if the particle moves with a velocity $\vec{v}$ parallel to the electric field? (d) The speed of an electron in an atom or molecule is typically about 10^6 m/s. What is the maximum magnitude of the ratio F_m/F_e on such an electron? (e) Does the electric field or the magnetic field in an electromagnetic wave exert the predominant force on

an electron in an atom or molecule? *(Answer: 1) (Suggestion: [s-2])*

SECT.

D SIGNIFICANCE OF MAXWELL'S THEORY

Maxwell's electromagnetic theory is one of mankind's greatest intellectual syntheses, of a scientific importance comparable to Newton's earlier great synthesis in the field of mechanics. Thus Maxwell's theory accounts successfully for all electromagnetic phenomena, including radio waves and light. Accordingly, the theory has a vast range of applications in all the natural sciences, engineering, and technology.

► *Role of fields*

Fields, both electric and magnetic, play a central role in Maxwell's theory. Thus the interaction between charged particles is viewed as being mediated by means of these fields. For example, consider the interaction between two distant charged particles 1 and 2. According to Maxwell's theory, an acceleration of particle 1 produces electric and magnetic fields which then travel away from this particle with the speed c of electromagnetic waves. Then these fields arrive some time later at the position of particle 2, they then produce on this second particle an electromagnetic force and thus a corresponding acceleration of this particle. Thus the acceleration of particle 2 occurs at a later time than the original acceleration of particle 1. In other words, the interaction between the two particles does not occur instantaneously, but is delayed in time.

► *Delayed interaction*

Note that in Newton's original theory (as discussed in text section B of Unit 408) the interaction between two particles was assumed to be instantaneous, i.e., the acceleration of one particle is assumed to produce an immediate effect on the other particle, no matter how distant this second particle might be. Actually, this assumption is never exactly true since some time is always required before the second particle feels the effect of the first particle. The fields in Maxwell's theory account explicitly and simply for the delay in the interaction: This delay is merely due to the fact that the fields, traveling with the speed c away from particle 1, require some finite time before they arrive at particle 2. Of course, if the distance between the particles is not too large, the time delay involved in the interaction may be negligibly small (because the speed c of electromagnetic waves is so large that a very short time is required for these waves to traverse the distance between the particles). But if the distance between the particles is large, or if precise measurements are made, the time delay can be quite important. (For example, it requires more than 1

second for electromagnetic waves to travel from the earth to the moon.)

► *Energy and momentum of fields*

Consider a moving charged particle which is brought to rest when it strikes an object. While the particle is decelerated, it produces electromagnetic fields which then travel through space as an electromagnetic wave. When these fields arrive at the position of some second particle originally at rest, they then produce a force accelerating this particle. The second particle thus acquires some energy and momentum. Where does this energy and momentum come from? The natural interpretation is that some of the energy and momentum originally possessed by the first particle is given to the electric and magnetic fields, which then transport this energy and momentum through space, and finally give them to the second particle. Thus the electric and magnetic fields in an electromagnetic wave not only transmit forces over long distances, but also carry with them energy and momentum.

► *Fields and particles*

We first introduced electric and magnetic fields as convenient auxiliary quantities facilitating the description of the interaction between charged particles. In Maxwell's theory, the role of these fields becomes essential since it would be exceedingly difficult to describe the time-delayed interaction between charged particles without introducing the notion of fields traveling between these particles. Furthermore, the traveling fields carry with them energy and momentum. Thus the electromagnetic fields in Maxwell's theory produce directly observable consequences fully as real as those produced by ordinary particles.

SECT.

E SUMMARY

IMPORTANT RESULTS

Induced magnetic field: Rule (A-1)

$\vec{B}_x$ produced by dE/dt, perpendicular to $\vec{E}$

Maxwell's electromagnetic theory: (Sec. B)

$\vec{E}$ produced by charged particles and dB/dt

$\vec{B}$ produced by moving charged particles and dE/dt

$\vec{F} = \vec{F}_e + \vec{F}_m$, where $\vec{F}_e = q\vec{E}$ and $\vec{F}_m = |qv_\perp B|$ (direction given by right-hand rule).

Speed of electromagnetic waves: Eq. (C-1), Eq. (C-2)

$c = \sqrt{k_e/k_m} = 3 \times 10^8$ m/s

USEFUL KNOWLEDGE

Properties of electromagnetic waves

SUGGESTIONS

s-1 *(Text problems C-1 and C-2): Any* electromagnetic wave travels in a vacuum with the same speed $c = 3.00 \times 10^8$ m/s. Radio waves, light, and X-rays are merely different kinds of electromagnetic waves.

s-2 *(Text problem C-3):* Remember that the electric and magnetic fields in an electromagnetic wave are perpendicular to each other. Also remember that the magnetic force, unlike the electric force, depends on the velocity of the particle in the manner discussed in text section B of Unit 426.

ANSWERS TO PROBLEMS

1. a. 0
   b. 0
   c. v/c
   d. 0.003
   e. electric field
2. a. 8.3 minute
   b. 8.3 minute
3. a. yes
   b. $\vec{E} = 0$, No.
4. a. 1.3 second
   b. same
   c. same

MODEL EXAM

$$1\,\text{mile} = 1609\,\text{m}$$
$$c = 3.0 \times 10^8\,\text{m/s}$$
$$e = 1.6 \times 10^{-19}\,\text{C}$$

1. See Output Skills K1-K4, in this module's *ID sheet*.
2. A radio station's antenna is one-quarter of a mile away from you. Calculate the time required for a radio wave to travel from the antenna to your position.
3. The electric field vector in an electromagnetic wave has a magnitude of $3.0 \times 10^3\,\text{N/C}$. If the wave is incident on an electron whose speed is $2.5 \times 10^6\,\text{m/s}$, what is the maximum magnitude of:
   a. the electric force on the electron,
   b. the magnetic force on the electron, and
   c. the electromagnetic force on the electron.

Brief Answers:

1. See this module's *text*.
2. $t = 1.3 \times 10^{-6}\,\text{s}$
3. a. $F_e = 4.8 \times 10^{-16}\,\text{N}$
   b. $F_m = 4.0 \times 10^{-18}\,\text{N}$
   c. $F = 4.8 \times 10^{-16}\,\text{N}$