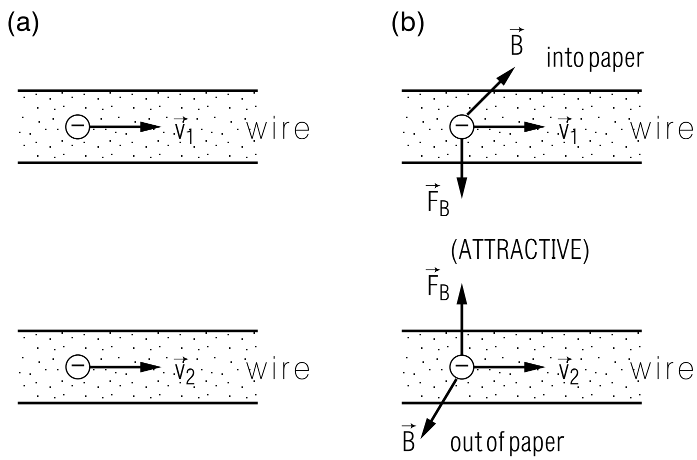
MISN-0-426

MAGNETIC FORCE AND FIELD



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MAGNETIC FORCE AND FIELD

by

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CONTENTS

Title: **Magnetic Force and Field**

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Input Skills:

1. Vocabulary: electric current (MISN-0-423).
2. State the magnitude and direction of the acceleration of a particle moving in a circle with uniform speed (MISN-0-376).

Output Skills (Knowledge):

K1. Vocabulary: ammeter, cyclotron, electric motor, electromagnetic force, gauss, magnetic field, magnetic moment, mass spectrometer, tesla.

K2. State the reason why magnetic forces do no work on moving charges.

K3. Derive expressions for the orbital speed and orbital period of a charged particle moving in circular motion in a uniform magnetic field.

Output Skills (Problem Solving):

S1. Determine the change in a given magnetic force or in a given magnetic field if: (a) the magnitude or sign of any charge is changed; (b) the magnitude or direction of any velocity is changed; (c) the magnitude or sense of any current is changed.

S2. Given the sense of the current in a coil, determine: (a) the direction of the resulting magnetic moment, and; (b) the direction the coil tends to rotate in a uniform magnetic field.

MISN-0-426

MAGNETIC FORCE AND FIELD

Abstract:
Experiments on the interaction between current-carrying wires lead to the discovery that the interaction between charged particles is more complex than that described by Coulomb's law. Thus it is found that a part of this interaction (called the "magnetic interaction") depends not only on the positions, but also on the velocities of the charged particles. Since the magnetic interaction is of great practical importance, we shall use the next few units to discuss this interaction and its many implications.

SECT.

A INTERACTION BETWEEN MOVING CHARGED PARTICLES

INTERACTION BETWEEN CURRENT-CARRYING WIRES

► *Observations*

Consider two straight wires, such as those shown in Fig. A-1, which can be connected to batteries so that steady currents I and I_1 flow in these wires. Then one observes that there is a force on each wire due to the other wire. This force has the properties: (1) The force depends on the currents in the wires and on the relative positions of these wires. (2) The direction of the force is reversed if the sense of either current is reversed. (3) The magnitude of the force is proportional to the magnitude of either current. (For example, if either current is 3 times as large, the magnitude of the force is also 3 times as large.) In particular, the force is zero if either current is zero.

► *Conclusions*

The force observed in the preceding experiment depends on the currents and is thus the result of interactions between moving charged particles. This force can certainly *not* be due to coulomb forces exerted on the charged particles in one wire by those in the other wire. For then a mere reversal of the sense of the current in one wire (i.e., a reversal of the directions of the velocities of the moving charged particles in this wire) would leave the observed force unchanged since all coulomb forces depend only on the positions of the interacting particles and remain thus unaffected irrespective of the velocities of these particles. *

* Furthermore, the coulomb force on the charged particles in one wire due to those in the other wire is negligibly small since the *total* charge of each current-carrying wire is zero.

Thus the observed force is a new kind of force which depends on the velocities of the interacting charged particles. This force is called the "magnetic force." (We shall see in the next unit how this force is related to the forces produced by "permanent magnets").

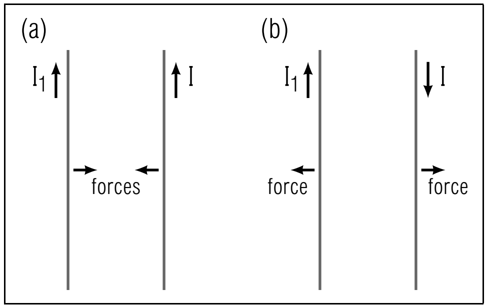


Fig. A-1: Observed forces on current-carrying wires with currents: (a) flowing in the same sense; (b) flowing in opposite senses.

MAGNETIC FORCE ON ONE PARTICLE DUE TO ANOTHER

In Unit 408 we assumed that all forces depend only on the positions of the interacting particles. Since the magnetic force depends also on the velocities of the interacting particles, this force is more complex than any of the forces previously studied by us. Thus we must try to find out how the magnetic force depends on the properties, positions, and velocities of the interacting charged particles. As usual, we shall proceed by making some basic assumptions suggested by a few experiments. The validity of these assumptions is then ultimately established by their success in predicting a wide range of experimental observations.

SUPERPOSITION PRINCIPLE

Suppose that we know the magnetic force exerted on one charged particle by another charged particle. Then we assume that the superposition principle is valid, i.e., that the magnetic force exerted on one set of moving charged particles (constituting one current) by another set of moving charged particles (constituting another current) is simply equal to the *vector sum* of the individual magnetic forces exerted on the particles in the first set by the particles in the second set.

► *Properties of* $\vec{F}$

A current in a wire is proportional to the charges and velocities of the moving particles. Hence the previously mentioned observations of the magnetic force on one wire due to another allow us to infer the following properties of the magnetic force $\vec{F}$ on one charged particle due to another: (1) The force $\vec{F}$ depends on the velocities as well as on the positions of the interacting particles. (2) The direction of $\vec{F}$ is reversed if the sign of the charge or the direction of the velocity of either particle is reversed.

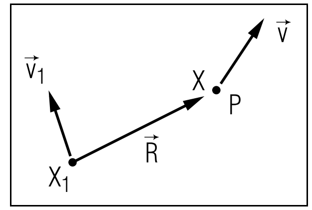


Fig. A-2: Magnetic interaction between two moving charged particles X and X_1.

(Thus $\vec{F}$ remains unchanged if the signs of the charges or the directions of the velocities of *both* particles are reversed.) (3) The magnitude of $\vec{F}$ is proportional to the magnitude of the charge and to the magnitude of the velocity of either particle. In particular, $\vec{F}$ is zero if the charge or the velocity of either particle is zero.

What then can we say about the magnetic force $\vec{F}$ exerted on a particle X, with charge q and velocity $\vec{v}$, by another particle X_1, with charge q_1 and velocity $\vec{v}_1$? *

> * Throughout the following units we shall assume that all velocities are specified relative to a particular inertial frame (e.g., the approximate inertial frame of a laboratory on the surface of the earth.) After one has obtained a full understanding of *all* the electric and magnetic interactions between charged particles, one can show that all observable results are independent of one's particular choice of reference frame.

(See Fig. A-2.) The *direction* of $\vec{F}$ should depend on the directions of the velocities $\vec{v}$ and $\vec{v}_1$ of these particles and on the direction of the vector $\vec{R}$ specifying the position of one particle relative to the other. The *magnitude* of $\vec{F}$ should depend on the magnitudes of the charges and of the velocities of both particles (so as to be proportional to all of these, in accordance with the third property in our list); it should depend on the distance R between the particles; and it should also depend on the various angles specifying the relative directions of $\vec{v}$, $\vec{v}_1$, and $\vec{R}$. Thus the magnitude of the magnetic force should be of the form

$$|\vec{F}| = k_m \left| \frac{q\, q_1\, v\, v_1}{R^2}\,(\text{angles}) \right| \qquad \text{(A-1)}$$

where k_m is some constant (the "magnetic force constant") and the parenthesis indicates some (as yet unspecified) combination of angles. The fact that the magnetic force, just like the electric coulomb force, decreases with distance so as to be inversely proportional to R^2 can be inferred from some simple experiments. *

> * For example, observations on the magnetic interaction between current-carrying wires show that the magnitude of the magnetic force on one particle due to another remains *unchanged* if the speed of each particle is twice as large and the separation R between them is also twice as large. This observation can only be true if $|\vec{F}|$ is proportional to $1/R^2$ [so that the 4-fold increase of vv_1 in the numerator of Eq. (A-1) is compensated by the 4-fold increase of R^2 in the denominator].

MAGNETIC FIELD

The problem of finding directly the magnetic force $\vec{F}$ on one particle due to another is thus quite complex. But this problem can be greatly simplified if we recall how we introduced the electric field $\vec{E}$ in order to simplify the analogous problem of finding the coulomb electric force on one particle due to another. Let us then use this analogy by introducing an auxiliary vector $\vec{B}$, called the "magnetic field," and let us use this vector to decompose the magnetic problem into two simpler successive problems. Instead of finding directly the magnetic force $\vec{F}$ exerted on the particle X by the particle X_1, we shall thus proceed as follows:

(1) We first find the magnetic field $\vec{B}$ produced by the particle X_1 at the position P of the particle X.

(2) We then use this magnetic field $\vec{B}$ to find the magnetic force $\vec{F}$ exerted on the particle X. Let us comment briefly on these two parts of the problem.

► *Production of $\vec{B}$*

The magnetic field $\vec{B}$ produced at a point P by a particle X_1 is defined so that $\vec{B}$ depends on the charge q_1 and velocity $\vec{v}_1$ of this particle, as well as on the position of the point P relative to this particle. [Thus $\vec{B}$ involves all the quantities in Eq. (A-1) except for the charge q and velocity $\vec{v}$ of the particle X acted on by the magnetic force.] The *direction* of $\vec{B}$ is assumed to reverse if either the sign of the charge q_1 or the direction of the velocity $\vec{v}_1$ of the particle X_1 is reversed. The *magnitude* of $\vec{B}$ is assumed to be proportional to the magnitudes of q_1 and $\vec{v}_1$. In particular, $\vec{B}$ is zero if either the charge or the velocity of the particle X_1 is zero. (Furthermore, by the superposition principle, the magnetic field $\vec{B}$ due

to several particles is just the vector sum of the magnetic fields produced by each of these particles separately.) We shall postpone until the next unit (Unit 427) a more detailed discussion of the production of magnetic fields.

► *Force produced by $\vec{B}$*

The magnetic force $\vec{F}$ exerted on a particle X by the magnetic field $\vec{B}$ (produced at its position by one or more other particles) depends then merely on $\vec{B}$ as well as on the charge q and velocity $\vec{v}$ of this particle. In this unit we shall first discuss the magnetic force produced by a magnetic field on a single charged particle. Then we shall examine the magnetic force produced on many charged particles constituting a current in a wire.

Dependence of $\vec{F}$ on Charges and Velocities (Cap. 1)

A-1 Suppose that a magnetic force $\vec{F}_m$ acts on the right wire in Fig. A-1a. What then would be the magnetic force on this wire if (a) the current in the right wire were twice as large; (b) the current in each wire were twice as large; (c) the current in the left wire were zero; (d) the current in the left wire had the opposite sense, (e) the currents in both wires had opposite senses? *(Answer: 5) (Suggestion: [s-1])*

A-2 At a particular instant, a magnetic force $\vec{F}_m$ is exerted on a particle X by another particle X_1. (See Fig. A-2.) What then would be the magnetic force exerted on X if (a) the charge of X_1 were 3 times as large; (b) the charge of each particle were 3 times as large; (c) the charge of X had the opposite sign, (d) the velocity of X were 5 times as large; (e) the velocity of each particle were 5 times as large; (f) the velocity of X had the opposite direction; (g) the velocity of X were zero? *(Answer: 2)*

A-3 A particle X_1 produces at some point P a magnetic field $\vec{B}$. What would be the magnetic field produced at this point if (a) the charge of the particle were 3 times as large; (b) the velocity of the particle were 2 times as large; (c) its velocity were zero; (d) its velocity had the opposite direction; (e) the charge of the particle had the opposite sign and its velocity had the opposite direction? *(Answer: 7)*

SECT.

B MAGNETIC FORCE ON A CHARGED PARTICLE

Consider a particle at a point where there is a magnetic field $\vec{B}$ produced by one or more other particles. (See Fig. B-1.) If the charge of the particle is q and its velocity is $\vec{v}$, what is the magnetic force $\vec{F}$ on this particle?

DIRECTION OF THE FORCE

How is the direction of the magnetic force $\vec{F}$ related to the direction of the particle's velocity $\vec{v}$ and to the direction of the magnetic field $\vec{B}$?

► *Argument*

We know that the direction of $\vec{F}$ is reversed if either the direction of $\vec{v}$ is reversed or if the direction of $\vec{B}$ is reversed (e.g., if the directions of the velocities of the particles producing $\vec{B}$ are reversed). Thus the direction of $\vec{F}$ should remain *unchanged* if *both* the direction of $\vec{v}$ and the direction of $\vec{B}$ are reversed. But one way of reversing both of these directions, and thus leaving $\vec{F}$ unchanged, is simply to rotate the whole system of interacting particles about an axis perpendicular to the plane containing both $\vec{v}$ and $\vec{B}$. (See Fig. B-2.) As this figure indicates, the effect of this rotation is also to reverse the direction of the component vector $\vec{F'}$ of the force $\vec{F}$ parallel to the plane containing $\vec{v}$ and $\vec{B}$, leaving the component vector of $\vec{F}$ perpendicular to this plane unchanged. But since the magnetic force $\vec{F}$ must be the same in Fig. B-2a and Fig. B-2b, we conclude that the component vector $\vec{F'}$ of the magnetic force parallel to this plane must be *zero*. Hence the magnetic force $\vec{F}$ must be *perpendicular* to this plane.

Consider now the special case where $\vec{v}$ is parallel to $\vec{B}$, as indicated in Fig. B-3. Then there are an infinite number of planes, of different orientations, containing both $\vec{v}$ and $\vec{B}$. By our preceding argument, the magnetic force $\vec{F}$ should then be perpendicular to all of these planes. But

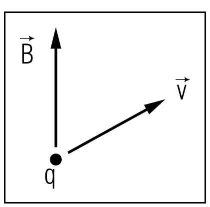


Fig. B-1: Particle moving with a velocity $\vec{v}$ in the presence of a magnetic field $\vec{B}$.

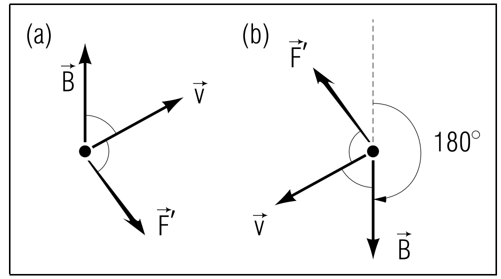


Fig. B-2: Rotation of a system of particles by 180° about an axis perpendicular to the plane of the paper.

this can only be true if $\vec{F}$ is zero.

► *Conclusion*

Thus we arrive at the conclusion:

The magnetic force $\vec{F}$ on a particle is perpendicular to both its velocity $\vec{v}$ and to the magnetic field $\vec{B}$, and is zero if $\vec{v}$ is parallel to $\vec{B}$.	(B-1)

► *Right-hand rule*

There is one remaining ambiguity. According to Rule (B-1), the magnetic force on the positively charged particle in Fig. B-4a must be perpendicular to the plane of the paper. But does this force point out of the paper (a direction conventionally indicated by the symbol $\odot$), or does it point into the paper (a direction indicated by the symbol $\otimes$)? *

> * The symbol $\odot$ is supposed to indicate the tip of an arrow pointing out of the paper, while the symbol $\otimes$ is supposed to indicate the tail feathers of an arrow pointing into the paper.

To remove this ambiguity, one conventionally defines the magnetic field in a way consistent with this "right-hand rule" (illustrated in Fig. B-

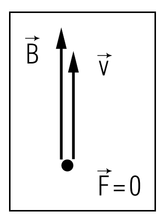


Fig. B-3: Particle moving with a velocity $\vec{v}$ parallel to a magnetic field $\vec{B}$.

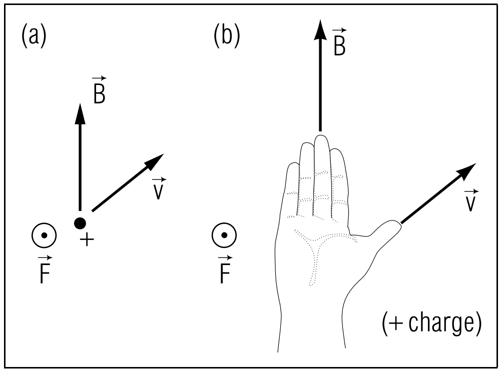


Fig. B-4: Direction of magnetic force. (a) Charged particle moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$. (b) Right-hand rule for magnetic force.

4b or Fig. B-5):

> *Right-hand rule for $\vec{F}$:* If the velocity $\vec{v}$ of a *positively* charged particle points along the thumb of the right hand and the magnetic field $\vec{B}$ points along the fingers of this hand, then the magnetic force $\vec{F}$ on the particle points *out* of the palm of this hand. (B-2)

The magnetic force on a *negatively* charged particle has the *opposite* direction.

MAGNITUDE OF THE FORCE

From our discussion in Sec. A, we know that the magnitude of the magnetic force $\vec{F}$ is proportional to the magnitude of the charge q and of the velocity $\vec{v}$. It should also be proportional to the magnitude of the magnetic field $\vec{B}$ (i.e., proportional to the magnitudes of the charges and velocities of the particles producing this field).

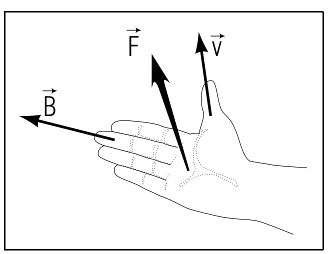


Fig. B-5: Right-hand rule for magnetic force.

► *Dependence on $\vec{v}_\perp$*

Our previous discussion of directions introduces one additional simplification. The velocity $\vec{v}$ of the particle can be imagined to consist of a component vector $\vec{v}_\parallel$ parallel to the magnetic field $\vec{B}$ and of a component vector $\vec{v}_\perp$ perpendicular to $\vec{B}$. But, by Rule (B-1), a particle moving with a velocity equal to $\vec{v}_\parallel$ parallel to $\vec{B}$ would experience *no* magnetic force. Hence we expect, in agreement with observations, that the magnetic force depends *only* on the component vector $\vec{v}_\perp$ of the velocity perpendicular to $\vec{B}$. Our preceding statements about proportionality can then be summarized by writing

$$|\vec{F}| = |q\, v_\perp\, B| \tag{B-3}$$

(if one defines $\vec{B}$ so that the constant of proportionality is simply 1).

SUMMARY

The magnetic field $\vec{B}$ at any point is defined in such a way that a particle, with charge q and velocity $\vec{v}$, located at this point is acted on by a magnetic force

> $\vec{F} = |qv_\perp B|$, direction given by right-hand rule of Fig. B-4b. (B-4)

Understanding Magnetic Force and Field (Cap. 2)

B-1 A positively charged particle moves with a velocity $\vec{v}$ at a point where the magnetic field $\vec{B}$ has the direction indicated in Fig. B-6. What is the direction of the magnetic force $\vec{F}$ on the particle in each of the following cases: (a) The velocity $\vec{v}$ is equal to the velocity $\vec{v}_1$ indicated in Fig. B-6. (b) $\vec{v} = \vec{v}_2$. (c) $\vec{v} = \vec{v}_3$. (d) The particle is negatively charged and moves with the velocity $\vec{v}_4$. *(Answer: 1) (Suggestion: [s-2])*

B-2 In Fig. B-7, where the magnetic field $\vec{B}$ points out of the paper, a particle moving with the indicated velocity $\vec{v}$ experiences a magnetic force $\vec{F}$ in the indicated direction. Is the charge of this particle positive or negative? *(Answer: 6) (Suggestion: [s-3])*

B-3 An electron moves with a velocity $\vec{v}$ in the presence of a magnetic field $\vec{B}$. When this velocity $\vec{v}$ is perpendicular to $\vec{B}$, the magnitude of the magnetic force on the electron is $\vec{F}_0$. (a) What then is the

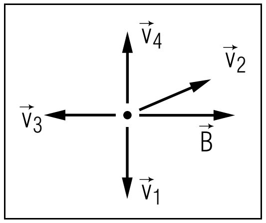


Fig. B-6.

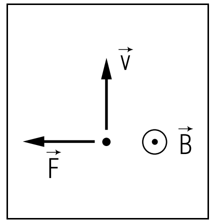


Fig. B-7.

magnitude of the magnetic force on the electron when its velocity $\vec{v}$ makes an angle of 30° relative to $\vec{B}$? (b) Suppose that the magnetic field were 5 times as large and the electron moved perpendicularly to this field with a speed 2 times as large as before. What then would be the magnitude of the magnetic force on the electron? *(Answer: 9) (Suggestion: [s-4]) (Practice: [p-1])*

SECT.

C PROPERTIES OF THE MAGNETIC FORCE AND FIELD

PROPERTIES OF THE MAGNETIC FIELD

► *Measurement of* $\vec{B}$

According to Rule (B-4), the magnetic field $\vec{B}$ at any point P can be measured by simply measuring the magnetic force $\vec{F}$ on any particle at P and then using Rule (B-4) to calculate $\vec{B}$ from the known charge q and known velocity $\vec{v}$ of this particle.

► *Unit of* $\vec{B}$

According to Eq. (B-3), the unit of $\vec{B}$ is the same as that of $F/(qv_\perp)$ so that

$$\text{unit of } \vec{B} = \frac{(\text{newton})(\text{second})}{(\text{coulomb})(\text{meter})} = \text{tesla} \qquad \text{(C-1)}$$

where the SI unit "tesla" (abbreviated "T") is merely a convenient abbreviation for the combination of units in Eq. (C-1). [This unit is named in honor of the Croatian-American engineer Nicola Tesla (1856-1943).]

Another common unit of magnetic field (*not* an SI unit) is the "gauss" [named in honor of the famous German mathematician and physicist Karl F. Gauss (1777-1855)]. These units are simply related so that

$$1\,\text{tesla} = 10^4\,\text{gauss} \qquad \text{(C-2)}$$

The magnitude of the magnetic field produced by the earth near its surface is about 0.3 gauss or 0.3×10^{-4} tesla. Magnetic fields as large as 5 tesla can readily be produced in a laboratory.

ELECTROMAGNETIC FORCE ON A CHARGED PARTICLE

The "electromagnetic force" $\vec{F}_{em}$ on a charged particle is the total force exerted on this particle because of its charge. This force consists of the electric force $\vec{F}_e = q\vec{E}$ (due to the electric field $\vec{E}$ at the position of the particle) and of the magnetic force $\vec{F}_m$ [due to the magnetic field $\vec{B}$ calculated by Rule (B-4)]. *

* As we shall see later, the electric field $\vec{E}$, or electric force $\vec{F}_e$ may be due to *coulomb* as well as to other electric forces.

Thus

$$\vec{F}_{em} = \vec{F}_e + \vec{F}_m \tag{C-3}$$

► *Comparison of $\vec{F}_e$ and $\vec{F}_m$*

The electric and magnetic forces have very different properties. (1) The electric force $\vec{F}_e = q\vec{E}$ does *not* depend on the velocity of the particle. But the magnetic force $\vec{F}_m$ *depends* on the velocity of the particle, and is zero if the velocity of the particle is zero. (2) The electric force $\vec{F}_e = q\vec{E}$ is *parallel* to the direction of the electric field $\vec{E}$. But the magnetic force $\vec{F}_m$ is *perpendicular* to the magnetic field $\vec{B}$.

WORK DONE BY A MAGNETIC FORCE

The magnetic force $\vec{F}_m$ on a moving charged particle is at any instant *perpendicular* to the velocity $\vec{v}$ of this particle. This magnetic force is therefore everywhere perpendicular to the path of the particle (i.e., perpendicular to any small displacement of the particle). Hence the work done on the particle by the magnetic force during any small displacement is always equal to zero. Thus we arrive at the conclusion:

The work done on a particle by a magnetic force is zero.	(C-4)

Understanding Magnetic Force and Field (Cap. 2)

C-1 *Finding magnetic field:* A proton, having a charge $e = 1.6 \times 10^{-19}$ C, moves perpendicularly to a magnetic field $\vec{B}$ with a velocity $\vec{v} = 1.0 \times 10^6$ m/s in the direction indicated in Fig. C-1. From the deflection of the proton, one infers that the magnetic force $\vec{F}_m$ on the proton is 4.8×10^{-15} N in the indicated direction. (a) What is the direction of the magnetic field $\vec{B}$ at the position of the proton? (b) What is the magnitude of $\vec{B}$? (c) What is this magnitude, expressed in the unit gauss? *(Answer: 4) (Suggestion: [s-5])*

C-2 *Finding magnetic force:* (a) What is the magnitude of the magnetic force exerted on an electron moving in an oscilloscope with a speed of 2×10^7 m/s perpendicularly to the earth's magnetic field of 0.5 gauss? The charge of the electron is -1.6×10^{-19} C and its mass is approximately 10^{-30} kg. (b) What is the approximate magnitude of the

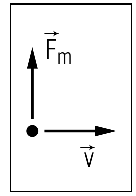


Fig. C-1.

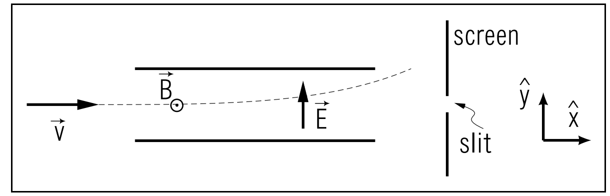


Fig. C-2.

resultant acceleration of the electron? *(Answer: 8)*

C-3 *Comparison of magnetic and electric forces:* (a) A proton, with charge $+e$, and an electron, with charge $-e$, have the same kinetic energies. Which particle has the larger speed? (b) If these particles move through an electric field, is the magnitude of the electric force on the proton larger than, equal to, or smaller than that on the electron? (c) If these particles move perpendicularly to a magnetic field, is the magnitude of the magnetic force on the proton larger than, equal to, or smaller than that on the electron? (d) If a charged particle moves through a non-zero electric field, can the electric force on the particle be zero? Under what conditions? (e) If a charged particle moves through a non-zero magnetic field, can the magnetic force on the particle be zero? Under what conditions? *(Answer: 3)*

C-4 *Electromagnetic force and velocity selector:* Figure C-2 shows a beam of charged particles, each having a charge q and moving with a velocity $\vec{v}$, entering a region between two parallel metal plates. In this region, a magnetic field $\vec{B}$, out of the plane of the paper, is produced by currents in some coils. An electric field $\vec{E}$, in the indicated direction perpendicular to $\vec{B}$, is produced by a potential difference maintained between these plates. (a) Suppose that q is positive. What then is the direction of the electric force $\vec{F}_e$ on a particle between the plates? What is the direction of the magnetic force $\vec{F}_m$ on this particle? (b) What must be the magnitude v_0 of the particle's velocity so that the total electromagnetic force on the particle is zero? (The particle then travels between the plates undeflected and passes through the slit in the screen.) (c) Would the value of v_0 be different if q were negative or if the particles had different mass? (d) Suppose that the speed of a positively charged particle is larger than v_0. In which direction is the particle between the plates deflected as a result of the electromagnetic force? Would the particle still pass through the slit? (e) Answer the same question if the speed v of the particle is smaller than v_0. (f) Suppose that a beam of Na^+ ions, moving with various speeds, entering the region between the plates. If $B = 0.08$ tesla and $E = 4 \times 10^3$ volt/meter, what is the speed of the Na^+ ions which emerge through the slit? *(Answer: 11) (Suggestion: [s-6])*

C-5 *Work done by electromagnetic forces:* Suppose that a positively charged particle moving between the plates in Fig. C-2 is deflected toward the upper plate. (a) Is the work done on the particle by the electric force between the plates positive, zero, or negative? (b) Is the work done on this particle by the magnetic force positive, zero, or negative? *(Answer: 14) More practice for this Capability: [p-4], [p-8]*

SECT. D MOTION OF CHARGED PARTICLES IN A MAGNETIC FIELD

What is the motion of a charged particle, with mass m and charge q, moving under the sole influence of a magnetic field? To be specific, we shall assume that the magnetic field $\vec{B}$ is uniform throughout the region in which the particle moves and that its direction is out of the plane of the paper in Fig. D-1.

► *Constant speed*

The acceleration $\vec{a}$ of the particle is, at any instant, related to the magnetic force $\vec{F}$ on the particle so that $m\vec{a} = \vec{F}$. Since the magnetic force is always perpendicular to the velocity $\vec{v}$ of the particle, the acceleration $\vec{a}$ is then also always perpendicular to $\vec{v}$. Hence the magnitude of $\vec{v}$ remains constant and only the direction of $\vec{v}$ changes. *

> * This conclusion is consistent with the fact that the magnetic force does no work. Consequently the particle's kinetic energy, and thus also the magnitude of its velocity, remains unchanged.

Furthermore, the magnitude of the magnetic force $\vec{F}$, and thus also that of the acceleration $\vec{a}$, remains constant.

► *Circular motion*

Suppose that the velocity $\vec{v}$ of the particle is initially perpendicular to the magnetic field $\vec{B}$ (i.e., in the plane of the paper in Fig. D-1). Then the magnetic force perpendicular to $\vec{v}$ tends to make the path of the particle curved. Indeed, since the particle moves with constant speed with an acceleration of constant magnitude perpendicular to its velocity, we expect (from text section F of Unit 406) that the particle moves simply around a circle.

Let us verify that this expectation is consistent with the equation of motion $ma = F$. As the particle moves around the circle, its acceleration $\vec{a}$ is at any instant directed toward the center of the circle. Hence the magnetic force $\vec{F}$ on the particle must also be toward the center. By the right-hand rule, this is true if a positively charged particle travels around the circle in Fig. D-1 in a clockwise sense (or a negatively charged particle in a counter-clockwise sense). Furthermore, the magnitudes of $\vec{a}$ and $\vec{F}$ must be related so that

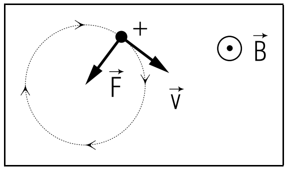


Fig. D-1: Circular path of a charged particle moving in a uniform magnetic field $\vec{B}$.

$$ma = F \tag{D-1}$$

But by Relation (F-4) of Unit 406, $a = v^2/r$ if the particle travels around a circle of radius r. Furthermore, by Eq. (B-3), $F = |q|vB$ since $v_\perp = v$ because $\vec{v}$ is perpendicular to $\vec{B}$. Thus Eq. (D-1) implies that

$$m\frac{v^2}{r} = |q|vB$$

Thus v and r are related so that

$$\boxed{v = \frac{|q|B}{m}r} \tag{D-2}$$

Let us now discuss some practical applications of these results.

MASS SPECTROMETER

► *Measurement of m*

The relation (D-2) allows one to find the mass m of the particle if all the other quantities are known. Indeed,

$$m = |q|B\frac{r}{v} \tag{D-3}$$

This relation is exploited in the "mass spectrometer," an important instrument used to make precise measurements of the masses of atoms or molecules, or to separate atomic particles according to their differing masses. In such a mass spectrometer, the molecules (or atoms) are first ionized by electron bombardment, thus acquiring a known charge such as e or $2e$ (where e is the magnitude of the electron charge). Then one can measure the speed v of the ionized molecule when it enters a region where there is a known magnetic field $\vec{B}$. Finally, one can measure the radius r of the molecule's circular path in this region. *

* For example, the speed of the molecule just before entering this region can be determined by the method of Problem C-4 , i.e., by determining what perpendicular electric and magnetic fields are required so that the particle moves without deflection.

This information can then be used in Eq. (D-3) to calculate the mass m of the molecule.

► *Applications*

Such mass spectrometers have provided physicists and chemists with a very precise knowledge of the masses of atoms and molecules. Furthermore, mass spectrometers can also be used for rapid chemical analysis (e.g., to determine the nature of a toxic substance swallowed by a patient). Such chemical analysis is achieved by using electron bombardment to disrupt the unknown molecules into several fragments whose masses can be measured by the mass spectrometer. A knowledge of the masses of the typical fragments resulting from electron bombardment of *known* molecules then allows one to identify the nature of the unknown molecules.

CYCLOTRON

► *Time per revolution*

The result Eq. (D-2) shows that the speed v of a charged particle, moving in a magnetic field $\vec{B}$ around a circle, is proportional to the radius r of this circle. If the particle moved around a circular path having a radius 3 times as large, the speed v of the particle would thus also be 3 times as large. But, since the particle would then move around a 3 times larger circumference with 3 times as large a speed, its period T (or time required to move once around the circular path) would remain *unchanged*. Indeed, since the circumference of the circular path is $2\pi r$, the time T required to go once around this circumference with speed v is simply

$$T = \frac{2\pi r}{v} = 2\pi\frac{m}{|q|B} \tag{D-4}$$

where we have used the result Eq. (D-3) for the speed v. The relation (D-4) shows explicitly that the period T is *independent* of the speed of the particle or of the radius of its circular path.

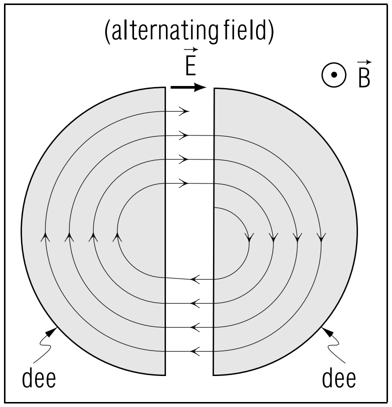


Fig. D-2: Path of a positively charged particle in a cyclotron.

► *Repetitive acceleration*

Figure D-2 shows a positively charged particle moving in a magnetic field $\vec{B}$ inside a pair of semi-circular metal boxes (called "dees," because each resembles the capital letter D). If a potential difference is applied between the dees so as to produce an electric field $\vec{E}$ in the gap between the dees, the particle moving across the gap in the direction of $\vec{E}$ is then accelerated while crossing the gap. But, by Eq. (D-4), the time $T/2$ required for the particle to traverse the next half of its circular path, and thus to arrive again at the gap, remains unaffected by the increase in the particle's speed. (Only the radius of its path increases.) Suppose then that the sign of the potential difference between the dees (and thus the direction of the electric field $\vec{E}$ between the dees) is regularly reversed so that the time between successive reversals is just equal to the time $T/2$ required for the particle to go around half a circle. Then the electric field reversals are so synchronized with the particle's motion that the particle arriving at the gap *always* encounters an electric field in a direction accelerating the particle. Thus the particle, moving around circles of ever increasing radius, always gains energy whenever it crosses the gap. These many successive gains of energy then result in a large total gain in the energy of the particle after the particle has gone through many revolutions to attain a circular path of radius as large as that of the dees. The entire device is called a cyclotron.

► *Applications*

The device which we have described is called a "cyclotron" and was invented by the American physicist, E. O. Lawrence (1901-1958) in 1930. Cyclotrons can be used to accelerate charged particles to very high kinetic energies. Such energetic particles can then be employed to bombard nuclei in order to obtain information about nuclear structures and nuclear forces; to produce various radioactive isotopes useful for chemical or biological research; and also to destroy cancerous cells.

Circular Motion In a Magnetic Field (Cap. 4)

D-1 A charged particle moves in a uniform magnetic field with a speed v_0 in a circular path of radius r_0. (a) If the particle had a charge of opposite sign, would the particle move around the circle in the same sense or the opposite sense? (b) If the speed v_0 of the particle were twice as large, what would be the radius of its path? Would the particle then require more time, the same time, or less time to go once around its circular path? *(Answer: 10)*

D-2 A charged particle, moving with a speed v_0, enters a region where there exists a uniform magnetic field $\vec{B}$. (a) Does the speed of the particle change as a result of the magnetic force acting on the particle? (b) If the magnetic field $\vec{B}$ were larger, would the radius of the particle's circular path be larger or smaller? (c) If the mass of the particle were larger, would the radius of the particle's path be larger or smaller? *(Answer: 16) (Suggestion: [s-7])*

D-3 *Mass measurement:* A magnesium (Mg) ion, having a charge $+e$, is accelerated in a mass spectrometer to a speed of 8.0×10^4 m/s before it enters a region where there is a uniform magnetic field of 0.10 tesla. The ion is then observed to travel in a circular path of radius 0.20 meter. (a) What is the mass m of this Mg ion? (b) What is the ratio m/M_H of the mass of this ion compared to the mass $M_H = 1.67 \times 10^{-27}$ kg of the hydrogen atom? *(Answer: 12)*

D-4 *Cyclotron:* A cyclotron is used to accelerate protons having a charge 1.6×10^{-19} C and mass 1.7×10^{-27} kg. (a) If the uniform magnetic field in this cyclotron is 1.0 tesla, what is the time required for the proton to complete one revolution? (b) What then should be the time between successive repetitions (the time of a single complete cycle) of the alternating potential difference applied between the dees? (c) If the potential difference between the dees is 800 volt whenever a proton traverses the gap between the dees, the proton gains an energy of 800 electron volt every time it crosses the gap. After how many revolutions then does the proton attain a final energy of 1.2×10^7 electron volt? (d) How long a time does the proton travel in the cyclotron before it attains this final energy? *(Answer: 13) (Suggestion: [s-8])*

More practice for this Capability: [p-2], [p-5], [p-6], [p-7]

SECT.

E MAGNETIC FORCE ON A CURRENT-CARRYING WIRE

Let us now use our knowledge of the magnetic force Eq. (B-3) on a single charged particle to find the magnetic force on a current-carrying wire.

► *Direction of force*

Consider a steady current I flowing through a wire. Focus attention on a short piece of this wire, of length ℓ small enough so that this piece is nearly straight and so that the magnetic field $\vec{B}$ is the same throughout this piece. (See Fig. E-1.) Then the magnetic force $\vec{F}$ on this piece of wire is simply the vector sum of the magnetic forces on the individual charged particles which move through the wire to produce the current I. The direction of the average velocity of these particles is along the wire, either in the sense of the current if the moving charged particles are positively charged, or opposite to the sense of the current if they are negatively charged. But, according to the right-hand rule of Fig. B-4b, the direction of the magnetic force $\vec{F}$ on these particles is the *same* in both cases. Thus the direction of the magnetic force $\vec{F}$ is most easily found from the right-hand rule by imagining the current to be due to *positively* charged particles moving along the wire.

► *Magnitude of force*

If the piece of wire contains N particles, each having a charge q and moving with a velocity $\vec{v}$ equal to its average velocity, the magnitude of the magnetic force $\vec{F}$ on this piece of wire is N times as large as the magnitude $|qv_\perp B|$ of the magnetic force on each particle. Thus

$$|\vec{F}| = |Nqv_\perp B| \qquad \text{(E-1)}$$

But we can relate this force directly to the current I in the wire since we know, from Relation (B-2) of Unit 423, that $I = (N/\ell)qv$ so that

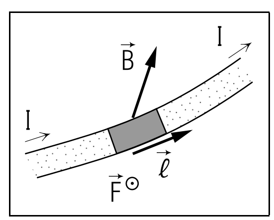


Fig. E-1: Magnetic force on a short piece of current-carrying wire.

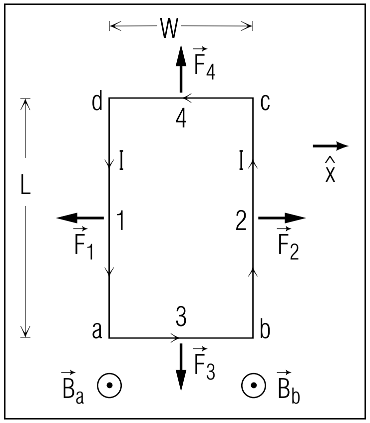


Fig. E-2: Current-carrying loop of wire in a magnetic field.

$$I\ell = Nqv \qquad \text{(E-2)}$$

Let us specify the direction of the piece of wire by the vector $\vec{\ell}$ having a magnitude ℓ and the direction in which positively charged particles would move to produce the current. Then Eq. (E-2) implies for the corresponding components of $\vec{\ell}$ and $\vec{v}$ perpendicular to $\vec{B}$ that $|I\ell_\perp| = |Nqv_\perp|$. Thus Eq. (E-2) becomes simply

$$|\vec{F}| = |I\ell_\perp B| \qquad \text{(E-3)}$$

► *Total force*

The total magnetic force on an *entire* current-carrying wire is then simply the vector sum of the magnetic forces on all the small pieces of this wire.

Example E-1: Magnetic force on a current-carrying loop

Figure E-2 shows a rectangular loop of wire, of length L and width W, through which there flows a current I in the counter-clockwise sense indicated in the diagram. What is the total external magnetic force exerted on this wire loop by a magnetic field produced by external moving charged particles (or currents)? *

> * As might be expected from Unit 413, and as we shall be able to verify explicitly in the next unit, the total *internal* magnetic force (exerted by the moving charged particles in the loop on each other) is zero.

In Fig. E-2, this magnetic field is perpendicular to the loop and directed out of the plane of the paper. The magnitude of this field may vary in the $\hat{x}$ direction (i.e., along the width of the loop) so that it has a magnitude B_a along the side ad of the loop and a magnitude B_b along the side bc of the loop.

By virtue of the right-hand rule, the magnetic forces on the four sides of the loop have the directions indicated. In particular, the directions of the magnetic forces on opposite sides of the loop are opposite since the senses of the currents through these sides are opposite. The magnitudes of the forces $\vec{F}_3$ and $\vec{F}_4$ on the sides ab and cd of the loop are equal (since the magnetic field has the same value at corresponding pieces of these wires.) Hence the vector sum of these forces is zero. The magnitudes of the forces $\vec{F}_1$ and $\vec{F}_2$ on the sides ad and bc of the loop are, by Eq. (E-3), $F_1 = ILB_a$ and $F_2 = ILB_b$. Hence the vector sum of these forces, and thus the total magnetic force $\vec{F}_{\text{tot}}$ on the loop (since $\vec{F}_3 + \vec{F}_4 = 0$), is

$$\vec{F}_{\text{tot}} = -(ILB_a)\,\hat{x} + (ILB_b)\,\hat{x} = -IL(B_a - B_b)\,\hat{x} \qquad \text{(E-4)}$$

If the magnetic field has the same value at all points of the loop so that $B_a = B_b$, the total magnetic force on the loop is thus zero. But if the magnitude of the magnetic field decreases along the $\hat{x}$ direction so that $B_a > B_b$ (i.e., so that $B_a - B_b$ is positive), the total magnetic force on the loop is non-zero and is *opposite* to the $\hat{x}$ direction.

► *Force on a coil*

Suppose that we are interested in finding the total magnetic force on a coil consisting of a wire wound N times around a rectangle of the size shown in Fig. E-2. If the current through this wire is I, the total magnetic force on this coil, consisting of N turns of wire, is then simply N times as large as the magnetic force Eq. (E-4) on a single loop of this wire. (This result is also obvious since such a coil is equivalent to a single loop of wire in which there flows a current N times as large as I.)

Understanding Magnetic Force on Currents (Cap. 3)

E-1 The ends a and b of the loop of wire in Fig. E-3 are connected to a battery so that a current $I = 3$ ampere flows through the wire in the indicated sense. The entire loop is located in a uniform magnetic field $\vec{B} = 0.05$ tesla directed into the paper. (a) What are the directions of the magnetic forces on the segments cd and ef of the wire? (b) What is

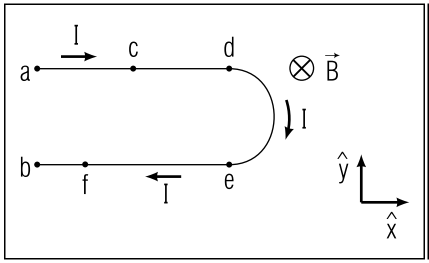


Fig. E-3.

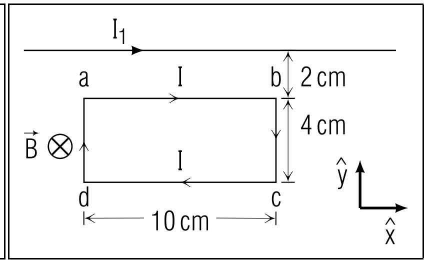


Fig. E-4.

the magnitude F_0 of the magnetic force on the segment cd if its length is 4 cm? (c) *Applicability:* The semi-circular segment de of the wire has also a length of 4 cm. Is the magnitude of the magnetic force on this segment equal to F_0? Why or why not? (d) *Dependence:* The segment ef is twice as long as the segment cd. Express the magnitude of the magnetic force on this segment in terms of F_0. (e) Suppose that the magnetic field were 3 times as large and the current through the wire were 2 times as large. What then would be the magnitude of the magnetic force on the segment cd? Express your answer in terms of F_0. *(Answer: 19) (Suggestion: [s-9])*

E-2 *Force on a coil:* The magnetic field produced by a long current-carrying wire decreases inversely with increasing distance from the wire. Thus the magnetic field at the position of the rectangular coil indicated in Fig. E-4 points into the paper and has a magnitude 3×10^{-4} tesla at the side ab of the coil and 1.0×10^{-4} tesla at the side cd of the coil. The coil consists of 10 turns of wire through which there flows a current I of 20 ampere in the indicated sense. (a) What are magnitudes and directions of the magnetic forces on the sides ab and cd of the coil? (b) Compare the magnetic force on the sides bc and da of the coil. (c) What is the total magnetic force on the coil? *(Answer: 15)*

E-3 *Measurement of B:* Figure E-5 shows a "current-balance," used for the accurate measurement of magnetic fields. This device consists merely of a sensitive equal-arms balance from one of whose sides there is suspended a rectangular coil of N turns of wire through which there flows a known current I. The lower end ab of the coil is placed in the region where there is a uniform magnetic field $\vec{B}$ of unknown magnitude (pointing out of the paper in Fig. E-5). The coil is so long that the magnetic field is negligible at the other end cd of the coil. (a) What is the magnetic force on the side ab of the coil? Express your answer

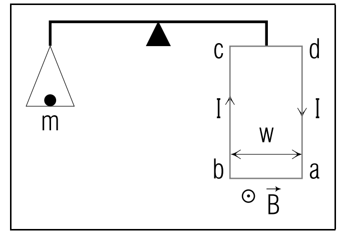


Fig. E-5.

in terms of N, I, B, and the width w of the coil between a and b. (b) What is the magnetic force on the side cd of the coil? (c) What is the downward component of the magnetic force on the side da or bc of the coil? (d) What then is the downward component of the *total* magnetic force on the coil? (e) To maintain the balance in a balanced condition when the current I flows through the coil, one places in the left pan a weight whose mass m is such that the downward gravitational force mg is equal to the downward magnetic force on the coil. Express the unknown magnitude B of the magnetic field in terms of the known quantities, m, g, N, I, and w. *(Answer: 17) (Suggestion: [s-10]) More practice for this Capability: [p-3], [p-9]*

SECT.

F ROTATION PRODUCED BY A MAGNETIC FIELD

► *Magnetic moment*

Consider a plane loop of wire, such as the rectangular loop in Fig. F-1a, around which there flows a steady current I. The spatial orientation of such a loop is most conveniently described by a vector $\vec{M}$ perpendicular to the plane of the loop. To be specific, this vector $\vec{M}$, called the "magnetic moment" of the loop, is defined as follows: (See Fig. F-1a.)

Def. **Magnetic moment**: The magnetic moment $\vec{M}$ of a current-carrying loop is a vector having a direction perpendicular to the plane of the loop, so as to point in the direction of the thumb of the right hand if the fingers of this hand are curled along the sense of the current in the loop. (The magnitude of $\vec{M}$ is defined as $M = IA$, where I is the current flowing around the loop of area A.) (F-1)

Note that, if the sense of the current around the loop is reversed, the direction of the magnetic moment is correspondingly reversed.

► *Rotation of loop*

Suppose that the rectangular loop of Fig. F-1a is located in a magnetic field $\vec{B}$ which is uniform (so that $\vec{B}$ has the same value at all points in the region of the loop). To examine the effect of this magnetic field, consider Fig. F-1b which shows an end-on view of the loop with the current I flowing out of the paper in side 1 of the loop, and flowing into the paper in side 2 of the loop. By the right-hand rule and Eq. (E-3), the magnetic forces $\vec{F}_1$ and $\vec{F}_2$ on these two sides have then opposite directions and equal magnitudes (proportional to the current I through the loop). Although the total magnetic force due to these forces is zero, the opposite component vectors $\vec{F}_1'$ and $\vec{F}_2'$ of these forces perpendicular to the plane of the loop produce a torque, i.e., they produce a rotation:

A current loop in a magnetic field tends to rotate so that its magnetic moment becomes aligned along the magnetic field.	(F-2)

When the magnetic moment has become so aligned (as shown in Fig. F-1c), the magnetic forces $\vec{F}_1$ and $\vec{F}_2$ are parallel to the plane of the loop

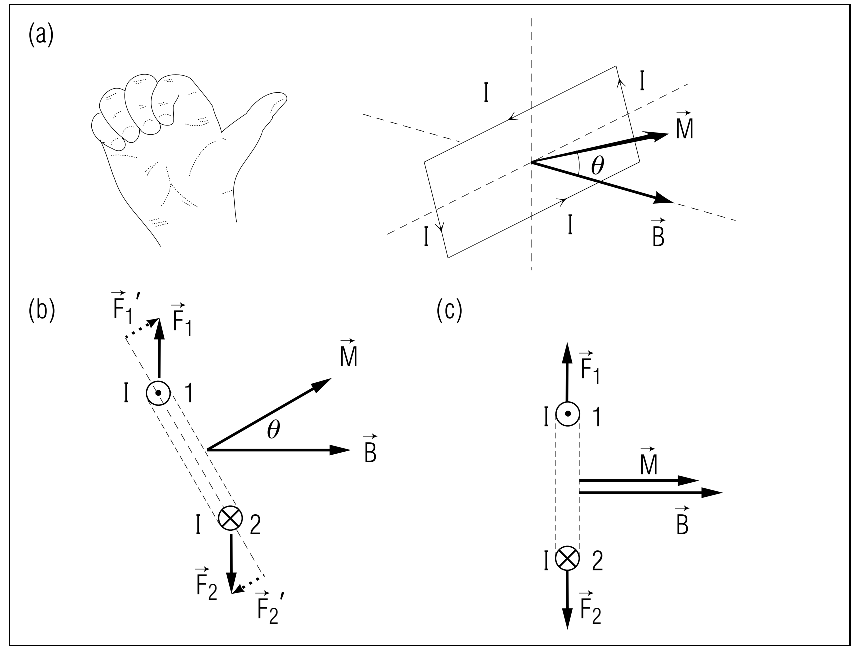


Fig. F-1: Current loop in a uniform magnetic field $\vec{B}$. (a) Three-dimensional view of the loop and comparison with a right hand specifying the direction of the magnetic moment $\vec{M}$. (b), (c) End-on views of the loop in two different orientations relative to $\vec{B}$.

so that they tend to stretch the loop, but produce no further rotation. *

* The magnetic forces $\vec{F}_3$ and $\vec{F}_4$ on the other two sides of the loop also have equal magnitudes, but their directions are into and out of the paper. Thus these forces do not tend to produce any rotation.

Let us discuss several applications.

AMMETER

A device used to measure currents is called an "ammeter" (i.e., an "ampere meter.") The most common type of ammeter consists of a rectangular coil of wire supported so as to be free to rotate in a magnetic

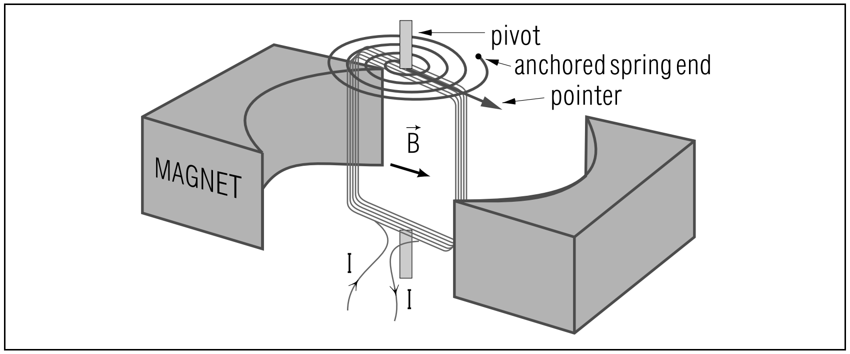


Fig. F-2: Basic parts of an ammeter.

field $\vec{B}$ produced by a permanent magnet. (See Fig. F-2.) The rotation of the coil is opposed by a spring and the orientation of the coil is indicated by a pointer attached to the coil. When a current I flows through the coil, the magnetic forces then tend to rotate the coil until it comes to rest because of the opposing spring forces. The net rotation of the coil (and of the attached pointer) then indicates the magnitude of the current I responsible for the magnetic forces on the coil.

ATOMIC AND NUCLEAR MAGNETIC MOMENTS

► *Orbital magnetic moment*

An electron moving around an orbit in an atom or molecule is equivalent to a current flowing around a loop of atomic size (about 10^{-10} meter, as indicated in Fig. F-3a). Hence an atom or molecule can have a magnetic moment as a result of the orbital motion of the electrons in this atom or molecule.

► *Spin magnetic moment*

The electron (and also the proton) is known to behave like a uniformly charged sphere spinning rapidly around an axis through its center. As indicated in Fig. F-3b, such a spinning charged sphere is equivalent to a current loop of a size comparable to that of the sphere (i.e., about 10^{-15} meter for an electron or proton). Thus both the electron and the proton have magnetic moments because of their spin. (The magnitude of the magnetic moment of the proton is, however, much smaller than that of the electron since the proton spins around its axis less rapidly because of its larger mass.)

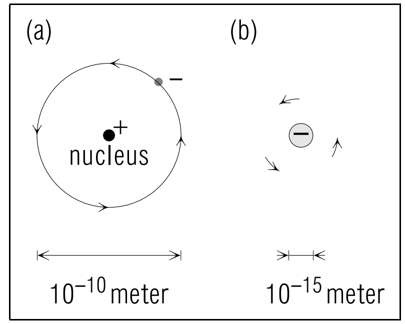


Fig. F-3: Magnetic moments produced by electron motion. (a) Orbital motion of an electron around the nucleus of an atom. (b) Spinning motion of an electron considered as a negatively charged sphere rotating about its axis.

► *Observed magnetic moments*

The observed magnetic moment of an atom (or molecule) is thus due *both* to the orbital motion and to the spin motion of its electrons. (A small part of this magnetic moment is also due to the magnetic moment of the atomic nucleus consisting of protons and neutrons.) In many cases the net magnetic moment of an atom or molecule is, however, zero because the magnetic moments produced by the motion of individual electrons have equal magnitudes but opposite directions, thus cancelling each other. (For example, this may happen if two electrons move with equal speeds in opposite senses so that they produce no net current.) Thus the hydrogen atom has a magnetic moment, but the helium atom has no net magnetic moment because of the cancellation of the magnetic moments of its two electrons.

► *Applications*

When an atom or molecule is placed in a magnetic field $\vec{B}$, the magnetic moment of this atom or molecule tends to rotate so as to become aligned along $\vec{B}$ (just as the magnetic moment of any current loop). For example, in the modern techniques of "electron spin resonance" and "nuclear magnetic resonance" the reorientation of electronic or nuclear magnetic moments within molecules in a magnetic field is used to obtain information about this field. Thus one can deduce important chemical or biological information about neighboring atoms since their electrons and nuclei contribute to the production of this field.

Magnetic Moment and Rotation of a Loop (Cap. 5)

F-1 What is the direction of the magnetic moment resulting from the orbital motion of the electron in the atom of Fig. F-3a? *(Answer: 22) (Suggestion: [s-11])*

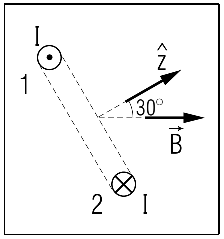


Fig. F-4.

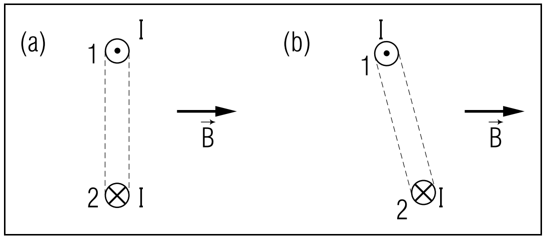


Fig. F-5.

F-2 Figure F-4 shows a cross-sectional view of a rectangular wire loop. The unit vector $\hat{z}$ indicates a particular direction perpendicular to the plane of this loop and makes an angle of 30° with the magnetic field $\vec{B}$. The current I in this loop flows out of the paper in side 1 of the loop and into the paper in side 2 of the loop. (a) What is the direction of the magnetic moment of the loop? (b) If the loop is free to rotate, through what angle does the loop rotate before it finally comes to rest? (c) Suppose that the current through the loop were flowing in the opposite sense. What then would be the direction of the magnetic moment of the loop? (d) Through what angle would the loop then rotate before finally coming to rest? *(Answer: 18) (Suggestion: [s-12])*

F-3 The preceding wire loop is again shown in Fig. F-5. (a) Suppose that the current flows out of the paper in side 1 of the loop and into the paper in side 2 of the loop. If the loop is oriented as in Fig. F-5a (so that its magnetic moment is along $\vec{B}$), indicate the directions of the magnetic forces on these sides of the loop. Do these forces tend to rotate the loop or not? (b) If the loop has the slightly different orientation shown in Fig. F-5b, do the forces on these sides of the loop tend to rotate the loop so as to restore it to its previous orientation in Fig. F-5a? (c) Suppose now that the current flows into the paper in side 1 of the loop and out of the paper in side 2 of the loop (so that the magnetic moment in Fig. F-5a is opposite to $\vec{B}$). Indicate the directions of the magnetic forces and answer question (a) in this case. (d) What is the answer to question (b) in this case? *(Answer: 21)*

F-4 In the ammeter shown in Fig. F-2, do the magnetic forces on the coil tend to make it rotate clockwise or counter-clockwise, when viewed looking down upon the pivot? *(Answer: 25) (Suggestion: [s-12])*

SECT.

G ELECTRIC MOTORS

An "electric motor" is a device which uses electric currents to move objects and to do useful work on them. All practical electric motors produce such motion as a result of magnetic forces on current-carrying wires placed in a magnetic field.

► *Linear motor*

Figure G-1 illustrates a very primitive motor consisting of a metal rod free to slide on two metal rails and located in a magnetic field $\vec{B}$ pointing out of the paper. When the rails are connected to a battery as shown in the diagram, a current I flows through the rod in the indicated sense. By the right-hand rule, a magnetic force $\vec{F}$ acts then on the rod in the $\hat{x}$ direction. As a result, the rod moves along the rails in the $\hat{x}$ direction and can do work on some system connected to the rod. *

* The work done is ultimately derived from energy supplied by the battery. A detailed analysis, using ideas discussed in Unit 428, shows that the magnetic forces act merely as intermediaries in the energy conversion process, but do not themselves do any net work.

► *Rotating motor*

Practical motors are designed to produce rotational motion. A simple motor of this kind can be constructed, as shown in Fig. G-2, by mounting a wire loop (or coil) so that it is free to rotate in a magnetic field $\vec{B}$ (ordinarily produced by a current flowing through some other stationary coils). When an emf source is connected to the loop so that a current I flows through it, the loop then rotates (as discussed in the preceding section) until its magnetic moment becomes aligned along $\vec{B}$. In order to prevent the motion of the loop from stopping, one needs then to reverse the current in the loop; for then the magnetic moment of the

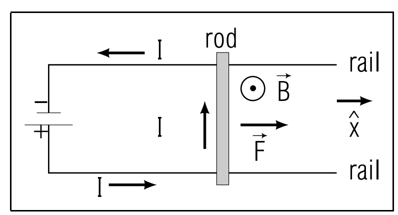


Fig. G-1: Linear motor consisting of a rod free to slide on rails. The magnetic field, indicated by the points, is directed out of the paper.

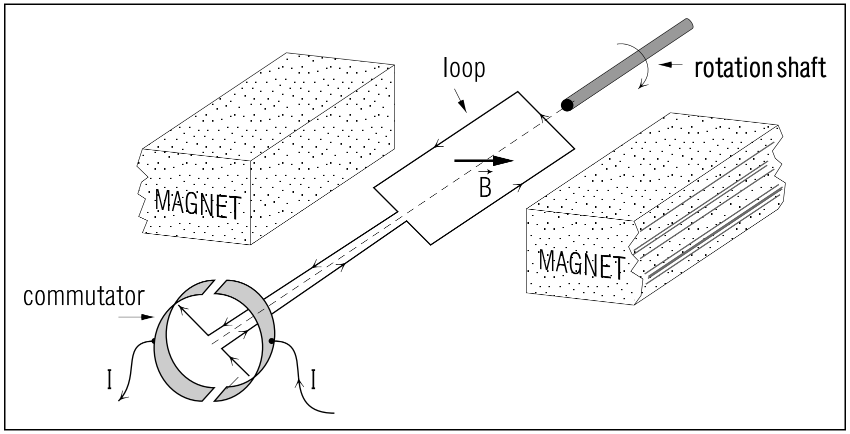


Fig. G-2: Motor consisting of a coil rotating in a magnetic field $\vec{B}$. The current to the coil is supplied through a commutator.

loop reverses so that the loop rotates another half revolution until its new magnetic moment becomes aligned along $\vec{B}$. If one keeps reversing the sense of the current in the loop every half revolution, the loop can then be kept rotating indefinitely.

► *Commutator*

In practice, such a repeated reversal of the sense of the current through the loop can be achieved by a "commutator," consisting of a split ring attached to the two ends of the wire on the rotating loop. By supplying the current to the loop through two contacts sliding along this split ring, as indicated in Fig. G-2, the current through the loop is then automatically reversed every time the loop rotates through half a revolution. (See Problem G-2.)

Knowing About Motors

G-1 Consider the linear motor illustrated in Fig. G-1 where a magnetic force $\vec{F}$ acts on the moving rod. (a) Would $\vec{F}$ be larger or smaller if the magnetic field $\vec{B}$ were larger? (b) if the emf source had a larger emf? (c) Would the direction of $\vec{F}$ be the same or opposite if the direction of the magnetic field $\vec{B}$ were reversed? (d) if the emf source were connected so that the sense of the current I were reversed? *(Answer: 20)*

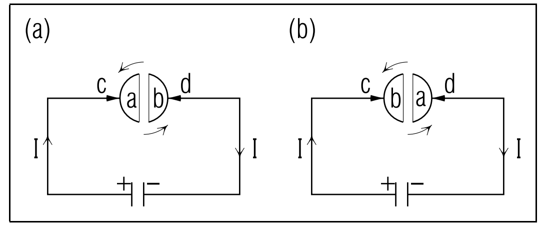


Fig. G-3.

G-2 *Commutator:* Figure G-3 shows an end-on view of the commutator of Fig. G-2. This commutator is merely a split ring whose two halves (separated by a dielectric) are rigidly attached to the two ends a and b of the rotating loop. The emf source is then connected to the loop by two *stationary* contacts ("brushes") c and d which remain in sliding contact with the commutator as this commutator rotates with the coil. (a) When the commutator is oriented as shown in Fig. G-3a, does the current flow into the end a of the loop and out of its end b, or does it flow around the loop in the opposite sense? (b) After the loop (and attached commutator) has rotated by 180° so as to be oriented as shown in Fig. G-3b, does the current flow into the end a of the loop and out of its end b, or does it flow in the opposite sense? *(Answer: 23)*

SECT.
H SUMMARY

DEFINITIONS

magnetic field; Rule (B-4)

tesla; Eq. (C-1)

gauss; Eq. (C-2)

magnetic moment; Def. (F-1)

IMPORTANT RESULTS

Magnetic force on a charged particle: Rule (B-4), Fig. B-4

$\vec{F} = |qv_\perp B|$, direction given by right-hand rule of Fig. B-4, ($\vec{v}$ along thumb, $\vec{B}$ along fingers, $\vec{F}$ out of palm for + charge).

Magnetic force on a short piece of wire: Eq. (E-3)

$|\vec{F}| = |I\ell_\perp B|$

Electromagnetic force: Eq. (C-3)

$\vec{F}_{em} = \vec{F}_e + \vec{F}_m$, where electric force $\vec{F}_e = q\vec{E}$.

Work done by magnetic force: Rule (C-4)

Work done by magnetic force is zero.

Rotation of a current loop in a magnetic field $\vec{B}$: Rule (F-2)

Loop rotates so that its magnetic moment becomes aligned along $\vec{B}$.

USEFUL KNOWLEDGE

mass spectrometer (Sec. D)

cyclotron (Sec. D)

ammeter (Sec. F)

electric motor (Sec. G)

NEW CAPABILITIES

(1) Find the change in the magnetic force exerted on a charged particle by another (or on a current-carrying wire by another), or find the change in the magnetic field produced by a charged particle (or by a current-carrying wire), if: (a) the magnitude or sign of any charge is changed, (b) the magnitude of any velocity is changed or its direction is reversed, (c) the magnitude or sense of any current is reversed. (Sec. A)

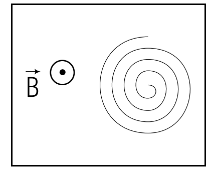


Fig. H-1.

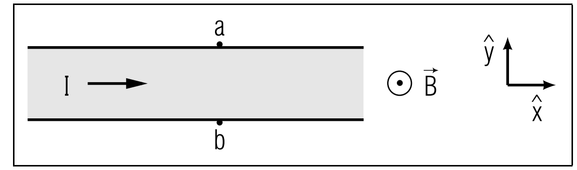


Fig. H-2.

(2) Understand the relation describing the magnetic force on a charged particle (i.e., the relation defining the magnetic field), $\vec{F} = |qv_\perp B|$, with direction given by the right-hand rule. (Sec. B, [p-1], [p-4], [p-8].)

(3) Understand the relation describing the magnetic force on a current-carrying wire $\vec{F} = |I\ell_\perp B|$, with direction given by the right-hand rule. (Sec. E, [p-3], [p-9].)

(4) Derive and use the relations $v = |qB/m|r$ and $T = 2\pi|m/qB|$, for the circular motion of a particle in a uniform magnetic field. (Sec. D, [p-4], [p-5], [p-6], [p-7].)

(5) Relate the direction of the magnetic moment of a current-carrying loop (or coil) to the sense of the current in the loop, and to the tendency of the loop to rotate in a magnetic field. (Sec. F)

H-1 *Bubble chamber (Caps. 2,4):* A "bubble chamber," which is a very useful device for making visible the paths of energetic charged particles, contains liquid hydrogen near its boiling point. A charged particle passing through this liquid then forms along its path a trail of visible bubbles. Fig. H-1 shows the path thus observed for an unknown particle when the bubble chamber is located in a magnetic field directed out of the paper. (a) Did the particle travel along this path in a clockwise or counter-clockwise sense? Why? (b) Does this particle have a positive or negative charge? Why? *(Answer: 27) (Suggestion: [s-13])*

H-2 *Hall effect (Cap. 2):* Figure H-2 shows a positive current flowing in the indicated sense through a conducting wire placed in a magnetic field pointing out of the paper. (a) Suppose that the current is due to moving *positively* charged particles. What then would be the direction of the magnetic force on these particles? Do the particles deflected by this force produce a net accumulation of positive charge on the upper or lower side of the wire in Fig. H-2? As a result, is the potential drop from a to b positive or negative? The existence of this non-zero potential drop is called the "Hall effect") (b) Suppose that the current is due to *negatively* charged particles. What then would be the answers to the preceding questions? (c) What is the sign of the charge of the particles responsible for the current if the observed potential at a is smaller than that at b? *(Answer: 24)*

Hint: In parts (a) and (b), first carefully determine the sense in which the particles must move if they are to give rise to a current in the sense indicated in Fig. H-2.

SECT.

I PROBLEMS

I-1 *Measurement of mass difference:* Two ions, of slightly different masses, have the same charge q and enter the magnetic field $\vec{B}$ of a mass spectrometer with the same speed v. As indicated in Fig. I-1, the ions, after traveling through half a circle, then arrive at two points separated by a distance s (equal to the difference of the diameters of the circular orbits of these ions). (a) Starting with the equation of motion, relate the mass of each ion to the diameter of its orbit. Then express the mass difference Δm of these ions in terms of the known quantities q, B, v, and s. (b) The preceding experiment is performed with the two naturally occurring types (or "isotopes") of chlorine atoms. Each Cl^- ion has a charge $-e = -1.6 \times 10^{-19}$ C. In a particular mass spectrometer, where $B = 0.50$ tesla, the ions enter the field with a speed of 2.0×10^5 m/s and the observed distance $s = 1.67$ cm. What then is the ratio $\Delta m/M_H$ of the mass difference Δm of the Cl^- ions compared to the mass $M_H = 1.67 \times 10^{-27}$ kg of a hydrogen atom? *(Answer: 29)*

I-2 *Energy measurement of electrons:* To determine the kinds and relative abundances of chemical elements contained in a sample, one can measure the energies of the electrons ejected by X-rays from molecules in the sample. The energy of the electron, of known charge $-e$ and mass m, can be readily determined by measuring the radius r of its orbit (in vacuum) in the presence of a known magnetic field $\vec{B}$. Find an expression for the kinetic energy of the electron in terms of e, m, B, and r. *(Answer: 26)*

I-3 *Mass spectrometer:* In a particular kind of mass spectrometer, the ion of charge q starts from rest and is first accelerated by passing between two plates between which there is a known potential difference V. The ion then enters a region of uniform magnetic field $\vec{B}$ where one can measure the radius r of the ion's circular orbit. Express the mass m of the ion in terms of the known quantities q, V, B, and r. *(Answer: 28)*

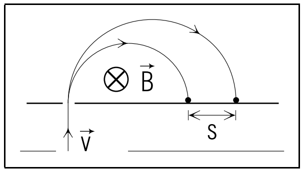


Fig. I-1.

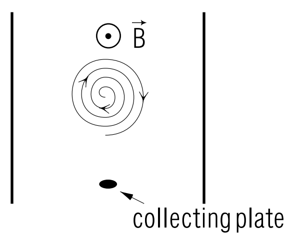
(Suggestion: [s-14])

Note: Tutorial section I contains additional problems.

TUTORIAL FOR I

ADDITIONAL PROBLEMS

i-1 *OMEGATRON MASS SPECTROMETER:* The "omegatron" is a very small mass spectrometer which is commonly used to identify and determine the relative amounts of the gases remaining in an evacuated chamber. The device consists basically of a parallel plate-capacitor placed in a magnetic field $\vec{B}$. When the gas molecules are ionized by electron bombardment, the resulting ions (usually with charge of magnitude e) move in circular orbits in the magnetic field with a frequency of ω revolutions per second. The ions can also be acted on by a small alternating electric field produced by an alternating potential difference applied between the capacitor plates. If the frequency of alternation of this alternating electric field is precisely equal to the frequency ω of revolution of the ions, these ions (just as in a cyclotron) then gain energy from this electric field and travel in circles of ever-expanding radius until they strike a small collecting plate at a distance r_0 from the center of the omegatron. The frequency of revolution of the ions in the magnetic field is thus simply determined by noting at what frequency of the alternating electric field ions arrive at the collecting plate.



(a) Express the mass m of an ion in terms of the measured frequency ω, the magnitude e of the ion charge, and B. (b) The mass of a nitrogen molecule (N_2) is about 28 times larger than that of the hydrogen atom (which has a mass of 1.67×10^{-27} kg). If $B = 0.10$ tesla, what would have to be the frequency of the alternating electric field in the omegatron to detect the presence of a nitrogen ion with charge of magnitude $e = 1.6 \times 10^{-19}$ C? *(Answer: 52) (Suggestion: s-15)*

i-2 *FLOATING WIRE:* A current I flows through a horizontal straight wire, of cross-sectional area A, which is perpendicular to a horizontal

magnetic field $\vec{B}$. (a) What must be the current per unit area I/A in this wire so that the magnitude of the upward magnetic force on the wire is equal to the magnitude of the downward gravitational force on the wire (i.e., so that there is no net force on the wire)? Express your answer in terms of B, the density ρ of the wire, and the magnitude g of the gravitational acceleration. (b) Suppose that $B = 1.0 \times 10^3$ gauss and that the wire, having a diameter of 2.0 mm, is made of copper having a density of $8.9 \times 10^3\,\mathrm{kg/m^3}$. What then must be the magnitude of the current in the wire so that no net force acts on it? ($g = 9.8\,\mathrm{m/s^2}$). *(Answer: 56) (Suggestion: s-16)*

i-3 *CYCLOTRON WITH DIFFERENT PARTICLES:* A proton (hydrogen nucleus) has a mass M_p and a charge e. A "deuteron" (nucleus of "heavy hydrogen") has a mass $2M_p$ and a charge e. An alpha particle (a helium nucleus) has a mass $4M_p$ and a charge $2e$. In a particular cyclotron, having some specified magnetic field, the proton requires a time T_p for one revolution in the magnetic field and finally emerges (at the radius corresponding to the edge of the dee) with a speed v_p and corresponding kinetic energy K_p. (a) In this cyclotron, what would be the period of revolution T_d of a deuteron and the period of revolution T_a of an alpha particle? Express your answers in terms of T_p. (b) With what speed v_d would a deuteron emerge from this cyclotron, and with what speed v_a would an alpha particle emerge? Express your answers in terms of v_p. (c) With what kinetic energy K_d would a deuteron emerge from this cyclotron, and with what kinetic energy K_a would an alpha particle emerge? Express your answers in terms of K_p. *(Answer: 55)*

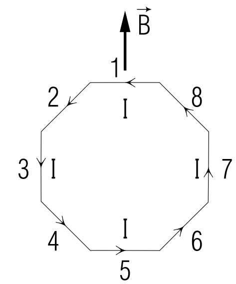
i-4 *MAGNETIC FORCE ON LOOP:* Suppose that the loop in Fig. E-2 has a small width $W = dx$ so that the magnetic field changes over this width by a corresponding small amount $dB = B_b - B_a$. Show that the result (E-4) for the total magnetic force on the loop is then simply equal to $\vec{F}_{\rm tot} = (M dB/dx)\,\hat{x}$ where $M = IA$ is the magnitude of the magnetic moment of the loop and A is the area of the loop.

PRACTICE PROBLEMS

p-1 *DIRECTION OF MAGNETIC FORCE (CAP. 2):* The diagram indicates a region where the magnetic field $\vec{B}$ points out of the paper. (a) If an electron in this region moves with a velocity $\vec{v}$ to the right, as indicated, what is the direction of the magnetic force on the electron? (b) If the electron moves with a velocity directed into the paper, what is the direction of the magnetic force on the electron? *(Answer: 53) (Suggestion: Review text problems B-1 and B-2.)*

p-2 *CIRCULAR MOTION IN MAGNETIC FIELD (CAP. 4): Cyclotron:* An alpha particle (i.e., helium nucleus) has a charge $2e = 3.2 \times 10^{-19}$ C and a mass 6.7×10^{-27} kg. (a) In a cyclotron where the uniform magnetic field is 0.75 tesla, what is the time required for such an alpha particle to go once around its circular orbit? (b) Use this time to find the radius of the final orbit attained by the alpha particle when it has a speed of 1.0×10^7 m/s. *(Answer: 51) (Suggestion: Review text problem D-4.)*

p-3 *MAGNETIC FORCE ON CURRENT-CARRYING WIRES (CAP. 3):* The diagram shows an octagonal loop of wire in which there flows a current I in the indicated sense. Each of the sides of the loop has the same length L and the loop is located in a uniform magnetic field $\vec{B}$ having the indicated direction. (a) What is the magnetic force on the side 1 of the loop? (b) What is the magnetic force on the side 3 of the loop? (c) What is the magnetic force on the side 2 of the loop? *(Answer: 54) (Suggestion: Review text problems E-1 and E-3)*



p-4 *MAGNETIC FORCE AND FIELD: Motion in perpendicular fields:* Charged particles move with a velocity $\vec{v}$ perpendicular to an electric field $\vec{E}$ and to a magnetic field $\vec{B}$ (where $\vec{E}$ and $\vec{B}$ are perpendicular to each other). (a) If $B = 1.0$ tesla, what must be the magnitude of $\vec{E}$ so that protons (with mass 1.67×10^{-27} kg), moving with a speed $v = 1.0 \times 10^5$ m/s, travel through these perpendicular fields without being deflected? (b) Would electrons (having a mass 9.1×10^{-31} kg), moving with the *same* velocity, be deflected when traveling through these fields? If so, in which direction would these electrons be deflected? *(Answer: 57) (Suggestion: Review text problem C-4.)*

p-5 *CIRCULAR MOTION IN MAGNETIC FIELD (CAP. 4): Magnetron:* In a "magnetron" (a device used to generate electromagnetic waves of microwave frequencies) electrons move perpendicular to a magnetic field of 0.10 tesla. (a) What is the period of revolution of an electron in this magnetic field? (b) What is the frequency of revolution (the number of revolutions per second) of an electron in this field? *(Answer: 59) (Suggestion: [s-15] and text problems D-3 and D-4.)*

p-6 *CIRCULAR MOTION IN MAGNETIC FIELD (CAP. 4): Mass spectrometer:* There are two naturally occurring types of carbon atoms (i.e., "isotopes") having the same chemical properties but different masses. The "atomic mass unit" (abbreviated as "u") is *defined* so that the more abundant carbon atom has a mass exactly equal to 12 u. Suppose that ionized carbon atoms enter a mass spectrometer with the same speed and then travel in the magnetic field of the mass spectrometer in circular orbits. The radius of the circular orbit of the more abundant isotope is observed to be 18.0 cm, while the radius of the circular orbit of the less abundant isotope is observed to be 19.5 cm. What then is the mass of the less abundant carbon isotope, expressed in atomic mass units? *(Answer: 58) (Suggestion: Review text problem D-3.)*

p-7 *CIRCULAR MOTION IN MAGNETIC FIELD (CAP. 4): Measurement of magnetic field:* Since time measurements can be made with very great precision, it is possible to measure a magnetic field with great precision by measuring the time T required for an electron in this magnetic field to travel (in a vacuum) once around a circular orbit in this field. (a) Starting from the equation of motion, and the magnetic force on a moving particle, derive an expression relating the magnitude B of the magnetic field to T, the magnitude e of the charge of the electron, and its mass m. (b) Suppose that the observed time $T = 5.00 \times 10^{-9}$ second. Using the values $e = 1.60 \times 10^{-19}$ C and $m = 9.11 \times 10^{-31}$ kg, what is B? *(Answer: 60) (Suggestion: Review text problems D-1 and D-4.)*

p-8 *MAGNETIC FORCE AND FIELD (CAP. 2): Velocity selector:* Consider a beam of charged particles traveling with a velocity $\vec{v}$ perpendicular to an electric field $\vec{E}$ and magnetic field $\vec{B}$ (where these fields are perpendicular to each other, as in text problem C-4). Suppose that these fields are such that the beam of particles moves without deflection. If the beam of particles moved through these fields with the same speed in the *opposite* direction, would the beam of particles be deflected or not? If so, in which direction? *(Answer: 62) (Suggestion: [s-17] and text problem C-4.)*

p-9 *MAGNETIC FORCE ON CURRENT-CARRYING WIRE (CAP. 3):* Is it possible that no magnetic force acts on a piece of wire located in a magnetic field, although a current flows through this wire? If so, under what conditions? *(Answer: 61) (Suggestion: Review text problems E-1 through E-3.)*

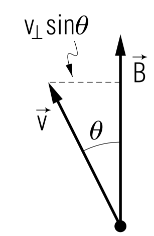
SUGGESTIONS

s-1 *(Text problem A-1):* If any one of the currents is 2 times as large, the magnetic force is 2 times as large. If *each* current is 2 times as large, the magnetic force is then $2 \times 2 = 4$ times as large.

s-2 *(Text problem B-1):* All these questions can be answered by applying the right-hand rule of Fig. B-4 or B-5. Remember that the force on a *negatively* charged particle is simply opposite to the force obtained by the right-hand rule for a positively charged particle.

s-3 *(Text problem B-2):* Assume that the particle is positively charged and find the direction of the magnetic force on it. If this direction is that of the actual force $\vec{F}$, the assumption that the particle is positively charged is correct. If this direction is opposite to that of $\vec{F}$, the particle must be negatively charged.

s-4 *(Text problem B-3):* Suppose that a particle moves with a velocity $\vec{v}$ making an angle θ relative to the magnetic field $\vec{B}$, as indicated in the diagram. Then the numerical component of the velocity perpendicular to $\vec{B}$ is $v_\perp = v \sin\theta$. Hence the magnitude of the magnetic force on the particle is, by the relation (B-4) in the text, equal to $F = qvB\sin\theta$.



s-5 *(Text problem C-1):* The magnetic field $\vec{B}$ can be directed either out of, or into, the paper. Assume that $\vec{B}$ has either of these directions and find the corresponding direction of the magnetic force. If the direction of this force is the same as $\vec{F}_m$ the assumed direction of $\vec{B}$ is correct. Otherwise, $\vec{B}$ must have the opposite direction.

s-6 *(Text problem C-4):* If the particle moves between the plates with a velocity $\vec{v}_0$ along the $\hat{x}$ direction, the total electromagnetic force on the particle is

$$\vec{F}_{em} = \vec{F}_e + \vec{F}_m = qE\hat{y} - qv_0B\hat{y} = q(E - v_0B)\,\hat{y}$$

since the magnetic force is opposite to the $\hat{y}$ direction.

s-7 *(Text problem D-2):* Since the magnetic force does no work on the particle, the kinetic energy of the particle must remain unchanged.

s-8 *(Text problem D-4):* Note that the proton crosses the gap between the dees twice during every revolution.

s-9 *(Text problem E-1):* Part c: Remember that the relation $F = |I\ell_\perp B|$ for the magnetic force on a segment of current-carrying wire applies only to a segment short enough so that it is nearly straight and so that the magnetic field is nearly the same at all points along the segment.

Further help on part (c) if needed: Break up a straight portion of the wire, *cd* for example, into perhaps five small segments and evaluate the direction of the magnetic force on each small segment. Then draw a diagram adding these five small-segment force vectors together to get the total magnetic force on the entire length (*cd* in our example). Finally, do the same job on the curved portion *de*: break it into small segments, evaluate the approximate direction of the magnetic force on each, then add the individual-segment forces (vectorially!) to to get the total force on the entire portion *de*. Comparing the two additions gives you the answer.

s-10 *(Text problem E-3):* Part c: Remember that the magnetic force on charged particles is always perpendicular to the direction of motion of these particles.

s-11 *(Text problem F-1):* Remember that negatively charged particles moving counter-clockwise give rise to a current flowing in a clockwise sense. By its definition, the magnetic moment is then related to this current.

s-12 *(Text problems F-2 and F-4):* Remember that the loop always tends to rotate so that its magnetic moment becomes aligned along the magnetic field.

s-13 *(Text problem H-1):* Part a: If the kinetic energy of the particle decreases, does its speed increase or decrease? Does the radius of the path of the particle in the magnetic field then increase or decrease?

Part b: If the particle travels around a circular path, must the force on it be directed radially inward or outward? If the particle is positively charged, and travels in the sense determined in part (a), is the magnetic force on it directed radially inward or outward? What then can one conclude about the sign of the charge of the particle?

s-14 *(Text problem I-3):* The kinetic energy acquired by the ion as a result of the potential difference can be found by relating the change of its kinetic energy to the particle's change of electric potential energy. But this kinetic energy involves the particle's speed, which is related to the radius of the particle's orbit in the magnetic field.

s-15 *(Tutorial frame [i-1], Practice problem [p-5]):* If the period of revolution of an ion is T, its frequency of revolution (or number of revolutions per second) is $\nu = 1/T$.

s-16 *(Tutorial frame [i-2]):* Assume that the wire has a length L. What then is its volume? What then is its mass and corresponding weight?

s-17 *(Practice problem [p-7]):* To be undeflected, the magnetic force on a particle must be opposite to the electric force. If the velocity has the opposite direction, what happens to the direction of the magnetic force? What happens to the direction of the electric force? What then is the direction of the magnetic force compared to that of the electric force? What then is the direction of the total force on a particle?

ANSWERS TO PROBLEMS

1. a. $\vec{F}$ out of paper
   b. into paper
   c. $\vec{F} = 0$
   d. out of paper
2. a. $3\vec{F}_m$; b. $9\vec{F}_m$
   c. $-\vec{F}_m$; d. $5\vec{F}_m$
   e. $25\vec{F}_m$; f. $-\vec{F}_m$ g. 0
3. a. electrons
   b. equal; c. smaller
   d. no
   e. yes, if particle velocity is parallel to $\vec{B}$.
4. a. into paper
   b. 3.0×10^{-2} tesla
   c. 3.0×10^{2} gauss
5. a. $2\vec{F}_m$
   b. $4\vec{F}_m$; c. 0
   d. $-\vec{F}_m$; e. $\vec{F}_m$
6. negative
7. a. $3\vec{B}$
   b. $2\vec{B}$; c. 0
   d. $-\vec{B}$; e. $\vec{B}$
8. a. 1.6×10^{-16} N
   b. $\approx 10^{14}$ m/s^2
9. a. $F_0/2$
   b. $10F_0$
10. a. opposite; b. $2r_0$, same time
11. a. $\vec{F}_e$ along $\hat{y}$, $\vec{F}_m$ opposite $\hat{y}$
    b. $v_0 = E/B$; c. no

    d. opposite $\hat{y}$
    e. along $\hat{y}$
    f. 5×10^4 m/s
12. a. 4.0×10^{-26} kg
    b. 24
13. a. 6.7×10^{-8} second; b. 6.7×10^{-8} second
    c. 7.5×10^{3}; d. 5.0×10^{-4} second
14. a. positive
    b. zero
15. a. on ab, 6×10^{-3} N; along $\hat{y}$; on cd, 2×10^{-3} N, opposite to $\hat{y}$
    b. forces are opposite
    c. 4×10^{-3} N along $\hat{y}$
16. a. no
    b. smaller
    c. larger
17. a. $NIwB$; b. 0
    c. 0 d. $NIwB$
    e. $mg/(NIw)$
18. a. along $\hat{z}$
    b. 30°
    c. opposite to $\hat{z}$
    d. 150°
19. a. along $\hat{y}$ on cd; opposite to $\hat{y}$ on ef
    b. $F_0 = 6 \times 10^{-3}$ N
    c. not equal
    d. $2F_0$; e. $6F_0$
20. a. larger; b. larger; c. opposite; d. opposite
21. a. no; b. yes; c. no; d. no
22. into the paper
23. a. into a and out of b

b. opposite sense

24. a. force opposite to $\hat{y}$, lower side, negative
    b. force opposite to $\hat{y}$, upper side, positive
    c. positive

25. clockwise

26. $[(eBr)^2/2]\,\mathrm{m}$

27. a. counterclockwise
    b. negative

28. $m = |qB^2r^2/2V|$

29. a. $(|q|B/2v)\,\mathrm{s}$
    b. 2.0

51. a. 1.75×10^{-7} second
    b. 0.28 meter

52. a. $meB/(2\pi\omega)$
    b. 5.4×10^4 second^{-1}

53. a. in $\hat{y}$ direction
    b. force is zero

54. a. ILB into paper
    b. 0
    c. $ILB/\sqrt{2}$

55. a. $T_d = T_a = 2T_p$
    b. $v_d = v_a = v_p/2$
    c. $K_d = K_p/2$, $K_a = K_p$

56. (not used)

57. a. $E = 1.0 \times 10^5$ volt/meter
    b. undeflected

58. $13\,\mathrm{u}$

59. a. 3.6×10^{-10} second
    b. 2.8×10^9 second^{-1}

60. a. $B = 2\pi m/eT$

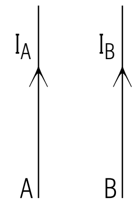
    b. 7.16×10^{-3} tesla $= 71.6$ gauss

61. yes, if the wire is parallel to the field

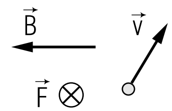
62. deflected in the direction of $\vec{E}$

MODEL EXAM

1. **Mutual forces on current-carrying wires.** Two current-carrying wires are arranged as shown in the diagram. If the senses of both currents are reversed, and the magnitudes of both currents are doubled, what are the changes (if any) in the magnitude and the direction of the force experienced by the wire labeled "A" in the diagram?



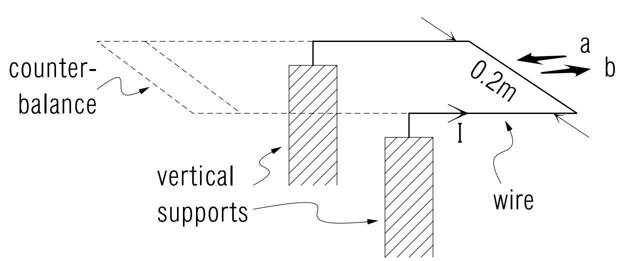
2. **Motion of charged particles in a magnetic field.** Charged particles moving in the direction shown in the diagram below experience a force into the paper. The magnetic field in the region shown has the direction indicated.



   a. What is the sign (+ or −) of the charge of the particles involved?

   b. If the particles have charge of magnitude of 1.6×10^{-19} coulomb, and have velocity components of magnitude 2×10^6 meter/second (parallel to $\vec{B}$) and 4×10^6 meter/second (perpendicular to $\vec{B}$), and the force they experience is of magnitude 3×10^{-15} newton, what is the magnitude of $\vec{B}$?

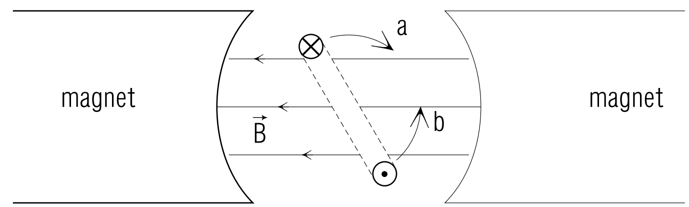
3. **Magnetic force on a current-carrying wire.** A U-shaped piece of stiff wire forms part of a framework lying in a horizontal magnetic field as shown. A magnetic force downward on the wire can be balanced by placing weights on the other end of the framework.



   a. If the current is in the direction shown in the diagram, what is the direction (a or b) of the magnetic field such that the force on the wire is downward, as desired?

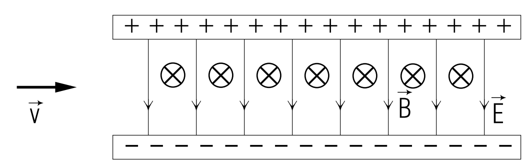
   b. If there is a current of 2 ampere in the wire (whose dimensions are as shown in the diagram), and the magnetic field has a magnitude of 5×10^{-4} tesla, what is the magnitude of the force on the wire?

4. **Rotation of an electric motor.** A simple electric motor is shown in this diagram:



   Given the directions of $\vec{B}$ and of the current flow through the coil, in which direction will the coil rotate (a or b)?

5. **Motion of negative ions in a velocity selector.** Negatively-charged ions pass through a velocity selector as shown in the diagram. Those ions having a particular speed v_0 proceed through the device without any deflection by the fields shown.



   In what direction are ions that move with speeds slower than v_0 deflected - up, down, into the paper, or out of the paper?

Brief Answers:

1. the magnitude increases by a factor of four; the direction is unchanged.
2. a. (negative)
   b. 5×10^{-3} tesla
3. a. b
   b. 2×10^{-4} newton
4. a
5. up