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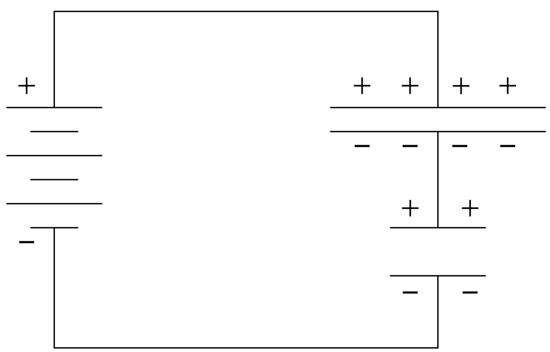
MISN-0-422

CAPACITANCE



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CAPACITANCE

by

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CONTENTS

Title: **Capacitance**

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Input Skills:

1. Vocabulary: conductor, potential difference (MISN-0-421).
2. Describe the electric field near a large uniformly charged plane (MISN-0-419).

Output Skills (Problem Solving):

S1. Given the geometric properties of a capacitor, qualitatively determine the dependence of its capacitance on these properties.

S2. Given a change in the distance between the plates, in the area of the plates, or in the substance between the plates of a parallel-plate capacitor, determine the resultant changes in the stored charge, potential difference, and capacitance of the system under these conditions: (a) each of the two plates is electrically insulated; (b) the plates are attached to the terminals of a battery.

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MISN-0-422

CAPACITANCE

Abstract:
If some of the mobile charged particles are transferred from one conductor to another, work must be done against the electric forces opposing such a charge separation. As a result, the separated charged particles acquire electric potential energy which can be stored and which can subsequently be used to do various kinds of work. Accordingly, we shall now discuss how the transfer of charge between conductors can be exploited for the storage of energy and or other practical applications.

SECT.

A CHARGE TRANSFER AND ENERGY STORAGE

Since a conductor contains charged atomic particles free to move throughout the conductor, it is analogous to an ordinary container filled with a gas whose uncharged molecules are free to move throughout the container. But containers of gas (such as tanks filled with compressed nitrogen gas) are commonly used to store gases and the energy associated with them. Hence this analogy suggests the conductors might similarly be used to store electric charge and energy.

CONTAINERS OF GAS

► *Mass transfer and pressures*
Figure A-1a shows two containers a and b originally filled with gas at the same pressure. Suppose that one now transfers a certain mass M of gas from the container b to the container a. Then the pressure p_a of the gas in container a will increase and the pressure p_b of the gas in container b will decrease. Thus the transfer of gas from b to a results in a corresponding positive pressure difference $p = p_a - p_b$.

In order to transfer the mass M of gas from the container at the lower pressure to the container at the higher pressure, some device (such as a pump) had to do work on the transferred gas against the opposing pressure forces. This work is then equal to the potential energy stored in the gases in the final situation after the gas transfer.

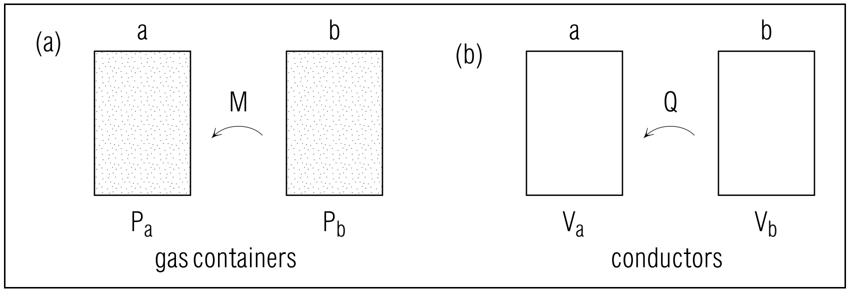


Fig. A-1: Analogy between gas containers and conductors.

► *Energy release*

Suppose now that one connects the two containers by a pipe. Then the gas will rush back through the pipe from the container a at the higher pressure to the container b at the lower pressure so as to equalize the pressures in the containers. In this process the potential energy stored in the gas is released and can be transformed into useful work done by the gas moving between the containers. For example, such a sudden release of energy is exploited in practical devices (such as air guns or pneumatic hammers) when initially compressed air is allowed to move toward lower pressure.]

CONDUCTORS

► *Charge transfer and potentials*

Figure A-1b shows two conductors a and b which are originally uncharged and at the same potential. Suppose that one now transfers a certain amount of positive charge Q from the conductor b to the conductor a so that the conductor a becomes positively charged and the conductor b becomes negatively charged. Then the potential V of the conductor a will increase and the potential V_b of the conductor b will decrease. Thus the transfer of positive charge Q from b to a results in a corresponding positive potential difference $V = V_a - V_b$. In order to transfer the positive charge Q from the conductor at the lower potential to the conductor at the higher potential, some device (such as a battery) had to do work on the transferred charge against the opposing electric forces. This work is then equal to the electric potential energy stored in the charged particles in the final situation after the charge transfer.

► *Energy release*

Suppose now that one connects the two conductors by a metal wire. Then positive charge will rush back through the wire, from the conductor a at the higher potential to the conductor b at the lower potential, so as to equalize the potentials of the containers. In this process the electric potential energy stored in the charged conductors is released and can be transformed into useful electric work done by the charged particles moving between the conductors. [For example, such a sudden release of electric energy is used to fire the electronic flash lamps employed in photography. It is also used in a device called a "defibrillator" to pass a large burst of charge through a patient's heart in order to stop dangerously uncoordinated heart contractions which might otherwise lead to death.]

Illustration: Relating Stored Charge and Potential Difference

A-1 If a larger mass M of gas is transferred from b to a in Fig. A-1a, is the magnitude of the pressure difference $p_a - p_b$ then larger or smaller than before? If a charge of larger magnitude Q is transferred from b to a in Fig. A-1b, is the magnitude of the potential difference $V_a - V_b$ larger or smaller than before? *(Answer: 3) (Suggestion: [s-5])*

SECT.

B CAPACITANCE OF A PAIR OF CONDUCTORS

DEFINITION OF CAPACITANCE

Let us look more closely at the transfer of charge between two conductors a and b which were initially uncharged. Suppose that a charge Q is transferred from conductor b to conductor a so that a acquires a charge Q and b is left with an opposite charge $-Q$. (See Fig. B-1.) Then a potential difference $V = V_a - V_b$ is produced between the conductors in equilibrium. *

> * Since all points of each conductor are at the same potential, the *same* potential difference V exists between *any* point on conductor a and *any* point on conductor b.

What is the relation between the transferred charge Q and the resulting potential difference V?

► *Relation between Q and V*

Suppose that every charged particle on the conductors had a charge 3 times as large. Then the total charge Q would also be 3 times as large. Furthermore, the electric field at every point between the conductors, and thus also the potential difference V, would also be 3 times as large. Hence the *ratio* Q/V would remain *unchanged.* Similarly, if the charge Q had the opposite sign, the resulting potential difference V would also have the opposite sign and the ratio Q/V would again remain *unchanged.* (Thus Q and V are said to be proportional to each other.) To express explicitly this relation between Q and V, let us denote the constant ratio Q/V by the symbol C which is called the "capacitance" of the pair of conductors and is thus defined as follows:

Def. **Capacitance**: $C = \dfrac{Q}{V}$ (B-1)

As we have seen, this capacitance $C = Q/V$ is independent of Q or V (although it may, of course, depend on other things, such as the positions and sizes of the conductors). Thus the capacitance of a pair of conductors is a quantity describing the relation between the charge transferred between the conductors and the resulting potential difference between them.

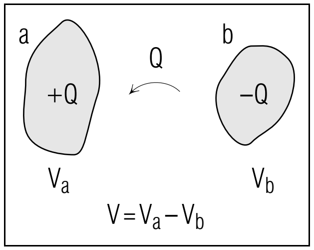


Fig. B-1: Transfer of charge between two conductors.

Note that the size of the capacitance of a pair of conductors is large if a given potential difference between them corresponds to a large transfer of charge between them.

► *Utility of C*

Suppose that we have determined the capacitance C of a pair of given conductors at specified positions (e.g., by measuring the potential difference produced when some particular known charge is transferred between these conductors). Then this knowledge of the capacitance C is very helpful because it allows us to use Def. (B-1) to find the potential difference $V = Q/C$ resulting from *any* amount of charge Q transferred between the conductors. Similarly, this knowledge of the capacitance C allows us to use Def. (B-1) to find the charge $Q = CV$ transferred between these conductors as a result of any applied potential difference V between them.

PROPERTIES OF CAPACITANCE

Let us look more closely at Def. (B-1) in order to examine the properties of capacitance.

► *C is positive*

If the charge Q transferred from conductor a to conductor b is positive, the resulting potential drop $V = V_b - V_a$ from a to b will increase. (Conversely, if Q is negative, V is < zero.) Thus the capacitance $C = Q/V$ of a pair of conductors is always *positive.*

► *Unit of C*

According to Def. (B-1), the unit of capacitance is

$$\text{unit of } C = \frac{\text{coulomb}}{\text{volt}} = \text{farad} \qquad \text{(B-2)}$$

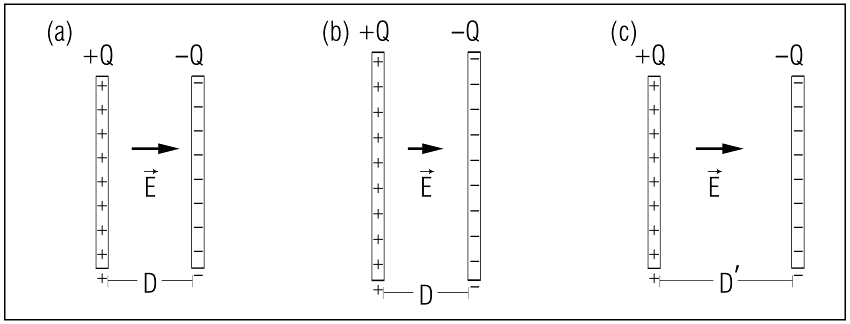


Fig. B-2: A pair of parallel metal plates. (a) Original plates. (b) Plates of larger size. (c) Plates with larger separation.

where the unit “farad” (abbreviated as “F”) is merely a conventional abbreviation for the combination coulomb/volt. [The “farad” is named in honor of Michael Faraday (1791-1867), the English experimental physicist who contributed enormously to our knowledge of electricity and magnetism.]

► *C and conductor size*

Consider a pair of conductors which are identical parallel conducting plates like those illustrated in Fig. B-2a (or which resemble such parallel plates, although their surfaces might be somewhat curved). How does the capacitance of such a pair of conductors depend qualitatively on their size and separation?

Let us first consider the two parallel plates separated by a fixed distance. How then does their capacitance depend on their size? To answer this question, suppose that the original plates in Fig. B-2a are replaced by larger plates, but that the same charge Q as before is transferred between them. (See Fig. B-2b.) Since the plates are larger, the same charge on each plate then spreads out over a larger surface so that the surface charge *density* (or charge per unit area) is *smaller*. Hence the electric field between the plates, and thus also the potential difference V between them is also *smaller*. Hence the capacitance $C = Q/V$ is *larger* than originally. Thus we see that larger conducting plates have a *larger* capacitance.

► *C and conductor separation*

If two parallel conducting plates have a fixed size, how does their capacitance depend on their separation? To answer this question, consider

the two plates of Fig. B-2a separated by a larger distance, as indicated in Fig. B-2c. Suppose that the same charge Q as before is transferred between these plates. Since the size of each plate is the same as before, the surface charge density is then also the same. Hence the electric field between the plates is also the same, but the potential difference $V = ED$ between the plates is *larger* because of the larger separation D' between the plates. Hence the capacitance $C = Q/V$ is *smaller* than originally. Thus we see that conducting plates separated by a larger distance have a *smaller* capacitance.

DISCUSSION

► *Fixed Q*

Suppose that, after a charge Q has been transferred between two conductors, each of them is electrically insulated. Then the charge on each of them remains subsequently unchanged. Under these conditions, the potential difference $V = Q/C$ between the two conductors changes if their capacitance is changed. (For example, if the capacitance of the conductors is increased by decreasing the separation between them, the potential difference V between them decreases.)

► *Fixed V*

On the other hand, suppose that the two conductors are respectively connected to the two terminals of a battery which maintains a fixed potential difference V between these conductors. Under these conditions, the charge $Q = CV$ transferred by the battery between the conductors changes if their capacitance is changed. (For example, if the capacitance of the conductors is increased by decreasing the separation between them, the charge Q transferred between the conductors increases.)

► *Relation between ΔQ and ΔV*

According to the definition of capacitance, Def. (B-1), the charge Q transferred between two conductors is always related to the potential difference V between them so that $Q = CV$, where C is the constant capacitance of the pair of conductors at their specified positions. Hence any *change* ΔQ in the charge transferred between these conductors must be related to the corresponding change ΔV of the potential difference between them so that $\Delta Q = C\Delta V$.

Understanding the Definition of Capacitance (Cap. 1a)

B-1 *Statement and example:* Two initially neutral metal spheres, supported on insulating stands, are connected to a 1.5 volt battery which causes 1.0×10^{-9} C of charge to be transferred from one sphere to the other. (a) What is the capacitance of the two spheres? (b) In general, a battery which produces a potential difference V between the two spheres causes a charge Q to be transferred from one sphere to the other. Express the capacitance C of the spheres in terms of Q and V. *(Answer: 10)*

B-2 *Relating quantities and dependence:* (a) If two conductors have a capacitance of 1×10^{-6} farad, what is the potential difference (produced by a battery connected between them) required to transfer 2×10^{-4} C of charge? (b) Suppose the potential difference between the conductors were twice as large as before. Would the total transferred charge now be twice as large or one-half as large as before? *(Answer: 7)*

B-3 *Relating changes in V and Q:* Two conductors have a capacitance of 3.0×10^{-8} F, an initial potential difference of V_0, and charges of $+Q_0$ and $-Q_0$. If the potential difference increases by 50 volt, what is the additional charge transferred from one conductor to the other? *(Answer: 5) (Suggestion: [s-3])*

More practice for this Capability: [p-1], [p-2]

Relating Capacitance to Geometric Quantities (Cap. 2)

B-4 Under each of the following conditions, does the capacitance, the transferred charge, or the potential difference remain the same for the two conductors? (a) The conductors remain in the same position (relative to each other). (b) The conductors are connected to the terminals of a battery. (c) Each conductor is electrically insulated from other objects. *(Answer: 1) (Suggestion: [s-7])*

B-5 Two parallel conducting plates are connected to the terminal of a battery, so that the potential difference V between them remains constant. For each of the following changes, state whether the transferred charge Q and the capacitance C of the two plates increases, decreases, or remains the same: (a) The plates are moved closer together. (b) The plates are moved farther apart. (c) The plates are replaced by new plates with larger area. *(Answer: 13) (Suggestion: [s-9])*

B-6 Two electrically insulated metal plates have charges of $+3 \times 10^{-8}$ C and -3×10^{-8} C If the two plates are moved closer together, does their capacitance increase, decrease, or remain the same? Does the potential difference between them increase, decrease, or remain the same? *(Answer: 8) (Practice: [p-3])*

SECT.

C CAPACITORS AND APPLICATIONS

A pair of conductors used as a device for storing charge is called a capacitor in accordance with this definition:

Def. **Capacitor**: A pair of conductors arranged so that the total charge on them remains constant. (C-1)

In other words, the charges on each of these conductors change only because of transfer of charge from one conductor to the other.

PARALLEL-PLATE CAPACITOR

Our qualitative comments in the preceding section suggest that a practical capacitor with a large capacitance can be made by using two parallel metal plates, each of large area A, separated by some small distance D. (See Fig. C-1.) What then is the capacitance C of such a "parallel-plate capacitor"?

► *Outline of calculation*

To answer this question, let us assume that a certain amount of charge Q has been transferred from one plate to the other so that plate a has a positive charge Q and plate b is left with a negative charge $-Q$. By finding the electric field $\vec{E}$ produced between the plates by these charges, we can then find the corresponding potential difference V between the plates. The capacitance of the pair of plate is then simply the ratio $C = Q/V$.

► $\vec{E}$ *between plates*

The electric field $\vec{E}$ at any point P between the plates can easily be found (as in text problem (E-5) of Unit 419 since it is merely the vector sum of the electric field $\vec{E}_a$ due to plate a and the field $\vec{E}_b$ due to plate b. Since the separation D between the plates is much smaller than their size, any such point between the plates (neglecting the relatively few points near their edges) is much closer to each plate than to their edges. Hence each such field can be found from the results of text section E of Unit 419.

The charge on each plate is spread uniformly over the plate (except for negligible effects near its edges). Thus each plate has a charge density of magnitude $|\sigma| = Q/A$. According to Relation (E-5) of Unit 419, the magnitude of the electric field due to each plate is then

$$|\vec{E}_a| = |\vec{E}_b| = 2\pi k_e|\sigma| = 2\pi k_e \frac{Q}{A} \qquad \text{(C-2)}$$

irrespective of the distance of P from the plates.

At the point P, the direction of the electric field $\vec{E}_a$ due to the positively charged plate a is *away* from this plate (i.e., to the right); the direction of the electric field $\vec{E}_b$ due to the negatively charged plate b is *toward* this plate (i.e., also to the right). Hence these fields are equal in direction as well as magnitude. The total field $\vec{E}$ between the plates is then

$$\vec{E} = \vec{E}_a + \vec{E}_b = 2\vec{E}_a$$

or

$$\vec{E} = 4\pi k_e \frac{Q}{A} \text{ to the right} \qquad \text{(C-3)}$$

► *V and C*

Since the parallel plates are separated by a distance D, the potential drop V from plate a to plate b is then

$$V = ED = \frac{4\pi k_e Q}{A} D = \frac{4\pi k_e D}{A} Q \qquad \text{(C-4)}$$

The capacitance C of the pair of plates is then

$$C = \frac{Q}{V} = \frac{Q}{\left(\frac{4\pi k_e D}{A}\right) Q}$$

If we simplify this expression by multiplying both numerator and denominator by A and then dividing both of them by Q, we obtain

$$C = \frac{A}{4\pi k_e D} \qquad \text{(C-5)}$$

This equation permits us to find the capacitance of a parallel-plate capacitor if we know the area A of its plates and their separation D.

Example C-1: Calculation of a typical capacitance

What is the capacitance of a parallel-plate capacitor with square plates, each of length and width 0.1 meter, separated by a distance of 1 mm?

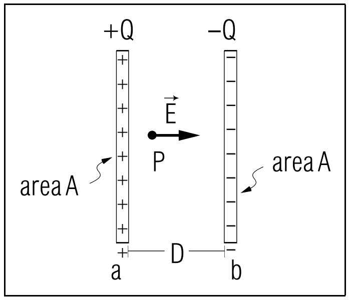


Fig. C-1: A parallel-plate capacitor.

The area A of each plate is $A = (0.1\,\mathrm{m})^2 = 0.01\,\mathrm{m}^2$. The separation between the plates is $D = 10^{-3}\,\mathrm{m}$. Using Eq. (C-5), we then obtain for the capacitance (to one digit of accuracy):

$$C \approx \frac{(0.01\,\mathrm{m}^2)}{(4)(3)(9\times10^9\,\mathrm{N\,m^2/C^2})(10^{-3}\,\mathrm{m})} = 9\times10^{-11}\,\mathrm{F} \qquad \text{(C-6)}$$

DISCUSSION

► *Size of farad*

The preceding example shows that ordinary capacitors have capacitances of the order of picofarads (i.e., of the order of 10^{-12} farad). The farad is thus an extremely large unit. Indeed, capacitors having capacitances of about 10^{-5} farad are considered to be quite large, and a capacitor with a capacitance of 1 farad would be so enormous that it is never encountered in ordinary life.

► *Dependence on A and D*

If the area A of the plates of a parallel-plate capacitor is made 3 times larger, Eq. (C-5) implies that the capacitance C becomes 3 times larger. (Thus the capacitance C is proportional to the area A.) But if the separation D between the plates is made 3 times larger Eq. (C-5) implies that the capacitance C becomes 3 times smaller. (Thus the capacitance C is *inversely* proportional to the separation D.) All these conclusions agree with our previous qualitative comments in Sec. B.

To store a large amount of energy Q for a fixed potential difference V (as might be provided by a battery), one needs a capacitor with a large capacitance $C = Q/V$. To obtain such a large capacitance, one needs a parallel-plate capacitor with plates having a large area and separated by a small distance.

► *Practical capacitors*

A practical way of making a capacitor with a large capacitance is to take two thin metal foils separated by a thin dielectric film of plastic. (This plastic film maintains the two conductors a small distance apart without allowing them to touch.) By rolling up the metal foils and interposed plastic film, one can then put this capacitor into a compact form.

CAPACITORS AS TRANSDUCERS

► *Transducers*

If the separation between the two conductors of a capacitor is changed, the capacitance C changes correspondingly. Hence it is possible to use a capacitor as a "transducer" in this sense:

Def. | **Transducer**: A device that translates ("transduces") from one physical quantity to another according to a known mathematical relationship (e.g., mechanical amplitude input, electrical voltage output). | (C-7)

Thus a capacitor can be used as a transducer to yield a capacitance corresponding to any distance between capacitor plates. For example, information about changes in a distance can then be obtained by measuring corresponding changes in a capacitance (e.g., by using an oscilloscope to measure changes is the potential difference of a capacitor with fixed charges on its conductors.)

► *Utility of transducers*

Transducers which yield electric quantities corresponding to non-electric quantities are tremendously useful for making all kinds of measurements. The reason is that electric quantities can be easily measured with great precision. Furthermore, information in electric form can be easily transmitted to remote locations through wires or by radio. (For example, the electric information about a person's electrocardiogram can be easily transmitted to a hospital monitoring station while the person goes about his normal business.)

► *Applications*

As a specific example, let us illustrate how a capacitor can be used as a transducer to measure pressure. Fig. C-2 shows a small hermetically sealed box consisting of a metal bottom, a top consisting of a thin flexible metal diaphragm, and of dielectric side walls. The top and bottom of this

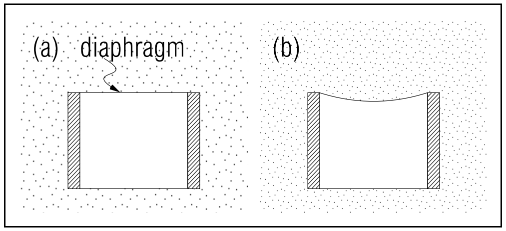


Fig. C-2: Capacitor used as a pressure transducer: (a) lower outside pressure; (b) higher outside pressure.

box constitute a capacitor. When the pressure outside the box increases, the resulting excess force causes the top diaphragm to flex inward (as indicated in Fig. C-2b). As a result, the separation between the top and bottom conductors decreases so that their capacitance increases. (For fixed charges on these conductors, the potential difference between them would then correspondingly decrease.)

Such a transducer can be used to measure atmospheric or other pressures continuously at remote locations, since the electric information can be readily transmitted over long distances to electronic recording equipment. Similarly, such a transducer can be used as a microphone, i.e., as a device to transform the pressure changes, produced in air by sound waves, into corresponding potential differences. (Such potential differences can then be electronically amplified and used to record magnetic tapes or to drive loudspeakers.)

Understanding Capacitances for Parallel-Plate Capacitors (Cap. 1b)

C-1 *Example:* What is the capacitance of two square parallel metal plates separated by 0.20 cm and having sides 1.0 meter long? *(Answer: 4)*

C-2 *Relating quantities:* Suppose that capacitors with capacitance 1 F and 10^{-6} F could be made from parallel metal plates separated by 1 cm. What would be the area of the plates in each capacitor? *(Answer: 12)*

C-3 *Organizing relations:* A 12 volt battery is connected to a capacitor made of two 0.01 m^2 metal plates separated by 0.1 cm. What is the charge transferred between the conductors of this capacitor? *(Answer: 9) (Suggestion: [s-2])*

C-4 *Dependence of capacitance on area and separation:* (a) Parallel-plate capacitors 1 and 2 are identical, except that the plates of capacitor 1 have four times the area of those of capacitor 2. What is the ratio C_1/C_2 of the capacitances of these capacitors? (b) Parallel-plate capacitors 3 and 4 are identical, except that the plates of 3 are separated by a distance three times as large as the distance separating the plates of 4. What is the ratio C_3/C_4 of the capacitance of these two capacitors? *(Answer: 11)*

Understanding the Definition of Capacitance (Cap. 1a)

C-5 *Properties:* Answer the following questions for each of the quantities charge, potential difference, and capacitance. (a) What algebraic symbol ordinarily represents each quantity? (b) What are the possible signs $(+, -, 0)$ of each quantity? (c) Which of the following is a typical magnitude of each quantity for an ordinary capacitor connected to an ordinary battery: 10^{-8} C, 1 C, 10^{8} C; 10^{-8} volt, 1 volt, 10^{8} volt; 10^{-8} F, 1 F, 10^{8} F? *(Answer: 2)*

Knowing About Capacitors As Transducers

C-6 Figure C-3 shows a transducer which changes information about the volume of a forearm into information about potential drop (which can then be processed by electronic equipment). The transducer is a capacitor made of the conducting skin surface of the forearm as one "plate" and a metal sleeve (insulated from the arm by a plastic coating) as the other "plate." The capacitor is connected to a voltage source through one wire to the sleeve on the arm and another wire to a metal cone segment in contact with the subject's skin. When a blood-pressure cuff (not shown) is tightened around the *upper* arm, a change in capacitance indicates how the volume of the forearm changes due to the blood held back by the blood-pressure cuff. (a) As the forearm swells toward the metal sleeve around it, does the capacitance increase or decrease? (b) If the charges Q and $-Q$ on the conductors of the capacitor remain constant, does the potential difference V increase or decrease? *(Answer: 6)*

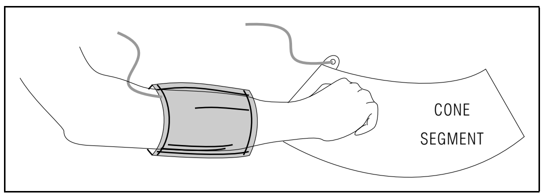


Fig. C-3.

SECT.

D CAPACITORS WITH DIELECTRICS

What happens to the capacitance of a capacitor if the entire space between the two conductors is filled with a dielectric material?

► *Increase of C*

To answer this question, let us consider a parallel-plate capacitor charged so that one of its plates has a charge Q and the other a charge $-Q$. (See Fig. D-1a.) For comparison, let us consider this capacitor with the *same* charges, but with the space between the plates entirely filled with a dielectric material. According to our discussion in text section F of Unit 421, the polarized molecules in the dielectric then produce on its surfaces charges which *decrease* the magnitude of the electric field between the plates. Hence the potential difference V between the plates is also decreased and the capacitance $C = Q/V$ is correspondingly increased.

► *Relation between capacitances*

To make these comments more quantitative, suppose that the dielectric material has a dielectric constant K. Then we know from text section E of Unit 421 that the electric field $\vec{E}$ between the plates (i.e., within the dielectric) is K times smaller than the electric field $\vec{E}_0$ between the plates of the capacitor without dielectric, so that $\vec{E} = \vec{E}_0/K$. Correspondingly, the potential difference V between the plates of the capacitor filled with dielectric is then also K times smaller than the potential difference V_0 between the plates of the capacitor without dielectric, so that $V = V_0/K$. Hence the capacitance $C = Q/V$ of the capacitor filled with dielectric is related to the capacitance $C_0 = Q/V_0$ of the capacitor without dielectric, and:

$$C = \frac{Q}{V} = \frac{Q}{V_0/K} = K\frac{Q}{V_0}$$

or

$$\boxed{C = KC_0} \qquad \text{(D-1)}$$

In other words, since the same transferred charge in the case of the capacitor with dielectric leads to a potential difference which is K times smaller, the capacitance of this capacitor is K times larger than that of the capacitor without dielectric.

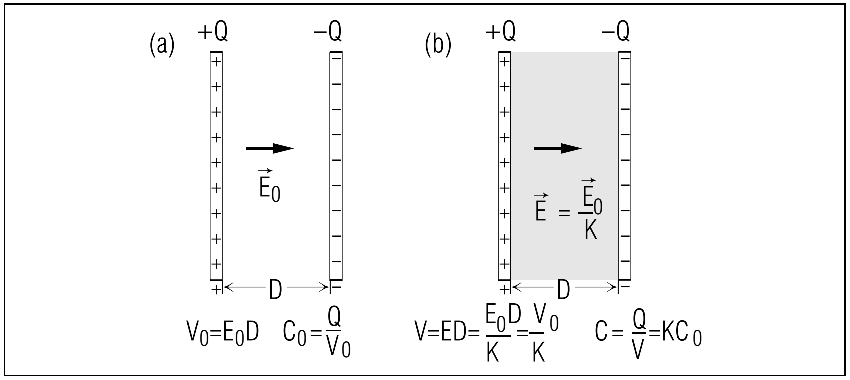


Fig. D-1: A charged capacitor. (a) Without dielectric. (b) With dielectric.

APPLICATIONS

► *Increasing capacitance*

The capacitance of a capacitor can be greatly increased by filling the region between its conductors by a material with a large dielectric constant. Practical capacitors, which are designed to obtain a large capacitance with a small size, are thus constructed by separating two metal foils (or films) by a thin dielectric material with large dielectric constant.

► *Measurement of K*

One of the easiest methods of measuring the dielectric constant of a material is to measure the capacitance of a capacitor before and after the space between its conductors is filled with this material. The ratio C/C_0 of the measured capacitances is then, by Eq. (D-1), just equal to the dielectric constant K of the material.

Relating Quantities Describing Capacitors (Cap. 3)

D-1 The two parallel plates of a capacitor are insulated so that the charges $+Q$ and $-Q$ of the plates remain constant. When the initially empty region between the plates is filled with a dielectric material, do each of the following quantities increase, decrease, or remain the same in magnitude? (a) Charge of one plate. (b) Charge of a region including one plate and the surface of the dielectric near it. (c) Electric field at a point between the plates. (d) Potential difference between the plates. (e) Capacitance of the two plates. *(Answer: 19) (Suggestion: [s-13])*

D-2 Suppose the two parallel plates of a capacitor are connected to a battery so that the potential difference V between them remains the same. When the initially empty region between the plates is filled by a dielectric material, do each of the five quantities listed in problem D-1 increase, decrease, or remain the same? *(Answer: 16) (Suggestion: [s-12]) (Practice: [p-4])*

Understanding $C = KC_0$ (Cap. 1c)

D-3 *Relating quantities:* An initially empty parallel plate capacitor has a capacitance of 9×10^{-11} F. When the capacitor is filled with the hydrocarbon butyl oleate, its capacitance is then 36×10^{-11} F. What is the dielectric constant of butyl oleate? *(Answer: 27)*

D-4 *Organizing relations:* (a) Write an expression for the capacitance C of a parallel plate capacitor which has plates of area A and separation D, and which is filled by a substance of dielectric constant K. (b) If the two plates of a 10^{-8} Farad capacitor are separated by 0.01 cm, what must be the area of the plates if the capacitor is empty and if the capacitor is filled with titanium dioxide ($K = 100$)? *(Answer: 22)*

SECT.

E ENERGY STORED IN A CAPACITOR

Consider a capacitor which has been charged so that one of its conductors has a charge Q and the other has a charge $-Q$. Then this capacitor has some electric potential energy U relative to a "standard state" where both conductors of the capacitor are uncharged. According to the general Definition (B-2) of Unit 416, this electric potential energy U is just equal to the total work done by the electric forces when the charge Q is transferred from conductor a to conductor b so that the capacitor is thereby "discharged" i.e., brought to its standard uncharged state. (See Fig. E-1.) *

> * Equivalently, the potential energy U is also equal to the work which must be done, by a battery or similar device, *against* the opposing electric forces in order to transfer the charge Q so as to bring the capacitor from its initial uncharged state to its final charged state.

► *Calculation of U*

Suppose that the capacitor is brought to its uncharged state by successively transferring equal small amounts of charge from conductor a to conductor b. In this process, the magnitude of the charge on each conductor decreases uniformly from Q to zero. Correspondingly, the potential difference between the conductors also decreases uniformly from its initial value $V = Q/C$ to its final value of zero.

If the total charge Q were transferred between a *fixed* potential difference equal to its initial value V, the electric work done would be simply QV. If the total charge were transferred between a fixed potential difference equal to its final value 0, the electric work would be simply zero. But since the total charge Q is actually transferred while the potential difference decreases uniformly from V to 0, the actual work is just the average value $QV/2$ between the previous two extremes. *

> * The preceding argument is analogous to that used in text section D of Unit 404 to find the displacement of a particle moving with uniformly changing velocity.

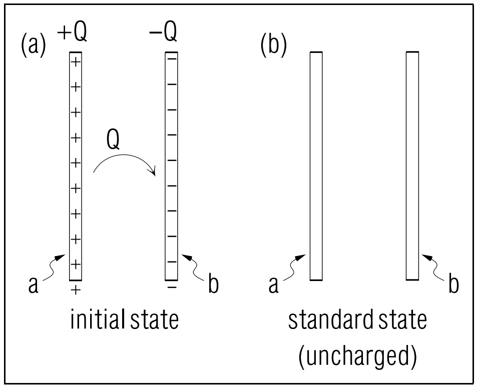


Fig. E-1: Discharge of an initially charged capacitor.

Since this work is, by definition, equal to the electric potential energy U initially stored in the charged capacitor, we thus obtain the result:

$$\boxed{U = \frac{1}{2}QV\,.} \qquad \text{(E-1)}$$

► *U related to C and V*

The relation U for the potential energy of a capacitor can be expressed in other equivalent forms. For example, the charge Q of the capacitor is related to its potential difference V so that $Q = CV$, where C is the capacitance of the capacitor. Hence the potential energy Eq. (E-1) is also equal to

$$U = \frac{1}{2}(CV)V = \frac{1}{2}CV^2 \qquad \text{(E-2)}$$

► *Dependence of U on V*

Suppose that the potential difference V between the conductors of the capacitor were 3 times as large. Then Eq. (E-2) implies that the corresponding electric potential energy in the capacitor would be $3 \times 3 = 9$ times as large. Thus the potential energy U is increased proportionately to the *square* of the potential difference. [This result is also directly apparent from Eq. (E-1). If V were 3 times as large, the charge Q of the capacitor would also be 3 times as large. Hence the potential energy $U = QV/2$ would be $3 \times 3 = 9$ times as large.]

Example E-1: Energy and power supplied by a defibrillator

When a person's heart has momentarily stopped or has gone into "fibrillation" (a condition of uncoordinated irregular contractions), the person may sometimes be saved by means of a "defibrillator." This device supplies a large amount of charge which is made to flow suddenly through the person's body (and thus partly through the heart). As a result, the heart may be jolted into resuming its normal rhythm.

Suppose that the capacitor in a defibrillator has a capacitance of 16×10^{-6} farad and is charged so that the potential difference between its two conductors is 7.0×10^3 volt. When the conductors are connected to a person's chest and back, the charge Q originally stored in the capacitor and then flowing through the person's body is then

$$Q = CV = (16 \times 10^{-6}\,\text{farad})(7.0 \times 10^3\,\text{volt}) = 0.11\,\text{coulomb}.$$

The electric energy originally stored in the capacitor and then delivered to the person during discharge is then

$$U = (1/2)QV = (1/2)(0.11\,\text{coulomb})(7 \times 10^3\,\text{volt}) = 3.9 \times 10^2\,\text{joule}.$$

If this energy is delivered during a discharge lasting about 0.01 second, the average electric power delivered to the person is then

$$\bar{\mathcal{P}} = (3.9 \times 10^2\,\text{joule})/(0.01\,\text{second}) \approx 4 \times 10^4\,\text{watt}$$

(i.e., equal to the power supplied to 400 light bulbs, each rated at 100 watt).

Understanding the Relation $U = QV/2$ (Cap. 1d)

E-1 *Statement and example:* (a) Use the relation $U = QV/2$, and the definition of capacitance to express the energy stored in capacitor in terms of capacitance C and potential difference V. (b) Express U in terms of capacitance and transferred charge Q. (c) What is the energy stored in a 4.0×10^{-10} farad capacitor connected to a 12 volt battery? *(Answer: 25)*

E-2 *Relating quantities:* An energy of 45 J is required to cause the "flash" of a photographic flash gun. (a) If the capacitor supplying this energy can be given a potential difference of 900 volt, what must be the capacitance of the capacitor in such a flash gun? (b) *Review:* If the light flash occurs in about 1×10^{-4} second, what is the power delivered to the flash bulb during this time? *(Answer: 20)*

E-3 *Dependence of energy on potential:* (a) For a given capacitor, if the potential difference V is made 5 times as large, is the corresponding magnitude of the charge Q transferred the same, 5 times as large, or 25 times as large? (b) On the basis of your answer to part (a) and on the relation $U = QV/2$, if the potential difference V is made 5 times as large, is the corresponding energy U stored 5 times as large or 25 times as large? (c) Is your answer to part (b) consistent with the expression $U = CV^2/2$? *(Answer: 17) (Practice: [p-5])*

Illustration: Partial Discharge of a Capacitor

E-4 The plates of a capacitor initially have a potential difference of V_0, and the corresponding transferred charge is Q_0. Suppose now that the capacitor is partially discharged so that afterwards only half the stored charge remains. (a) What is the potential difference after this partial discharge? (b) During the discharge, a charge of $Q_0/2$ moves from one plate to the other. What is the average (or middle) potential drop through which this charge moves? (c) What is the electric work done on the charge which flows from one plate to the other during the partial discharge? (d) What then is the remaining electric potential energy stored in the capacitor after this discharge? (e) Does this answer agree with the potential energy calculated directly from the final transferred charge $Q_0/2$ and the corresponding potential difference? *(Answer: 14) (Suggestion: [s-10])*

SECT.

F LARGE CONDUCTORS AND GROUNDING

Let us finally consider the special simple case of a *single* conductor. What is the relation between any charge placed on such a conductor and the resulting potential of this conductor?

► *V of large conductor*

In particular, consider a conductor which is large and which is initially either uncharged or charged. What then happens to the potential V of this conductor if some additional charge Q is placed on it? Since the charge Q spreads out over the large surface of this large conductor, the resulting change in the charge *density* (or charge per unit area) of the conductor is small. Accordingly, the electric field near the conductor changes only slightly. Thus the potential V of the conductor changes correspondingly also only slightly.

► *Potential of the earth*

The preceding comments are particularly interesting when applied to the earth which is an extremely large conductor. Indeed, the earth is so large that any charge Q placed on the earth leads to negligibly small change in the potential of the earth. Accordingly, the potential of the earth remains nearly constant, irrespective of how much charge is ordinarily put on the earth.

Because the potential of the earth remains constant in terrestrial experiments, it is often convenient to define the potential so that the potential of the earth is zero (i.e., to define the potential V relative to the earth as the standard position where $V = 0$.) *

* In fact, the potential of the earth is not zero relative to a standard position *very far* from the earth since the earth has a net charge.

GROUNDING

Any conductor which is in contact with the earth must, in equilibrium, have the same potential as the earth. Since the potential of the earth remains essentially constant it is thus possible to maintain the potential of any other conductor constant (equal to that of the earth) by "grounding" the conductor, i.e., by simply connecting it by a metal wire

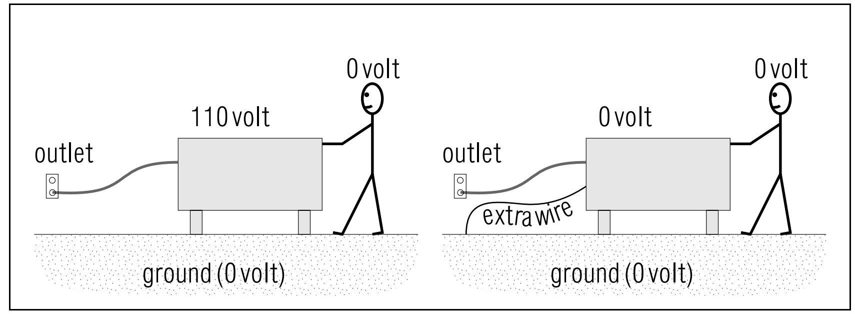


Fig. F-1: Defective electric appliances with metal case. (a) Appliance ungrounded. (b) Appliance grounded.

to the earth. *

* To make sure that a good conducting contact is made with the earth, it is advisable to connect the wire to a metal cold-water pipe since such a pipe is ultimately well buried in the ground.

► *Avoiding electrocution*

The grounding of electric appliances is an extremely simple, although too often neglected, precaution for avoiding possible electrocution of persons handling such appliances. To illustrate the possible danger, consider an electric appliance, such as a washing machine which has an exterior case made of metal. The electric power for the appliance is provided by connecting the appliance by electric wires to an electric power outlet in the wall. Thus some of the wires in the appliance are at a high potential (say, 110 volt) relative to the ground. As long as the wires are properly insulated by a dielectric covering, there is no danger to anyone touching the metal case of the appliance. Suppose, however, that the insulation of the wires is defective so that a bare wire at a potential of 110 volt touches the metal case, as illustrated in Fig. F-1a. Then the case itself has a potential of 110 volt relative to the ground. If a person in good contact with the ground, and thus having the same potential as the ground, now touches the metal case of the appliance, the potential difference will result in a flow of charge from the metal case through the person to the ground. Some of the charge flowing through the heart of the person may then cause death by electrocution.

Suppose, however, that one had taken the precaution of grounding the metal case of the appliance, as illustrated in Fig. F-1b. If the insulation of the wire becomes defective, the metal case connected to the ground now remains nevertheless at the constant potential of the earth, no matter how much charge flows from the wire through the case into the earth. Thus a person touching the metal case remains safe.

► *Electric shielding*

Suppose that a metal enclosure is maintained at a constant potential by being connected to the ground. Then it can be shown that the potential at all points inside this enclosure has the same constant potential, irrespective of the presence of any stationary charged particles outside the enclosure. Thus the inside of the enclosure is a region unaffected by (or "shielded from") electric disturbances originating from outside the enclosure. Instruments designed to measure small potential differences (such as those arising from biochemical processes of the heart or brain) are thus commonly enclosed in grounded metal boxes to avoid interfering electric effects from other electric equipment in the environment.

Note: Tutorial section F contains a simple illustration of grounding.

Illustrations

F-1 *Effects of grounding:* (a) *Review:* Express the potential V of a conducting sphere in terms of its charge Q and radius R. (b) What potential is acquired by an initially neutral conducting sphere when a charge of 10^{-5} C is placed on it, if this sphere has a radius of 10 meter and if this sphere has a radius of 10,000 meter? (c) Which of these is the safer grounding procedure? Grounding appliances in a building to metal drain pipes (which may be insulated from the earth because they are connected to plastic drainage pipes); or grounding appliances to a metal stake driven deep into the ground. Explain your answer. *(Answer: 23)*

F-2 *Relating charge and potential of a sphere:* (a) *Review:* Write an expression for the potential V at the surface of a uniformly charged sphere in terms of its radius R and total charge Q. (b) A sphere has a large uniformly distributed charge Q, but only a small potential V. Does this sphere have a large or a small radius? (c) The earth has a radius of 6.4×10^3 km $= 6.4 \times 10^6$ meter. If a large charge of 0.010 coulomb were uniformly distributed over the earth, what would be the potential of the earth? *(Answer: 21)*

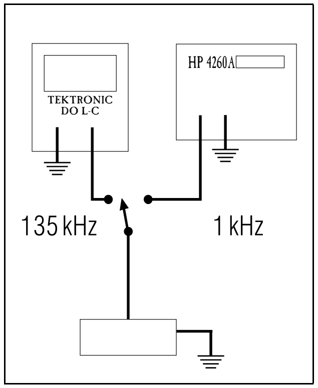


Fig. F-2: Diagram of electronic equipment, with (⏚) symbols indicating that cases should be grounded.

Knowing About Shielding

F-3 Diagrams describing sensitive electronic equipment used for biological measurements often indicate that the metal cases of the equipment should be "grounded" (Fig. F-2). Why is such grounding of the cases suggested even when the potentials involved are not dangerously large? *(Answer: 15)*

SECT.
G SUMMARY

DEFINITIONS

Capacitance; Def. (B-1)

farad; Eq. (B-2)

capacitor; Def. (C-1)

transducer; Def. (C-7)

IMPORTANT RESULTS

Relation between V and transferred Q for two conductors: Def. (B-1)

$Q = CV$

Capacitance of a parallel-plate capacitor: Eq. (C-5)

$C = A/(4\pi k_e D)$

Capacitance of capacitor filled with dielectric: Eq. (D-1)

$C = KC_0$

Electric energy stored in a capacitor: Eq. (E-1)

$U = (1/2)QV$

USEFUL KNOWLEDGE

Capacitors used as transducers (Sec. C)

Grounding (Sec. F)

Shielding (Sec. F)

NEW CAPABILITIES

Be able to:

(1) Understand these relations:
- (a) The definition $C = Q/V$ of capacitance (Sects. B and C, [p-1], [p-2]),
- (b) The relation $C = A/(4\pi k_e D)$ describing the capacitance of a parallel-plate capacitor (Sec. C),
- (c) The relation $C = KC_0$ describing the capacitance of a capacitor filled with dielectric (Sec. D),
- (d) The relation $U = (1/2)QV$ describing the electric energy stored in a capacitor. (Sec. E, [p-5]).

(2) Describe qualitatively the dependence of capacitance on the geometric properties of a capacitor (Sec. B, [p-3]).

(3) Relate changes in the transferred charge, potential difference, and capacitance of a parallel-plate capacitor to changes in the distance between the plates, in the area of these plates, or in the substance between the plates, under these conditions: (a) each of the two plates is electrically insulated; (b) the two plates are attached to the terminals of a battery (Sec. D, [p-4]).

Tutorial study aids:

Sec. F: A simple illustration of grounding

Sec. G: Applying relations describing capacitors

Sec. H: Additional problems

Applying Relations Describing Capacitance (Cap. 1,2,3)

G-1 The cylindrical membrane of a nerve axon separates two conducting solutions which thus form a capacitor. Measuring the capacitance of this capacitor provides an accurate way of determining the thickness of the membrane. Typically, the capacitance per unit area of the solutions separated by a membrane is 1 microfarad/cm^2. That is, a capacitor made of two solutions separated by a membrane of area 1.0 cm^2 would have a capacitance of 1.0×10^{-6} farad. The membrane is composed of "lipid," a dielectric with dielectric constant of about 4.0. What is the thickness of a typical nerve membrane? *(Answer: 18)*

G-2 Suppose the potential difference V between the plates of a capacitor is reduced by 3 (i.e., made 1/3 as large). Describe the corresponding change in the transferred charge Q, in the capacitance C, and in the stored energy U. *(Answer: 24)*

G-3 Does the insertion of a dielectric material into a capacitor ever *decrease* its capacitance? Explain why or why not. *(Answer: 29)*

G-4 A parallel-plate capacitor can be used as a microphone if one plate is a thin diaphragm free to vibrate in response to sound, thus causing a change in the distance between the two plates. (This changing potential difference can be used to send an electric signal.) If the charge of each plate has constant magnitude Q, and the space between the plates is empty, write an expression for the potential difference V in terms of the distance d between the plates. *(Answer: 33) (Suggestion: [s-1])*

Now: Go to tutorial section G.

SECT.

H PROBLEMS

H-1 *Charges on capacitor plates:* The initially uncharged plates of a capacitor are connected to the terminals of a battery. Are the charges acquired by the plates always equal in magnitude? Why or why not? Does your answer depend on the plates being the same size? *(Answer: 31)*

H-2 *Work done on a capacitor:* Two parallel plates have area A, separation d, and charges $+Q$ and $-Q$. They have insulating handles and each is insulated from all other objects. (a) To move these plates apart so that they are at rest with a final separation of $2d$, would you do positive or negative work on them? (b) As the plates are moved apart, does their electric potential energy increase or decrease? (c) Write an expression for the final electric potential energy of the plates in terms of the transferred charge Q and the initial potential difference V between the plates. Is this energy larger or smaller than the initial potential energy of the plates? *(Answer: 26) (Suggestion: [s-11])*

H-3 *Quantities describing a capacitor:* A 10^{-8} farad capacitor consists of foil plates of area 0.5 meter^2 separated by oil of dielectric constant 4.5 (actually by oil-soaked paper). The capacitor plates have a potential difference of 50 volt. (a) What are the magnitudes of the charge and charge density of each plate? (b) What would be the magnitude of the electric field between the plates if this region were empty? What is the magnitude of the actual electric field in the oil? *(Answer: 28)*

H-4 *Ions crossing a nerve membrane:* Two conducting solutions separated by 1 cm^2 of nerve membrane would have a capacitance of 1.0 microfarad $= 1.0 \times 10^{-6}$ farad (see problem G-1). When a nerve is inactive, the potential difference between the two sides of the membrane is about 0.1 volt. (a) Use these numbers to estimate the magnitude of the charge on each side of a nerve membrane of area 5.0×10^{-7} meter^2. (b) How many ions (K^+ or Na^+) each with a charge of 1.6×10^{-19} C make up this charge? (c) How many such ions must migrate across the nerve membrane when the nerve "depolarizes" so that both sides of the membrane become (very nearly) uncharged? *(Answer: 32)*

H-5 *Capacitor to power a car?:* To assess the practicality of using the energy stored in a capacitor to run a car, let us estimate the size of the *smallest* capacitor which could possibly be used for this purpose.

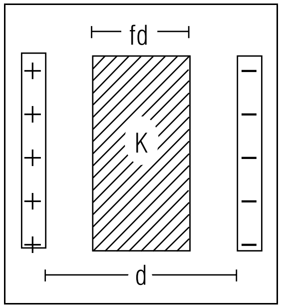


Fig. H-1.

We therefore neglect the energy which must be supplied to overcome any friction. (a) What energy must then be supplied to a 1,000 kg car as it moves with constant speed up a hill of height 100 meter? (b) Express the energy stored in a parallel-plate capacitor in terms of the area A and separation D of its plates, the dielectric constant K of the material filling the plates, and the potential difference V between the plates. (c) To make-this energy as large as possible, suppose that the plates are separated by only 0.1 cm, that they are filled with titanium dioxide ($K = 100$), and that $V = 10,000$ volt. What is the area of the plates of a capacitor storing enough energy to move the car up the hill? (d) What is the volume of this capacitor? Compare this volume with the approximate volume of a small car with dimensions of length about 3 meter, width 2 meter, and height 2 meter. *(Answer: 34)*

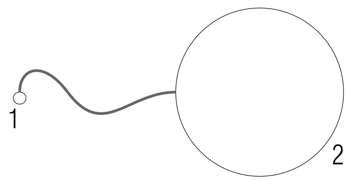
H-6 *A partly filled capacitor:* A fraction f of the space between two capacitor plates is filled with a material of dielectric constant K (Fig. H-1). Thus the thickness of the dielectric slab is fd, where d is the separation of the plates. Express the capacitance C of the partially filled capacitor in terms of the fraction f and the capacitance C_0 of the unfilled capacitor. Check that for $f = 1$, your expression corresponds to what you know about entirely filled capacitors. *(Answer: 30) (Suggestion: [s-8])*

Note: Tutorial section H contains additional problems.

TUTORIAL FOR F

GROUNDING

f-1 *A SIMPLE ILLUSTRATION OF GROUNDING:* To illustrate simply why grounding is useful, consider a metal sphere #1 of radius $R = 0.1$ meter which (like the appliance case in Fig. F-1) initially has a potential of 110 V. (a) *Review:* Express the potential V of this sphere in terms of its charge Q and radius R. What is the charge of this sphere? Now suppose this sphere is "grounded" by using a conducting wire to connect it to a large, initially uncharged metal sphere #2 of radius 100 meter.



(b) Which of the following best describes the charges and potentials of the two spheres after they are connected? The charges of the spheres are both equal to $Q/2$ but the potential of 1 is much larger than the potential of 2. The potentials of the two spheres are equal, but the charge of 2 is much larger than the charge of 1 (so that 2 has a charge very nearly equal to Q). (c) Use your answers to parts (a) and (b) to find the potential of each sphere *after* they are connected. (d) What is the effect on the potential of a small object of "grounding" it by connecting it to a large conductor with a potential of 0? *(Answer: 56)*

Now: Go to text problems F.

TUTORIAL FOR G

APPLYING RELATIONS DESCRIBING CAPACITORS

g-1 *RELATING POTENTIAL AND CHARGE TO ENERGY:* The plates of a capacitor initially have a potential difference of 30 volt and each has a charge of magnitude 1.2×10^{-8} C. (a) What is the energy stored in the capacitor? (b) If the capacitor partially discharges so that its final stored energy is one fourth its initial energy what then is the final potential difference between the plates and the magnitude of the charge of each plate? *(Answer: 62)*

g-2 *RELATING ELECTRIC FIELD TO CHANGES IN CAPACITANCE (review):* Suppose the parallel plates of a capacitor are initially separated by a distance D but then are moved so that they have a final separation $3D$. (a) Write an expression for the final capacitance C of these plates in terms of their initial capacitance C_0. (b) If each plate is insulated from other objects what is the magnitude E of the final electric field in terms of the magnitude E_0 of the initial field? (c) If the plates are connected to a battery so that the potential difference between them remains constant, what is the magnitude E of the final electric field in terms of the magnitude E_0 of the initial field? *(Answer: 60)*

TUTORIAL FOR H

ADDITIONAL PROBLEMS

The following problems provide additional practice in applying capacitance and related quantities and ideas.

Capacitance and Properties of Capacitors

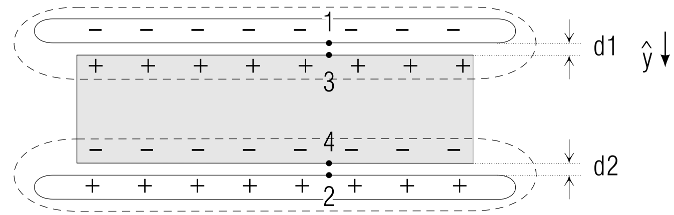
h-1 *BREAKDOWN STRENGTH AND DESIGN OF CAPACITORS:* A capacitor is filled with a ceramic material which has a dielectric constant of S and a breakdown strength of 2×10^6 V/m. When the plates acquire a potential difference $V = 500$ volt the electric field inside the ceramic is sufficiently large that breakdown occurs allowing charge to flow through the ceramic from one plate to the other. How can the area and separation of the plates in this capacitor be changed so as to keep the capacitance the same but so as to reduce the magnitude of the electric field corresponding to $V = 500$ volt? *(Answer: 54) (Suggestion: [s-)]*4

h-2 *DESIGNING FOR LARGE CAPACITANCE:* To make a capacitor with as large a capacitance as possible one must consider not only the dielectric constants of materials for filling the capacitor but also the thickness of available slabs of the material. For example, mica ($K = 6$) can easily be cleaved into sheets with a thickness of only 0.1 mm $= 1 \times 10^{-4}$ meter. However typical glass ($K = 7$) is too fragile for easy use if its thickness is any less than 2 mm $= 2 \times 10^{-3}$ m. Which of these slabs when sandwiched between two metal plates will produce the capacitor with the larger capacitance? *(Answer: 58)*

h-3 *A CAPACITOR WITH A METAL SLAB:* Metals have an infinitely large dielectric constant K, and $C = KC_0$ for a dielectric-filled capacitor. Thus it is easy to wrongly conclude that capacitors of very large capacitance might be made simply by using metal to fill the region between the plates. Let us look more closely at this apparent possibility.

By answering the following questions, summarize the differences between a capacitor filled with a dielectric and a capacitor filled with a slab of metal: (a) Why can not capacitors be completely filled with metal but instead must include gaps between each plate and the inserted metal slab (see following diagram)? (b) Why does each region indicated by a dotted line in the following diagram have a charge of zero? (c) Why is

the potential drop from plate 1 to plate 2 equal just to the sum of the potential drops from 1 to 3 and from 4 to 2 across the two gaps?



Suppose that the capacitor plates have area A and separation D and are insulated so that their charges ($+Q$ and $-Q$) remain the same. (d) What is the electric field in each gap? (e) What is the potential difference V between the two plates with the slab inserted? (Express V in terms of the distances d_1 and d_2 shown in the diagram.) (f) What is the capacitance C of the capacitor with the slab in place? (g) Suppose we just made a simple capacitor with two plates of area A and separation $d_1 + d_2$. What is the capacitance of this capacitor? How does it compare with your answer to (f)? *(Answer: 52)*

Electric Effects Inside a Metal Shell

As mentioned in Sec. F if a metal enclosure is maintained at a constant potential then all points inside the enclosure also have this same constant potential.

h-4 *MICHAEL FARADAY S CUBE:* Michael Faraday an early investigator of electric effects built a large metal cube which he supported on insulating feet and maintained at a fixed large potential. Then according to his own records:

> "I went into the cube and lived in it, and using lighted candles, electrometers, and all other tests of electrical states, I could not find the least influence upon them . . . though all the time the outside of the cube was very powerfully charged, and large sparks and brushes were darting off from every part of its surface."

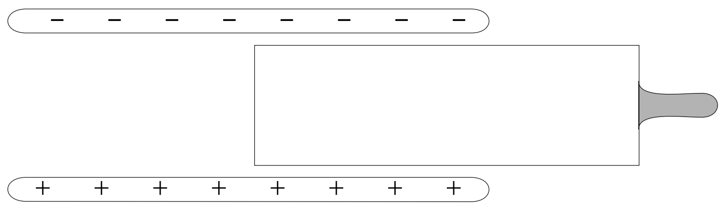
Briefly explain why Faraday could detect no electric effects inside his cube. *(Answer: 55)*

h-5 *SAFETY OF A CAR IN A THUNDERSTORM:* You may have heard that even if a car is struck by lightning (so that a very large amount of charge is placed on the car) the people inside are perfectly safe. (a) Is this true? Why or why not? *(Answer: 63)*

Forces due to Capacitors

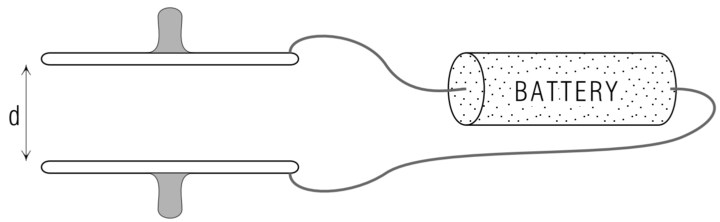
h-6 *FORCE ON A METAL SLAB:* The following diagram shows a metal slab held by an insulating handle and inserted half way into a charged capacitor. (a) Sketch + and − signs on the slab to indicate roughly its induced charge distribution.

►



(b) Is there a force on the slab due to the capacitor? If so does this force pull the slab inward or thrust it out? *(Answer: 61) (Suggestion: s-6)*

h-7 *SEPARATING PLATES CONNECTED TO A BATTERY:* The two parallel plates shown in the following diagram are supported on insulating handles and connected to a battery producing a potential difference V_0. (a) To move these plates so that they have a final separation of $2d$ (twice the separation shown) would you have to do positive or negative work on them?



(b) What is the final capacitance of the plates in terms of their initial capacitance C_0? (c) Write expressions in terms of V_0 and C_0 for the electric energy stored in these plates before and after they are separated. What is the increase or decrease in the potential energy of the plates as they are separated?

Because positive work is done on these plates you may wonder how their potential energy can *decrease*. Remember that the plates are connected to the battery. The potential of the battery does increase because it acquires energy from charged particles flowing through it as the plates partially discharge. (d) What is the magnitude of the charge flowing through the battery? (e) What is the electric work done by the plates on this charge? (This work is then stored as chemical energy in the battery).

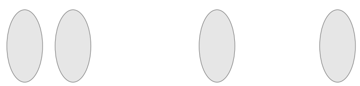
The total change in the energy of the system, including both the plates and the battery, is equal to the work done on this system by the forces separating the plates. (f) What is this work? *(Answer: 64)*

PRACTICE PROBLEMS

p-1 *DEFINITION OF CAPACITANCE: (CAP. 1A):* The charge transferred between two conductors is initially Q_0 and corresponds to a potential difference V_0 between these conductors. To transfer three times as much charge ($3Q_0$), what must be the potential difference between the conductors? *(Answer: 53) (Suggestion: Review text problems B-1, B-2, and B-3.)*

p-2 *DEFINITION OF CAPACITANCE: RELATING CHANGES (CAP. 1A):* Two conductors with capacitance 2.5×10^{-8} F initially have charges of $+Q_0$ and $-Q_0$. What must be the *change* in the potential difference between these conductors to cause an additional 2.0×10^{-6} C of charge to be transferred from one conductor to the other? *(Answer: 57) (Suggestion: Review text problems B-1, B-2, and B-3.)*

p-3 *RELATING CAPACITANCE TO GEOMETRIC PROPERTIES (CAP. 2):* (a) Which of the pairs of conductors shown in the following diagram has the larger capacitance?



(b) If the transferred charge Q is the same for each pair, for which pair is the potential difference V larger (or is V the same for both)? (c) If the potential difference V is the same for both pairs, for which pair is the transferred charge Q larger (or is Q the same for both)? *(Answer: 59) (Suggestion: Review text problems B-5 and B-6.)*

p-4 *RELATING QUANTITIES DESCRIBING CAPACITORS (CAP. 3):* Complete the following chart by writing *increases*, *decreases* or *same* to summarize how the magnitudes of various quantities change when the initially empty region between the capacitor plates is filled with a dielectric. We consider two cases: (a) each plate of the capacitor is insulated. (b) The two plates are connected to the terminals of a battery producing a constant potential difference.

►

	Plates insulated	*Plates connected to a battery*
Charge transferred (Q)		
Potential difference (V)		
Capacitance (C)		

(Answer: 51) (Suggestion: Review text problems D-1 and D-2.)

p-5 *UNDERSTANDING $U = QV/2$: DEPENDENCE (CAP. 1D):* (a) For a given capacitor if the transferred charge Q is made (1/3) as large ($Q_{new} = Q_{old}/3$), is the new potential difference between the plates (1/3) or (1/9) as large as the original potential difference? (b) Is the new stored energy (1/3) or (1/9) as large as the original stored energy? *(Answer: 65) (Suggestion: Review text problems E-1, E-2, and E-3.)*

SUGGESTIONS

s-1 *(Text problem G-4):* Use the definition Q/V of capacitance and the expression for the capacitance of a parallel plate capacitor in terms of the area and separation of its plates.

s-2 *(Text problem C-3):* Use both the definition of capacitance Q/V and the relation $C = A/(4\pi k_d)$ describing the capacitance of a parallel plate capacitor.

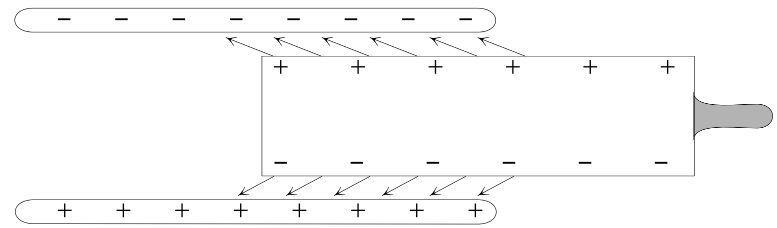
s-3 *(Text problem B-3):* The total charge Q transferred from one conductor to the other is always proportional to the potential difference V producing this charge transfer. Thus $Q = CV$ where C is a constant (for two given conductors). Therefore any *change* in Q is also proportional to the corresponding *change* in V i.e. $\Delta Q = C\Delta V$.

s-4 *(Tutorial frame [h-1]):* Because $C = AK/(4\pi k_e d)$, the capacitance can remain the same only if both A and d are increased or if both A and d are decreased.

The electric field $\vec{E}$ in the ceramic is related to the potential difference V by $E = V/d$ and to the charge of the plate by $E = 4\pi k_e \sigma/K$. ($\sigma = Q/A$ is the charge density of the plate.)

s-5 *(Text problem A-1):* In Fig. A-1b, if the magnitude of the transferred charge Q is now larger than before, then the electric field $\vec{E}$ at each point between the conductors is also larger than before. Then, because the potential drop along each small path segment is $V_1 - V_2 = ED_E$, the total potential drop from a to b must also be larger in magnitude.

s-6 *(Tutorial frame [h-6]):* The following diagram shows the charge distribution induced on the slab by the parallel plates. On each small region of the slab's charged surfaces there is a force directed roughly towards the oppositely charged metal plate.

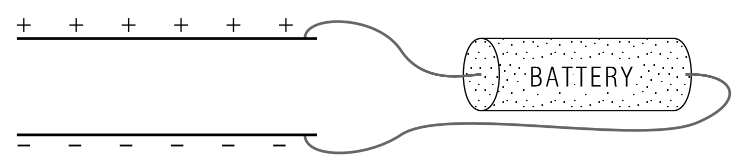


s-7 *(Text problem B-4):* Two objects which are fixed in position (relative to each other) always have the same capacitance. Thus the ratio $Q/V = C$ always has the same value (although both Q and V can change proportionately). When two objects move relative to each other their capacitance C ordinarily changes. A battery produces a constant potential difference between conductors connected to its terminals.

s-8 *(Text problem H-6):* The capacitance of the unfilled capacitor is $C_0 = Q_0/V_0$. Suppose that the plates are insulated so that the charge Q remains constant. When the dielectric is inserted the potential difference between the plates has then some new value V such that $C = Q_0/V$ is the capacitance of the partly filled capacitor.

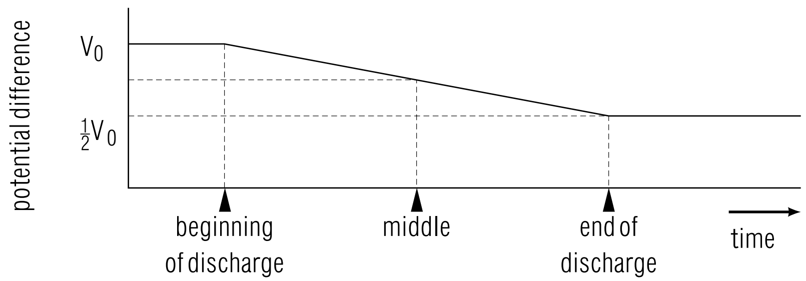
To relate C and C_0 recall from text section F of Unit 421 that the potential drop across a dielectric is $(1/K)$ times as large as the potential drop along the same path when the dielectric is not there. The potential difference between the top and bottom of the dielectric is thus $(1/K)(fV_0)$, while the potential difference between the top of the dielectric and the upper plate is just $(1-f)(V_0)$. The total potential difference between the plates is the sum of these two.

s-9 *(Text problem B-5):* (a) The following diagram shows two plates connected to a battery.



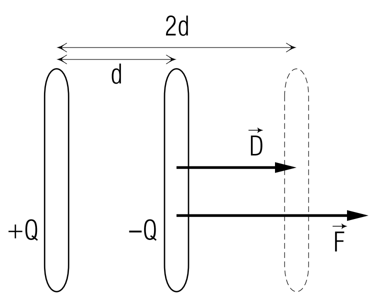
If the plates are moved closer together (while the potential difference remains constant), then the electric field between the plates must increase. But physically, this increased electric field must be produced by increased amounts of charge on each plate. Thus, as the plates move closer together, the transferred charge Q, and correspondingly the capacitance $C = Q/V$ increases.

s-10 *(Text problem E-4):* As the capacitor discharges, the stored charge decreases from Q_0 to $Q_0/2$. The potential difference correspondingly decreases from V_0 to $V_0/2$ as indicated in this graph:



As discussed in the text, while the charge flows from one plate to the other, the potential difference between the plates decreases. Thus the work done on the flowing charge is equal to this charge times the *average* (or middle) potential difference of the plates during the time the charge moves.

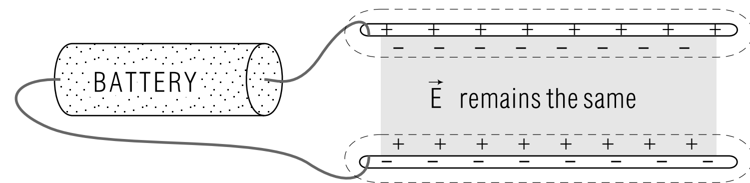
s-11 *(Text problem H-2):* (a) To move one plate away from the other you would have to exert on it a force $\vec{F}$ in the same direction as the displacement D so as to overcome the attractive force between the oppositely charged plates. Thus you would do positive work on the plates.



(b) Since the kinetic energy of the plates does not change the work done on the plates must result in an increase in their electric potential energy.

(c) The charges Q and $-Q$ on each plate remain the same. Therefore the electric field between the plates remains the same. Thus when the separation of the plates is increased from d to $2d$ the potential difference between then becomes twice as large (increases from ED to $E2D$).

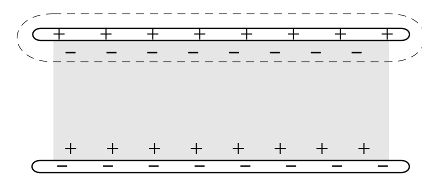
s-12 *(Text problem D-2):* Because the potential difference between the plates (and the distance between them) remain the same the electric field at a point between the plates must remain the same. But then the charges producing this field (i.e. in the regions indicated by dotted lines in the following diagram) must also remain the same.



As discussed in text section F of Unit 421 when a dielectric slab is placed in an electric field it acquires an induced charge distribution as indicated in the preceding diagram. Therefore if the charge in the regions indicated by dotted lines remains the same when this slab is inserted then the charges of the plates themselves must *increase* in magnitude.

(Note: If you found this problem or the preceding one difficult, [p-4] provides further practice.)

s-13 *(Text problem D-1):* Because the capacitor plates are insulated, the charge of each plate must remain the same. As discussed in text section F of Unit 421 when a dielectric slab is placed in the electric field due to these charged plates it acquires an induced charge distribution as indicated in this diagram:



Consider the total charge of the region indicated by a dotted line (including the upper plate and the top surface of the dielectric). This charge is smaller in magnitude than the original charge of the plate.

For this reason the electric field within the dielectric is smaller in magnitude than the electric field between the plates before the dielectric was inserted.

ANSWERS TO PROBLEMS

1. a. capacitance
   b. potential difference
   c. transferred charge

2.

	charge	potential difference	capacitance
symbol	Q	V	C
signs	$+, -, 0$	$+, -, 0$	$+$
magnitude	10^{-8} C	1 volt	10^{-8} F

3. larger, larger

4. 4.4×10^{-9} F

5. 1.5×10^{-6} C

6. a. increase
   b. decrease

7. a. 2×10^{2} volt
   b. twice as large

8. capacitance increases, potential difference decreases

9. 1×10^{-9} C

10. a. 6.7×10^{-10} farad
    b. $C = Q/V$

11. a. 4
    b. 1/3

12. 1×10^{9} meter^2, 1×10^{3} meter^2

13. a. C and Q increase
    b. C and Q decrease
    c. C and Q increase

14. a. $V_0/2$
    b. $(V_0 + V_0/2)/2 = 3V_0/4$
    c. $(Q_0/2)(3V_0/4) = 3Q_0V_0/8$
    d. $(Q_0V_0/2) - (3Q_0V_0/8) = Q_0V_0/8$

e. yes, $(1/2)(Q_0/2)(V_0/2) = Q_0V_0/8$

15. equipment inside grounded case is then unaffected by electric disturbances from outside.

16. (a) increases; (b), (c), and (d) remain the same; (e) increases.

17. a. 5 times as large

    b. 25 times as large

    c. yes, if V is 5 times as large, then U is 25 times as large

18. 3.5×10^{-9} meter or about 35 Å

19. (a) remains the same; (b), (c), and (d) decrease; (e) increases.

20. a. 1×10^{-4} F

    b. 4.5×10^5 watt

21. a. $V = k_eQ/R$

    b. large

    c. 14 volt

22. a. $C = KA/(4\pi k_eD)$

    b. 0.1 meter^2; 0.001 meter^2 = 10 cm^2

23. a. $V = k_eQ/R$

    b. 9×10^3 volt, 9 volt

    c. grounding to the stake, because a large charge could appreciably change the potential of the pipes, while it would not appreciably change the potential of the very large earth.

24. Q: 1/3 as large, C: the same, U: 1/9 as large

25. a. $U = CV^2/2$

    b. $U = Q^2/(2C)$

    c. 2.9×10^{-8} J

26. a. positive

    b. increase

    c. U = larger than initial U

27. 4

28. a. 5×10^{-7} C, 1×10^{-6} C/meter^2

b. 1×10^5 V/m, (2 or 3) $\times 10^4$ V/m

29. No, see problems D-1, D-2. When a dielectric is inserted, if V is constant, Q increases; if Q is constant, V decreases. Either way, $C = Q/V$ *increases*.

30. $C = C_0[K/(K - Kf + f)]$. If $f = 1$, $C = C_0|K|$

31. Yes, because battery *transfers* charge making one plate positive and the other negative; no.

32. a. 5.0×10^{-10} C

    b. 3.1×10^9

    c. 3.1×10^9

33. $V = 4\pi k_e dQ/A$

34. a. 10^6 J

    b. $U = (KAV^2)/(8\pi k_e d)$

    c. 2×10^4 meter^2

    d. 20 meter^3, larger than 12 meter^3 volume of car

51.

	Plates insulated	Plates connected to a battery
Q	same	increases
V	decreases	same
C	increases	increases

52. a. If no gaps, charge would flow from one plate to the other.

    b. Electric field within slab is zero. Thus charge producing this field is zero in each indicated region.

    c. Potential drop from 3 to 4 is zero (where field is zero).

    d. $(4\pi k_eQ/A)\hat{y}$

    e. $V = 4k_eQ(d_1 + d_2)/A$

    f. $C = A/|4\pi k_e(d_1 + d_2)|$

    g. same as (f)

53. $3V_0$

54. Increase both area and separation of plates by the same factor.

55. Electric potential is constant inside. Thus electric field (and hence all electric forces and their observable effects) are zero.

56. a. $V = k_e Q/R$, $Q = 1 \times 10^{-9}$ C
    b. potentials equal, charge of 2 has a charge nearly equal to Q and much larger than charge of 1.
    c. for both, $V = 0.09$ volt
    d. reduces it greatly in magnitude

57. 80 volt

58. mica

59. a. 1
    b. 2
    c. 1

60. a. $C = C_0/3$
    b. $E = E_0$
    c. $E = E_0/3$

61. left end of slab is positively charged towards the top, negatively charged towards the bottom

    b. inward

62. a. 1.8×10^{-7} J
    b. 15 volt, 6.0×10^{-9} C

63. a. Probably true. Charge delivered to car remains largely on the surface, so that ultimately the car and its interior have a constant potential, which produces no observable effects inside.

64. a. positive
    b. $C_0/2$
    c. $C_0V_0^2/2$, $C_0V_0^2/4$, *decrease* of $C_0V_0^2/4$
    d. C_0V_0, $C_0V_0/2$, $C_0V_0/2$
    e. (charge)(potential difference) $= (C_0V_0/2)(V_0) = C_0V_0^2/2$
    f. $C_0V_0^2/2 - C_0V_0^2/4 = C_0V_0^2/4$

65. a. (1/3) as large
    b. (1/9) as large

MODEL EXAM

USEFUL INFORMATION

$k_e = 9.0 \times 10^9$ newton meter^2/coulomb^2

THE QUESTIONS

1. **Relation between charge and potential difference in capacitors.** Two capacitors, C_1 and C_2, have the same capacitance, 2×10^{-6} F. C_1 and C_2 are charged in such a way that the charge transferred for capacitor C_2 is twice the charge transferred for capacitor C_1.

   Express the potential difference V_2 between the plates of C_2 as a number times the potential difference V_1 between the plates of C_1.

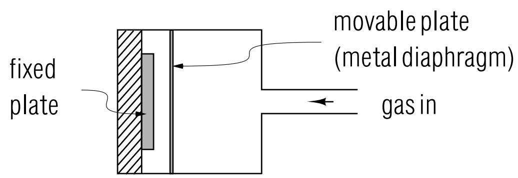
2. **Dielectric constant of an oil used in capacitors.** The capacitance of a parallel-plate capacitor is determined with air ($K = 1.0$) between the plates, and is found to be $C_{air} = 2.0 \times 10^{-10}$ F. When pyranol (an oil commonly used in capacitors as a dielectric material) is poured between the plates, the capacitance is found to be $C_{oil} = 1.06 \times 10^{-9}$ F.

   What is the dielectric constant of pyranol?

3. **Energy stored in a capacitor.** In a circuit designed to make "flashcubes" flash, a capacitor is charged so that 2.0×10^{-6} coulomb of charge is transferred from one plate to another. The potential difference between the plates is then 22 volt.

   What is the energy stored in the capacitor?

4. **Behavior of a capacitive pressure transducer.** In a capacitive pressure transducer, the pressure to be measured deflects a flexible metal diaphragm which forms one plate of a parallel-plate capacitor. See the diagram below. (None of the details in the diagram are crucial.)



When gas pressure slightly decreases the distance between the movable and fixed plates, does the capacitance of the transducer increase, decrease, or remain the same?

Brief Answers:

1. $V_2 = 2V_1$
2. 5.3
3. 2.2×10^{-5} joule
4. increase