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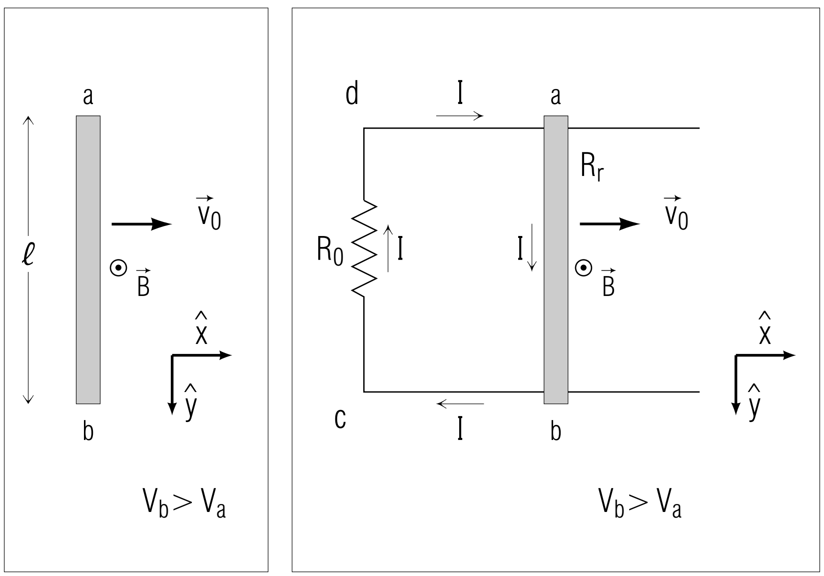
MISN-0-428

MAGNETICALLY INDUCED EMF



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MAGNETICALLY INDUCED EMF

by

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CONTENTS

Title: **Magnetically Induced EMF**

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Input Skills:

1. Vocabulary: magnetic field (MISN-0-426).
2. State the expression for the non-coulomb power in a two-terminal system (MISN-0-425).
3. Calculate the magnitude and direction of the magnetic force on a moving charged particle (MISN-0-426).

Output Skills (Knowledge):

K1. Vocabulary: betatron, electric generator, electric transformer, magnetic flux.

K2. State Faraday's law for the magnitude and direction of the emf induced around a closed path.

K3. State at least two practical applications of magnetically induced emf's.

Output Skills (Problem Solving):

S1. Given the velocity of a conductor moving through a constant, uniform magnetic field, determine the emf induced in this conductor, the resulting potential difference between the ends of the conductor, and the induced current flowing through the conductor.

S2. Given a changing magnetic field perpendicular to a conducting loop (or a closed path), determine the emf induced in this loop (or path) and the resulting current flowing around the loop.

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MISN-0-428

MAGNETICALLY INDUCED EMF

Abstract:
Magnetic force and field (Unit 426)

By examining further the applications of the magnetic interaction, we shall find that this interaction can be used to produce an emf. Thus we shall be led to the fundamentally important conclusion that magnetic and electric interactions are very closely connected. This close connection reveals not only the existence of a whole new range of remarkable phenomena, but has also immediate practical applications for the generation and transmission of electric energy.

SECT.
A EMF INDUCED IN A MOVING ROD

MAGNETICALLY INDUCED CHARGE SEPARATION IN A MOVING CONDUCTOR

When a conductor moves through a magnetic field, the charged particles in the conductor move along with it through the magnetic field and they thus experience magnetic forces. These magnetic forces then cause those of the charged particles that are mobile to move through the conductor and thus to produce effects that would not exist if the conductor was at rest or if the magnetic field was not present.

ROD MOVING THROUGH $\vec{B}$

A simple illustration of magnetic effect inside a moving conductor is the accumulation of charge that occurs on the ends of a metal rod which is moving sideways through a magnetic field. Such a rod is shown in Fig. A-1, with the rod moving with a velocity $\vec{v}_0$ that is perpendicular to the length of the rod. The rod is moving through a uniform magnetic field that is perpendicular to both the direction along the rod's length and the direction of the rod's velocity. In the figure, the magnetic field is pointing out of the plane of the paper.

Any mobile electron in the metal rod must also be moving with a velocity $\vec{v}_0$ through the magnetic field $\vec{B}$ and thus it must experience a

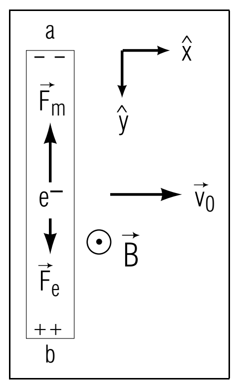


Fig. A-1: Forces on an electron in a metal rod moving through a magnetic field.

magnetic force $\vec{F}_m$ directed along the rod toward end a in Fig. A-1.[1] The mobile electrons respond to this force by drifting through the rod toward its end a, eventually producing at this end a an accumulation of negative charge. The other end b of the rod is then left with a deficiency of negative charge, i.e., with a net positive charge.

The opposite charges that accumulate at the opposite ends of the rod produce a coulomb electric force $\vec{F}_e$ on any electron inside the rod. This electric force is directed away from the negatively charged end a and toward the positively charged end b.[2] Thus this electric force $\vec{F}_e$ on an electron, caused by the accumulations of charge at the rod ends, has a direction opposite to that of the magnetic force $\vec{F}_m$ that caused the charge accumulation. Furthermore, the electric force increases as time goes on and more charge accumulates at the ends due to the magnetic force. Eventually the charge accumulated at the ends of the rod becomes so large that the magnitude of the electric force $\vec{F}_e$ is equal to that of the oppositely directed magnetic force $\vec{F}_m$. Then the total force on any electron in the rod, $\vec{F}_m + \vec{F}_e$, is zero and there is no further motion of electrons along the rod. The rod has then reached a steady state where there is a constant accumulated charge at each end of the rod. The fact that there is an accumulation of charge at each end means that there is a potential difference between the ends of the rod, with the electric potential being larger at the positively charged end b than at the negatively charged end a.)

We can summarize by saying that the sideways motion of a metal rod through a magnetic field produces the same kind of charge separation as would be produced by any other kind of emf source, such as a battery, that was inserted into the middle of the rod.

MAGNETICALLY INDUCED EMF AND CURRENT

► *Emf along the rod*

We are now in a position to determine the actual amount of emf[3]

[1]Since the electron is *negatively* charged, the direction of this force is *opposite* to that specified by the right-hand rule for a *positively* charged particle. The rule itself, for the force on a charged particle moving through a magnetic field, is given and discussed at some length in Unit 426.

[2]Recall that a negatively charged particle is attracted toward positive charges and is repelled by other negative ones.

[3]Informally, the emf between the ends of the rod is called the "Voltage difference" between the ends of the rod. Recall that it is the amount of work per unit charge that would be done in moving a charge from one end of the rod to the other.

between the ends of a conducting rod moving sideways through a magnetic field. Consider a single particle in the rod, moving with velocity $\vec{v}_0$ in the $\hat{x}$ direction of Fig. A-2a. By Relation (B-4) of Unit 426, the magnetic force on this particle is equal to qv_0B along the $\hat{y}$ direction (the charge q will be a negative number for an electron). The work done by this force to move a charged particle from end a of the rod to its other end b is then simply $(qv_0B)\ell$ where ℓ is the length of the rod.[4]

Since the emf $\mathcal{E}_{ab}$ between ends a and b is, by definition, simply the work done done by this force, per unit charge, in moving a charge from a and b, we obtain *

$$\boxed{\mathcal{E}_{ab} = \frac{(qv_0B)\ell}{q} = v_0B\ell} \qquad \text{(A-1)}$$

> * Notice that the size of the emf produced between the ends, Eq. (A-1), does not depend on the sign or the magnitude of the charge q that we moved from one end to the other in our "thought experiment" in order to determine the emf's size.

In short, the magnetic force along the rod in the $\hat{y}$ direction produces, along the rod in this direction, a positive emf $\mathcal{E}_{ab} = v_0B\ell$.[5]

► *Current and potential drop*

Recall that when the rod first starts moving through the magnetic field there is an electric current along the rod as the electrons move away from one end, toward the other, and start to accumulate there. This current dies away with time. However, a steady current can result if there is an external conductor connecting the two ends. Such a steady current I through the rod should then be related to the potential drop V_{ab} across the ends of the rod and to the emf $\mathcal{E}_{ab}$. By the relation Relation (F-4) of Unit 423,

$$R_rI = V_{ab} + \mathcal{E}_{ab} \qquad \text{(A-2)}$$

if R_r is the resistance between the ends of the rod.

[4]Recall that the work done while moving an abject is the amount of the force times the distance through which the force moves the object.

[5]The magnetic force on a particle moving along the rod has also a component perpendicular to the rod. Since this component is not along the rod, it does not contribute to the emf along the rod. However, the existence of this other component assures that the *total* work done by the magnetic forces is properly zero and that the conservation of energy is properly satisfied. (See the later discussion of energy transformations.)

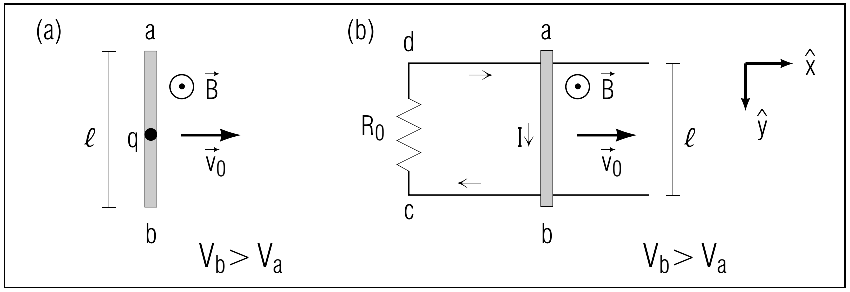


Fig. A-2: Emf produced in a metal rod moving through a magnetic field. (a) Unconnected rod. (b) Rod sliding on rails connected to a resistor.

► *Isolated rod*

As an example of a steady current in the sideways moving rod, suppose the rod is unconnected to anything else as in Fig. A-2a. Then the current I through the rod in the steady state is zero and Eq. (A-2) implies that $V_{ab} + \mathcal{E}_{ab} = 0$. Then $(V_a - V_b) + \mathcal{E}_{ab} = 0$ or

$$V_b - V_a = \mathcal{E}_{ab}\,. \qquad \text{(A-3)}$$

Thus between the two ends of the rod the magnetically induced emf produces a potential difference equal in magnitude but opposite in sign to that of the emf itself and this is consistent with our previous qualitative comments about the potential difference produced by the redistribution of electrons along the rod.

► *Rod connected to circuit*

Suppose now that the rod slides along two metal rails connected to a resistor having a resistance R_0 as shown in Fig. A-2b. As indicated in this figure, the magnetically induced emf in the rod produces then a steady current I flowing in the indicated sense through the rod, the rails, and the resistor R_0. This current I can be found from Eq. (A-2).

Indeed, suppose that the resistance of the rails is negligibly small so that the potential drop along the rails is negligible. Then the potential drop V_{ab} across the rod must be the same as the potential drop V_{dc} across the resistor. Thus $V_{ab} = V_{dc} = -R_0 I$ and Eq. (A-2) implies that the current I is such that

$$(R_r + R_0)I = \mathcal{E}_{ab}\,, \qquad \text{(A-4)}$$

where $\mathcal{E}_{ab}$ is given by Eq. (A-1).

► *Electric generator*

As a result of its motion through the magnetic field, the conducting rod acts then as an emf source analogous to a battery. Thus the rod is a simple generator of electric current:

Def.	**Electric generator**: A device which produces an emf as a result of the motion of conductors through a magnetic field.	(A-5)

In the next section, we shall discuss more practical generators of the kind used to generate the electric power supplied to our homes.

► *Similarity to motor*

Despite its simplicity, the moving metal rod in Fig. A-2 illustrates all the basic principles involved in practical generators. For example, the arrangement shown in Fig. A-2b is quite similar to that of the very simple motor illustrated in Figure (G-1) of Unit 426. In both cases the rod is located in a magnetic field. In the case of the motor, an emf source (such as a battery) is used to supply the electric energy needed to produce a current through the rod. Hence the rod moves because of magnetic forces and can thus do mechanical work. In the case of the generator, the situation is reversed. Thus there is no other source of emf, but mechanical work is done to move the rod through the magnetic field. Hence an emf is produced in the rod and can produce a current to supply energy in electric form.

In summary, the metal rod can be used as a motor to convert electric energy into mechanical work. But the same metal rod can also be used in a reverse way as a generator to convert mechanical work into electric energy.

ENERGY TRANSFORMATIONS IN A GENERATOR

Let us look in somewhat greater detail at the energy transformations occurring in the electric generator illustrated in Fig. A-2b.

► *Force needed to move rod*

When a current I flows through the rod in Fig. A-2b as a result of the induced emf, the mobile charged particles in the rod have a velocity component parallel to the rod, in addition to the component $\vec{v}_0$ perpendicular to the rod. Hence an additional magnetic force acts on the particles in the rod when the current I flows through the rod along the $\hat{y}$ direction. By the right-hand rule, the direction of the magnetic force acting on the rod because of this current I is opposite to the $\hat{x}$ direction, i.e., opposite to the direction of the velocity $\vec{v}_0$ with which the rod moves. Hence one must apply an external force along the $\hat{x}$ direction in order to keep the rod moving with the constant velocity $\vec{v}_0$ despite this opposing magnetic force.

► *Energy conversion*

Suppose that one wants to supply more electric energy to the resistor R' by passing a larger current I through it. The larger current I through the rod then causes a larger magnetic force opposing the motion of the rod through the magnetic field. Hence one needs to apply a larger external force, and thus also to supply more mechanical work, in order to keep the rod moving with the constant velocity despite the opposing magnetic force. Thus energy is properly conserved because more mechanical work must be done to supply more electric energy. *

> * The *total* work done by all magnetic forces in this energy conversion process is zero, i.e., the magnetic forces act merely as intermediaries in the total energy-conversion process.

EMF Induced In a Moving Conductor (Cap. 1)

A-1 Suppose that Fig. A-1 illustrates a steel rod, 0.40 m long, moving with a velocity of 10 m/s along the $\hat{x}$ direction through a magnetic field of 5.0×10^{-2} tesla directed out of the paper. (a) As a result of this motion, what is the magnetic force $\vec{F}_m$ on an electron in the rod? (The charge of the electron is -1.6×10^{-19} C.) (b) What is the work done by this magnetic force on the electron moving from the end a to the end b of the rod? (c) Use this result to find the emf $\mathcal{E}_{ab}$ produced by the magnetic force along the rod from a to b. (d) Does this result agree with that obtained from the relation Eq. (A-1)? *(Answer: 4) (Suggestion: [s-1])*

A-2 (a) In the preceding example, what must be the electric force exerted on an electron in the rod by the accumulated charges at the ends of the rod so that the electron has no net motion along the rod? (b) What then is the electric field in the rod? (c) What then is the resultant potential drop V_{ab} along the rod from a to b? (d) How is this potential drop related to the emf $\mathcal{E}_{ab}$ along the rod? *(Answer: 7)*

A-3 *Dependence of induced emf:* In Fig. A-1 an emf $\mathcal{E}_{ab}$ is induced between the ends of the rod moving through the magnetic field. (a) What would be the induced emf if the rod were moving with a 3 times larger velocity? (b) If the magnetic field were 3 times larger? (c) If the

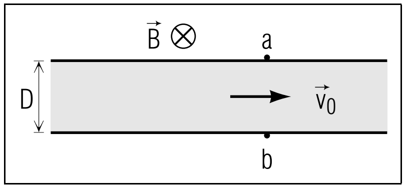


Fig. A-3.

rod were 3 times longer? (Express all your answers in terms of the original emf $\mathcal{E}_{ab}$.) *(Answer: 2)*

A-4 *Current induced in a circuit:* In Fig. A-2b the current I, flowing around the entire circuit *abcda*, should be related to the total potential drop V_{aa} around this circuit and the total emf $\mathcal{E}_{ab}$ around this circuit by the general relation $RI = V_{aa} + \mathcal{E}_{aa}$, where R is the total resistance of this circuit. (a) What is the potential drop V_{aa} from a to a? (b) How is the emf $\mathcal{E}_{aa}$ around the entire circuit related to the emf $\mathcal{E}_{ab}$ induced in the moving rod? (c) How is the total resistance related to the resistance R_r of the rod and the resistance R_0 the resistor (if the resistance of the rails is negligible)? (d) How then is the current I in the circuit related to $\mathcal{E}_{ab}$ and the preceding resistances? Compare this result with the relation Eq. (A-4). *(Answer: 5) (Suggestion: [s-3])*

A-5 *Electromagnetic flowmeter:* An electromagnetic flowmeter is a useful device for measuring the flow rate of blood in an artery without needing to insert any measuring instrument in the blood itself. It is only necessary (as illustrated in Fig. A-3) to place the artery in a known magnetic field $\vec{B}$ perpendicular to the artery. An emf is then induced in the moving blood so that a measurable potential drop V_{ab} is detected between two points a and b on opposite sides of the artery wall. (a) In Fig. A-3, where the magnetic field points into the paper, is the potential at the point a larger or smaller than that at the point b? (b) Express the speed v_0 of the flowing blood in terms of the magnitude of the measured potential drop V_{ab}, the magnitude B of the magnetic field, and the diameter D of the artery. (c) The inner diameter of the main human artery (the aorta) is about 2 cm. When this artery is placed in a magnetic field of 300 gauss, the measured potential drop is 1.8×10^{-4} volt. What is the speed of the blood flowing in this artery? *(Answer: 1) (Suggestion: [s-5]) More practice for this Capability: [p-1], [p-2], [p-3]*

SECT.
B ELECTRIC GENERATORS

A practical electric generator is obtained if one uses a rotating motor (of the kind described in text section G of Unit 426) in a reverse way so as to convert mechanical work into electric energy. Such a generator consists, basically, of just a wire loop (or coil) rotating in the presence of a magnetic field.

► *Emf in a rotating loop*
The sketch labeled (a) in Fig. B-1 shows a cross-section through a rectangular loop of metal wire. The two circles, one with a dot and the other with a cross, are cross-sections through each of the two sides of the loop that are perpendicular to the page. The dotted line represents the bottom and the top of the rectangular loop that connect the two sides, where the bottom is below the surface of the page and parallel to the surface while the top part is above the surface and parallel to it. The loop is rotating about about its central axis, which you can think of as a line that is perpendicular to the paper and half way between the two sides shown in the sketch.

As you look from left to right in Fig. B-1 you see sketches of the cross-sections at four successive times as the loop rotates about its central axis. Imagine the "plane of the loop" as an imaginary flat sheet that cuts through the centers of all four sides of the loop (it is perpendicular to the plane of the page and goes left-right in Fig. B-1a). Imagine the plane of the loop rotating a quarter of a complete rotation as you advance through the four sketches across the page. Note the changing positions of the sides labeled 1 and 2. Finally, notice that the two sides of the loop always have

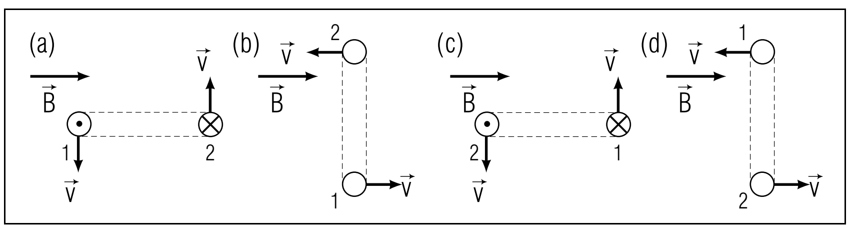


Fig. B-1: Rectangular wire loop rotating in a magnetic field. The diagrams show cross-sectional views of the loop at various times. The symbols ⊙ and ⊗ indicate currents flowing out of, or into, the paper.

opposite velocities (this is because they are equidistant from the axis of rotation).

The loop is in a constant magnetic field, $\vec{B}$, as shown in the sketches. In Fig. B-1a the two sides of the loop are shown when the plane of the loop (out of the page) is parallel to the magnetic field. In Fig. B-1b the plane of the loop has rotated and is now perpendicular to the magnetic field. In the two succeeding sketches the loop plane is again parallel to the field and then perpendicular to it.

In Fig. B-1a a positively charged partice in the downward-moving side 1 will experience a magnetic force that is *out* of the paper. If we call the direction up (out of the page) the positive direction, then a positive emf will exist in this side of the loop at this instant and there will be a positive current. On side 2 of the loop, at this same instant, the velocity is opposite so the magnetic force and induced emf are opposite so the current on thios side flows into the page. Thus the rotation of the loop, at this instant, results in a current flowing around the loop in the direction indicated by the spot and the cross in Fig. B-1a, as described in the figure's caption.

Figure B-1b shows the loop after it has rotated by 90° so its plane is perpendicular to $\vec{B}$. Note that at this instant the velocities of each of the two sides of the loop are parallel to $\vec{B}$ so *no* magnetic force is produced on a charged particle in these sides at this instant. This is a time when the current in the loop is zero.

Figure B-1c shows the loop after it has rotated another 90° so the plane of the loop is again parallel to $\vec{B}$. Then the situation is the same as in Fig. B-1a except that the current now flows out of the paper in side 2 of the loop and into the paper in side 1 of the loop. In other words, the direction of the current flowing through the loop is now *opposite* to the direction of the current in Fig. B-1a.

Finally, Fig. B-1d shows the loop after it has rotated another 90°. Here the situation is exactly the same as that in Fig. B-1b, with no current flowing around the loop.

In summary, as a conducting loop of wire rotates in a constant magnetic field, a current will go around the loop in one direction, then will pass through zero as the loop rotates, then will go around the loop in the other direction, will again go through zero, etc. Thus the current in the wire reverses its direction every half turn of the loop. Since all changes must be continuous as time progresses, the rotation of the loop must cause the current to rise to a peak, then smoothly start to descend to zero and

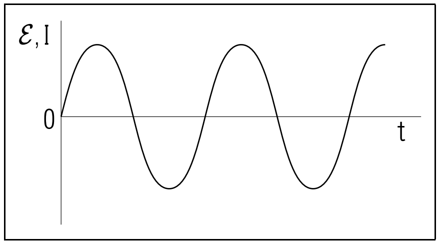


Fig. B-2: Illustration of how the emf or current produced in the rotating loop of Fig. B-1 varies with time.

then peak in the opposite direction, etc.

► *Alternating emf and current*

We have seen that an emf (and corresponding current) is produced around a loop as this loop is rotated in a magnetic field $\vec{B}$. As indicated in Fig. B-1, the sign of this emf (or current) around a particular sense of the loop keeps on reversing as the loop is rotated. Thus the rotation of the loop produces an *alternating* emf or current and this actual emf and current are shown in Fig. B-2.

A practical generator used for the production of large currents for distribution to homes and factories consists of a coil, having many loops of wire, rotating in a magnetic field. The emf and corresponding current produced by such a generator is then alternating. This Alternating Current is distributed to our homes and is said to be "AC" current.

► *DC generators*

If one is interested in producing a "direct current", a current whose sense does not vary in time, one can use the generator of Fig. B-1 together with a "commutator" of the kind described for the motor in text section G of Unit 426. Such a commutator is merely an automatic switch which reverses the sense of the current (in the circuit connected to the generator) every time the loop rotates by half a revolution. The current in the outside circuit has then always the same sign (although it still fluctuates in magnitude) and approximates more nearly a direct current.

► *Energy conversion*

The process of energy conversion in a rotating generator is quite similar to that for the simple generator discussed in the preceding section. Thus mechanical forces must be applied to keep the loop rotating in the magnetic field (despite opposing magnetic forces acting on this loop as a result of the current produced in the loop). For example, the gravitational potential energy provided by water in Niagara falls is used to provide the mechanical work necessary to keep generators rotating in the power

station. The current thus produced can then be transmitted over long distances to deliver electric energy in New York City.

EMF Induced In a Moving Conductor (Cap. 1)

B-1 The rectangular wire loop shown in Fig. B-1 rotates in a magnetic field of 7.0×10^{-2} tesla so that the sides 1 and 2 of the loop move with a speed of 40 m/s. Each of these sides has a length of 0.50 m, while each of the other two sides 3 and 4 of the loop has a length of 0.20 m. (a) At the instant when the loop is in the position shown in Fig. B-1a, what is the magnitude of the emf induced in the side 1 of the loop? (b) What is the magnitude of the emf induced in the side 2 of the loop? (c) What is the magnitude of the emf induced in each of the remaining two sides of the loop? (d) What then is the magnitude of the emf induced around the entire loop? (e) If the loop has a resistance of 0.10 ohm, what is the magnitude of the current flowing around the loop? *(Answer: 8) (Suggestion: [s-2])*

B-2 Answer all the preceding questions for the instant when the loop is in the position shown in Fig. B-1b. *(Answer: 3) (Suggestion: [s-7])*

B-3 Suppose that the loop in Fig. B-1 were rotating with the same speed in the opposite sense. (a) Would the answers to the preceding two questions then be the same or different? (b) At any instant, would the sense of the induced current flowing in the loop be the same or opposite? *(Answer: 10)*

SECT.
C EMF INDUCED IN A MOVING LOOP

► *Description of situation*

Let us now look more closely at the simple situation where a loop of metal wire moves through a magnetic field *without* rotation. As illustrated in Fig. C-1a, we assume that this loop is rectangular, with a length ℓ and a width w. The loop moves with the velocity $\vec{v}_0$ in the $\hat{x}$ direction so that the plane of the loop remains always perpendicular to the magnetic field which points out of the paper. We assume that this field is not necessarily uniform, but may vary in magnitude along the direction of motion of the loop. At the particular instant illustrated in Fig. C-1a, the magnetic field has then a magnitude B_1 at the side dc of the loop and a magnitude B_2 at the other side ab of the loop. As a result of the motion of the loop through the magnetic field, what is the emf $\mathcal{E}$ induced around the *entire loop* in the clockwise sense (i.e., going around the loop through the successive points a, b, c, d, a)?

► *Emf around the loop*

The emf $\mathcal{E}$ around the entire loop $abcda$ is simply the sum of the emfs around the successive sides of the loop. By Eq. (A-1), the emf along

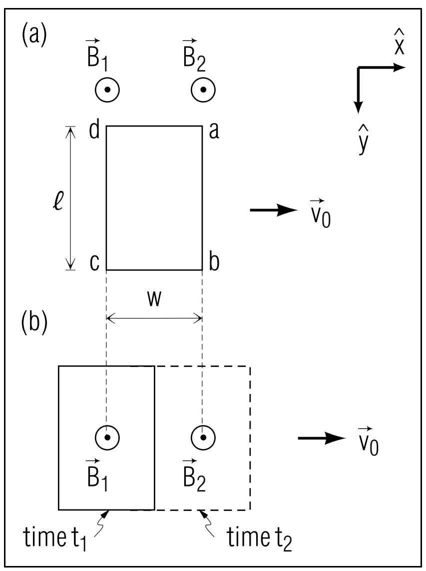


Fig. C-1: A wire loop moving through a magnetic field. (a) Loop at a particular instant of time. (b) Loop at a slightly earlier time t_1 and at a slightly later time t_2.

the loop from a to b is $\mathcal{E}_{ab} = v_0 B_2 \ell$. Similarly, the emf along the loop from d to c is $\mathcal{E}_{dc} = v_0 B_1 \ell$. The emf along the loop in the opposite direction from c to d is thus $\mathcal{E}_{cd} = -\mathcal{E}_{dc} = -v_0 B_1 \ell$. On the other hand, the emf along the sides bc and da of the loop is zero since the magnetic force on a particle moving along such a side is perpendicular to this side and thus does no work. Thus the emf $\mathcal{E}$ around the entire loop $abcda$ consists only of the non-zero emfs along the sides ab and dc so that

$$\mathcal{E} = \mathcal{E}_{ab} + \mathcal{E}_{cd} = v_0 B_2 \ell - v_0 B_1 \ell$$

or

$$\mathcal{E} = v_0 \ell (B_2 - B_1) \tag{C-1}$$

► *Discussion*

Suppose that the loop moves through a *uniform* magnetic field so the value of this field is everywhere the same. Then $B_2 = B_1$ and the emf $\mathcal{E}$ induced around the loop, and thus also the corresponding current flowing around the loop, is zero. But if the loop moves through a magnetic field which is *not* uniform so that $B_2 \neq B_1$, the induced emf $\mathcal{E}$ around the loop is not zero and thus produces a current flowing around the loop.

INDUCED EMF AND CHANGE OF MAGNETIC FLUX

The emf induced in a wire moving through a magnetic field varies according to the size of the wire, the strength of the magnetic field, and the speed and orientation of the wire relative to the magnetic field. The mathematical description of those relationships leads naturally to the introduction of a pictorial construct called "magnetic flux." Almost all professionals use the concept of magnetic flux when faced with the need to quickly deduce properties of induced currents in a device.

► *$\mathcal{E}$ and dB/dt*

The result Eq. (C-1) for the emf induced around the loop can be expressed in a much more useful form by focusing attention on how the magnetic field through the loop changes with time because of the motion of the loop. For simplicity, we assume that the loop is small. Let us then consider the value of the magnetic field existing at the center of the loop at successive times. Thus Fig. C-1b shows the loop at a particular time t_1 when the center of the loop is at a position where the magnetic field has a magnitude B_1; it also shows this loop at a slightly later time t_2 when the center of the loop is at a position where the magnetic field has a magnitude B_2. During the small time $dt = t_2 - t_1$, the loop has then moved along the $\hat{x}$ direction by a distance equal to the small width w of the loop. (See Fig. C-1.) Thus the velocity of the loop has a magnitude v_0 related to the time dt so that $v_0 = w/dt$. Hence the result Eq. (C-1) for the emf can be written as

$$\mathcal{E} = \frac{w}{dt}\ell(B_2 - B_1) = \frac{w\ell}{dt}dB$$

or

$$\mathcal{E} = A\frac{dB}{dt} \qquad \text{(C-2)}$$

where $A = w\ell$ is the area of the loop and $dB = B_2 - B_1$ is the change of the magnetic field at the center of the loop during the small time dt. Thus we see that the emf $\mathcal{E}$ produced around the loop is related quite simply to dB/dt, the rate of change with time of the magnetic field at the center of the loop.

► *Magnetic flux*

Since the area A is just a constant, Eq. (C-2) can also be written as $\mathcal{E} = d(AB)/dt$. This result can be expressed particularly simply by introducing $B_\perp$, the component of the magnetic field perpendicular to the plane of the loop. Then we make this general definition of a quantity called the "magnetic flux" ϕ through a small loop:

Def. | **Magnetic flux**: $\phi = AB_\perp$. | (C-3)

In our simple case where the magnetic field is perpendicular to the plane of the loop, $B_\perp$ is just the magnitude B of the field so that $\phi = AB$. Hence Eq. (C-2) can be written as:

$$\boxed{\mathcal{E} = \frac{d\phi}{dt}} \qquad \text{(C-4)}$$

Thus the emf induced around the loop is simply equal to the rate of change of the magnetic flux through the loop. For example, if the loop moves through a uniform field, the magnetic flux through the loop remains unchanged as the loop moves. Hence the emf induced around the loop is then zero. But if the magnetic field is different at the successive positions of the moving loop, the magnetic flux through the loop changes with time. Hence the emf induced around the loop is correspondingly *not* zero and produces a current flowing around the loop. [All these conclusions are, of course, consistent with those implied by Eq. (C-1).]

► *Sign of emf*

Let us recall that $\mathcal{E}$ is the emf induced around the entire loop in the *clockwise* sense in Fig. C-1, i.e., in the sense opposite to that of the curled fingers of the right hand if the thumb of this hand points along the magnetic field. According to Eq. (C-1) or Eq. (C-4), the sign of this emf is then positive if $B_2 > B_1$, i.e., if the loop moves so that the magnitude of the magnetic field increases through the loop. This conclusion can be summarized:

If the magnetic field (or flux) through a loop increases along the direction of the thumb of the right hand, a positive emf is induced around the loop in a sense *opposite* to the curled fingers of this hand.	(C-5)

LENZ'S LAW

Since a positive emf is induced around the clockwise sense of the loop in Fig. C-1a, positive current flows in this sense around the loop. According to the right-hand rule, this current produces then a magnetic field directed *into* the paper in Fig. C-1a, i.e., directed *opposite* to the original magnetic field. Thus the statement (C-5) implies also this conclusion, called Lenz's law: The sign of the emf induced in a loop is such that the current produced by this emf produces a magnetic field which tends to *oppose* the change of the magnetic flux through the loop.

EMF INDUCED IN A LARGE LOOP

Any large loop can be imagined to be subdivided into many small squares, as illustrated in Fig. C-2. The emf $\mathcal{E}$ around the entire loop is then equal to the sum of the emfs around each of the small squares. If the total flux ϕ through the large loop is defined as the sum of the fluxes through all the small squares, the result $\mathcal{E} = d\phi/dt$ of Eq. (C-4) is then also true for the large loop.

EMF Induced In a Moving Conductor (Cap. 1)

C-1 *Sense of current in loop:* Suppose that the loop of Fig. C-1 moves through a magnetic field which points out of the paper, the magnitude of this field being larger at the side dc than at the side ab of the loop. (a) Is the emf $\mathcal{E}_{ab}$ from a to b along the side ab of the loop positive

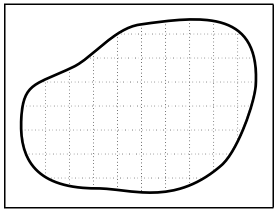


Fig. C-2: A large loop subdivided into many small squares.

or negative? (b) Is the emf $\mathcal{E}_{dc}$ along the side dc of the loop positive or negative? Is the magnitude of this emf larger or smaller than that of the emf $\mathcal{E}_{ab}$? (c) What is the emf along each of the other two sides of the loop? (d) Is the emf around the entire loop *abcda* then positive or negative? (e) Does the induced current in the loop then flow around the loop in the clockwise sense *abcda* or in the opposite sense? *(Answer: 6) (Suggestion: [s-8])*

C-2 *Magnitude of current:* In Fig. C-1 the loop moves with a velocity $\vec{v}_0 = 10\,\text{m/s}$ to the right. At a particular instant, the magnetic fields at the two sides dc and ab, each of length 1.0 m, are $B_1 = 0.040$ tesla and $B_2 = 0.035$ tesla. (a) What is the magnitude of the emf $\mathcal{E}_{ab}$ and that of the emf $\mathcal{E}_{dc}$? (b) What then is the magnitude of the emf around the entire loop? (c) If the loop has a resistance of 0.10 ohm, what is the magnitude of the induced current flowing around the loop? *(Answer: 11) (Suggestion: [s-6])*

EMF Induced by Changing Magnetic Field (Cap. 2)

C-3 *Sense of current in loop:* (a) In the situation described in the preceding two problems, does the magnetic field through the center of the moving loop change so as to increase in a direction out of the paper or into the paper? (b) According to the rule, (C-5), should the induced emf around the loop then be positive along the sense *abcda* or along the opposite sense? (c) In which sense should the induced current then flow around the loop? (d) Do these conclusions agree with those obtained in Problem C-1? *(Answer: 13) (Suggestion: [s-11])*

C-4 *Magnitude of emf:* If the loop of Problem C-2 has a width of 0.60 m and moves with a velocity of 10 m/s to the right, it takes a time of 0.06 second for the magnetic field at the center of the loop to change from 0.040 tesla to 0.035 tesla. (a) What is the area of the loop if its length is 1.00 m? (b) What then is the magnitude of the magnetic flux through the loop at the beginning and at the end of the preceding

0.06 second interval? (c) What then is the magnitude of the rate of change $d\phi/dt$ of the magnetic flux through the loop? (d) According to Eq. (C-4), what then should be the magnitude of the emf induced around the loop? (e) Does this result agree with that obtained in Problem C-2? *(Answer: 9)*

SECT.

D EMF INDUCED BY A CHANGING MAGNETIC FIELD

The mathematical relation between an emf around a loop and a changing magnetic flux through that loop allows us to describe a wide variety of situations and remarkable phenomena.[6] To examine these various situations, let us discuss the interaction of a wire loop L with another wire loop L' under several possible conditions. In the process we will make use of the fact that a current always produces a magnetic field and that field is strongest near the current and fades away as one goes far from the current. Recall that the magnetic field produced by a loop of current is somewhat similar to the magnetic field of a bar magnet.

► *L moving, L' at rest*

Let us determine the emf induced in a conducting loop L when we move it, relative to the laboratory, near a current-carrying loop L' which is at rest in the laboratory (see Fig. D-1a). Suppose the current flowing through the stationary loop is I', maintained by a battery. This current through the stationary loop L' produces a magnetic field in the vicinity of this loop. As the moving loop L moves through this field, the magnetic flux ϕ through this loop L changes. According to Eq. (C-4), this change of flux causes an emf $\mathcal{E} = d\phi/dt$ to be induced around the loop L and and this in turn causes an induced current to flow around this moving loop. We will call this induced current I and its presence is confirmed by experimental observation.

► *L at rest, L' moving*

We now determine the emf induced in a stationary loop by a moving current-carrying loop. Suppose the loop L is at rest relative to the laboratory while L' is a moving, current-carrying loop (see Fig. D-1b.). This case is very similar to the previous one since it again involves a relative motion of two loops. An observer sitting on the loop L' could describe this situation as one where the current-carrying L' is at rest and L is moving. Hence this observer on the current-carrying L' would conclude that the changing magnetic flux ϕ through the loop L should produce an induced emf $\mathcal{E} = d\phi/dt$ in L and a corresponding induced current I.

An observer at rest relative to the loop L should see the same result seen by the observer that is at rest relative to L' (if one sees a current, the

[6] This mathematical relationship is shown in Eq. (C-4).

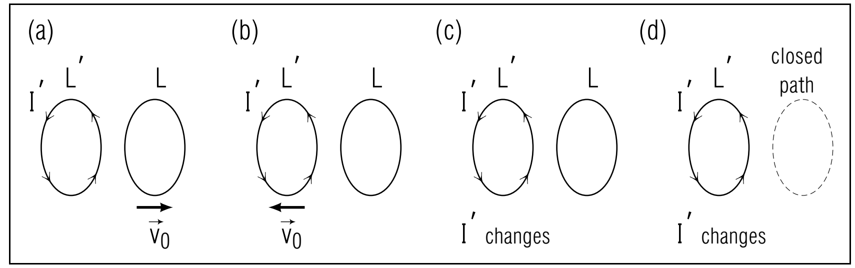


Fig. D-1: Emf induced around a wire loop or closed path under various conditions. The two loops or the loop and path are in a single plane.

other should also see a current). Hence we expect that an emf $\mathcal{E} = d\phi/dt$ is produced in a *stationary* loop if the magnetic flux ϕ through this loop changes because of the motion of some other current-carrying loop. This conclusion is confirmed by experimental observation.

► *L and L' at rest, I' changing*

We now turn our attention to the case where the two loops are at rest but the electric current is changing with time in one of them (this couls be due to a battery that is wearing out or it could be part of an alternating current. We will use a roundabout argument to get the emf, starting by again considering the situation where the loop L is at rest and a changing magnetic flux is produced through this loop as a result of the motion of a current-carrying loop L'. An observer sitting on the loop L would say that an emf $\mathcal{E} = d\phi/dt$ is induced around the loop L because of the changing magnetic flux ϕ through L. But suppose the observer does not look at the other loop L'. Then the observer on L could not tell whether the magnetic flux through L changes because the other loop L' moves while the current in it, I', remains constant, or because L' is stationary while the current I' in that loop changes (see Fig. D-1c.). The two descriptions would be indistinguishable to the observer as long as the rate of change of magnetic flux is the same. Thus the observer would in both cases expect to observe around loop L an induced emf $\mathcal{E} = d\phi/dt$ and a corresponding induced current I.

The expectation that an induced emf is produced in a stationary loop as a result of a changing current in a nearby stationary loop is confirmed by experimental observations. This important effect was first discovered by the English physicist, Michael Faraday (1791-1867) in 1831.

► *Induced electric field*

In the previous situation an emf $\mathcal{E}$, and corresponding current I, is induced in a stationary loop L as a result of a changing magnetic flux through this loop. Such a current is, of course, a measure of the motion of charged particles in the loop. However, any such charged particle can only move if an electric force acts on it. When that electric force is produced as a result of a changing magnetic flux, we call it the "induced" electric force, $\vec{F}_{induced}/q$. Correspondingly, we call the force *per unit charge* $\vec{F}_{induced}/q$ the "induced electric field" $\vec{E}_{induced}$ produced by the changing magnetic flux. *

> * We know that this induced force can produce a current in a loop L in which there was originally no current. Hence the induced force acts on charged particles irrespective of their velocity (i.e., even if their velocity is zero.) This is why this force is properly called an electric (rather than a magnetic) force, in accordance with our discussion in text section C of Unit 426.

The loop L of metal wire is merely a special example of systems consisting of charged particles. Nevertheless, the induced electric force acting on any particle in such a system ought to be the same irrespective of the nature of the system. Hence we are led to the general conclusion, illustrated in Fig. D-1d:

> *Faraday's law:* A changing magnetic field produces in space an induced electric field $\vec{E}_x$ such that the emf resulting from this field around any closed path (whether occupied by a wire loop or not) is related to the magnetic flux ϕ through this path so that $\mathcal{E} = d\phi/dt$ [with sign specified by rule (C-5)]. (D-1)

If a particle with charge q happens to be located at a point where the induced electric field is $\vec{E}_x$, this particle experiences then an induced electric force $\vec{F}_x = q\vec{E}_x$. In particular, our previous example of the wire loop L corresponds merely to the case where some special closed path in space happens to be occupied by a metal wire. The induced electric field produced along this wire loop then exerts induced electric forces on the charged particles in the loop. Hence the mobile charged particles (e.g., electrons) in the loop move and give rise to an induced current.

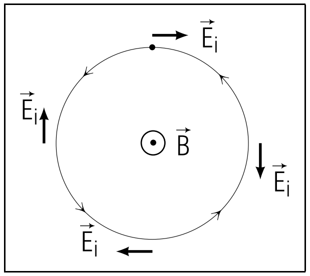


Fig. D-2: Motion of an electron in a betatron. The negatively charged electron moves in a circle counterclockwise around the magnetic field $\vec{B}$. If $\vec{B}$ is increased, an induced electric field $\vec{E}_x$; is produced along clockwise directions around the circle. The oppositely directed electric force on the negatively charged electron then accelerates the electron.

BETATRON

The induced electric field produced by a changing magnetic field is strikingly illustrated by the "betatron," a machine for accelerating electrons to high-energies. These electrons are commonly used in medicine to produce high-energy X-rays for the treatment of certain forms of cancer.

In such a betatron, electrons injected into a vacuum in the presence of an applied magnetic field $\vec{B}$ move around circular orbits perpendicular to $\vec{B}$ (as previously discussed in text section D of Unit 426 and illustrated in Fig. D-2). If the magnetic field is now rapidly increased, the induced electric field (produced along the orbit of the electrons by the changing magnetic flux through this orbit) accelerates the electrons to high energies. The betatron is a relatively simple device because the acceleration of the electrons is accomplished *solely* by means of a magnetic field. Thus the magnetic field itself keeps the electrons moving in circles, while the *rate of change* of this field accelerates the electrons to high energies.

SUMMARY

The preceding discussion shows that the relation

$$\mathcal{E} = \frac{d\phi}{dt} \tag{D-2}$$

describes not only the emf induced around a moving loop when the magnetic flux through this loop changes. It also describes the emf due to an induced electric field produced in a stationary reference frame whenever a magnetic flux changes. This last effect, described by Faraday's law, rule (D-1), shows that there exists a very close connection between electric and magnetic fields. Thus an electric field cannot only be produced directly by charged particles in accordance with Coulomb's law, but can also be produced by a changing magnetic field.

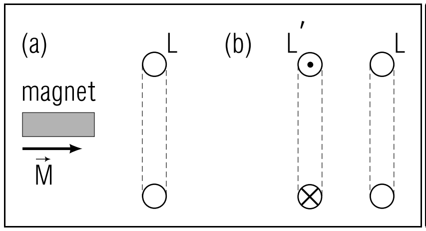


Fig. D-3.

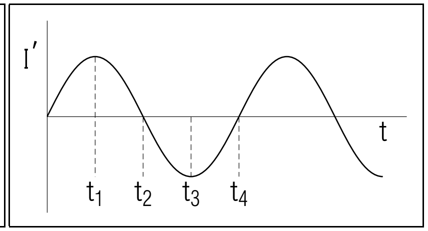


Fig. D-4.

Note that an induced electric field is only produced by a magnetic field which changes with time, and not by a magnetic field which remains constant. Note also that, as illustrated in Fig. D-2, the induced electric field is perpendicular to the direction of the changing magnetic field.

EMF Induced by Changing Magnetic Field (Cap. 2)

D-1 *Current induced in a loop:* Figure D-3 shows a cross-sectional view of a stationary circular wire loop L. In each of the following cases, state whether there is an induced current flowing in this wire loop or not. (a) A permanent magnet, having a magnetic moment $\vec{M}$, is at rest near the loop L, as shown in Fig. D-3a. (b) This magnet moves toward the loop L. (c) This magnet moves away from the loop L. (d) Another wire loop L' is at rest near the loop L, as shown in Fig. D-3b. This loop L' is now connected to a battery so that a current starts flowing in this loop in the indicated sense (clockwise, when looking upon this loop from the left). (e) The current in the loop L' has attained a constant value. (f) The loop L' is now disconnected from the battery so that the current in this loop decreases to zero. *(Answer: 15) (Suggestion: [s-10])*

D-2 For each of the cases mentioned in the preceding problem, state either the sense of the current induced in the loop L (whether this current flows clockwise or counter-clockwise, when looking upon this loop from the left) or whether this induced current is zero. *(Answer: 12) (Suggestion: [s-12])*

D-3 *Wire loop in uniformly changing field:* A circular wire loop, having an area of $0.15\,\mathrm{m}^2$ and a resistance of 0.30 ohm, is located in a uniform magnetic field $B = 0.020$ tesla, perpendicular to the plane of the loop. The magnetic field is now gradually reduced at a constant rate so that it becomes equal to zero after 10 seconds. What is the magnitude of the induced current flowing in the wire loop during the time that the

magnetic field is being reduced? *(Answer: 16) (Suggestion: [s-9])*

D-4 *Emf induced by AC current:* Suppose that the loop L' in Fig. D-3b is connected to the alternating emf supplied by a wall outlet. The resulting alternating current I' flowing through this loop varies then with the time t in the manner indicated in Fig. D-4. (a) At the time t_1 when I' has its maximum value, what is the magnitude of the emf induced in the loop L? (b) At the time t_3 when I' has its minimum value, is the magnitude of the emf induced in the loop L larger than, equal to, or smaller than the magnitude of the induced emf at the time t_1? (c) At the time t_2 when $I' = 0$, is the magnitude of the emf induced in the loop L larger than, equal to, or smaller than the magnitude of the induced emf at the time t_1? (d) At which of the times t_1, t_2, t_3, and t_4 is the *magnitude* of the emf induced in L largest and at which times is it smallest? (e) How is the sign of the emf induced in L at the time t_4 related to the sign of this emf at the time t_2? *(Answer: 14) (Suggestion: [s-15])*

D-5 *Betatron:* A magnetic field used in a betatron is usually alternating. Figure D-2 then illustrates the situation when electrons are injected into the betatron during the time when the magnetic field $\vec{B}$ is directed out of the paper and increases in magnitude. What would happen if electrons were injected somewhat later when the magnetic field $\vec{B}$ is still directed out of the paper but decreasing in magnitude? *(Answer: 18) More practice for this Capability: [p-4], [p-5]*

SECT.

E TRANSFORMERS AND OTHER APPLICATIONS

► *Emf produced by ac current*

Transformers have two coils of wire within them, arranged so that an alternating current sent into one coil induces an (output) alternating current in the other coil. The purpose is usually to produce an output current with characteristics that are different from the input current. This is typically achieved by varying the ratio of the number of loops in one coil to the number of loops in the other.[7]

To see how the transformer works, imagine a coil of metal wire that is connected to an alternating source of emf (such as an ordinary wall outlet), an alternating current flows through this coil. This current then produces inside and near this coil an alternating magnetic field. Hence the magnetic flux through this coil, and through any other neighboring coil, also changes in an alternating way. By Faraday's law, an alternating emf is then induced in any other coil C_2 in the vicinity of the first coil C_1 and produces in this second coil C_2 an alternating current. *

> * The alternating emf induced in the coil C_1 itself tends to oppose the externally supplied emf and thus results in a reduced current through the coil.

► *Power transfer without contact*

The induced emf may be used to transmit electric power from one coil C_1 to another coil C_2 although these coils are not in contact with each other and may be some distance apart. For example, electric power can be supplied to a cardiac pacemaker implanted in a patient's chest by supplying an alternating current to a coil outside the patient's chest since a current can thus be induced in a coil in the pacemaker.

► *Transformer*

Suppose that a coil C_2 consists of N_2 turns of insulated metal wire wrapped around a first coil C_1 consisting of N_1 turns of insulated metal wire. (See Fig. E-1.) Then each coil has almost the same cross-sectional area and the magnetic field (and thus also the magnetic flux) passing through each coil is always the same. If this magnetic flux changes, the

[7] In a transformer a loop is commonly called a "winding", so we talk about the ratio of the windings on the "primary" and "secondary" coils. The primary coil is connected to the input soutce while the secondary coil is hooked to the output terminals.

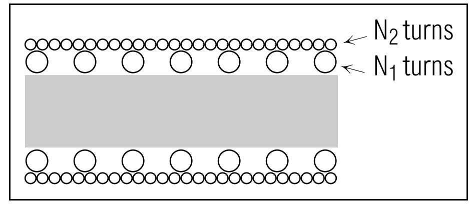


Fig. E-1: Two coils consisting of insulated wire wound on top of each other around a common core.

same emf $\mathcal{E}'$ is thus induced in a single loop of either coil. The total emf $\mathcal{E}_1$ induced around all the N_1 turns of wire of the first coil C_1 is then N_1 times as large as the emf $\mathcal{E}'$ induced in a single turn. Similarly the total emf $\mathcal{E}_2$ induced around all the N_2 turns of wire of the second coil C_2 is N_2 times as large as the emf $\mathcal{E}'$ induced in a single turn. Thus $\mathcal{E}_1 = N_1\mathcal{E}'$ and $\mathcal{E}_2 = N_2\mathcal{E}'$ so that

$$\boxed{\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1}} \qquad \text{(E-1)}$$

In other words, the ratio of the emfs induced in the coils is simply proportional to the number of turns of wire in these coils.

The preceding arrangement of two coils is called a "transformer" for the following reason: Suppose that an alternating current is made to flow in the first coil C_1 (the "primary coil") and thus produces in this coil an emf $\mathcal{E}_1$. Then a corresponding alternating emf $\mathcal{E}_2$ is induced in the second C_2 (the "secondary coil"). But the magnitude of this second emf $\mathcal{E}_2$ can be made either larger or smaller than that of the original emf $\mathcal{E}_1$ if the ratio N_2/N_1 of the number of turns of the coils has been appropriately chosen. Thus the transformer can be either used to produce an emf $\mathcal{E}_2$ larger than the original emf $\mathcal{E}_1$ (if it is a "step-up" transformer for which $N_2 > N_1$), or to produce an emf $\mathcal{E}_2$ smaller than the original emf $\mathcal{E}_1$ (if it is a "step-down" transformer for which $N_2 < N_1$).

A transformer is a very simple and widely used device for transforming an alternating emf into another emf of either larger or smaller magnitude. Note that such a transformer can only be used with *alternating* emfs since the operation of the transformer depends entirely on the induced emf produced by a *changing* magnetic field.

► *Electric power transmission*

The utility of transformers is well illustrated by the transmission of electric power over long distances. The wires connecting the generators in an electric power plant to the ultimate consumers of this power may be hundreds of kilometers long. Thus the resistance of these wires can be appreciable. Hence it is important to keep the electric current in these wires as small as possible in order to minimize the electric power dissipated into random internal energy of the wires. In order to transmit large power with the smallest possible current, it is then necessary that the emf used for transmission be as large as practical. *

> * Since the power is equal to the product of the current multiplied by the emf (text section C of Unit 425), the same power can be transmitted either by using a large current and a small emf, or a small current and a large emf.

But the alternating emf produced by an electric generator is typically only several hundred volts. Hence one uses transformers to change this emf to one which may be as much as 1000 times larger (i.e., as large as 5×10^5 volt). By conservation of energy, the current transmitted over the wires with this emf is then correspondingly about 1000 times smaller than that originally produced by the generator. Finally, at the point where the power is to be delivered to the consumer, transformers are again used to change the emf to a magnitude small enough (about 100 volt) to be used safely in homes and factories. Indeed, the primary reason why alternating (rather than direct) currents are used for the transmission of electric power to our homes is the great ease with which transformers can be used to change alternating emfs from one magnitude to another.

Induced EMF and Number of Turns (Cap. 3)

E-1 The emf supplied to American homes is alternating, varying in time so that its maximum value is about 160 volt. If this emf is directly used to light lamps in a garden, where persons can be in contact with wet ground, the danger of accidental electrocution can be appreciable. Hence lamps for such garden environments are sometimes made to work with a low alternating emf (e.g., one varying in time so that its maximum value is only about 16 volt). A transformer can readily be used to transform the household emf to the emf required by such a "low-voltage" lamp. (a) If the primary coil of such a transformer has 500 turns, what must be the number of turns in the secondary coil of this transformer? (b) Is the maximum value of the current flowing through the lamp larger or smaller than that flowing through the primary coil of the transformer? *(Answer: 19) (Suggestion: [s-14])*

E-2 To ionize the gas in a neon sign, a manufacturer wishes to use an alternating emf having a maximum value of 11.2×10^3 volt. (a) What then must be the ratio N_2/N_1 of the number of turns of a step-up transformer to be used to supply this emf by starting with the household emf having a maximum value of 160 volt? (b) Why can the secondary coil of this wire be made of very thin wire? *(Answer: 17)*

SECT.

F SUMMARY

DEFINITIONS

electric generator; Def. (A-5)

magnetic flux; Eq. (C-4)

IMPORTANT RESULTS

Emf induced in a moving rod; Eq. (A-1)

$\mathcal{E} = v_0 B \ell$

Emf induced around a moving loop or stationary closed path; Eq. (C-4), rule (D-1), rule (C-5)

$\mathcal{E} = d\phi/dt$

(positive opposite to curled fingers of right hand if $\vec{B}$ increases along the thumb)

Transformer; Eq. (E-1)

$\mathcal{E}_2/\mathcal{E}_1 = N_2/N_1$

USEFUL KNOWLEDGE

practical electric generators (Sec. B)

betatron (Sec. D)

applications of transformers (Sec. E)

NEW CAPABILITIES

(1) Relate the velocity of a conductor moving through a magnetic field to the emf induced in this conductor, and correspondingly to the resulting potential difference between the ends of this conductor or the induced current flowing through this conductor. (Sec. A; [p-1], [p-2], [p-3])

(2) Relate a changing magnetic field perpendicular to a loop (or closed path) to the emf induced around this loop (or path) or to the resulting current flowing around this loop. (Sects. C and D, [p-4], [p-5])

(3) If the magnetic flux through two coils is the same, relate the emfs induced in these coils to the number of turns of these coils. (Sec. E)

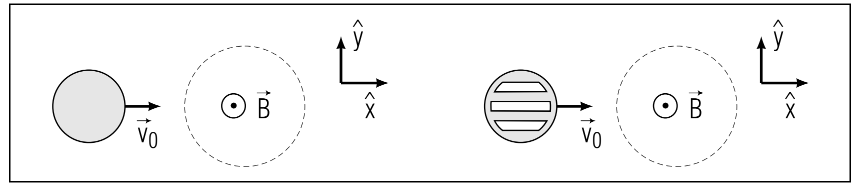


Fig. F-1.

F-1 *Metal disk moving through a field (Cap. 1):* Figure F-1a shows a metal disk moving with a velocity $\vec{v}_0$ in the $\hat{x}$ direction. As the disk enters a region in which there is a magnetic field $\vec{B}$ directed out of the paper, induced currents (so-called “Eddy currents”) start flowing within the metal disk as a result of the magnetic forces acting on the mobile charged particles in the disk. (a) Along what direction do these currents flow? (b) What is the direction of the magnetic forces acting on these currents? (c) Do these magnetic forces tend to accelerate the disk, decelerate the disk, or leave its velocity unchanged? (d) Suppose that the disk is supported by a string and thus forms a pendulum swinging back and forth. What would happen to the motion of this pendulum if a magnetic field is turned on (e.g., by setting up a current in a coil) in a region through which the disk is swinging? *(Answer: 22) (Suggestion: [s-17])*

F-2 *Electromagnetic brake (Cap. 1):* The preceding problem shows that a magnetic field acts like a useful brake which can slow down a moving metal object without relying on frictional forces due to any mechanical contact. (a) As the moving disk is slowed down, into what form of energy is its initial kinetic energy converted? (b) What would happen to the braking action, illustrated in parts (c) and (d) of the preceding problem, if the magnetic field in Fig. F-1 had the opposite direction into the paper? (c) What would happen to the braking action if the disk were made of a dielectric rather than of a conducting material? Why? (d) What would happen to the braking action if the metal disk contained many slots parallel to the velocity of the disk, as illustrated in Fig. F-1b? Why? (e) What would happen if the disk had slots perpendicular to the velocity of the disk? Why? *(Answer: 20)*

F-3 *Metal locator (Cap. 2):* A device useful in medicine to detect and locate metallic foreign bodies (e.g., in the eye) can be made by winding a pair of insulated coils on a small probe. When an alternating current is sent through one of these coils, the alternating emf induced in the second coil can be measured. If the probe is brought close to a metallic object, the magnitude of the induced emf then changes and this change indicates the presence of the metal object. Explain qualitatively why this change comes about. *(Answer: 24)*

SECT.

G PROBLEMS

G-1 *Force on induced current in a loop:* Figure G-1 illustrates a wire loop 2 supported above a similar loop 1. When the loop 1 is connected to a battery, the current flowing in it increases and a current is correspondingly induced in the loop 2. Is the resultant magnetic force exerted on loop 2 by loop 1 repulsive or attractive? What happens to loop 2 if it is free to move? *(Answer: 21) (Suggestion: [s-16])*

G-2 *Current induced in the absence of a field:* Figure G-2 shows a cross-sectional view of a long circular current-carrying coil. A circular wire loop L, concentric with the coil but of larger diameter, is located in the central plane of the long coil. In this plane the current flowing around the long coil produces a large magnetic field *within* the long coil, but a negligibly small magnetic field *outside* the coil where the loop is located. When the current in the coil changes, is there a current induced in the loop despite the fact that the magnetic field at the position of the loop is negligibly small? Why or why not? *(Answer: 23)*

G-3 *Emf produced by a generator:* A generator consists of a rectangular coil, consisting of 100 turns of wire, whose sides 1 and 2 (as illustrated in Fig. B-1) are 0.70 m long and whose width is 0.30 m. This coil rotates in a magnetic field of 0.050 tesla at a constant rate of 60 revolutions per second. What then is the maximum magnitude of the emf induced around this coil during its rotation? *(Answer: 27) (Suggestion: [s-19])*

G-4 *Current and potential difference induced in a loop:* Figure G-3a shows a circular wire loop, of area $0.50\,\mathrm{m}^2$ and resistance $0.10\,\Omega$, located in a uniform magnetic field (perpendicular to the loop) which changes at the constant rate of 3.0×10^{-3} tesla/second. (a) What is the magnitude of the induced emf around this loop? (b) What is the magnitude of the steady current flowing in this loop? (c) Fig. G-3b shows the same situation, but with the loop cut so that its two ends a and b are very close together. What then is the emf around this loop from a to b? (d) What is the magnitude of the steady current flowing in this cut loop? (e) What is the magnitude of the potential difference between the ends a and b of the loop? *(Answer: 25) (Suggestion: [s-2])*

G-5 *Measurement of magnetic field:* A circular coil, consisting of N turns of wire, is connected by flexible wires to an instrument which can measure the current or charge flowing through the coil. Initially the coil is placed in an unknown uniform magnetic field $\vec{B}_0$ perpendicular to the coil. The coil is then pulled out into a field-free region. (See Fig. G-4.) (a) While the coil is pulled out, the induced current in the coil is observed to flow around the coil in a clockwise sense. What then is the direction of $\vec{B}$? (b) At any instant while the coil is being pulled out, how is the current I through the coil related to N, the area A of the coil, the resistance R of the coil, and the rate of change dB/dt of the magnitude of the magnetic field through the coil? (c) Use this result to express the initial magnitude B_0 of the magnetic field in terms of N, A, R and the measured total charge Q_0 which flows through the coil during the entire process while the coil is being pulled out of the field. *(Answer: 28) (Suggestion: [s-18])*

G-6 *Induced electric field:* A uniform magnetic field $\vec{B}$, directed out of the paper, exists in the circular region indicated in Fig. G-5. (a) If the magnitude B of this field changes, what is the magnitude and direction of the electric field induced at the points P_1, P_2, and P_3 at a distance R from the center of the region of magnetic field? Express your answer in terms of R and dB/dt. (b) What would be the resultant electric force on an electron, with charge $-e$, located at the point P_1? *(Answer: 26) (Suggestion: [s-20])*

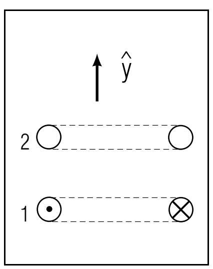


Fig. G-1.

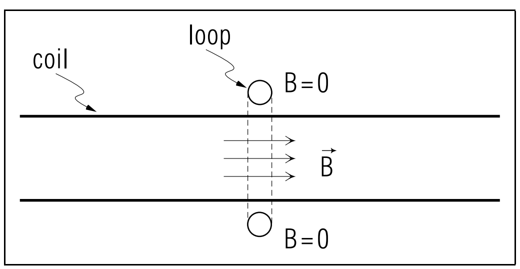


Fig. G-2.

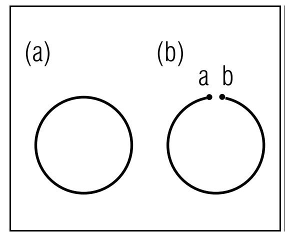


Fig. G-3.

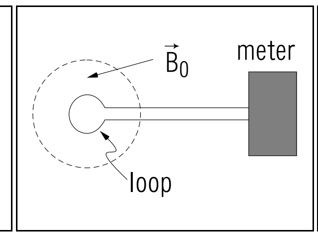


Fig. G-4.

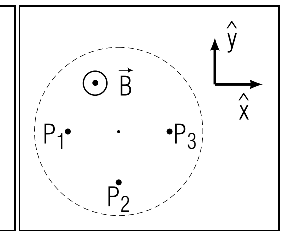


Fig. G-5.

TUTORIAL FOR G

ADDITIONAL PROBLEMS

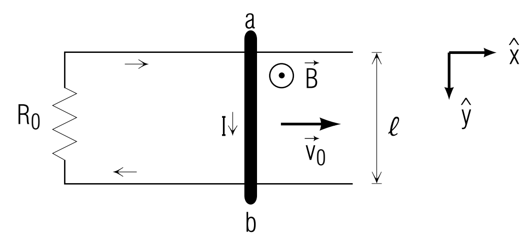
g-1 *UNITS OF $d\phi/dt$:* The unit of magnetic flux is that of a magnetic field multiplied by an area, i.e., tesla meter^2. (This unit is sometimes also called "weber.") By expressing the unit tesla in terms of more basic SI units (such as newton, kg, ...), show that the unit of $d\phi/dt$ is properly equal to volt.

g-2 *MEASUREMENT OF EMF INDUCED BY MOTION:* When a rod moves perpendicularly to a magnetic field, an emf $\mathcal{E}$ is induced between the two ends of the rod which has a resistance R_r. To measure the resultant potential difference between the ends of the rod, a voltmeter (having an internal resistance R_v) is connected between the ends of the rod. (a) What is the magnitude of the potential difference between the ends of the rod (and thus the potential difference measured by the voltmeter) when the voltmeter is connected to the ends of the rod? Express your answer in terms of $\mathcal{E}$, R_r, and R_v. (b) How is this measured magnitude of the potential difference related to the induced emf $\mathcal{E}$ if the resistance R_v of the voltmeter is very much larger than the resistance R_r of the rod? (c) How is the measured magnitude of the potential difference related to the induced emf if R_v is very much smaller than R_r? (d) Use these results to explain why it is difficult to measure the induced potential difference between the ends of the rod if this rod is a poor conductor (i.e., if its resistance is very large). *(Answer: 54) (Suggestion: s-23)*

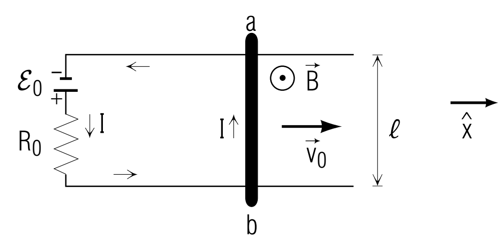
g-3 *ELECTROMAGNETIC FLOWMETER:* Consider again the electromagnetic flowmeter described in Problem A-5 of the text. Express the volume flow rate of the blood (i.e., the volume of blood flowing through the artery per second) in terms of the diameter D of the artery, the magnitude B of the magnetic field, and the measured magnitude of the potential drop V between opposite sides of the artery. *(Answer: 59) (Suggestion: s-24)*

g-4 *ENERGY CONVERSION IN A GENERATOR:* To examine more quantitatively the process of energy conversion in an electric generator, consider again the simple linear generator described in Sec. A of the text (and illustrated in the diagram) where a conducting rod of length ℓ slides on metal rails with a velocity $\vec{v}_0$ perpendicular to a magnetic field $\vec{B}$. As

a result of the emf $\mathcal{E}$ induced in the rod, a current I then flows around the circuit. (a) The power supplied by the induced emf to the circuit is then $\mathcal{E}I$. Express this power in terms of I, B, ℓ, and v_0. (b) Because of the current I flowing in the rod, a magnetic force acts on the rod in a direction perpendicular to the rod. To keep the rod moving with the constant velocity $\vec{v}_0$, a force $\vec{F}$ opposite to this magnetic force must then be used to pull the rod through the magnetic field. What are the direction and magnitude of this force? (c) What is the power supplied by this force $\vec{F}$ if the rod moves with the velocity $\vec{v}_0$? Express your result in terms of F and v_0. Express it also in terms of the quantities B, I, ℓ, and v_0. (d) Compare this power, supplied by the force $\vec{F}$ pulling the rod, with the power supplied to the circuit by the induced emf $\mathcal{E}$ (as calculated in part (a)). *(Answer: 57) (Suggestion: s-22)*



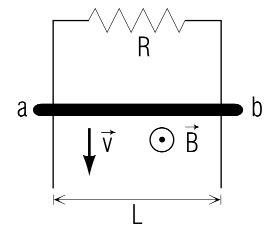
g-5 *ENERGY CONVERSION IN A MOTOR:* The diagram shows a simple linear motor in which a metal rod, of length ℓ ,is free to slide on two rails. The rod is located in a magnetic field perpendicular to the rod and pointing out of the paper. When a battery with emf $\mathcal{E}_0$ is connected to the rails as shown, a current I flows through the rails and the rod. The resistance of the entire circuit is R (and the resistance of the rails and the rod are assumed to be negligible).



(a) What is the magnitude and direction of the magnetic force $\vec{F}_m$ acting on the rod? (b) If the rod moves with a velocity $\vec{v}$ along the $\hat{x}$ direction as a result of this force, what is the power (i.e., work per unit time) delivered by this force to the rod? (c) When the rod moves with a velocity $\vec{v}$ in the $\hat{x}$ direction, the charged particles in the rod move not only along the rod, but have also a velocity component along the $\hat{x}$ direction. As a result of the motion of the rod, an emf $\mathcal{E}_x$ is then induced along the rod from its end a to its end b. Is this emf positive or negative? What is the magnitude of this emf? (d) Does this induced emf do positive or negative work on the current I flowing in the circuit? (e) To keep the magnitude of the current I in the circuit unchanged, would the emf of the battery have to be increased or decreased? By what amount? (f) What then would be the additional power supplied by the battery emf to the current I in the circuit? Express your answer in terms of I, B, v, and $\mathcal{E}_0$. (g) How does this increased power supplied by the battery compare to the power supplied by the motor to the rod (as found in part (b))? *(Answer: 61)*

g-6 *CURRENT FLOWING IN A MOTOR:* Consider again the simple motor of the last problem, but in the usual situation where the emf $\mathcal{E}_0$ of the battery remains fixed. (a) When the rod moves with the velocity $\vec{v}$ so that an emf $\mathcal{E}_1$ is induced in the moving rod, what is the current I in the rod? Express your answer in terms of $\mathcal{E}_0$, $\mathcal{E}_x$, and the resistance R of the circuit. Express it also in terms of $\mathcal{E}_0$, B, v, ℓ, and R. Is this current larger than, equal to, or smaller than when the rod is stationary? (b) When the rod moves, is the power dissipated in the resistance R_0 larger than, equal to, or smaller than when the rod is stationary? If the power is different, where does the extra power show up to account for the conservation of energy? (c) In the case of practical motors, it is found that the fuse in the circuit supplying the electric power to the motor is most likely to blow immediately after the motor is switched on, i.e., before it has attained its operating speed. Furthermore, there is a danger that a motor can overheat and burn out if the motor is run at too slow a speed (e.g., if it is forced to move too large a mechanical load). Explain these observations. *(Answer: 58)*

g-7 *METAL ROD FALLING THROUGH A MAGNETIC FIELD:* The two ends a and b of a horizontal metal bar, of length L and mass M, make electrical contact with two vertical metal rails so that the metal rod is free to slide vertically without friction. (See the diagram.) The ends of the rails are connected by a resistor of resistance R, while the resistance of the rails and the metal bar are negligibly small. A magnetic field $\vec{B}$, directed out of the paper, exists in the region between the rails.



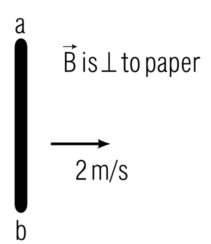
(a) Consider the rod at some instant when it is falling with a downward velocity $\vec{v}$. As a result of the emf induced in the falling rod, what is the magnitude and sense of the current I flowing through the rod? (b) What is the magnitude and direction of the vertical magnetic force which acts on the falling rod when the current I flows through it? Express your answer in terms of B, L, v, and R. (c) Under what conditions will the rod no longer be accelerated downward, but fall with a *constant* downward velocity $\vec{v}_0$? (d) Express the magnitude of this final constant velocity in terms of B, L, R, M, and g. (e) Would this final speed v_0 be larger or smaller if the magnetic field were larger? (f) Would v_0 be larger or smaller if the resistance R of the circuit were larger? (g) How would the answers to all questions (b) through (f) be changed if the magnetic field had the opposite direction? *(Answer: 60)*

g-8 *MOTION ALONG INCLINED RAILS IN A FIELD:* The effects discussed in the preceding problem can be demonstrated more conveniently by letting a metal rod of mass M slide down with negligible friction along a pair of parallel metal rails (separated by a distance L and inclined at an angle θ with the horizontal) in the presence of a vertical magnetic field $\vec{B}$. The rod and rails are assumed to have negligible resistance, but the rails are connected at the bottom by a resistor with a resistance R. What then is the final constant speed acquired by the sliding rod? *(Answer: 63)*

PRACTICE PROBLEMS

p-1 *EMF INDUCED IN A MOVING CONDUCTOR (CAP. 1):* In Fig. A-1 of the text, suppose that the moving rod contained positively charged mobile particles instead of electrons. (a) What then would be the direction of the magnetic force on such a particle? (b) As a result of the motion of these positively charged particles, would an excess of positive charge accumulate at the end a or the end b of the rod? (c) What then would be the direction of the resulting electric field inside the rod? (d) At which of the two ends of the rod would the resulting electric potential then be larger? (e) How do the answers to parts (b), (c), and (d) differ from the case where the mobile charged particles in the rod are negatively charged electrons? *(Answer: 53) (Suggestion: Review text problems A-1 and A-2.)*

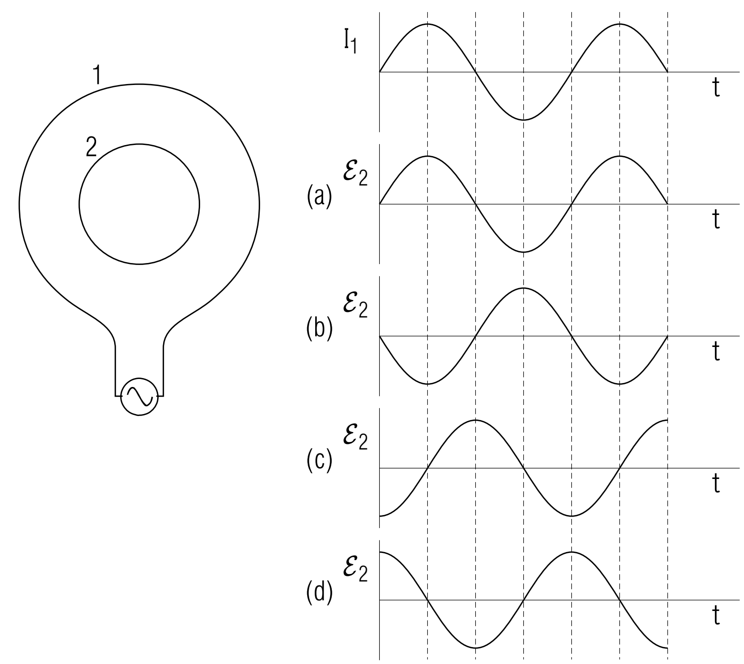
p-2 *EMF INDUCED IN A MOVING CONDUCTOR (CAP. 1): Measurement of $\vec{B}$*: In order to measure the magnetic field $\vec{B}$ existing in some region, a metal rod of length 0.10 m is moved with a velocity of 2 m/s perpendicular to the direction of $\vec{B}$, as illustrated in the diagram. The potential drop from the end a to the end b of the rod is then found to have a value of 1.0×10^{-4} volt. (a) What is the direction of the magnetic field? (b) What is the magnitude of this field, expressed in gauss? *(Answer: 56) (Suggestion: [s-4] and text problems A-1 and A-2.)*



p-3 *EMF INDUCED IN A MOVING ROD (CAP. 1): Potential difference induced in a flying plane:* An airplane, having a wingspan of 100 meter, is flying south with a speed of 220 m/s in a region where the vertical component of the earth's magnetic field is 5.0×10^{-5} gauss. What then is the magnitude of the potential difference induced between the wingtips of the plane? *(Answer: 52) (Suggestion: Review text problems A-1 and A-2.)*

p-4 *EMF INDUCED BY A CHANGING MAGNETIC FIELD (CAP. 2):* A wire loop has a radius of 0.10 m and a resistance of 0.125 ohm. At what rate must a magnetic field perpendicular to this loop change so as to produce a current of 1.0×10^{-3} ampere in the loop? *(Answer: 55) (Suggestion: Review text problem D-3.)*

p-5 *EMF INDUCED BY A CHANGING MAGNETIC FIELD (CAP. 2):* The diagram shows a circular loop 1 connected to an alternating emf source so that there flows in this loop an alternating current I_1 which, specified with respect to a counter-clockwise sense, varies with the time t in the manner indicated in the graph of I_1 versus t. Which of the four graphs (a), (b), (c), or (d) then specifies correctly the emf $\mathcal{E}_2$ induced around the smaller circular loop 2 in the counter-clockwise sense? *(Answer: 51) (Suggestion: Review text problem D-4 and look at [s-13] if necessary.)*



SUGGESTIONS

s-1 *(Text problems A-1 and A-2):* Remember that the charge $-e$ of the electron is negative. Thus the work W done on the electron and the work per unit charge $W/(-e)$ have opposite signs. Similarly the electric force on the electron and the electric field have opposite directions.

s-2 *(Text problem B-1):* Part (c): The other two sides of the loop are always parallel to the plane of the paper, i.e., to the plane containing the magnetic field $\vec{B}$. Hence the magnetic force on any charged particle in these sides is *perpendicular* to these sides and thus does not contribute to the work per unit charge on a particle moving along this side. Hence the emf along such a side is always zero.

s-3 *(Text problem A-4):* Part (a): Note that $V_{aa} = V_a - V_a = 0$. Or more directly since the coulomb work per unit charge is independent of the path, V_{aa} is just equal to the coulomb work per unit charge done on a particle which remains at the point a. Hence $V_{aa} = 0$.

Part (b): Note that all parts of the circuit, other than the moving rod, are stationary. Hence there is an emf along the moving rod only.

Part (c): The resistors are connected in series.

s-4 *(Practice problem [p-2]):* Assume that the magnetic field $\vec{B}$ has one of the two possible directions (either out of the paper or into the paper). Then deduce the corresponding sign of the induced potential drop. If this sign agrees with the observed sign of the potential drop, the assumed direction of $\vec{B}$ is correct. Otherwise, it is opposite.

s-5 *(Text problem A-5):* The blood is an ionic solution and thus a reasonably good conductor. The manner in which an emf is induced in the moving blood is thus completely analogous to the manner in which an emf is induced in a conducting rod moving through a magnetic field.

Part (a): Does the magnetic force tend to move positively charged particles in the moving blood toward the point a or the point b? As a result of the resulting accumulation of charge, is the potential at the point a larger or smaller than that at the point b?

Part (b): If negligible current flows through the voltmeter used to measure the potential drop from a to b, how is the magnitude of this potential drop related to the magnitude of the emf induced between these points?

s-6 *(Text problem C-2):* Part (c): Review text problem A-4.

s-7 *(Text problem B-2):* At this instant the sides 1 and 2 of the loop move parallel to the magnetic field. Remember that no magnetic force acts on a charged particle moving parallel to the magnetic field.

s-8 *(Text problem C-1):* Part (a): What would be the direction of the magnetic force on a positively charged particle? Would the work per unit charge done by this force on the particle moving from a to b then be positive or negative?

Part (d): Remember that the emf from c to d has the opposite sign (but the same magnitude) as the emf from d to c.

s-9 *(Text problem D-3):* What is the magnitude of the initial and final magnetic flux ϕ through the loop? What then is the magnitude of $d\phi/dt$? The induced current can be very easily found from the induced emf (as previously done in text problem A-4).

s-10 *(Text problem D-1):* In each case, determine whether the magnetic flux through the wire loop L changes or remains constant. An emf (and resulting current) is induced in the loop L only when this flux changes with time.

s-11 *(Text problem C-3):* Part (a): The magnetic field in the direction *out* of the paper *decreases* from 0.040 tesla to 0.035 tesla. Correspondingly, the magnetic field in the direction *into* the paper *increases* from -0.040 tesla to -0.035 tesla.

Part (b): If the thumb of the right hand points *into* the paper (along the direction along which the magnetic field increases) the curled fingers of this hand point in the sense *abcda*.

s-12 *(Text problem D-2):* To determine the sense of the induced current, determine the sign of the induced emf by the rule (C-5) in the text. If the magnetic moment of the permanent magnet points to the right (as indicated in Fig. D-3a), does the magnetic field produced by this magnet at the center of the loop L point to the right or to the left? If the current in loop L' has the sense indicated in Fig. D-3b (clockwise, as viewed from the left), does the magnetic field produced by this current at the center of the loop L point to the right or to the left?

s-13 *(Practice problem [p-5]):* What should be the emf $\mathcal{E}_2$ at the instant when the current I_1 is maximum or minimum, so that its rate of change with time is zero (and the corresponding rate of change of the magnetic flux produced by this current is also zero)? When the current I_1 is increasing, what should be the sign of the emf $\mathcal{E}_2$? Which of the four graphs is then the correct one?

s-14 *(Text problem E-1):* Part (b): The power supplied to the lamp must be the same as the power supplied to the primary coil of the transformer (neglecting the small amount of power dissipated into random internal energy of the transformer itself). Since the emf supplied to the lamp is much smaller than that supplied to the primary coil of the transformer, what then can one conclude about the size of the current supplied to the lamp? (Remember that the power supplied depends on the product of the emf and the current.)

s-15 *(Text problem D-4):* Note that the magnetic flux through the loop L is proportional to the magnetic field produced by the current I' in the other loop, and that this field is proportional to this current I'. Hence the graph in Fig. D-4 also illustrates qualitatively how the magnetic flux through the loop L varies with the time t. What then can you say about the rate of change $d\phi/dt$ at the various times? (If you have difficulty with rates, review text section B of Unit 404.)

s-16 *(Text problem G-1):* Suppose that the current in loop 1 flows in a clockwise sense (when looking along the $\hat{y}$ direction from below) and increases. What then is the sign of the emf induced along this sense in loop 2? What is the sense of the resulting induced current flowing in loop 2? As already discussed in Unit 427, what then is the direction of the magnetic force on the current in loop 2 by the current in loop 1?

s-17 *(Text problem F-1):* Parts (a) and (b): What would be the direction of the magnetic force on a positively charged particle moving with the disk through the field? In what direction would such a particle move? If it actually does move in this direction, what would be the direction of the resultant magnetic force on the particle?

Alternatively, you can answer the same questions about negatively charged particles, such as the electrons which are the mobile charged particles in the metal disk. The answers are the same as if positively charged particles were mobile.

s-18 *(Text Problem G-5):* Part (c): During any small time dt, show that the small amount of charge $dQ = Idt$ flowing through the coil is related to the change dB in the magnetic field through the coil so that $dQ = (NA/R)dB$. How then is the total charge ΔQ flowing through the coil during any time related to the corresponding change ΔB in the magnetic field?

s-19 *(Text problem G-3):* Review problem B-1. What is the speed of the sides 1 and 2 if the coil rotates at 60 revolutions per second? (Review text section F of Unit 406.) How is the emf induced around the entire coil of 100 turns related to the emf induced around a single turn of this coil?

s-20 *(Text problem G-6):* By symmetry, the induced electric field should have the same magnitude E at all points at the same distance from the center and should have at these points directions pointing along the same sense (either clockwise or counter-clockwise). Consider then a circular path of radius R. How is the magnitude of the emf around this path related to the magnitude of E along this path and to the radius R of this path? How is the magnetic flux through this path related to B and the radius R of this path? What then are the implications of Faraday's law (D-1) applied to this path?

s-21 *(Text problem G-4):* In all cases, the steady current I in the loop is related to its resistance R and to the potential drop V from a to b and the emf $\mathcal{E}$ from a to b so that $RI = V + \mathcal{E}$. When the points a and b are joined, what is V? How then is R related to $\mathcal{E}$? When the points a and b are *not* connected to each other, the emf $\mathcal{E}$ around the loop is essentially the same as before. What then, however, is the steady current I in the loop? Hence, how must V be related to $\mathcal{E}$?

s-22 *(Tutorial frame [g-4]):* Part (c): What is the displacement of the rod during some small time dt? What then is the work done on the rod by the force $\vec{F}$ during this displacement? What then is the power supplied by this force (remembering that power is defined as work done per unit time)?

s-23 *(Tutorial frame [g-2]):* What current flows through the rod and the voltmeter when the voltmeter is connected to the rod? How does the potential drop produced by this current flow affect the potential difference which would exist between the ends of the rod in the absence of this current flow?

s-24 *(Tutorial frame [g-3]):* If the blood moves in the artery with a speed v_0, what is the volume of blood which crosses any cross-sectional surface of the artery during some time t? (You might wish to review text section B of Unit 418).

ANSWERS TO PROBLEMS

1. a. larger
   b. $v_0 = |V_{ab}|/BD$
   c. $0.3\,\mathrm{m/s}$
2. (a), (b), (c) $3\mathcal{E}_{ab}$
3. 0 for all questions
4. a. $8 \times 10^{-20}\,\mathrm{N}$ opposite $\hat{y}$
   b. $-3.2 \times 10^{-20}\,\mathrm{J}$
   c. $0.20\,\mathrm{volt}$; d. yes
5. a. $V_{aa} = 0$
   b. $\mathcal{E}_{aa} = \mathcal{E}_{ab}$
   c. $R = R_r + R_0$
   d. $I = \mathcal{E}_{ab}/(R_r + R_0)$, same
6. a. positive; b. positive, larger
   c. zero; d. negative; e. opposite
7. a. $8 \times 10^{-20}\,\mathrm{N}$ along $\hat{y}$
   b. $0.50\,\mathrm{V/m}$ opposite $\hat{y}$
   c. $-0.20\,\mathrm{volt}$
   d. opposite
8. a. 1.4 volt; b. 1.4 volt
   c. 0; d. 2.8 volt
   e. 28 ampere
9. a. $0.60\,\mathrm{m}^2$
   b. $0.024\,\mathrm{T\,m}^2$, $0.021\,\mathrm{T\,m}^2$
   c. $0.05\,\mathrm{T\,m}^2/\mathrm{s} = 0.05\,\mathrm{volt}$
   d. 0.05 volt; e. yes
10. a. same
    b. opposite
11. a. $\mathcal{E}_{ab} = 0.35\,\mathrm{V}$, $\mathcal{E}_{dc} = 0.40\,\mathrm{V}$
    b. 0.05 volt; c. 0.5 ampere

12. a. 0; b. counter-clockwise
    c. clockwise; d. counter-clockwise
    e. 0; f. clockwise
13. a. decreases out of paper, increases into paper
    b. opposite
    c. around *adcba* (counter-clockwise); d. yes
14. a. 0; b. equal; c. larger
    d. largest at t_2 and t_4, smallest at t_1 and t_3
    e. opposite
15. a. no; b. yes; c. yes
    d. yes; e. no; f. yes
16. 1.0×10^{-3} ampere
17. a. 70
    b. much smaller current than in the primary coil
18. Electrons would be decelerated
19. a. 50; b. larger
20. a. converted into increased random internal energy of disk because of currents induced in disk
    b. braking the same
    c. no braking (no induced currents in dielectric)
    d. braking much reduced (induced currents along $\hat{y}$ direction interrupted by slots)
    e. braking action only slightly reduced (induced currents are not interrupted by slots)
21. repulsive: loop 2 jumps up
22. a. opposite $\hat{y}$; b. opposite $\hat{x}$
    c. decelerate
    d. motion is rapidly stopped
23. Yes. The magnetic flux through the loop changes in the region within the coil.

24. The changing magnetic field produced by the AC current in the first coil produces induced currents in the metal object. The emf in the second coil is then induced not only by the magnetic field produced by the currents in the first coil, but also by the currents in the metal object.
25. a. 1.5×10^{-3} volt; b. 0.015 ampere
    c. 1.5×10^{-3} volt; d. 0
    e. 1.5×10^{-3} volt
26. a. magnitude: $(R/2)(dB/dt)$, directions: $\hat{y}$, $-\hat{x}$, $-\hat{y}$
    b. $(eR/2)(dB)(dt)$, opposite $\hat{y}$
27. 4.0×10^2 volt
28. a. into paper
    b. $I = (NA/R)(dB/dt)$
    c. $B_0 = RQ_0/NA$
51. (c)
52. 1.1×10^{-4} volt
53. a. along $\hat{y}$; b. end b
    c. opposite $\hat{y}$; d. end b
    e. same
54. a. $|\mathcal{E}|R_V/(R_r + R_V)$
    b. $|\mathcal{E}|$
    c. $|\mathcal{E}|(R_V/R_r)$
    d. measured $|V|$ is very much smaller than $|\mathcal{E}|$
55. 4.0×10^{-3} T/s
56. a. into paper
    b. 5 gauss
57. a. $(v_0 B\ell)I$
    b. $\vec{F} = (IB\ell)$ along $\hat{x}$
    c. $Fv_0 = (IB\ell)v_0$
    d. powers are equal
58. a. $I = (\mathcal{E}_0 - \mathcal{E}_x)/R = (\mathcal{E}_0 - B\ell v)/R$, smaller

b. Smaller. Extra power is delivered to the moving rod.

c. Slower motion results in larger, and possibly excessive, current through the motor.

59. $(\pi/4)|VD/B|$

60. a. BLv/R through rod from b to a

b. B^2L^2v/R

c. When the rod moves so fast that the upward magnetic force becomes equal in magnitude to the downward gravitational force.

d. $v_0 = MgR/B^2L^2$

e. smaller; f. larger; g. unchanged

61. a. $IB\ell$

b. $F_mv = IB\ell v$

c. $\mathcal{E}_x = B\ell v$ (positive)

d. negative

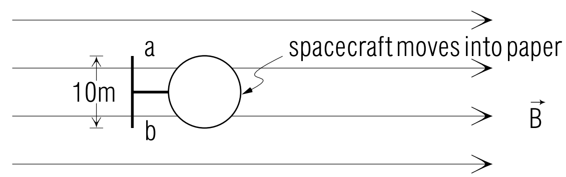
e. increased by $\mathcal{E}_x$

f. $\mathcal{E}_xI = (B\ell v)I$

63. $MgR\sin\theta/(BL\cos\theta)^2$

MODEL EXAM

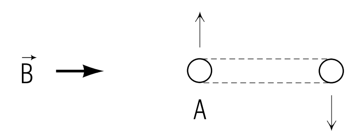
1. **Emf induced in a spacecraft antenna.** A spacecraft in orbit around the earth has a speed of 800 meter/second. It moves through the earth's magnetic field, which has an intensity of 1×10^{-4} tesla, as shown in the following diagram. The spacecraft antenna is 10 meter long.



a. Which end of the antenna (a or b) is at the higher potential?

b. What is the emf induced in the antenna?

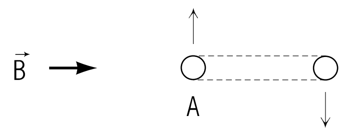
2. **Emf induced in a coil.** A steady current flows in the coil C_1 shown in the following diagram. The magnetic field through the small coil C_2, whose area is 4×10^{-4} meter^2, is 5×10^{-3} tesla.



a. Is the sense of the current induced in C_2 the same as that flowing in C_1, or opposite?

b. If C_2 is moved to the right in the diagram so that the field through it decreases to 2×10^{-3} tesla in the short enough time 1 second, what is the magnitude of the emf induced in C_2?

3. **Sense of emf induced in a generator.** A loop of wire is used as a simple generator, as shown in this diagram:



What is the sense of the current which flows in the portion of the loop labeled A in the diagram, into the paper or out of the paper?

4. **Design requirements for a transformer.** It is desired to step down an emf from 3.00×10^2 to 25 volt, and to have 120 turns of wire on the primary side of the transformer.

How many turns of wire should there be in the secondary side of this transformer?

Brief Answers:

1. a. b

   b. 0.8 volt

2. a. no current in C_2

   b. 12×10^{-7} volt

3. into the paper

4. 10