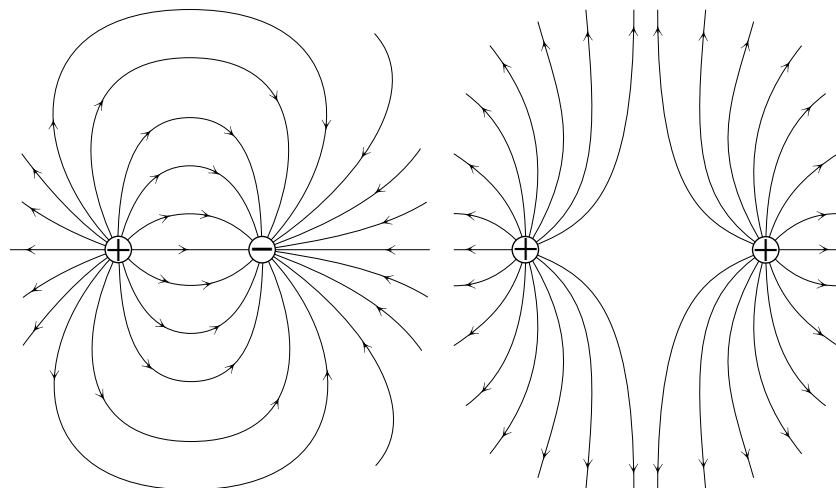


## THE ELECTRIC FIELD



## THE ELECTRIC FIELD

by

F. Reif, G. Brackett and J. Larkin, University of California at Berkeley

### CONTENTS

- A. Electric Force and Field
- B. Electric Field Produced by Charged Particles
- C. Electric Field Lines
- D. Effects of Electric Fields
- E. Electric Field Near a Charged Plane
- F. Electric Field due to a Charged Sphere
- G. Summary
- H. Problems
- I. Calculation of Field due to a Uniformly Charged Plane

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**Input Skills:**

1. State the superposition principle for the forces on a particle (MISN-0-408).
2. State Coulomb's force law (MISN-0-411).

**Output Skills (Knowledge):**

- K1. Vocabulary: electric field, electric field lines, surface charge density.
- K2. State an expression for the electric field due to a single charged particle.
- K3. State the superposition principle for electric fields due to several particles.
- K4. Describe the electric field: (a) near the center of a uniformly charged plane; (b) due to a uniformly charged sphere or spherical surface.
- K5. State three properties of electric field lines.

**Output Skills (Problem Solving):**

- S1. Given the electric fields at a point due to many charges, determine the total electric field there.
- S2. Given a drawing of electric field lines, determine or compare the directions of the electric forces on charged particles placed in the field.
- S3. Determine the electric force on an object consisting of fixed charged particles, given information about nearby charged particles or the electric field due to them.

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SECT.

# A

 ELECTRIC FORCE AND FIELD

## MISN-0-419

### THE ELECTRIC FIELD

- A. Electric Force and Field
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#### Abstract:

We shall begin our study of the interaction between charged particles by examining the implications of Coulomb's electric force law which describes the interaction between charged particles at rest. In particular, we shall use the present unit to introduce the concept of "electric field" which simplifies appreciably the process of finding the electric force on any charged particle due to other charged particles. Then we shall find the electric field (or correspondingly the electric force) due to several commonly occurring arrangements of charged particles.

### COULOMB ELECTRIC FORCE

As discussed in Unit 411, every particle can be characterized by a quantity called its charge  $q$ , measured in terms of the SI unit "coulomb" (abbreviated "C"). If two particles are at rest relative to some inertial frame, the electric force on one particle due to the other is called the "Coulomb electric force." This Coulomb force  $\vec{F}$  depends on the charges  $q$  and  $q_1$  of the interacting particles and on the distance  $R$  between them. In accordance with Coulomb's law discussed in text section B of Unit 411, the direction and magnitude of  $\vec{F}$  are such that

Coulomb electric force: $\vec{F}$ is: repulsive if charges have same signs, is: attractive if charges have opposite signs. is: $F = k_e \frac{ q  q_1 }{R^2}$ , where: $k_e \approx 9.0 \times 10^9 \text{ N m}^2/\text{C}^2$	(A-1)
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### DEPENDENCE OF ELECTRIC FORCE ON CHARGE

Consider some point  $P$  near one or more charged particles at specified positions. If some other particle is placed at  $P$ , how does the Coulomb electric force  $\vec{F}$  on this particle depend on the charge  $q$  of this particle? (See Fig. A-1.)

#### ► Point near single particle

Consider first the simple case of a point  $P$  near a single particle with charge  $q_1$ . Then the electric force  $\vec{F}$  on a particle with charge  $q$  placed at  $P$  is the Coulomb electric force on this particle due to the particle with charge  $q_1$ . How does this force  $\vec{F}$  depend on the charge  $q$  of the particle placed at  $P$ ? Suppose, for example, that the charge  $q$  of this particle were 3 times as large. Then the force  $\vec{F}$  on this particle would also be 3 times as large. Thus the ratio  $\vec{F}/q$  has always the same value, irrespective of the *magnitude* of the charge  $q$  of the particle at  $P$ . Similarly, suppose that the charge  $q$  of the particle at  $P$  had the opposite sign. Then the force  $\vec{F}$  on the particle would have the opposite direction. Thus the ratio

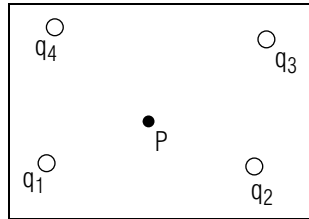


Fig. A-1: A point  $P$  near several charged particles.

$\vec{F}/q$  has always the same value, irrespective of the *sign* of the charge of the particle at  $P$ .

► *Point near several particles*

Consider now the case where a point  $P$  is near several particles, for example, near *two* particles with charges  $q_1$  and  $q_2$ . Then the electric force  $\vec{F}$  on a particle with charge  $q$  placed at  $P$  is, by the superposition principle, simply equal to

$$\vec{F} = \vec{F}_1 + \vec{F}_2 \quad (\text{A-2})$$

where  $\vec{F}_1$  is the electric force on this particle due to the particle with charge  $q_1$ , and  $\vec{F}_2$  is the electric force on this particle due to the particle with charge  $q_2$ . How then does this force  $\vec{F}$  depend on the charge  $q$  of the particle at  $P$ ? This question is readily answered since we already know from the preceding paragraph how each of the separate forces  $\vec{F}_1$  and  $\vec{F}_2$  depends on the charge  $q$ . Indeed, suppose that the charge  $q$  on the particle at  $P$  were 3 times as large. Then each of the forces  $\vec{F}_1$  and  $\vec{F}_2$ , and thus also the total force  $\vec{F}$  given by Eq. (A-2), would also be 3 times as large. Hence the ratio  $\vec{F}/q$  has always the same value, irrespective of the *magnitude* of the charge  $q$  of the particle at  $P$ . Similarly, suppose that the charge  $q$  of the particle at  $P$  had the opposite sign. Then each of the forces  $\vec{F}_1$  and  $\vec{F}_2$ , and thus also the total force  $\vec{F}$  given by Eq. (A-2), would have the opposite direction. Hence the ratio  $\vec{F}/q$  has always the same value, irrespective of the *sign* of the charge of the particle at  $P$ .

► *General conclusion*

Thus we arrive at this general conclusion: If a particle with charge  $q$  is placed at a point  $P$ , the electric force  $\vec{F}$  on this particle, due to any number of other charged particles at specified positions, is such that the ratio  $\vec{F}/q$  is *independent* of the magnitude and sign of  $q$ . (Accordingly, the force  $\vec{F}$  on the particle is said to be “proportional” to its charge  $q$ .)

## DEFINITION OF ELECTRIC FIELD

The preceding conclusion can be summarized by writing

$$\boxed{\frac{\vec{F}}{q} = \vec{E}} \quad (\text{A-3})$$

where the quantity  $\vec{E}$  is *independent* of the value of  $q$ . This quantity  $\vec{E}$  is called the electric field in accordance with this definition:

Def.	<p><b>Electric field:</b> The electric field <math>\vec{E}</math> at a point <math>P</math>, due to charged particles at specified positions other than <math>P</math>, is the vector <math>\vec{E} = \vec{F}/q</math> where <math>\vec{F}</math> is the electric force produced by these charged particles on <i>any</i> particle with charge <math>q</math> placed at <math>P</math>.</p>	(A-4)
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Thus the electric field at a point is a quantity describing the relation between the electric force and the charge on any particle at this point. According to its definition  $\vec{E} = \vec{F}/q$ , the electric field is a *vector* which has the units of newton/coulomb.

Note that, although the electric field  $\vec{E} = \vec{F}/q$  at a point  $P$  does *not* depend on the charge  $q$  of the particular particle placed at  $P$ , it *does* depend on the point  $P$  as well as on the charges and positions of all the other charged particles producing the force  $\vec{F}$ .

► *Finding  $\vec{E}$*

By its definition  $\vec{E} = \vec{F}/q$ , the electric field  $\vec{E}$  at a point  $P$  is simply the electric force “per unit charge” (i.e., the electric force divided by the corresponding charge) of any particle placed at  $P$ . To find the electric field at a point  $P$ , we need thus merely consider *any* convenient charged particle at  $P$ , find the electric force  $\vec{F}$  on this particle, and then divide  $\vec{F}$  by the charge  $q$  of this particle.

In particular, to *calculate* the electric field  $\vec{E}$  at a point  $P$ , we need only *imagine* placing at  $P$  a particle having some convenient charge  $q$ . (For example, we might choose  $q$  to be simply +1 coulomb.) Then we can calculate the electric force  $\vec{F}$  on this particle to find the electric field  $\vec{E} = \vec{F}/q$  at  $P$ .

To *measure* the electric field  $\vec{E}$  at a point  $P$ , we must *actually* place at  $P$  a particle having some convenient charge  $q$ . Then we can measure the electric force  $\vec{F}$  on this particle to find the electric field  $\vec{E} = \vec{F}/q$  at  $P$ . (In this actual situation the charge  $q$  of the particle placed at  $P$  should be

sufficiently small so that it exerts negligible forces on all the other charged particles and thus does not cause these to move appreciably from their original positions.)

### Example A-1: Measurement of electric field

Suppose that a small plastic ball, having a charge of  $-10^{-8}$  C, experiences an electric force of  $10^{-2}$  N in the  $\hat{x}$  direction when it is placed at a point  $P$  near a charged rubber rod. Then the electric field  $\vec{E}$  at  $P$  is  $\vec{E} = \vec{F}/q = (10^{-2} \text{ N}\hat{x})/(-10^{-8} \text{ C}) = -(10^6 \text{ N/C})\hat{x}$ , or  $10^6 \text{ N/C}$  opposite to the  $\hat{x}$  direction.

#### ► Finding force from $\vec{E}$

Suppose that we know the electric field  $\vec{E}$  at a point  $P$  from information about the electric force on some particular convenient particle (with charge  $q'$ ) placed at  $P$ . Then we can use this knowledge of the electric field  $\vec{E}$  to find immediately the electric force  $\vec{F}$  on *any other* particle with charge  $q$  placed at  $P$ . Indeed, the ratio of electric force divided by charge is the same for *any* particle at  $P$  and just equal to the electric field at  $P$ . Thus  $\vec{F}/q = \vec{E}$  or

$$\vec{F} = q\vec{E} \quad (\text{A-5})$$

Thus the electric force  $\vec{F}$  on *any* particle at a point  $P$  (due to other charged particles at specified positions) is simply found by multiplying the charge  $q$  of this particle by the electric field  $\vec{E}$  at  $P$ . Note that the force  $\vec{F}$  has the same direction as  $\vec{E}$  if the charge  $q$  of the particle is positive, but has a direction opposite to  $\vec{E}$  if the charge  $q$  is negative.

### REMARK: ANALOGOUS USE OF PROPORTIONALITY IN DAILY LIFE

The electric field  $\vec{E} = \vec{F}/q$  is introduced as a useful quantity describing the fact that the electric force on a particle is proportional to its charge. Analogous quantities are commonly used in everyday life. For example, the cost  $C$  of gasoline is proportional to the amount  $A$  of gasoline purchased, i.e., the ratio  $C/A$  is *independent* of the amount  $A$  purchased. Hence it is convenient to introduce the quantity  $C/A = U$  which is independent of the amount and is called the “unit cost” (e.g., the cost per gallon). Once one knows this unit cost, one can immediately find the cost of *any* amount of gas by using the fact that  $C = AU$ , i.e., by simply multiplying the amount purchased by the unit cost.

Note that the unit cost  $U = C/A$  depends on the particular station where gasoline is purchased (just as the electric field  $\vec{E} = \vec{F}/q$  depends on the particular point under consideration). Note also that the unit cost  $U$  of gasoline at a station is the same irrespective of whether gasoline is actually purchased at this station (just as the electric field at a point is the same irrespective of whether a charged particle is actually located at this point).

### Understanding the Definition of Electric Field (Cap. 1a)

**A-1** *Statement and example:* In order to find the electric field  $\vec{E}$  at a point  $P$ , imagine a particle which is at the point  $P$ , which has a charge of  $q_a$ , and which is acted on by an electric force  $\vec{F}_a$ . (a) Use the symbols provided to state the definition of the electric field at  $P$ . (b) If  $q_a = 3.0 \times 10^{-8}$  coulomb and  $\vec{F}_a = 4.5 \times 10^{-3} \text{ N}\hat{x}$ , what is the value of  $\vec{E}$  at  $P$ ? (c) Now imagine at the point  $P$  a different particle of charge  $q_b = -q_a = -3.0 \times 10^{-8}$  C. The electric force on this particle is then  $\vec{F}_b = -4.5 \times 10^{-3} \text{ N}\hat{x}$ . What now is the value of  $\vec{E}$  at  $P$ ? (d) Does the value of  $\vec{E}$  at  $P$  depend on whether the particle imagined to be at  $P$  has a positive or a negative charge? (*Answer: 6*)

*Now: Go to tutorial section A.*

**A-2** *Comparing electric field with electric force:* At a point  $P$ , the electric field has the value  $\vec{E} = 3.0 \times 10^6 \text{ N/C}$  toward the right. (a) Imagine a particle with charge 1.0 coulomb at the point  $P$ , and compare  $\vec{E}$  with the force  $\vec{F}$  on this particle. Are the magnitudes of  $\vec{E}$  and  $\vec{F}$  the same? Are their directions the same? (b) Now imagine a particle of charge  $-1.0$  C at the point  $P$ . Compare the electric field  $\vec{E}$  at  $P$  with the force  $\vec{F}$  on this particle by answering the questions from part (a). (*Answer: 1*)

**A-3** *Relating quantities and dependence:* (a) What is the electric force  $\vec{F}_A$  on a particle  $A$  with a charge  $1.2 \times 10^{-8}$  C located at a point  $P$  where the electric field  $\vec{E} = 2.0 \times 10^6 \text{ N/C}$  toward the left? (b) Suppose we replace the particle  $A$  at  $P$  with a new particle  $B$  of charge  $-2.4 \times 10^{-8}$  C. Is the electric field at  $P$  equal to the value for  $\vec{E}$  given in part (a)? If not, what is the new value for  $\vec{E}$ ? Is the electric force  $\vec{F}_B$  on the new particle equal to the value for  $\vec{F}_A$  found in part (a)? If not, what is  $\vec{F}_B$ ? (c) If there is *no* particle at the point  $P$ , what then is the value of the electric field at  $P$ ? (d) Does the value of  $\vec{E}$  at  $P$  depend on the charge

of any particle which may be at  $P$ ? Does the value of the electric force on a particle at  $P$  depend on the charge of this particle? (*Answer: 13*) (*Suggestion: [s-14]*)

### Relating Proportional Quantities and Their Ratios (Cap. 2)

**A-4** At two garden supply stores, the price  $P$  charged for topsoil is proportional to the volume  $V$  of topsoil bought. (a) If George pays 120 dollars for 8 meter<sup>3</sup> of topsoil at one store  $A$ , what is the “price per unit volume” (i.e., the ratio  $R = P/V$ )? (b) What must Fred pay to buy 6 meter<sup>3</sup> of topsoil from this same store  $A$ ? (c) Consider the ratio  $R = P/V$  paid by Jill if she buys topsoil at the same store  $A$  or at a different store  $B$ . In each case, is the value of  $R$  for Jill *the same*, *different*, or *either the same or different* than the value of  $R$  found for George in part (a)? (*Answer: 10*) (*Suggestion: [s-12]*)

**A-5** (a) At a point  $A$ , a particle of charge  $2.0 \times 10^{-8}$  C is acted on by an electric force of  $-2.4 \times 10^{-2}$  N  $\hat{y}$ . What is the electric field  $\vec{E} = \vec{F}/q$  at  $A$ ? (b) What is the electric force on a particle with charge  $-3.0 \times 10^{-8}$  C if it is placed at  $A$ ? (c) Consider the ratio  $\vec{E} = \vec{F}/q$  for a third charged particle which can be placed at the point  $A$  or at a different point  $B$ . In each case compare this value of  $\vec{E}$  with the value found in part (a) by saying if these values are the *same*, *different*, or might be *either the same or different*. (*Answer: 2*) (*Suggestion: [s-9]*) (*Practice: [p-1]*)

SECT.

## **B** ELECTRIC FIELD PRODUCED BY CHARGED PARTICLES

The electric field produced at any point by one or more charged particles can be readily calculated by using the definition of the electric field and Coulomb’s electric force law.

### FIELD DUE TO A SINGLE CHARGED PARTICLE

Consider a single particle with charge  $q_1$ . What is the electric field  $\vec{E}$  at a point  $P$  at a distance  $R$  from this charged particle? (See Fig. B-1.)

To find the electric field at  $P$ , we need only imagine that any conveniently chosen particle (for example, a particle with some positive charge  $q$ ) is placed at the point  $P$ . The electric force  $\vec{F}$  on this particle is then the Coulomb force due to the particle with charge  $q_1$ .

#### ► Direction of $\vec{E}$

The electric force  $\vec{F}$  on the positively charged particle at  $P$  is repulsive (i.e., directed *away* from the particle with charge  $q_1$ ) if  $q_1$  is positive; but this force  $\vec{F}$  is attractive (i.e., directed *toward* the particle with charge  $q_1$ ) if  $q_1$  is negative. But the electric field  $\vec{E} = \vec{F}/q$  has the same direction as  $\vec{F}$  for our positive charge  $q$ . Hence we arrive at this conclusion about the direction of the electric field  $\vec{E}$ :

Direction of $\vec{E}$ : away from $q_1$ if $q_1$ is positive, toward $q_1$ if $q_1$ is negative.	(B-1)
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#### ► Magnitude of $\vec{E}$

By Rule (A-1), the Coulomb electric force on the particle at  $P$  has the magnitude  $F = k_e q|q_1|/R^2$  since  $|q| = q$  for our positive charge  $q$ . Correspondingly, the magnitude of the electric field at  $P$  is equal to  $E = F/q$  so that

$E = k_e \frac{ q_1 }{R^2}$	(B-2)
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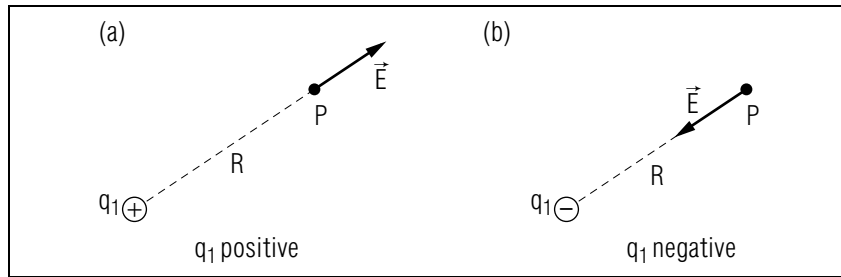


Fig. B-1: Electric field due to a single charged particle.

**Example B-1: Force on a particle at  $P$** 

Suppose that a particle with a *negative* charge  $q'$  is placed at  $P$ . Then the electric force  $\vec{F}'$  on this particle is equal to  $\vec{F}' = q'\vec{E}$ , so that  $\vec{F}'$  is *opposite* to the electric field  $\vec{E}$  (since  $q'$  is negative). According to Rule (B-1),  $\vec{F}'$  is thus directed *toward* the particle with charge  $q'$  if  $q_1$  is positive, and is directed *away* from this particle if  $q_1$  is negative. Also, by Eq. (B-2),  $|\vec{F}'| = |q'|\vec{E}| = k_e|q'||q_1|/R^2$ . These conclusions are, of course, consistent with those obtained by using Coulomb's law directly to calculate the electric force  $\vec{F}'$  on the particle with negative charge  $q'$  due to the particle with charge  $q_1$ .

**FIELD DUE TO SEVERAL CHARGED PARTICLES**

Consider several charged particles, e.g., two particles 1 and 2, with charges  $q_1$  and  $q_2$ , located at specified positions. What then is the electric field  $\vec{E}$  at some point  $P$  due to these particles? (See Fig. B-2a.)

**► Superposition of fields**

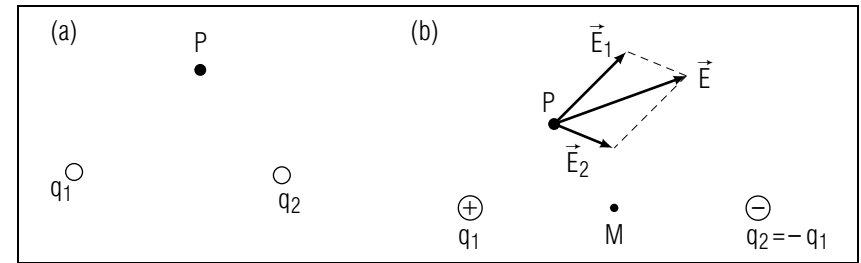
To find the electric field at  $P$ , we again imagine that any conveniently chosen particle with charge  $q$  is placed at  $P$ . By the superposition principle, the electric force  $\vec{F}$  on this particle is then

$$\vec{F} = \vec{F}_1 + \vec{F}_2 \quad (\text{B-3})$$

where  $\vec{F}_1$  is the force due to particle 1 and  $\vec{F}_2$  is the force due to particle 2. But the electric field at  $P$  is, by its definition, simply equal to  $\vec{E} = \vec{F}/q$ . Hence Eq. (B-3) implies that  $\vec{E} = \vec{F}/q = \vec{F}_1/q + \vec{F}_2/q$ , or

$$\vec{E} = \vec{E}_1 + \vec{E}_2 \quad (\text{B-4})$$

Thus we see that the superposition principle Eq. (B-3) for forces implies

Fig. B-2: Electric field due to two charged particles. (a) Point  $P$  near two particles. (b) Electric field at points near a dipole.

this superposition principle for electric fields:

Superposition principle for fields: The electric field at a point due to any number of charged particles is the vector sum of the electric fields produced at this point by all these particles separately. (B-5)

By Rule (B-1) and Eq. (B-2), we already know how to find the electric field produced at a point  $P$  by any single charged particle separately. Hence Rule (B-5) tells us that we need simply add these fields vectorially in order to find the electric field produced at  $P$  by any number of such charged particles present simultaneously.

**Example B-2: Electric field due to a “dipole”**

A system of two particles, having charges of equal magnitudes and opposite signs, is called an “electric dipole.” Fig. B-2b shows such a dipole consisting of two particles 1 and 2 having a positive charge  $q_1$  and a negative charge  $q_2 = -q_1$ .

At the point  $M$  midway between the two particles, the electric field  $\vec{E}_1$  due to the charge  $q_1$  has the same magnitude as the electric field  $\vec{E}_2$  due to the charge  $q_2$ . Furthermore,  $\vec{E}_1$  is directed away from the positive charge  $q_1$  and  $\vec{E}_2$  is directed toward the negative charge  $q_2$ , so that both  $\vec{E}_1$  and  $\vec{E}_2$  have the same direction toward the right. Hence the electric field at  $M$  is  $\vec{E} = \vec{E}_1 + \vec{E}_2 = 2\vec{E}_1$  and is directed toward the right.

The particular point  $P$ , indicated in Fig. B-2b, is closer to particle 1 than to particle 2. Hence the magnitude of the electric field  $\vec{E}_1$  produced at  $P$  by the positive charge  $q_1$  is larger than the magnitude of the electric

field produced at  $P$  by the negative charge  $q_2 = -q_1$ . The electric field  $\vec{E}$  at  $P$  is then the vector sum  $\vec{E} = \vec{E}_1 + \vec{E}_2$  where  $\vec{E}_1$  is larger than  $\vec{E}_2$ .

### UTILITY OF THE ELECTRIC-FIELD CONCEPT

#### ► *Methods of finding electric force*

Consider a particle with charge  $q$  at a point  $P$ . In order to find the electric force  $\vec{F}$  exerted on this particle by other charged particles, one can either (1) calculate this force directly by applying Coulomb's law, or (2) first calculate the electric field  $\vec{E}$  produced at  $P$  by the other particles, and then use  $\vec{F} = q\vec{E}$  to find the desired force  $\vec{F}$ . These two equivalent methods of calculation correspond to slightly different descriptions of the interaction between the particle at  $P$  and the other charged particles. According to the first description, the other charged particles produce *directly* the electric force on the particle at  $P$ . According to the second description, the other charged particles produce at  $P$  an electric field  $\vec{E}$  which, in turn, produces the electric force  $\vec{F}$  on the particle at  $P$ . This second description thus views the interaction as occurring *indirectly* via the electric field  $\vec{E}$  as an intermediary.

#### ► *Advantage of using fields*

What is the advantage of this second point of view which introduces the electric field as a convenient auxiliary concept? The virtue is that the problem of describing the interaction between charged particles is thus decomposed into two separate parts - one part describing the production of the electric field and the other part describing the production of the force by this field. Hence there is a gain in simplicity since it is usually easier to break a complex problem into separate parts than to deal with the entire problem at once. Furthermore, it becomes easier to make general predictions. (For example, once one has completed the first part of the problem and found the electric field at a point  $P$ , one can immediately find the electric force on any charged particle placed at  $P$ .)

The preceding advantages of introducing the electric field are relatively minor in the simple situations which we have considered up to now. But, when we shall discuss more complex interactions between charged particles, the use of "fields" to describe the interaction in two stages becomes almost essential and leads to enormous advantages.

### Understanding the Field due to a Charged Particle (Cap. 1b)

**B-1** *Statement and example:* (a) State the expression for the magnitude of the electric field  $\vec{E}$  produced at a distance  $R$  from a particle of charge  $q$ . (b) What is the electric field at the point  $P$  due to the particle shown in Fig. B-3? (*Answer: 7*)

**B-2** *Interpretation of  $\vec{E}$ :* (a) Make a sketch showing an isolated positively charged particle and five points each 2 cm from this particle. Beginning at each point, draw an arrow indicating the direction of the electric field at that point. (b) Make a sketch, like the one described in part (a), for a negatively charged particle. (*Answer: 4*)

**B-3** *Comparing electric field and force:* (a) Use the definition  $\vec{E} = \vec{F}/q$  and your answer to problem B-1 to find the electric force on a particle of charge  $-2 \times 10^{-7}$  C at the point  $P$  in Fig. B-3. Find this same force by using Coulomb's law  $F = k_e q_1 q_2 / R^2$ . (You should get the same value from both calculations.) (b) Which of the following phrases describes the electric field produced by one particle and which describes the electric force exerted on one particle by another particle? (i) Can be found at a point where there is no particle. (ii) Depends on the charges of two particles. (iii) Depends on the charge of just one particle. (*Answer: 16*)

**B-4** *Relating quantities:* At a distance of  $10 \text{ \AA}$  ( $1.0 \times 10^{-9}$  meter) from an ionized atom of copper, the electric field due to this ion has a magnitude of  $2.88 \times 10^9$  N/C and a direction away from the ion. What is the charge of this ion? Is the ion  $\text{Cu}^+$  (which has a charge of  $e = 1.6 \times 10^{-19}$  C) or  $\text{Cu}^{++}$  which has a charge of  $2e$ ? (*Answer: 12*) (*Practice: [p-2]*)

### Describing the Electric Field due to Several Particles (Cap. 3)

**B-5** A particle of charge  $-2 \times 10^{-8}$  C is located at a point  $P$  on the line joining two other charged particles (1 and 2). At  $P$  the electric field due to particle 1 is  $4 \times 10^6$  N/C  $\hat{x}$  and the electric field due to particle 2 is  $-3 \times 10^6$  N/C  $\hat{x}$ . (a) What are the forces  $\vec{F}_1$  and  $\vec{F}_2$  on the particle at  $P$  due to particle 1 and due to particle 2? (b) What is the total force  $\vec{F}$  on the particle at  $P$  due to both particles 1 and 2? (c) What is the electric field  $\vec{E}$  at  $P$  due to particles 1 and 2? (d) Use your answer to part (c) and the definition  $\vec{E} = \vec{F}/q$  to find the force on the particle at  $P$



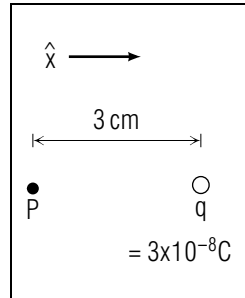
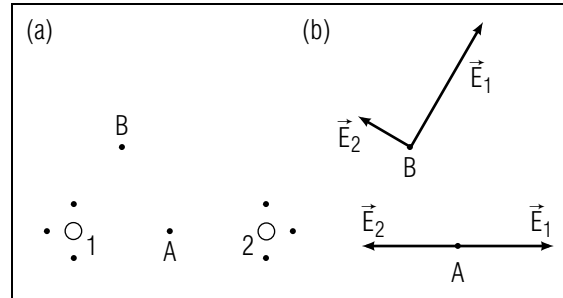


Fig. B-3.

Fig. B-4: At  $A$ :  $E_1 = E_2 = 14 \times 10^3 \text{ N/C}$ . At  $B$ :  $E_1 = 12 \times 10^3 \text{ N/C}$ ,  $E_2 = 5 \times 10^3 \text{ N/C}$ .

due to both particles 1 and 2. Does this value for  $\vec{F}$  check with the value found in part (b)? (*Answer: 3*)

**B-6** Figure B-4a shows two particles (1 and 2) each of charge  $4 \times 10^{-9} \text{ C}$ . At the points  $A$  and  $B$ , the electric fields  $\vec{E}_1$  and  $\vec{E}_2$  due to particle 1 and to particle 2 have the values indicated in Fig. B-4b. (a) What is the magnitude of the total electric field  $\vec{E}$  due to particles 1 and 2 at the point  $A$  and at the point  $B$ ? Describe the direction of each field. (b) Unlabeled small dots in Fig. B-4a indicate six points near the charged particles. At which of these points is the electric field due to particle 1 much larger than the electric field due to particle 2, i.e., at which points is the total electric field  $\vec{E}$  nearly equal to the field  $\vec{E}_1$ ? At which of these points is  $\vec{E}$  nearly equal to  $\vec{E}_2$ ? At which points is  $\vec{E}$  equal to the sum of two vectors  $\vec{E}_1$  and  $\vec{E}_2$  of approximately equal magnitude? (c) On a sketch like that shown in Fig. B-4a, draw an arrow from each small dot indicating the approximate direction of the field  $\vec{E}$  at that point. (*Answer: 15*) (*Suggestion: [s-17]*) (*Practice: [p-3]*)

SECT.

## C ELECTRIC FIELD LINES

A knowledge of the electric field in the vicinity of charged particles is very useful since it allows one to find immediately the electric force on any other charged particle and thus to predict how it will move. Hence it is helpful if the information about the electric field near charged particles can be described in a way which can be simply visualized.

To achieve such a visual description, let us focus attention on the *direction* of the electric field at every point near some specified charged particles. For example, as indicated in Fig. C-1, we consider some point  $A$  and draw from this point a very short arrow in the direction of the electric field at  $A$ . Then we consider the neighboring point  $B$  at the tip of this arrow and draw from  $B$  another very short arrow in the direction of the electric field at  $B$ . Then we consider the neighboring point  $C$  at the tip of this last arrow and draw from  $C$  another very short arrow in the direction of the electric field at  $C$ . By continuing in this way, we draw a succession of very short arrows and thus generate a line which is called an “electric field line” in accordance with this definition:

Def.	<b>Electric field line:</b> A line every small segment of which is directed along the electric field at the position of this segment.	(C-1)
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After drawing the field line passing through  $A$ , we can draw a different field line by simply starting at another point which is not on the field line already drawn. Thus we can draw a whole *set* of electric field lines. Such a diagram of field lines then allows us to visualize immediately the direction of the electric field at any point in the vicinity of the charged particles producing the field.

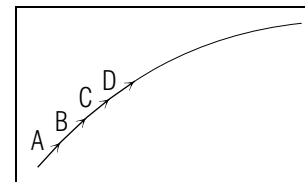


Fig. C-1: Generation of an electric field line by a succession of short arrows along the field.

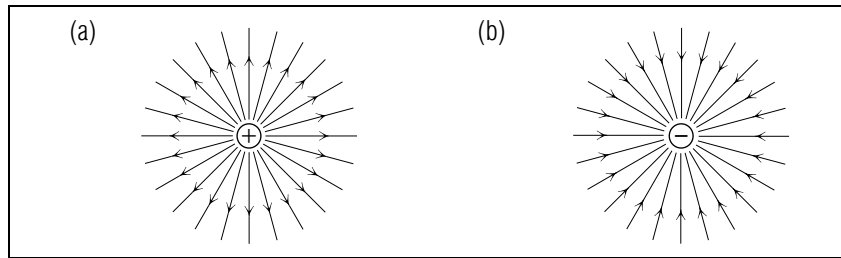


Fig. C-2: Electric field lines due to a single charged particle. (a) Positively charged particle. (b) Negatively charged particle.

### EXAMPLES OF ELECTRIC FIELD LINES

#### ► *Single particle*

Figure C-2 shows electric field lines in the vicinity of a single charged particle. In accordance with Rule (B-1), the electric field lines point everywhere away from a positively charged particle, but point everywhere toward a negatively charged particle.

#### ► *Two particles*

Figure C-3a shows the electric field lines near two particles having equal positive charges. Fig. C-3b shows the electric field lines near two particles having charges of the same magnitude, but of opposite signs (i.e., near an “electric dipole”). In these diagrams the direction of an electric field line at any point is the direction of the total electric field  $\vec{E}$  at that point, i.e., the direction of the vector sum  $\vec{E} = \vec{E}_1 + \vec{E}_2$  of the electric fields produced at this point by each of the charged particles.

Note that, at a point very near any charged particle, the electric field produced by this particle is much larger than the electric field produced by more distant particles. Hence the electric field at such a point is predominantly due to the nearby particle.

Correspondingly, the electric field lines very near this particle are nearly the same as if they were produced by this single particle alone

#### ► *Electric fish*

Figure C-4 shows the electric field lines in the vicinity of the fish *Gymnarchus niloticus*. Biochemical processes in this fish result in a separation of some of the positively and negatively charged particles in the fish and thus in the production of an electric field in the vicinity of the fish. (This field is thus similar to that in Fig. C-3b.) The fish has sense

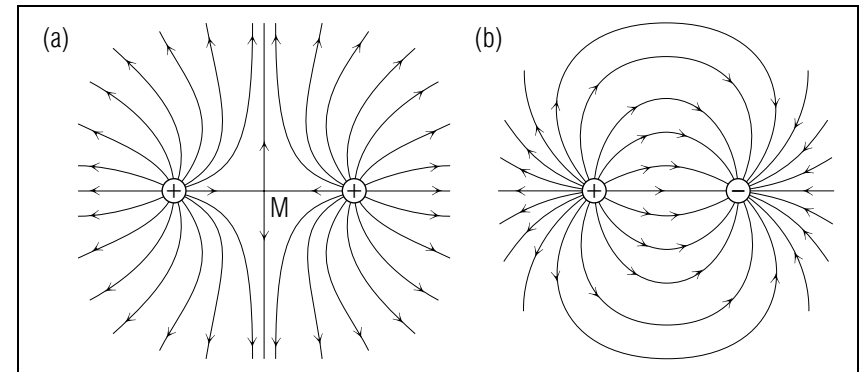


Fig. C-3: Electric field lines due to two particles having charges of the same magnitude. (a) Both charges positive. (b) Charges of opposite signs.

organs which can detect small changes in this electric field, e.g., changes produced by nearby prey or other objects. Thus this fish obtains most information about the outside world not through its fairly poor eyesight, but predominantly through its electric sense organs.

### PROPERTIES OF FIELD LINES

#### ► *No intersection*

Can two electric field lines ever intersect (i.e., cross each other)? To answer this question, suppose that two field lines  $L_1$  and  $L_2$  did intersect, as indicated in Fig. C-5. Then the electric field at  $P$  should have a direction along the line  $L_1$  (as indicated by  $\vec{E}_1$ ), but should also have

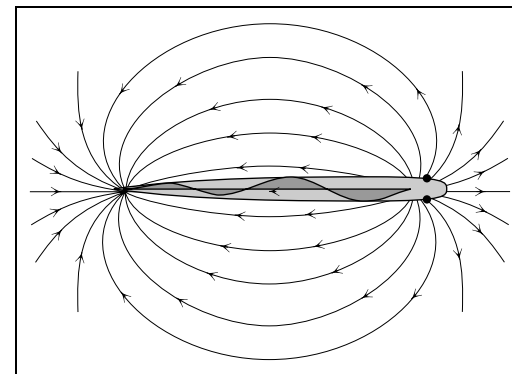


Fig. C-4: Electric field lines in the vicinity of the electric fish *Gymnarchus niloticus*. [Diagram from H. W. Lissmann, *Scientific American*, vol. 208, p. 53 (March 1963).]

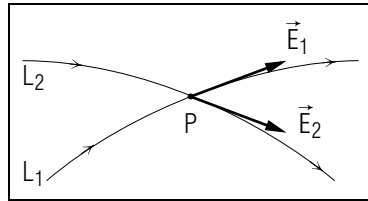


Fig. C-5: Hypothetical intersection of two electric field lines.

a direction along the line  $L_2$  (as indicated by  $\vec{E}_2$ ). But these conclusions are inconsistent with the fact that the electric field  $\vec{E}$  at any point has a unique magnitude and direction (unless  $\vec{E} = 0$ , when the field has no direction). Accordingly we see that two electric field lines *cannot* intersect except at a point where the electric field is zero. (For example, in Fig. C-3a the electric field lines intersect only at the point M where the electric field is zero.)

► *General properties*

We mention, without formal proof, the general properties of electric field lines, properties illustrated by the field lines in the diagrams of Fig. C-2 and Fig. C-3. (a) Electric field lines neither intersect (except where  $\vec{E} = 0$ ), nor do they form closed curves. (b) Electric field lines always start and end on charged particles (or very far away). (c) The magnitude of the electric field is larger where the spacing between adjacent field lines is smaller (e.g., close to the charged particles in Fig. C-2 and Fig. C-3); conversely its magnitude is smaller where the spacing between adjacent field lines is larger.

**Using Electric Field Lines to Describe Electric Forces (Cap. 4)**

**C-1** Figure C-6 shows the electric field lines describing the electric field due to charged particles on the membrane of an active nerve

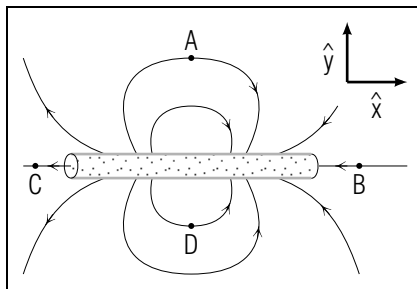


Fig. C-6.

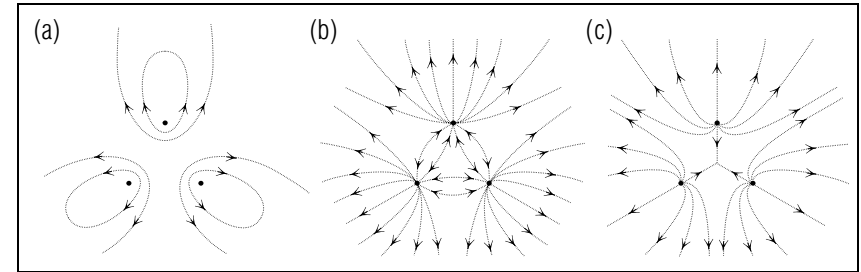


Fig. C-7.

cell. [This electric field causes nearby charged particles to move, part of the process through which an impulse propagates along the cell.] At which of the points  $A$ ,  $B$ ,  $C$ , and  $D$  is the electric field directed toward the right, and at which of the points toward the left? (*Answer: 9*)

**C-2** (a) What is the direction of the electric force on a positively charged ion at the point  $A$  in Fig. C-6? Suppose this ion is initially at rest. In what direction would it move because of this electric force? (b) Answer the preceding questions for a negatively charged ion. (*Answer: 14*)

**Knowing About Electric Field Lines**

**C-3** Which of the diagrams in Fig. C-7 best indicates the electric field due to three particles with equal positive charges which are located at the corners of an equilateral triangle? (*Answer: 5*) (*Suggestion: [s-15]*)

SECT.

## **D** EFFECTS OF ELECTRIC FIELDS

Electric fields can produce important observable effects, especially when the fields are large. All such effects are due to the fact that charged particles in the presence of an electric field experience an electric force and are thus accelerated.

► *Electric breakdown*

For example, suppose that a gas (such as air) is in the presence of a large electric field. In such a gas of uncharged molecules there is almost always at least one free electron.\*

\* Such an electron may result, for example, from the disruption (i.e., “ionization”) of a gas molecule by the highly energetic particles (called “cosmic rays”) which constantly arrive at the earth from outer space. Thus it is highly likely that at least one gas molecule, out of the about  $10^{20}$  molecules in a cubic centimeter of an ordinary gas, is ionized at any time.

Since the electron is acted on by the electric force due to the electric field, the electron is accelerated so that its speed, and correspondingly its kinetic energy, increases. If the electric field is sufficiently large, the electron attains so large a kinetic energy before encountering a molecule in the gas that it can disrupt (i.e., ionize) this molecule, thus forming another electron and a positively charged ion. As a result, there are now 2 free electrons in the gas, the original one as well as the one produced by the ionization of the molecule. But *each* of these 2 electrons is acted on by the large electric field and thus attains a sufficiently large kinetic energy to produce another free electron by ionizing a gas molecule. As a result, there are now 4 electrons in the gas. Each of these is again acted on by the electric field so that it attains a sufficiently large kinetic energy to produce another free electron by ionizing a gas molecule. As a result, there are now 8 free electrons in the gas. As the process goes on, the number of ionized molecules, and of newly freed electrons, continues to be doubled. Thus this process constitutes a chain reaction which results in a very rapid ionization of many molecules. Such a rapid ionization of a substance by a large electric field is called “electric breakdown” of the substance. \*

\* The positive ions resulting from the disruption of the molecules are also accelerated by the large electric field and can also contribute to the ionization of molecules.

The chain reaction produced by the large electric field does not lead to complete ionization of all the molecules, since the liberated free electrons and residual positive ions also tend to recombine to form neutral molecules. In this recombination process some energy is released in the form of light. Hence the electric breakdown of a substance is usually accompanied by visible effects, such as sparks. (Lightning produced in a thunderstorm is a spectacular example of such a spark produced by the electric breakdown of air in the atmosphere.) The familiar neon sign used in advertising displays is another familiar example where light is produced by applying a large electric field to cause the electric breakdown of neon gas in a glass tube.

► *Breakdown strength*

The electric “breakdown strength” of a substance is the maximum magnitude of the electric field in which the substance can be placed without becoming ionized by the field. The breakdown strength of dry air at room temperature and atmospheric pressure is about  $3 \times 10^6$  newton/coulomb. The breakdown strength of liquids and solids is considerably higher.

► *Producing high-energy particles*

If a large electric field is applied to charged particles in a vacuum, the particles can attain very high kinetic energies (since the particles then don’t lose their energy by collisions with any molecules). Such energetic charged particles can then be used to bombard various materials. To mention just a few practical applications, highly energetic protons or electrons can be used to bombard and thus destroy cancerous cells, or to bombard foods in order to sterilize them. Similarly, when highly energetic electrons are made to bombard a material with massive nuclei (such as tungsten), the electrons are stopped very rapidly in the material and produce X-rays as a result of their rapid deceleration. The X-ray machines used so extensively in medicine all use large electric fields to accelerate electrons and thus to produce X-rays in the manner just described.

► *Producing large fields*

In principle, one of the simplest ways of producing large electric fields is to accumulate a large amount of charge on an object. A practical device for producing large electric fields in this way is the “Van de Graaff generator” (so-called in honor of its inventor). In this device (illustrated schematically in Fig. D-1) charged particles are continually deposited onto a moving conveyor belt near its bottom and are then transported by the belt to the top where they are transferred to a metal sphere. In this way

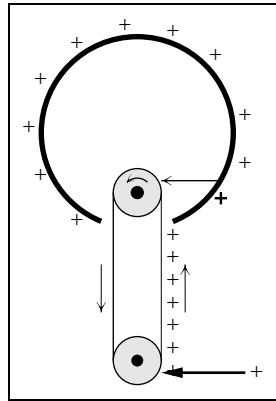


Fig. D-1: Schematic diagram of a Van de Graaff generator.

a large amount of charge is accumulated on the sphere so that the electric field produced in the vicinity of the sphere becomes very large (easily exceeding the breakdown strength of air). Van de Graaff generators are, in fact, used in many practical applications requiring the acceleration of charged particles to very high energy.

### TOTAL ELECTRIC FORCE ON A SYSTEM

Consider a system of charged particles, 1, 2, 3, ..., for example, nuclei and electrons in a molecule or in a piece of metal. (See Fig. D-2.) In the presence of an electric field due to charged particles outside the system, the total electric force  $\vec{F}$  on the system is simply the total *external* force due to the particles outside the system (since we know from Unit 413 that the total internal force, due to the mutual forces between the particles within the system, is equal to zero). How can we find this total external force  $\vec{F}$  on the system?

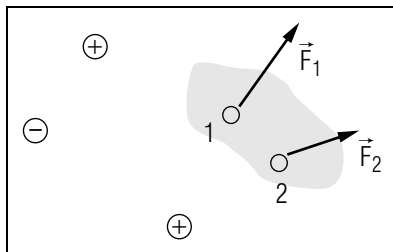


Fig. D-2: Electric force on a system of two charged particles.

Suppose that the external particles produce an electric field  $\vec{E}_1$  at the position of particle 1 having a charge  $q_1$ ; that they produce an electric field  $\vec{E}_2$  at the position of particle 2 having a charge  $q_2$ ; ... Then the external electric force acting on particle 1 is  $\vec{F}_1 = q_1\vec{E}_1$ ; the external electric force acting on particle 2 is  $\vec{F}_2 = q_2\vec{E}_2$ ; ... Hence the *total* external electric force  $\vec{F}$  on the entire system of charged particle is simply

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \dots = q_1\vec{E}_1 + q_2\vec{E}_2 + \dots \quad (\text{D-1})$$

#### ► Force due to uniform field

The calculation of the electric force on a system is especially simple if the electric field produced by the outside particles is uniform (i.e., has the *same* value  $\vec{E}$ ) throughout the system. For then  $\vec{E}_1 = \vec{E}_2 = \dots = \vec{E}$  so that

$$\vec{F} = q_1\vec{E} + q_2\vec{E} + \dots = (q_1 + q_2 + \dots)\vec{E}$$

or

$$\boxed{\text{for uniform field, } \vec{F} = Q\vec{E}} \quad (\text{D-2})$$

where  $Q = q_1 + q_2 + \dots$  is the total charge of the system. In particular, if a system of charged particles is of sufficiently small size (e.g., if it is an atom), the electric field produced by outside particles has ordinarily nearly the same value at the positions of all particles in the system. Then the result Eq. (D-2) is applicable so that the total electric force on the system depends only on the total charge  $Q$  of the system and on the electric field  $\vec{E}$ . **Example D-1: Force on an ion**

A chlorine  $\text{Cl}^+$  ion results when an ordinary Cl atom has lost an electron. Such a  $\text{Cl}^+$  ion consists of 17 protons, each having a charge  $e = 1.6 \times 10^{-19}$  coulomb, and of 16 electrons, each having a charge  $-e$ . The total charge of this ion is then  $Q = 17e + 16(-e) = +e$ , and so is simply equal to the charge of a single electron. If such a Cl ion is located in an electric field  $\vec{E}$  uniform throughout the ion, the electric force on this ion is then  $\vec{F} = +e\vec{E}$ .

### Knowing About Electric Breakdown of Matter

**D-1** Due to the electric activity of nerve cells, the membranes enclosing such cells are constantly exposed to electric forces. These electric forces could cause “breakdown” of the fatty “lipid” material making up the membrane if the magnitude of the electric field in this membrane had a value larger than  $5 \times 10^7$  N/C. Is the magnitude of this field

in which lipids break down much larger, much smaller, or about the same in magnitude as the following “typical” electric fields? (a) The electric field in which air breaks down. (b) The electric field 3 cm from a particle with charge  $2 \times 10^{-8}$  C. (Answer: 11)

### Describing Electric Forces on Fixed Charge Distributions (Cap. 5)

**D-2** A simple model of an ionized hydrogen molecule consists of two protons and an electron (each a particle with a charge of magnitude  $e = 2 \times 10^{-19}$  C). This ionized molecule is located in a uniform electric field  $\vec{E} = 4 \times 10^6$  N/C  $\hat{x}$  due to *other* nearby charged objects. (a) What is the external electric force on each proton and on the electron? (b) What is the total external electric force on the ion? (c) What is the charge  $Q$  of the ion? (d) Does  $\vec{F} = Q\vec{E}$  correctly relate the uniform field  $\vec{E}$  to the external electric force on the ion? (Answer: 24)

**D-3** Consider a hydrogen *molecule* (composed of two protons and *two* electrons) in the presence of an electric field. The molecule can be considered a particle because the electric field in the region occupied by this molecule has a uniform value of  $2 \times 10^6$  N/C  $\hat{x}$ . What is the external electric force on this molecule? (Answer: 21)

**D-4** A dipole for study in the laboratory consists of two small plastic balls which are connected by a stick and which have charges  $q$  and  $-q$ , where  $q = 3 \times 10^{-8}$  C (Fig. D-3). (a) If this dipole is suspended in the non-uniform electric field indicated in Fig. D-3, what are the directions of the external electric forces on particle 1 and on particle 2? (b) Which of these forces is larger in magnitude? (c) What is the direction of the external electric force  $\vec{F}$  on the entire dipole? (d) If the magnitude of  $q$  were larger, would this force  $\vec{F}$  be larger or smaller? (e) The total charge of this dipole is  $Q = q + (-q)$ . Thus the relation  $\vec{F} = Q\vec{E} = 0$  is *not* correct. Why does the relation  $\vec{F} = Q\vec{E}$  not apply to this situation? (Answer: 27)

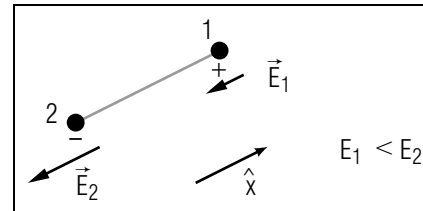


Fig. D-3.

SECT.

## **E** ELECTRIC FIELD NEAR A CHARGED PLANE

In this section and the next, we shall discuss some common situations where an electric field is produced by many charged particles. By the superposition principle, such an electric field is simply the vector sum of the electric fields due to all the individual charged particles. This vector sum can be found quite easily in the simple situations which we shall discuss, those where the charged particles are uniformly distributed over a plane or over a sphere.

### ► *Surface charge density*

The distribution of charged particles over a surface can be described by introducing the quantity “surface charge density” or “charge per unit area” in the same way as the distribution of people in a certain geographical area can be described by the “population density” or “number of people per unit area”). To be specific, consider a portion of surface of area  $A$ , surrounding some point  $P$ , and suppose that a total amount of charge  $Q$  is contained on this portion of surface. Then the “surface charge density” at the point  $P$  is defined as follows

$$\text{Def. } \left| \begin{array}{l} \text{Surface charge density: } \sigma = \frac{Q}{A} \text{ (} A \text{ small} \\ \text{enough)} \end{array} \right| \quad (\text{E-1})$$

where  $A$  is small enough so that the ratio  $Q/A$  would remain unchanged if the portion of surface surrounding the point  $P$  had any smaller area  $A$ . The charge density thus defined is just the “charge per unit area” (i.e., charge divided by the area containing this charge) and has accordingly the units of coulomb/meter<sup>2</sup>.

### ► *Uniformly charged surface*

We say that charge is *uniformly* distributed over a surface if all portions of surface with equal area have the *same* amount of charge. Then the ratio  $\sigma = Q/A$  has the same value for all portions of the surface (and is thus also equal to the ratio obtained by dividing the total charge on the entire surface by the area of the entire surface).

### ► *Field near uniformly charged plane*

Consider a plane (e.g., a flat disk) which is uniformly charged so that it has everywhere the same charge density  $\sigma$ . What then is the electric field  $\vec{E}$  at a point  $P$  near the center of this plane (but not on it), i.e., at a point whose distance  $D$  from the plane is much less than the

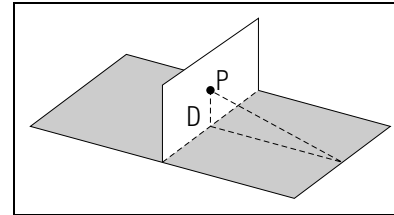


Fig. E-1: Point  $P$  near the center of a charged plane.

distance  $L$  of this point from the nearest point at the edge of this plane? (See Fig. E-1.) This situation is particularly simple because the electric field produced at  $P$  by the charged particles near the edges of the plane is then negligibly small compared to the electric field produced at  $P$  by the charges closer to the center of the plane. \*

\* The charged particles near the edges of the plane are far from  $P$  and produce at  $P$  electric fields which are nearly parallel to the plane. Furthermore, these fields nearly cancel each other since the fields produced by charged particles at one edge of the plane have directions opposite to the fields produced by the charged particles at the opposite edge of the plane.

Hence the electric field  $\vec{E}$  at a point  $P$  near the center of the plane does not depend significantly on the charged particles far from the center of the plane. \*

\* The neglect of contributions to the field  $\vec{E}$  due to charges near the edges of the plane should lead to relative errors of the order of  $D/L$ , small since  $D \ll L$ .

Thus the field  $\vec{E}$  can depend only on the distance  $D$  of  $P$  from the plane, but not on any other distances specifying the position of  $P$  relative to the edges of the plane.

Because of the preceding properties, most interesting features of the electric field  $\vec{E}$  can be easily found without any detailed calculations.

## DIRECTION OF FIELD

Because the electric field  $\vec{E}$  at the point  $P$  does not depend on the position of  $P$  relative to the edges of the plane,  $\vec{E}$  cannot point toward any particular point on the edges of the plane (since  $\vec{E}$  then would depend on the position of  $P$  relative to the edges of the plane). Hence the electric field  $\vec{E}$  must be along a direction *perpendicular* to the plane.

To determine whether  $\vec{E}$  points perpendicularly away from or toward the plane, we remember that  $\vec{E}$  is the vector sum of the fields due to the charged particles on all portions of the plane. But these fields point away from each portion if it is positively charged, and point toward each portion if it is negatively charged. Hence the resultant electric field  $\vec{E}$  must correspondingly point away from the plane if the surface charge density  $\sigma$  is positive, and must point toward the plane if  $\sigma$  is negative. Thus we arrive at this conclusion:

$$\boxed{\vec{E} \text{ is perpendicular to plane, away from plane if } \sigma \text{ is positive (+), toward plane if } \sigma \text{ is negative (-).}} \quad (\text{E-2})$$

## REMARK ON CANCELLATION OF PARALLEL COMPONENTS OF THE FIELD

Figure E-2 illustrates in greater detail why the resultant field  $\vec{E}$  must be perpendicular to the plane. This figure shows that, for every small portion 1 of the plane on one side of  $P$ , there is a corresponding small portion 2 of the plane on the other side of  $P$  (both portions having the same small area, and hence the same charge, and being at the same distance  $R$  from  $P$ ). The electric fields  $\vec{E}_1$  and  $\vec{E}_2$  due to these corresponding portions have the same magnitude, but their component vectors parallel to the plane have opposite directions. Hence the total field  $\vec{E}_1 + \vec{E}_2$  due to any such pair of corresponding small portions of the plane has *no* net component vector parallel to the plane. Thus only the component vectors *perpendicular* to the plane contribute to the total electric field due to the plane.

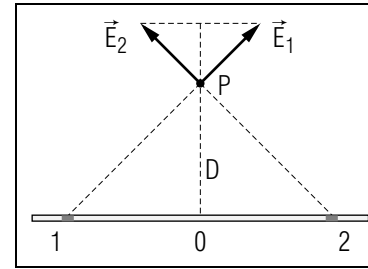


Fig. E-2: Edge-on view of a uniformly charged plane and of the electric fields produced at  $P$  by charges on two symmetrically located equal small portions of the plane.

## MAGNITUDE OF FIELD

### ► Dependence on $k_e$ and $\sigma$

Imagine that the whole plane is subdivided into very small squares, each having the same small area  $A_s$  and thus correspondingly the same charge  $\sigma A_s$ . The total field  $\vec{E}$  at  $P$  is then the vector sum of the electric fields due to the charges on all these small squares. The magnitude of the field at  $P$  due to any small square, at a distance  $R$  from  $P$ , is by Eq. (B-2) equal to  $k_e |\sigma A_s| / R^2$ , so that it is proportional to  $k_e$  and to the magnitude  $|\sigma|$  of the charge density of the plane. Hence the total electric field  $\vec{E}$  at  $P$  must also be proportional to these quantities. (For example, if the charge density on the plane were 3 times as large, the magnitude of the electric field  $\vec{E}$  would also be 3 times as large).

### ► Dependence on distance from plane

How does the electric field  $\vec{E}$  at  $P$  depend on the position of  $P$ ? We already know that  $\vec{E}$  can only depend on the distance  $D$  of  $P$  from the plane. To find just how the magnitude  $E$  of the field depends on  $D$  let us compare the original situation with a new one where the field at  $P$  is due to a plane with the same charge density  $\sigma$ , but where all new lengths are multiplied by the same number (e.g., with a situation where the new plane is 3 times as long and wide as the original plane, and where the distance  $D'$  of the new plane from the point  $P$  is 3 times as large as the distance  $D$  of the original plane from  $P$ ).

As indicated in Fig. E-3, there corresponds then to every small square on the original plane a small square on the new plane. The distance  $R'$  of this new small square from the point  $P$  is 3 times as large as the distance  $R$  of the corresponding small square on the original plane. By Eq. (B-2), the magnitude of the electric field produced at  $P$  by a charged particle on the new small square is then  $1/3^2 = 1/9$  times as large as the magnitude of the electric field produced by this same particle on the original square.



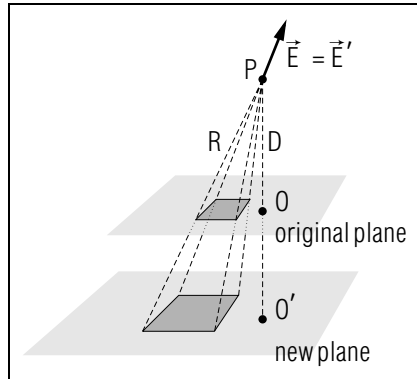


Fig. E-3: Comparison of field produced by charged plane with field produced in same situation but with all lengths multiplied by the same number.

But since the length and the width of the new small square are each 3 times as large as the length and width of the original small square, the area of the new small square is  $3 \times 3 = 3^2 = 9$  times as large as the area of the original square. Thus the new small square contains also 9 times as much charge as the original square (since the charge *density* is the same on both planes). Thus the field produced at  $P$  by *all* the charged particles on the new small square is  $9 \times (1/9) = 1$  times as large (i.e., is the *same*) as the field produced at  $P$  by all the charged particles on the original small square. Hence the total field produced at  $P$  by all the small squares (i.e., by the entire plane) is also the *same* for the new plane as for the original plane. [As the preceding argument shows, the reason is the compensation of two competing effects, the electric field due to a single charge decreasing with increasing distance just as fast as the area of a plane (and thus its charge) increases with increasing size.]

Suppose now that each of the preceding planes is so large that the distance of  $P$  from each plane is much smaller than the distance of  $P$  to the nearest edge of each plane. Then the electric field produced at  $P$  by each plane is due only to the central portion of each plane - since the charges far from the center of each plane contribute negligibly to the field. Hence we can conclude that, irrespective of how large the planes are or what shapes they have near their edges, the field produced at  $P$  by both of these planes is the same despite the different distance of  $P$  from these planes. In short,

The magnitude of the electric field at a point near the center of a uniformly charged plane is *independent* of the distance of this point from the plane.

 (E-3)

► *Magnitude of field*

Since we already know that the magnitude  $E$  of the electric field near the center of a uniformly charged plane is proportional to  $k_e$  and  $|\sigma|$ , the result, Rule (E-3), implies that  $E$  is independent of  $D$  and simply equal to

$$E = (\text{constant}) k_e |\sigma| \quad (\text{E-4})$$

where the constant is some number without units. Indeed, as we show in Section I, the constant is just equal to  $2\pi$ . Thus

$E = 2\pi k_e |\sigma|$

 (E-5)

The results, Rule (E-2) and Eq. (E-5), specify completely the electric field  $\vec{E}$  at any point  $P$  near the center of a uniformly charged plane.

### Understanding the Field due to a Charged Plane (Cap. 1c)

**E-1** *Statement and example:* (a) A point  $P$  is a distance  $h$  from a plane (and not too near its edges). If the plane has a uniform charge density  $\sigma$ , what is the magnitude  $E$  of the electric field at  $P$ ? (b) If each square of area  $1.0 \text{ cm}^2 = 1.0 \times 10^{-4} \text{ meter}^2$  on a large horizontal plane has a charge of  $3.0 \times 10^{-10} \text{ unitC}$  what is the electric field  $\vec{E}$  at a point 2 cm above the center of this plane? What is  $\vec{E}$  at a point 2 cm below this plane? (*Answer: 8*) (*Suggestion: [s-10]*)

**E-2** *Properties of surface charge density:* (a) What is the SI unit of surface charge density? Is it a number or a vector? What are its possible signs? What symbol is ordinarily used to represent surface charge density? (b) A  $1 \text{ cm}^2$  square of metal foil acquires a total charge of  $1 \times 10^{-8} \text{ C}$  when it is touched with a charged rubber rod. If the surface charge density of the foil is uniform, what is its value? (*Answer: 20*) (*Suggestion: [s-18]*)

**E-3** *Interpretation:* Which of the drawings in Fig. E-4 best describe the electric field lines due to a plane with a uniform negative charge density? Explain why each of the other drawings does not correctly describe these field lines. (*Answer: 25*)

**E-4** *Organization of relations:* Figure E-5 shows a point  $P_1$  very near a uniformly charged plane, and a point  $P_2$  very far away from all parts of the *same* charged plane. The plane has a uniform charge density

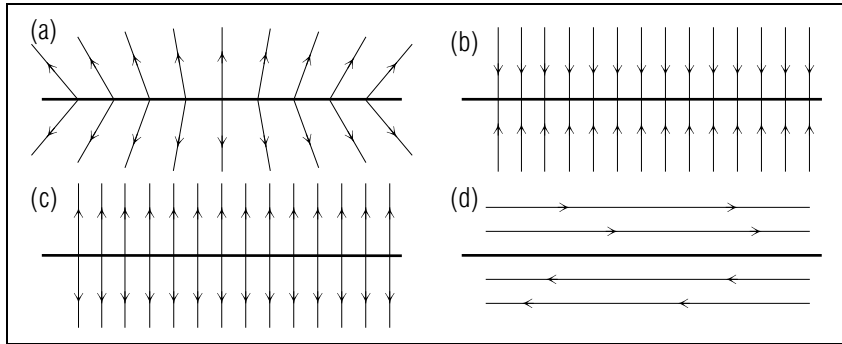


Fig. E-4.

$\sigma$  and a total charge  $Q$ . (a) At  $P_1$  and at  $P_2$ , which of the relations  $E = 2\pi k_e |\sigma|$  and  $E = k_e |Q|/R^2$  best describes the electric field at that point? (b) Consider a point  $P'$  slightly farther from the plane than  $P_1$ . Is the electric field at  $P'$  larger, smaller, or the same in magnitude as the electric field at  $P_1$ ? (c) Answer the question in part (b) for the point  $P_2$ . (Answer: 17) (Suggestion: [s-4]) More practice for this Capability: [p-4], [p-5]

### Finding the Electric Field due to Several Objects

**E-5** Two vertical deflecting plates in an oscilloscope each have a length and width which are large compared to the distance between the plates. Because the plates are charged by transferring electrons from one plate to the other, they have uniform charge densities of equal magnitude but opposite signs (Fig. E-6). Thus the electric fields  $\vec{E}_1$  and  $\vec{E}_2$  due to each of these plates (at any point nearby) have the same magnitudes,  $E_1 = E_2 = E_0$ . Answer these questions for the points  $A$ ,  $B$ , and  $C$  shown in Fig. E-6: (a) What are the directions of the electric fields due to plate 1 and due to plate 2? What is the direction of the total field due to both plates? (Draw arrows to indicate these directions.) (b) What is the magnitude of the total field due to both plates? (Express your answer in terms of the magnitude  $E_0$ .) (Answer: 23) (Suggestion: [s-13])

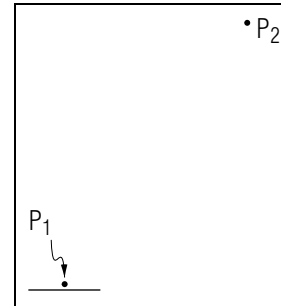


Fig. E-5.

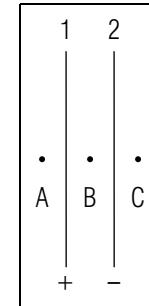


Fig. E-6.

SECT.

## **F** ELECTRIC FIELD DUE TO A CHARGED SPHERE

Consider a sphere uniformly charged over its surface or uniformly charged throughout its volume. What then is the electric field  $\vec{E}$  at some point  $P$  outside this sphere? (See Fig. F-1.) The electric field  $\vec{E}$  is again the vector sum of the electric fields due to all the charged particles distributed over the sphere. The properties of  $\vec{E}$  are quite simple because the charge distribution is “spherically symmetric”, i.e., is the same along any direction through the center of the sphere. Accordingly, the electric field  $\vec{E}$  produced at any point  $P$  by the charged particles on the sphere can depend only on the distance  $R$  of  $P$  from the center of the sphere.

► *Direction of  $\vec{E}$*

The direction of  $\vec{E}$  must be along the radial line passing through the center of the sphere and the point  $P$ , since the direction of  $\vec{E}$  would otherwise depend not only on the distance  $R$ , but also on other geometrical features. Thus  $\vec{E}$  can either point along the radial line away from the sphere (if the charge on the sphere is positive), or toward the sphere (if the charge on the sphere is negative).

► *Magnitude of  $\vec{E}$*

The Coulomb electric force and the gravitational force are both inversely proportional to the square of the distance between the interacting particles. This common property of these two different forces implies some corresponding consequences. For example, we already mentioned in Unit 410 that the gravitational force due to a uniform sphere is the same as if its entire mass were concentrated at its center.

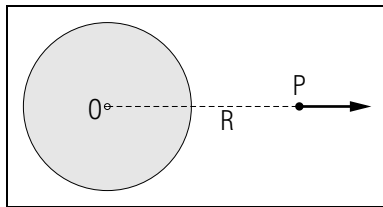


Fig. F-1: Electric field produced by a uniformly charged sphere.

A similar conclusion holds then also for the Coulomb electric force due to a uniformly charged sphere and implies this simple result:

The electric field due to a uniformly charged sphere (or spherical surface) at a point  $P$  on or outside the sphere is the same as if the entire charge of the sphere were concentrated at its center. (F-1)

(This result can be proven directly by vector addition of the electric fields produced at  $P$  by all the charged particles on the sphere.)

Suppose that the total charge of a uniformly charged sphere is  $Q$ . According to Rule (F-1), the electric field  $\vec{E}$  at a point a distance  $R$  from the center of the sphere (where  $R$  is equal to or larger than the radius of the sphere) is then simply equal to

$$\vec{E} = k_e \frac{|Q|}{R^2} \text{ with direction: away from center if } Q \text{ is } +, \text{ toward center if } Q \text{ is } -$$

2)

### Understanding the Relation $E = K_e|Q|/R^2$

**F-1** *Example:* A particle of charge  $q$  produces an electric field  $\vec{E} = 3.4 \times 10^5 \text{ N/C } \hat{x}$  at a point  $P$  which is 5 cm from the charged particle. Suppose that this same amount of charge  $q$  is instead uniformly distributed over the surface of a sphere of radius 3 cm with its center at the position of the original particle. What then is the electric field at  $P$ ? (*Answer: 19*) (*Suggestion: [s-2]*)

**F-2** *Relating quantities:* The sphere of a Van De Graaff generator has a radius of 10 cm and produces at its surface an electric field equal in magnitude to  $3 \times 10^6 \text{ N/C}$ , the breakdown strength of air. What is the magnitude of the total charge on the sphere's surface? [This is the largest charge which can remain on the sphere without causing breakdown of any surrounding air which then conducts charge away from the sphere.] (*Answer: 26*)

### Finding the Electric Field due to Several Objects (Cap. 3)

**F-3** Two spherical shells, of radius 2 cm and 3 cm, are placed one inside the other with their centers at the same point. The spheres each have a uniformly distributed total charge of magnitude  $Q = 3 \times 10^{-7}$  C, the smaller sphere having a charge of  $-Q$ , and the larger sphere a charge of  $+Q$ . (a) What are the electric fields due to each of these shells at a point 6 cm above their center? (b) What is the total field at this point due to both spheres? (*Answer: 18*) (*Suggestion: [s-7]*)

SECT.

## **G** SUMMARY

### DEFINITIONS

electric field; Def. (A-4)

electric field line; Def. (C-1)

surface charge density; Def. (E-1)

### IMPORTANT RESULTS

Electric field due to a single charged particle: Rule (B-1), Eq. (B-2)

$$\vec{E} = k_e |q_1| / R^2 \text{ with direction:}$$

away from  $q_1$  if  $q_1$  is +, toward  $q_1$  if  $q_1$  is  $-$ .

Superposition principle for fields due to several particles: Rule (B-5)

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots$$

Electric force on a system of several particles: Eq. (D-1), Eq. (D-2)

$$\vec{F} = q_1 \vec{E}_1 + q_2 \vec{E}_2 + \dots$$

$\vec{F} = Q\vec{E}$  if  $\vec{E}$  is uniform throughout system

Field near center of a uniformly charged plane: Def. (E-1), Eq. (E-5)

$$\vec{E} = 2\pi k_e |\sigma| \text{ with direction:}$$

away from plane if  $\sigma$  is +, toward plane if  $\sigma$  is  $-$ .

Field due to a uniformly charged sphere or spherical surface: Rule (F-1)

$\vec{E}$  outside sphere is same as if entire charge were concentrated at center of sphere.

### USEFUL KNOWLEDGE

Qualitative properties of electric field lines

The meaning of breakdown strength

### NEW CAPABILITIES

(1) Understand these relations:

(a) The definition of electric field (Sec. A);

(b) The relation  $E = k_e |q| / R^2$  toward or away from a charged particle, or spherically symmetric charge distribution (Sects. B and F, [p-2], [p-6]);

- (c) The relation  $E = 2\pi k_e |\sigma|$  towards or away from a relatively large, uniformly charged plane (Sec. E).
- (2) If the ratio  $R = A/B$  of two quantities remains constant under certain conditions, relate values for  $A$ ,  $B$ , and  $R$ , and identify situations in which  $R$  is constant (Sec. A, [p-1]).
- (3) At a given point, use the values of the electric field due to each of several objects to describe the electric field due to all of these objects (Sects. B, E, and F, [p-3]).
- (4) Use a drawing of electric field lines to describe or compare the directions of the electric forces on charged particles (Sects. C and D).
- (5) Describe the electric force on an object from information about nearby charged particles or the electric field due to them, if the object consists of fixed charged particles (Sec. D).

Study aids are available in:

Tutorial section A: Understanding the definition of electric field;

Tutorial section H: Additional problems.

### Relating Electric Field to Other Quantities (Cap. 1)

**G-1** The surface of each object shown in Fig. G-1 has a uniform charge density  $\sigma$ , and each object has a total charge  $Q$ . At each of the points  $A$  through  $F$ , is the electric field described by the relation  $E = k_e |Q|/R^2$ , by the relation  $E = 2\pi k_e |\sigma|$  or by neither of these relations? (*Answer: 28*) (*Suggestion: [s-11]*)

**G-2** Two horizontal deflecting plates in an oscilloscope each have an area of  $1 \text{ cm}^2 = 1 \times 10^{-4} \text{ meter}^2$ . The upper plate has a charge of  $Q = 2 \times 10^{-9} \text{ C}$ , while the lower plate has a charge of  $Q = -2 \times 10^{-9} \text{ C}$ . (a) What is the uniform charge density of each of these plates? (b) At a point  $P$  between the plates, what are the electric fields due to the upper plate and due to the lower plate? What is the total field at  $P$  due to both of these plates? (c) What is the electric force due to these plates on an electron (of charge  $-2 \times 10^{-19} \text{ C}$ ) located at the point  $P$ ? As the electron moves between the two plates towards the oscilloscope screen, is it deflected upwards or downwards? (*Answer: 22*) *More practice for this Capability: [p-6], [p-7]*

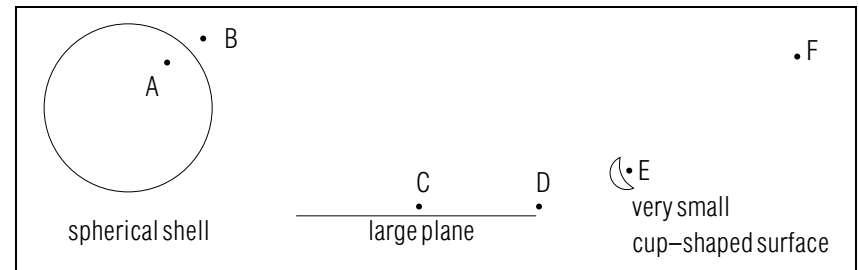


Fig. G-1.

SECT.

## H

 PROBLEMS

**H-1** *Forces on a dipole:* Consider any object (e.g., a “polar” molecule) which is a dipole, that in one end has a positive charge and the other end a negative charge of equal magnitude. This dipole is in a uniform electric field  $\vec{E}$  as shown in Fig. H-1. (a) What is the total electric force on the dipole? If it is initially at rest, will its center of mass move? If so in what direction? (b) If the dipole is initially as shown in Fig. H-1, will it rotate? If so in what direction? (*Answer: 30*) (*Suggestion: [s-5]*)

**H-2** *Field due to a dipole:* Figure H-2 shows a dipole, made up of two particles of charge  $+q$  and  $-q$  separated by a distance  $d$ . We wish to find the electric field due to this dipole at the point  $P$  which is a distance  $R$  from each of the particles making up the dipole. (a) What is the magnitude of the electric field at  $P$  due to each of the particles individually? Draw arrows indicating the directions of these fields. (b) Draw arrows showing the two individual fields,  $\vec{E}_1$  and  $\vec{E}_2$ , and their vector sum, the electric field  $\vec{E}$  at  $P$  due to both particles. Then use similar triangles to find an expression for  $\vec{E}$  in terms of  $q$ ,  $d$ , and  $R$ . (c) Consider the electric field  $\vec{E}'$  at a point  $P'$  to the right of  $P$  in Fig. H-2, and located at a distance  $3R$  from each particle. Express  $\vec{E}'$  as a number times  $\vec{E}$ . (d) Answer the question in part (c) if the dipole is replaced by a single particle of charge  $q$  located at the position of particle 1. (*Answer: 29*)

*Note: Tutorial section H includes additional problems.*

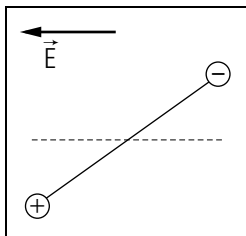


Fig. H-1.

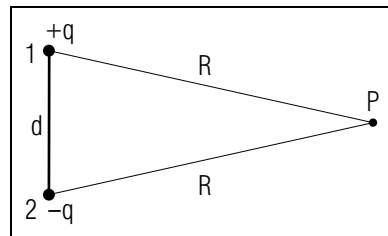


Fig. H-2.

SECT.

## I

 CALCULATION OF FIELD DUE TO A UNIFORMLY CHARGED PLANE

► *Field due to portion of plane*

Let us calculate the electric field produced at a point  $P$  at a distance  $D$  from a plane having a uniform positive charge density  $\sigma$ . Consider first the electric field  $\vec{E}_s$  produced at  $P$  by the charge on a very small square portion of the plane, a portion having an area  $A_s$  and at a distance  $R_s$  from the point  $P$ . (See Fig. H-3.) Then the charge of this small portion is  $\sigma A_s$  and produces at  $P$  an electric field  $\vec{E}_s = k_e(\sigma A_s)/R_s^2$  along the indicated direction. The numerical component of this field along a direction perpendicularly away from the plane is then  $E'_s = E_s \cos \theta$ , where  $\theta$  is the angle between  $\vec{E}_s$  and the direction perpendicular to the plane. Thus

$$E'_s = k_e \frac{\sigma A_s \cos \theta}{R_s^2} \quad (\text{I-1})$$

The numerical component  $E'$ , perpendicular to the plane, of the *total* electric field  $\vec{E}$  at  $P$  due to *all* portions of the charged plane is then obtained by adding the components  $E'_s$  for all portions of the plane. (We already know from Sec. E that the sum of the components of the electric fields *parallel* to the plane is equal to zero. Hence the total field  $\vec{E}$  at the point  $P$  is perpendicular to the plane and has a magnitude simply equal to that of its total perpendicular component  $E'$ .)

► *Simplification*

To simplify our addition of components, let us simplify the expression (1) by noting its geometrical interpretation. In Fig. H-3, consider the “cone” formed by the set of lines emanating from the point  $P$  to the edges of the small square portion of the plane. Then  $A_s \cos \theta = A_\perp$  is simply the area  $A_\perp$  of the surface perpendicular to the cone and contained within this cone at the distance  $R_s$  from  $P$ . But the ratio  $A_s \cos \theta / R_s^2 = A_\perp / R_s^2$  in (1) can be compared with the small area  $A_o$  cut out by the cone on the sphere of radius  $D$  and with center at  $P$ . (See Fig. H-3.) Indeed, since the areas of corresponding portions of spheres of different radii are proportional to the squares of their radii,  $A_\perp / R_s^2 = A_o / D^2$ . Thus (1) can be written simply

$$E_s = k_e \frac{\sigma A_\perp}{R_s^2} = k_e \frac{\sigma A_o}{D^2} = \frac{k_e \sigma}{D^2} A_o \quad (\text{I-2})$$

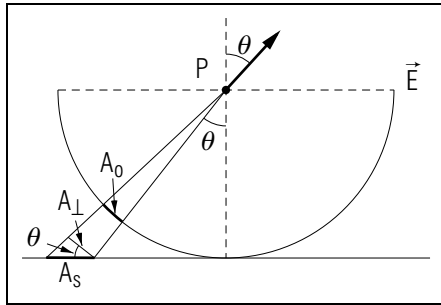


Fig.H-3: Edge-on view of a uniformly charged plane, showing the electric field produced at  $P$  by the charge on a small square of this plane.

► *Addition*

If we add the components  $E'_s$  for all portions of a very large plane (much larger than the distance  $D$  of  $P$  from the plane), these portions of the plane correspond to all the portions of area  $A_o$  of the hemisphere of radius  $D_o$  indicated in Fig. H-3. But the sum of all these areas is just the area  $1/2(4\pi D^2) = 2\pi D^2$  of the entire hemisphere. Thus (2) implies that the sum of all the field components of  $E$  (i.e., the total component  $E'$  of the field due to the entire plane) is simply

$$E' = \frac{k_e \sigma}{D^2} (2\pi D^2) = 2\pi k_e \sigma \quad (\text{I-3})$$

Since  $E'$  has a magnitude equal to that of the field  $\vec{E}$  itself, we thus obtain the result quoted in Eq. (E-4).

## TUTORIAL FOR A

### UNDERSTANDING THE DEFINITION OF ELECTRIC FIELD (Cap. 1a)

**a-1** *PURPOSE:* Although the definition  $\vec{E} = \vec{F}/q$  of the electric field is easily stated, this relation is not so easy to understand. The following frames should help you in acquiring an understanding of this very important relation.

**a-2** *PROPERTIES:*

- (a) Complete the following table summarizing the properties of the quantities force, charge, and electric field.

	force	charge	electric field
Number or vector:			
Possible signs:			
SI unit:			
Usual algebraic symbol:			

- (b) Rubbing with fur or silk causes a rubber rod and a plastic ball (of weight 0.01 N) to acquire charges with opposite signs. When held above the ball, the rod exerts on the ball an electric force barely large enough to lift it against the force of gravity.

Which of the following is a typical magnitude for the electric force on the ball due to the rod?

- 1 N, 0.01 N

Which of the following is a typical magnitude for the charge of the plastic ball?

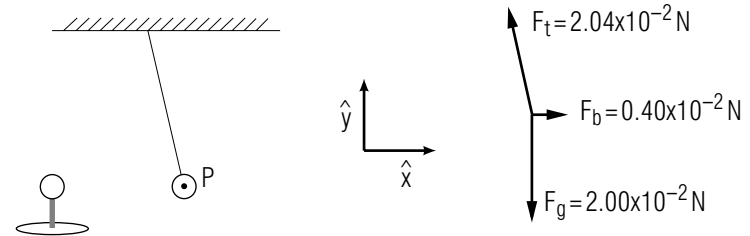
- 1 C, 0.01 C,  $1 \times 10^{-8}$  C

- (c) On the basis of your answer to part b, what is a typical magnitude for the electric field due to a charged rubber rod at a point nearby?

► \_\_\_\_\_

(Answer: 55)

**a-3** *MEANING OF THE QUANTITY  $\vec{F}$  IN  $\vec{E} = \vec{F}/q$ :* A small plastic ball of charge  $q = 3.0 \times 10^{-8}$  C is suspended from a thread and hangs at rest at a point  $P$ . The ball is acted on by the forces  $\vec{F}_g$  due to gravity,  $\vec{F}_t$  due to the thread, and  $\vec{F}_b$  due to a second charged ball. These forces have the magnitudes and directions indicated in this drawing:



The total force  $\vec{F}$  on the ball is zero because the ball remains at rest.

Which of the following is the correct value for the electric field at the point  $P$ ?

(a)  $\vec{F}_g/q = -6.7 \times 10^5$  N/C  $\hat{y}$

(b)  $\vec{F}_b/q = 1.3 \times 10^5$  N/C  $\hat{x}$

(c)  $\vec{F}/q = 0$

- (a), (b), (c)

Now: Check answer 51. Suggestion [s-8].

**a-4** *COMPARING CHARGE, ELECTRIC FIELD, AND ELECTRIC FORCE:* The electric field in a nerve membrane is related to the charge of an ion in that membrane and to the electric force on that ion.

Which of the following values is the electric field?

Which is the electric force? Which is the charge of the ion?

(a)  $1.6 \times 10^{-11}$  N  $\hat{x}$

(b)  $3.2 \times 10^{-19}$  C

(c)  $5.0 \times 10^7$  N/C  $\hat{x}$



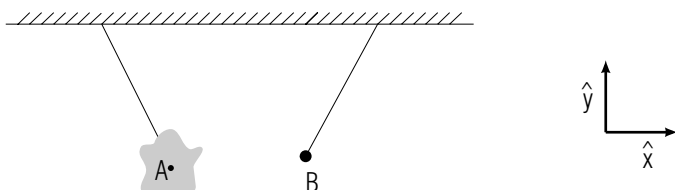
► Field: (a), (b), (c)

Force: (a), (b), (c)

Charge: (a), (b), (c)

Now: Check answer 63.

**a-5** *APPLICABILITY:* A charged piece of plastic sheet and a small charged plastic particle are suspended from threads as shown in this drawing: Both objects are at rest, the sheet with its center at the point  $A$ , and the particle at the point  $B$ .



The sheet has a total charge of  $2.0 \times 10^{-7}$  C while the particle has a charge of  $-4.0 \times 10^{-8}$  C. The electric force on the particle due to the sheet is  $-0.08$  N  $\hat{x}$ , while the electric force on the sheet due to the particle is  $0.08$  N  $\hat{x}$ .

Either find the electric field at  $A$  or describe why the information provided cannot be used to find this field.

► \_\_\_\_\_

Either find the electric field at  $B$ , or describe why the information provided cannot be used to find this field.

► \_\_\_\_\_

Now: Check answer 59. Then go to text problem A-2.

## TUTORIAL FOR H

### ADDITIONAL PROBLEMS

The following problems involve sketching electric field lines due to charged objects ([h-1], [h-2]), use of electric fields to explain several interesting situations ([h-3], [h-4]), and methods for describing quantitatively the electric field due to objects ([h-5] through [h-10]).

### SKETCHING FIELD LINES

Rough sketches of field lines, even due to complicated systems, can be made by using this procedure: (a) Sketch the field very near each particle or plane region. This field is almost equal to the field due to that particle or plane alone. (b) Sketch the field very far away from the object. This field is almost equal to the electric field of a particle of charge equal to the total charge of the system. (c) Very roughly sketch smooth curves joining the nearby fields to the far-away fields.

**h-1** *THE ELECTRIC FIELD DUE TO A CHARGED DISK:* A positively charged disk is shown from the edge in this drawing: Sketch the electric field lines due to this disk.

►

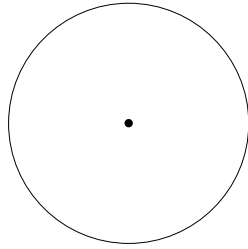


(Answer: 52)

**h-2** *THE ELECTRIC FIELD INSIDE A GEIGER TUBE:* A Geiger tube is a negatively charged metal tube filled with gas and having a positively charged wire along its axis. [When rapidly moving charged particles ionize the gas within the tube, the electric field due to the wire and tube causes “breakdown” of the gas and motion of ions. The result is a pulse of “charge current,” which is then detected by electronic devices attached

to the Geiger tube. Thus the Geiger tube is a detector of rapidly moving particles such as those coming from “radioactive” materials.]

Because the wire and tube have charges of equal magnitude and opposite sign, the electric field outside the tube is zero. Sketch the electric field inside the Geiger tube which is shown from the end in this drawing:



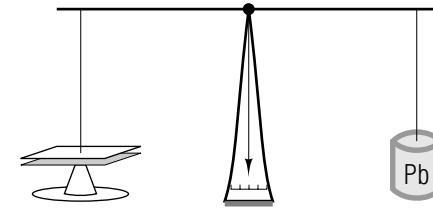
(Answer: 61)

### Applying Electric Fields to Describe Various Phenomena

**h-3** *MAXIMUM CHARGE DENSITIES:* In many common situations, two long parallel plates have uniform charge densities with equal magnitudes but opposite signs. (a) Express the magnitude  $E$  of the electric field between two such plates in terms of the magnitude  $|\sigma|$  of the charge densities of each plate. (b) The maximum charge density which can occur on these plates is that density which produces between the plates an electric field equal in magnitude to the “breakdown strength” of the material between the plates. [Any larger charge density causes this material to break down and to conduct charge away from the plates.] What is the maximum magnitude  $|\sigma|$  of charge density which can occur on two plates separated by 0.1 cm of air? What is the maximum value of  $|\sigma|$  for the charge density near each side of a 50 Å sheet of lipid forming a nerve cell wall? [Such lipid has a breakdown strength of about  $5 \times 10^7$  N/C.] (Answer: 67)

**h-4** *MEASURING CHARGE WITH A SCALE:* One arm of the balance-type scale shown in the following drawing supports a lead “weight” of mass  $m$ . The other arm suspends a uniformly charged plate (of mass  $M$ , total positive charge  $Q$ , and area  $A$ ) just above a similar uniformly charged plate which has a charge of  $-Q$ . In this situation the charge  $Q$  (difficult to measure) can be related to easily measured

quantities such as mass and area by the following reasoning.



(a) What objects exert forces on the upper plate? (b) What is the electric field due to the lower plate at each point on the upper plate (except those very near the edge)? (c) Consider on the upper plate a very small square which has an area  $a$  and which is not too near the edge of this plate. What is the charge of this small square? (d) Considering this square as a particle, what is the force on it due to the lower plate? (e) If both plates are large, so the area of all small squares near the edges is negligible, what is the total force on the upper plate due to the lower plate? (f) If the upper plate is suspended at rest (under the influence of all forces acting on it), what is the charge  $Q$  in terms of known quantities and the easily measured quantities  $M$ ,  $m$ , and  $A$ ? (Answer: 64)

### Applying Unit Consistency

Correct relations have consistent units, i.e., all terms in the relation have the same unit. The following two frames illustrate the use of unit consistency to find properties of the electric field.

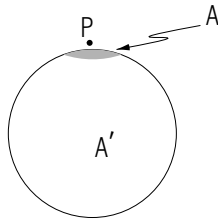
**h-5** *FIELD DUE TO A CHARGED PLANE:* Consider the electric field  $\vec{E}$  at a point  $P$  near the center of a large plane with uniform charge density  $\sigma$ . As discussed in Sec. E, the magnitude  $E$  can depend only on  $|\sigma|$ , on the constant  $k_e$ , and on the distance  $h$  of  $P$  from the plane. (a) What are the units (in terms of kg, m, s, and C) of  $E$ ,  $|\sigma|$ ,  $h$ , and  $k_e$ ? (b) Show that the units of the correct relation  $E = 2\pi k_e |\sigma|$  are consistent by finding the unit of each side of this equation. (c) Briefly explain why each of these expressions cannot be correct.  $E = k_e |\sigma|/h$ .  $E = k_e |\sigma|^2/h^2$ .  $E = (\text{constant with a unit } k_e |\sigma|)$  (Answer: 70)

**h-6** *FIELD OF A CHARGED LINE:* Consider a point  $P$  a distance  $h$  from a long uniformly charged line (e.g., a wire). Then the ratio of the charge  $q_\ell$  of some length  $\ell$  of the line divided by  $\ell$  always has the same

value,  $q_\ell/\ell = \lambda$  where  $\lambda$  is called the “charge per unit length.” If  $P$  is far from the ends of the line, then (according to reasoning like that applied to a plane in Sec. E) the electric field  $\vec{E}$  at  $P$  can be directed only towards or away from the line, and its magnitude can depend only on  $h$ ,  $k_e$  and  $|\lambda|$  (a) What is the direction of  $\vec{E}$  if the line is positively charged and if it is negatively charged? (h) Use unit consistency to find an expression for  $E$  at  $P$  in terms of  $h$ ,  $k_e$ ,  $|\lambda|$ , and an unknown constant with *no* unit. (Answer: 65) (Suggestion: [s-3])

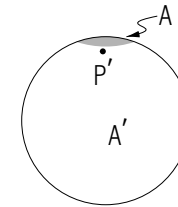
### The Electric Field due to Various Charged Objects

**h-7** *FIELDS NEAR A CHARGED SPHERE AND NEAR A CHARGED PLANE:* Any sufficiently small region of a spherical surface is nearly a plane. (See the following diagram, and consider the region of the earth’s surface which is near you.) This observation suggests that at a point  $P$  near its surface, the electric field due to a uniformly charged sphere is related to the electric field due to a uniformly charged plane.



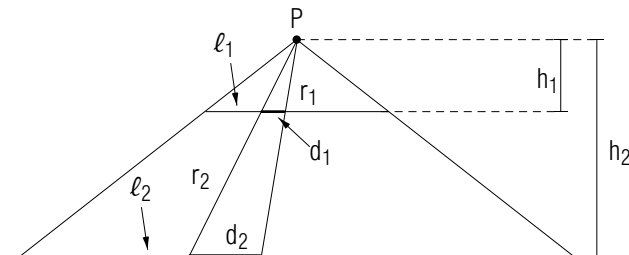
(a) The surface area of a sphere of radius  $R$  is  $4\pi R^2$ . What is the uniform charge density  $\sigma$  of a spherical surface having a radius  $R$  and a total positive charge  $Q$ ? (b) Use the charge density  $\sigma$  to express the electric field  $\vec{E}$  due to this sphere at a point  $P$  just above its surface. (c) Let us consider separately a plane region  $A$  near  $P$  and the rest of the spherical surface  $A'$ . (See the preceding diagram.) What is the electric field at  $P$  due to the plane region  $A$ ? (d) On the basis of your answer to part (b), what is the field at  $P$  due to the remainder of the sphere  $A'$ ? (Answer: 56)

**h-8** *THE FIELD JUST INSIDE A UNIFORMLY CHARGED SPHERE:* Consider a point  $P'$  just inside a sphere with a uniform positive charge density  $\sigma$ .



(a) What is the electric field at  $P'$  due to the plane region  $A$ ? (b) On the basis of your answers to [h-7], what is the electric field at  $P$  due to the remainder of the sphere  $A'$ ? (Notice that the point  $P'$  and the point  $P$  described in [h-7] are at almost the same position relative to the charged particles making up  $A'$ .) (c) What is the field at  $P'$  due to the entire sphere? (Answer: 60)

**h-9** *FIELD DUE TO A CHARGED LINE: DEPENDENCE ON DISTANCE:* We found the electric field due to a uniformly charged plane by first determining the dependence of this field on other quantities, and then using the geometric argument in Section I to find the exact relation between the field and these quantities. In this problem and the next we apply this procedure to find the electric field at a point  $P$  near a long “uniformly charged” line (e.g., a charged wire). A line is uniformly charged if the ratio of the charge  $q_\ell$ , of any segment, to the length  $\ell$  of that segment, always has the same value,  $q_\ell/\ell = \lambda$ , no matter what segment is chosen. If  $P$  is a distance  $h$  from the line, and not near either end of the line, then (according to reasoning like that applied to the plane in Sec. E) the electric field  $\vec{E}$  at  $P$  can be directed only towards or away from the line, and its magnitude can depend only on  $h$ ,  $k_e$ , and  $|\lambda|$ .



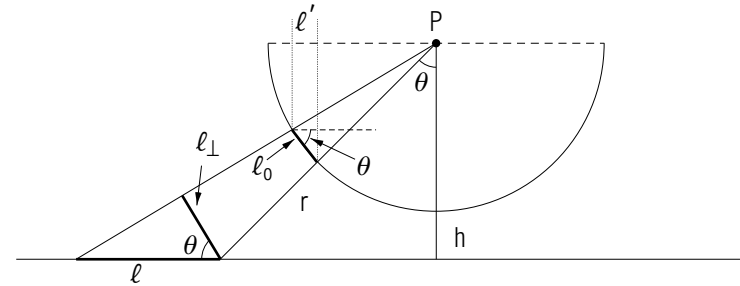
The preceding drawing shows two lines ( $\ell_1$  and  $\ell_2$ ) each with the same charge per unit length  $\lambda$ . Line  $\ell_2$  is 3 times as far from  $P$  as line  $\ell_1$

( $h_2 = 3h_1$ ) and  $\ell_2$  is 3 times as long as  $\ell_1$ . To see how  $E$  at  $P$  depends on the distance  $h$  from  $P$  to a charged line, we shall compare the fields at  $P$  due to  $\ell_1$  and due to  $\ell_2$ . We first consider the electric fields due to the small lengths  $d_1$  and  $d_2$  which can be considered as particles. (a) Express the magnitudes  $E_1$  and  $E_2$  of the electric fields at  $P$  due to the charged line segments  $d_1$  and  $d_2$  in terms of the charges  $q_1$  and  $q_2$  of the two line segments, the distances  $r_1$  and  $r_2$  shown on the diagram, and known quantities. (b) For each of the following pairs of quantities, is the first quantity 3,  $1/3$ , 9, or  $1/9$  times the second? ( $q_2, q_1$ ) ( $r_2, r_1$ ) ( $E_2, E_1$ ) (c) Express the electric field  $\vec{E}_{2,total}$  due to the entire line  $\ell_2$  as a number times the electric field  $\vec{E}_{1,total}$  due to the entire line  $\ell_1$ . (d) Suppose line  $\ell_2$  were 5 times as far from  $P$  as  $\ell_1$ , and that  $\ell_2 = 5\ell_1$ . What then is the expression for  $\vec{E}_{2,total}$  as a number times  $\vec{E}_{1,total}$ ?

Now consider two very long uniformly charged lines at the locations of  $\ell_1$  and  $\ell_2$ . As in the case of a large uniformly charged plane described in Sec. E, the charged particles near the ends of these lines produce an electric field which is negligibly small. (e) Is the electric field due to a long uniformly charged line *independent of the distance  $h$*  from the line, or is it proportional to  $h$ , to  $h^2$ , to  $1/h$ , or to  $1/h^2$ ? (*Answer: 53*)

**h-10** *FIELD DUE TO A CHARGED LINE: EXACT EXPRESSION:* In frame [h-9] we found that the electric field near a long, uniformly charged line is proportional to  $1/h$ , where  $h$  is the distance from this line. In this frame, we shall find an exact expression for this field by using a geometric procedure like that applied in Section I to a uniformly charged plane—a procedure for directly adding the electric fields due to the particles making up the line. Before working this problem, study Section I carefully.

The following diagram shows a long line with a uniform positive charge per unit length,  $\lambda = q_\ell/\ell$ . We shall find the electric field at  $P$  due to this line.



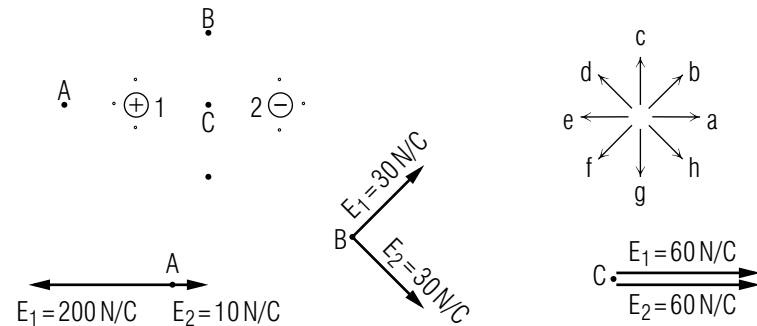
We first consider the electric field at  $P$  due to the small segment of length  $\ell$  shown in the diagram. Use reasoning similar to that in Section I to find the electric field  $\vec{E}$  at  $P$  due to all such small lengths making up the line. Your work should include answers to these questions: (a) Why are the three angles labeled  $\theta$  all equal? (b) Why need we consider only the component of  $\vec{E}$  perpendicular to the line? (c) What is this component of the electric field due the small segment of length  $d$  in terms of  $\lambda$ ,  $k_e$ ,  $\ell$ ,  $\theta$ , and  $r$ ? What is this component in terms of  $\lambda$ ,  $k_e$ ,  $\ell_0$ ,  $r$ , and  $r_0$ . [The small length  $\ell$  is at a distance  $r$  from  $P$ . The semicircle with center at  $P$  has a radius  $r_0$ . The length  $\ell_0$  on this hemisphere corresponds to  $d$  as shown.] (d) By using the relation  $\cos \theta = h/r$ , rewrite the expression found in part (c) so that this expression includes  $h$  and  $\theta$ , but does not include  $r$ . What is this expression for the component field in terms of the small length  $d'$  along the diameter of the semicircle? (e) What is the electric field at  $P$  due to the entire charged line? Express this result in terms of  $\lambda$ ,  $k_e$  and  $h$ . (*Answer: 57*) (*Suggestion: [s-6]*)

## PRACTICE PROBLEMS

**p-1** *RELATING PROPORTIONAL QUANTITIES AND THEIR RATIOS (CAP. 2):* The fluid near a point exerts on surface of small enough area  $A$  a force  $\vec{F}$  which is proportional to  $A$ . The pressure  $p$  of the fluid at this point is then the ratio  $p = F/A$ . Compare each of the following pairs of quantities by stating whether these quantities *must be the same*, *must be different*, or *might be either the same or different*. (a) The forces exerted by the atmosphere on the small enough top surfaces of a soup can and of a coffee can when each horizontal can top is placed at the same location. (b) The pressures due to the atmosphere on each of the top surfaces of each can described in part (a). (c) The pressures due to the atmosphere on the top surfaces of each of two identical soup cans when one is at sea level and the other on a very high mountain. (*Answer: 54*) (*Suggestion: Review text problems A-4 and A-5.*)

**p-2** *ELECTRIC FIELD DUE TO A CHARGED PARTICLE: DEPENDENCE (CAP. 1B):* (a) An electron in a hydrogen atom can move with an average distance from the nucleus of  $r_1$  or of  $r_2 = 4r_1$ . Compare the magnitudes  $E_1$  and  $E_2$  of the electric field produced by the hydrogen nucleus at these two distances. Express  $E_2$  as a number times  $E_1$ . (b) In a beryllium ion, the nucleus has a charge equal to four times the charge of a hydrogen atom, and an electron moves with an average distance from the nucleus of  $r_2$ . Compare the electric field  $\vec{E}_B$  produced by the beryllium nucleus at this distance with the electric field  $E_2$  produced by the hydrogen nucleus at this same distance. Then express  $\vec{E}_B$  as a number times  $E_2$ . (*Answer: 68*) (*Suggestion: [s-16] and review text problems B-1 through B-4.*)

**p-3** *DESCRIBING THE ELECTRIC FIELD DUE TO SEVERAL PARTICLES (CAP. 3):* Consider two particles with charges of equal magnitude but opposite sign, as indicated in the following diagram. [This system of charged particles, called a “dipole,” produces an electric field which is similar in many ways to the electric fields of interesting systems including the heart, nerve cells, and electric fish.]



The preceding vector diagrams show the electric fields  $\vec{E}_1$  and  $\vec{E}_2$  due to particle 1 and to particle 2 at the points  $A$ ,  $B$ , and  $C$ . (a) For each of the points  $A$ ,  $B$ , and  $C$ , what is the magnitude  $E$  of the total electric field due to particles 1 and 2? Which of the labeled arrows best indicates the direction of each total field? (b) From each dot on the preceding diagram, draw an arrow indicating the direction of the total field  $\vec{E}$  at that point. (*Answer: 62*) (*Suggestion: Review text problems B-5 and B-6.*)

**p-4** *THE FIELD DUE TO A CHARGED PLANE: COMPARISONS (CAP. 1C):* (a) A metal plate of area  $0.60 \text{ meter}^2$  has a charge density of  $4.0 \times 10^{-6} \text{ C/meter}^2$ . What is the total charge  $Q$  of this plate? (b) Suppose this same charge  $Q$  is distributed uniformly over a plate with one third the area of the original plate. What is the charge density of this plate? (c) Answer question (b) for a plate with three times the area of the original plate. (*Answer: 66*) (*Suggestion: Review text section E and text problems E-1, E-2, and E-3.*)

**p-5** *THE FIELD DUE TO A CHARGED PLANE: DEPENDENCE (CAP. 1C):* Consider a point  $P$  located near the center of a very large plane with a uniform charge distribution. After each of the following changes, is the magnitude of the electric field  $E$  at  $P$  *larger*, *smaller*, or the *same* in magnitude as its initial value? (a) The charge density of the plane is made larger in magnitude. (b) The point  $P$  is moved to a location farther from the plane. (c) The charge density of the plane is made smaller in magnitude. (d) The plane is moved to a location nearer to the point  $P$ .



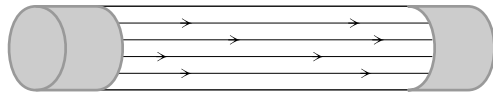
(Answer: 69) (Suggestion: [s-1] and review text section E.)

**p-6** RELATING ELECTRIC FIELD TO OTHER QUANTITIES

(CAP. 1): The positively charged sphere of a Van de Graaff generator operated in air has a maximum possible charge  $Q$ . This charge  $Q$  is that charge which produces at the surface of the sphere an electric field equal in magnitude to the breakdown strength  $E_o$  of air. [Any larger charge will result in dissociated air molecules, including free electrons which move to the sphere and so reduce its charge. (a) Express this maximum charge  $Q$  in terms of  $E_o$  and the radius  $r$  of the sphere. (b) What is the value of  $Q$  for a sphere of radius 10 cm? (c) Which of the following procedures could be used to increase the maximum charge on the sphere: using a sphere of larger radius, using a sphere of smaller radius, placing the sphere in a gas with a higher breakdown strength? (Answer: 58) (Suggestion: Review text problems G-1 and G-2.)

**p-7** RELATING DESCRIPTIONS OF THE ELECTRIC FIELD

(CAP. 4): The following diagram shows the electric field lines in a fluorescent tube for a desk lamp. (a) What are the directions of the electric force on positively charged mercury ions and on negatively charged electrons in this tube? (b) When the lamp is in operation, each (ion or electron) moves with an average velocity which has the same direction as the electric force on that particle. In what direction does each kind of particle move? (Answer: 71)



## SUGGESTIONS

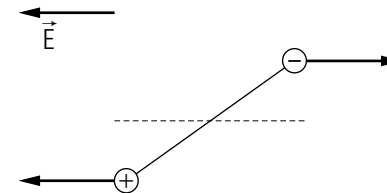
**s-1** (Practice problem [p-5]): According to the relation  $E = 2\pi k_e \sigma$ , the magnitude of the electric field due to a plane does not depend appreciably on the distance from the plane to the point  $P$  at which  $E$  is measured (so long as  $P$  is near the charged plane and not too near its edge). Thus the position of  $P$  relative to the plane (or the position of the plane relative to  $P$ ) can change without affecting the value of  $\vec{E}$  at  $P$ .

**s-2** (Text problem F-1): The main result of text section F is: The electric field outside a spherically symmetric charge distribution is exactly the same as if the entire charge were concentrated on a particle located at the sphere's center.

**s-3** (Tutorial frame [h-6]): (b) State the units of all quantities of interest. Then you need to find an expression which has the same units as  $\vec{E}$ , and which involves only  $h$ ,  $k_e$ , and  $|\lambda|$ . (Multiplied by a constant with no unit).

**s-4** (Text problem E-4): Compared to the distance between the plane and  $P_2$ , the length of the plane is very small. Thus in determining the electric field at  $P_2$ , the plane can be considered as a particle.

**s-5** (Text problem H-1): The following drawing shows the electric forces on the dipole shown in Fig. H-1. Because the field is uniform, these forces have equal magnitude and opposite direction. However, they can cause the dipole to rotate.



**s-6** (Tutorial frame [h-10]): Notice that  $\ell_{\perp}$  and  $\ell_0$  on the diagram are lengths. Then because  $\ell_{\perp}$  and  $\ell_0$  are corresponding sides of similar triangles:

$$\frac{\ell_0}{\ell_{\perp}} = \frac{r_0}{r}$$

This ratio is not the same as the relation between areas in Section I, because the ratio of these areas is equal to  $(r_0/r)^2$ , not just  $r_0/r$ .

**s-7** (*Text problem F-3*): Try making a sketch of the spheres and the point  $P$  at which  $\vec{E}$  is to be found. Each sphere produces at  $P$  an electric field of magnitude  $E = k_e|Q|/R^2$  where  $|Q|$  is the magnitude of that sphere's charge, and  $R$  is the distance from  $P$  to the center of that sphere. The direction of the electric field due to each sphere depends on the sign of the charge of that sphere. The total electric field is the vector sum of the individual fields due to each sphere.

**s-8** (*Tutorial frame [a-3]*): In the definition  $\vec{E} = \vec{F}/q$ , the symbol  $\vec{F}$  means the *electric* force on a particle of charge  $q$ . Thus answers (a) and (c) are incorrect because either the gravitational force or the total force has been substituted for  $\vec{F}$ .

Note: Be very careful of symbols. Very often the symbol  $\vec{F}$  is used to mean the total force on a particle. Nonetheless, in the relation  $\vec{E} = \vec{F}/q$ ,  $\vec{F}$  means the electric force on a particle of charge  $q$ .

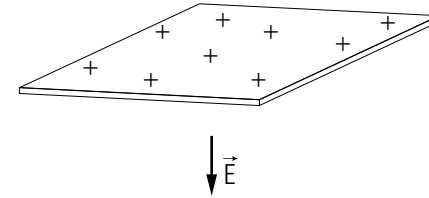
**s-9** (*Text problem A-5*): This problem is very similar to problem A-4. For any particle at the point  $A$ , the electric force  $\vec{F}$  on this particle is proportional to the particle's charge  $q$ , and so the ratio  $\vec{E} = \vec{F}/q$  must always have the same value. However, at a different point  $B$ , the electric force  $\vec{F}$  on a particle of charge  $q$  might be different from the electric force on this same particle at the point  $A$ . Thus the ratio  $\vec{F}/q$  may be different at  $B$  than it is at  $A$ .

**s-10** (*Text problem E-1*): Recall that the charge density  $|\sigma|$  is the ratio  $q_s/A_s$ , where  $A_s$  is the area of a small square and  $q_s$  is the charge of this square.

According to the relation  $E = 2\pi k_e|\sigma|$ , the electric field  $\vec{E}$  near a charged plane does *not* depend on the distance  $h$  from the plane at which  $\vec{E}$  is measured.

The direction of  $\vec{E}$  for a charged plane (like  $\vec{E}$  for a charged particle) is towards a negatively charged object and away from a positively charged object. Thus, for example, below a horizontal positively charged plane,

$\vec{E}$  is directed away from the plane, or downward.



**s-11** (*Text problem G-1*): Let us review the applicability of the two relations considered here:

$E = k_e|Q|/R^2$  describes the magnitude of the electric field due to a particle (an object which is very small compared with other distances of interest). This relation also describes the electric field due to a spherically symmetric charge distribution at points outside this charge distribution.

$E = 2\pi k_e|\sigma|$  describes the magnitude of the electric field due to a uniformly charged plane at points which are near the plane's surface but far from the edges of the plane.

**s-12** (*Text problem A-4*): If the price  $P$  is proportional to the volume  $V$ , then the ratio  $R = P/V$  has the same value for *any* amount of topsoil bought by *any one* at store  $A$ . However, for a different store, the price  $P$  for a certain volume  $V$  may be higher or lower than the price for this volume at store  $A$ . Thus  $R = P/V$  may have different values for topsoil bought at different places.

**s-13** (*Text problem E-5*): As an example, let us consider first the point  $A$ . The electric fields  $\vec{E}_1$  and  $\vec{E}_2$  due to plates 1 and 2 have the *same* magnitude  $E_o$ . [The distance of a plate from point  $A$  does not affect the magnitude of the electric field due to that plate.] Because plate 1 is positively charged,  $\vec{E}_1$  is directed away from plate 1, or towards the left. Because plate 2 is negatively charged,  $\vec{E}_2$  is directed towards plate 2, or towards the right. The total field  $\vec{E} = \vec{E}_1 + \vec{E}_2$  is then the sum of two vectors with equal magnitude and opposite direction. Thus  $\vec{E}$  is zero.

**s-14** (*Text problem A-3*): The value of the electric field  $\vec{E}$  at a point  $P$  does *not* depend on the charge of any particle which may be at  $P$ , although it does depend on the charges and positions of all other particles. (Reviewing text problem A-1 may help to make this clear.) Thus in all

the situations described in problem A-3, the value of  $\vec{E}$  at  $P$  has the same value,  $2.0 \times 10^6$  N/C towards the left. The force  $\vec{F}$  on a particle of charge  $q$  located at  $P$  can be found from the definition  $\vec{E} = \vec{F}/q$  rearranged to give  $\vec{F} = q\vec{E}$ .

**s-15** (Text problem C-3): At points very near each particle, the electric field lines should be similar to electric field lines for a single positively charged particle. (See Fig. C-2a.) Of the three drawings of field lines in Fig. C-7, only (c) correctly shows that, near each positively charged particle, the electric field is directed away from that particle.

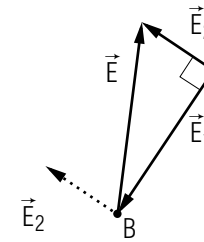
**s-16** (Practice problem [p-2]): In using the relation  $E = k_e|q|/R^2$  to describe dependence, it is helpful to group separately quantities with values remaining the same and quantities with values which change. Thus for parts (a) and (b) of this problem, we rewrite the relation in these ways:

$$E = (k_e|q|) \left( \frac{1}{R^2} \right) \quad (\text{Part a})$$

$$E = \left( \frac{k_e}{R^2} \right) (|q|) \quad (\text{Part b})$$

(Note: If you need further help, review Suggestion [s-6] of Unit 410 and Suggestion [s-7] of Unit 410.)

**s-17** (Text problem B-6): To find the total field  $\vec{E} = \vec{E}_1 + \vec{E}_2$ , just add the individual fields. Remember that electric fields are vectors and are always added as vectors, by using arrow diagrams. Thus the following diagram shows  $\vec{E}_1$ ,  $\vec{E}_2$  and  $\vec{E}$  at the point  $B$ .  $\vec{E}$  is directed roughly upward and towards the right. The magnitude of  $\vec{E}$  can be found by using the Pythagorean theorem.



(Note: If you need further help in using vectors, review text section D of Unit 405.)

Since particles 1 and 2 have equal charges, the magnitudes  $E_1$  and  $E_2$  depend just on the distance from each particle. The electric field due to a particle is largest in magnitude for points near that particle.

**s-18** (Text problem E-2): Remember that the charge covers the entire *surface* of the foil. The distinguishing feature of a foil is that its thickness is negligible compared to its other dimensions, so its total surface area is extremely close to being just twice the area of either side. Nevertheless, this very small thickness is still much larger than the thickness of the layer of charge near the surface. Thus the charge distributes itself over the *entire surface* (both sides) of the metal foil.



## ANSWERS TO PROBLEMS

1. a. magnitudes different (different units), directions same  
b. magnitudes different, directions different (opposite)
2. a.  $-1.2 \times 10^6 \text{ N/C } \hat{y}$   
b.  $3.6 \times 10^{-2} \text{ N } \hat{y}$   
c. At  $A$ ,  $\vec{E}$  must have the same value. At  $B$ , the value of  $\vec{E}$  might be the same or different.
3. a.  $\vec{F}_1 = -8 \times 10^{-2} \text{ N } \hat{x}$ ,  $\vec{F}_2 = 6 \times 10^{-2} \text{ N } \hat{x}$   
b.  $\vec{F} = -2 \times 10^{-2} \text{ N } \hat{x}$   
c.  $\vec{E} = 1 \times 10^6 \text{ N/C } \hat{x}$   
d.  $\vec{F} = -2 \times 10^{-2} \text{ N } \hat{x}$ , yes
4. a. At each of the five points,  $\vec{E}$  is directed from the point *away* from the positively charged particle.  
b.  $\vec{E}$  is directed from each of the points *towards* the negatively charged particle.
5. (c)
6. a.  $\vec{E} = \vec{F}_a/q_a$   
b.  $1.5 \times 10^5 \text{ N/C } \hat{x}$   
c.  $1.5 \times 10^5 \text{ N/C}$   
d. no
7. a.  $E = k_e q/R^2$   
b.  $\vec{E} = -3 \times 10^5 \text{ N/C } \hat{x}$
8. a.  $E = 2\pi k_e |\sigma|$   
b.  $1.7 \times 10^5 \text{ N/C}$  upward,  $1.7 \times 10^5 \text{ N/C}$  downward
9. Right:  $A, D$ . Left:  $B, C$ .
10. a. 15 dollar/meter<sup>3</sup>  
b. 90 dollar  
c. At store  $A$ ,  $R$  must be the same. At store  $B$ ,  $R$  might be the same or might be different.
11. a. much larger than  $3 \times 10^6 \text{ N/C}$ , the breakdown strength of air

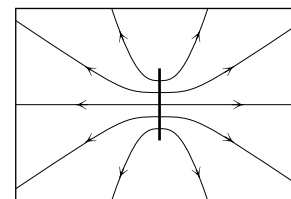
- b. much larger than  $2 \times 10^5 \text{ N/C}$ , a typical magnitude of the electric field due to an ordinary charged particle.
12.  $+3.2 \times 10^{-19} \text{ C}$ ,  $\text{Cu}^{++}$
13. a.  $\vec{F}_a = 0.024 \text{ N}$  toward the left  
b.  $\vec{E}$  is the same,  $\vec{F}_b = 0.048 \text{ N}$  toward the *right*.  
c.  $\vec{E} = 2.0 \times 10^6 \text{ N/C}$  toward the left  
d. no, yes
14. a. along  $\hat{x}$ , along  $\hat{x}$   
b. opposite to  $\hat{x}$ , opposite to  $\hat{x}$
15. a. At  $A$ ,  $\vec{E} = 0$ ; at  $B$ ,  $\vec{E} = 13 \times 10^3 \text{ N/C}$  upward and slightly toward the right.  
b. At points very near particle 1 (or 2),  $\vec{E}$  is nearly equal to  $\vec{E}_1$  (or  $\vec{E}_2$ ). At points equidistant from 1 and 2,  $\vec{E} = \vec{E}_1 + \vec{E}_2$  is the sum of two vectors with equal magnitude.  
c.



16. a.  $\vec{F} = \vec{E}q = (-3 \times 10^5 \text{ N/C}) \hat{x} (-2 \times 10^{-7} \text{ C}) = 6 \times 10^{-2} \text{ N } \hat{x}$ ;  
 $|F| = k_e |q_1| |q_2| / R^2 = (9 \times 10^9)(3 \times 10^{-8}) \times (2 \times 10^{-7}) / (0.03)^2 \text{ N} = 6 \times 10^{-2} \text{ N}$ ,  $\Rightarrow \vec{F} = 6 \times 10^{-2} \text{ N } \hat{x}$ .  
b. (i) and (iii) describe electric field, (ii) describes electric force.
17. a. at  $P_1$ ,  $E = 2\pi k_e \sigma$ ; at  $P_2$ ,  $E = k_e Q/R^2$   
b. same  
c. smaller
18. a. smaller, sphere,  $(7 \text{ or } 8) \times 10^5 \text{ N/C}$  *downward*; larger sphere,  $(7 \text{ or } 8) \times 10^5 \text{ N/C}$  *upward*  
b. zero
19.  $3.4 \times 10^5 \text{ N/C } \hat{x}$
20. a.  $\text{C/m}^2$ ; number; +, -, 0;  $\sigma$

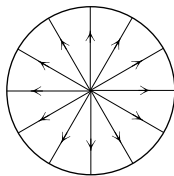
- b.  $5 \times 10^{-5} \text{ C/m}^2$
21. zero
22. a. upper,  $2 \times 10^{-5} \text{ C/m}^2$ ; lower,  $-2 \times 10^{-5} \text{ C/m}^2$
- b. upper,  $1 \times 10^6 \text{ N/C}$  downward; lower,  $1 \times 10^6 \text{ N/C}$  downward; total field,  $2 \times 10^6 \text{ N/C}$  downward
- c.  $(4 \text{ or } 5) \times 10^{-13} \text{ N}$  upward, upward
23. a. At A:  $\vec{E}_1$ , left;  $\vec{E}_2$ , right;  $\vec{E}$ , zero.  
At B:  $\vec{E}_1$ , right;  $\vec{E}_2$ , right;  $\vec{E}$ , right.  
At C:  $\vec{E}_1$ , right;  $\vec{E}_2$ , left;  $\vec{E}$ , zero.
- b. At A,  $E = 0$ ; at B,  $E = 2E_0$ ; at C,  $E = 0$
24. a. on each proton,  $8 \times 10^{-13} \text{ N } \hat{x}$ ;  
on the electron,  $-8 \times 10^{-13} \text{ N } \hat{x}$
- b.  $\vec{F} = 8 \times 10^{-13} \text{ N } \hat{x}$
- c.  $Q = 2 \times 10^{-19} \text{ C}$
- d. yes
25. b. drawings (a) and (d) show an electric field which is not perpendicular to the plane at points very near this plane; drawing (c) shows the electric field due to a positively charged plane.
26.  $3 \times 10^{-6} \text{ C}$
27. a. 1, opposite to  $\hat{x}$ : 2, same direction as  $\hat{x}$
- b. force on 2
- c. along  $\hat{x}$
- d. larger
- e.  $\vec{F} = Q\vec{E}$  applies only when  $\vec{E}$  is uniform near system of charge  $Q$ .
28. A. neither
- B.  $E = k_e|Q|/R^2$
- C.  $E = 2\pi k_e|c|$
- D. neither
- E. neither
- F. probably  $E = k_e|Q|/R^2$ , if the small object shown can be regarded as a particle. (If wrong, go to tutorial frame [s-11].)

29. a. due to each particle;  $k_e q/R^2$ ; due to 1, direction is from P, away from 1; due to 2, direction is from P, towards 2
- b.  $E/E_1 = d/R$ , thus  $E = k_e qd/R^3$
- c.  $E' = (1/27)E$
- d.  $E' = (1/9)E$
30. a. zero, no.
- b. yes, clockwise, i.e., the dipole will tend to become parallel to  $\vec{E}$ .
51. b. If wrong, go to Suggestion [s-8].
- 52.



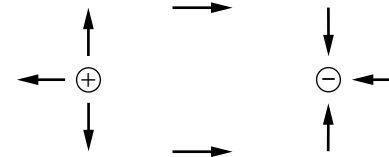
53. a.  $E_1 = k_e q_1/r_1^2$ ,  $E_2 = k_e q_2/r_2^2$
- b.  $q_2 = 3q_1$  (because  $d_2 = 3d_1$ ),  $r_2 = 3r_1$ ,  $\vec{E}_2 = (1/3)\vec{E}_1$ .
- c.  $\vec{E}_{2, \text{total}} = (1/3)\vec{E}_{1, \text{total}}$
- d.  $\vec{E}_{2, \text{total}} = (1/5)\vec{E}_{1, \text{total}}$
- e.  $\vec{E}$  is proportional to  $1/h$ .
54. a. must be different
- b. must be the same
- c. according to information in this problem, might be the same or different. [However, Unit 417 indicates that a fluid always exerts *different* pressures at different heights above the earth's surface.]
55. a.
- |        | Force     | Charge     | Elec. field |
|--------|-----------|------------|-------------|
| Kind   | vector    | number     | vector      |
| Signs  |           | +, -, 0    |             |
| Unit   | newton(N) | coulomb(C) | N/C         |
| Symbol | $\vec{F}$ | $q$        | $\vec{E}$   |
- b.  $0.01 \text{ N}$ ,  $1 \times 10^{-8} \text{ C}$

- c.  $1 \times 10^6 \text{ N/C}$
56. a.  $\sigma = Q/A = Q/(4\pi R^2)$   
 b.  $\vec{E} = k_e Q/R^2$  upward =  $4\pi k_e \sigma$  upward  
 c.  $\vec{E} = 2\pi k_e \sigma$  upward  
 d.  $2\pi k_e \sigma$  upward
57. a. In each case, the sides of one angle are perpendicular to the sides of another angle.  
 b. Because the component of  $\vec{E}$  perpendicular to this line is zero.  
 c.  $k_e \lambda l \cos \theta / r^2$ ,  $k_e \lambda l_0 / (r_0 r)$ ,  $k_e \lambda l_0 \cos \theta / (r_0 h)$ ,  $k_e \lambda l' / (r_0 h)$   
 e. Since the entire diameter is  $2r_0$ , this field has a magnitude  $2r_0$  times the preceding answer:  $2k_e \lambda / h$  upward
58. a.  $Q = E_0 R^2 / k_e$   
 b.  $3 \times 10^{-6} \text{ C}$   
 c. Using a sphere with larger radius or placing it in a gas with a higher breakdown strength.
59. At A, cannot be found because we don't know the charge of a *particle* at A. At B,  $\vec{E} = 2.0 \times 10^6 \text{ N/C } \hat{x}$ .
60. a.  $2\pi k_e \sigma$  downward  
 b.  $2\pi k_e \sigma$  upward  
 c. zero
- 61.



62. a. At A,  $\vec{E} = 190 \text{ N/C}$ , with direction e. At B,  $\vec{E} = 42 \text{ N/C}$ , with direction a. At C,  $\vec{E} = 120 \text{ N/C}$ , with direction a.

b.



63. field, c; force, a ; charge, b
64. a. the lower plate, the suspending arm of the scale, and the earth (through the gravitational force)  
 b.  $2\pi k_e Q/A$  downward  
 c.  $a(Q/A)$   
 d.  $\vec{F} = q\vec{E} = 2k_e \pi a Q^2 / A^2$  downward  
 e. The answer to part (d) multiplied by the number  $(A/a)$  of small squares on the plate,  $2\pi k_e Q^2 / A$  downward  
 f.  $Q = \sqrt{(m - M)gA / (2\pi k_e)}$
65. a. away from a positively charged line and towards a negatively charged line  
 b.  $E = (\text{constant with no unit}) k_e |\lambda| / h$
66. a.  $2.4 \times 10^{-6} \text{ C}$   
 b.  $12 \times 10^{-6} \text{ C/m}^2$   
 c.  $1.3 \times 10^{-6} \text{ C/m}^2$
67. a.  $E = 4\pi k_e |\sigma|$   
 b.  $3 \times 10^{-5} \text{ C/m}^2$ ,  $4 \times 10^{-4} \text{ C/m}^2$
68. a.  $E_2 = E_1 / 16$   
 b.  $E_B = 4E_2$
69. a. larger  
 b. same  
 c. smaller  
 d. same
70. a.  $E$ ,  $(\text{kg m}) / (\text{s}^2 \text{ C})$ ;  $|\sigma|$ ,  $\text{C/m}^2$ ;  $h$ , m;  $k_e$ ,  $\text{kg m}^3 / (\text{s}^2 \text{ C}^2)$   
 b. left side,  $(\text{kg m}) / (\text{s}^2 \text{ C})$ ; right side,  $[(\text{kg m}^3) / (\text{s}^2 \text{ C}^2)] \cdot (\text{C/m}^2) = \text{kg m} / (\text{s}^2 \text{ C})$ .

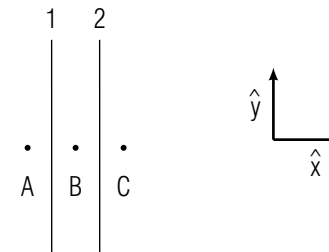
## MODEL EXAM

## USEFUL INFORMATION

$$k_e = 9.0 \times 10^9 \text{ newton meter}^2/\text{coulomb}^2$$

- c. For none of these expressions is the unit of the right side correctly equal to the unit of  $E$ .
71. a. ions, right; electrons, left  
b. ions, right; electrons, left

1. **Relations between field and force at a point.** A particle 1 with charge  $q_1 = 2 \times 10^{-8} \text{ C}$ , placed at a point  $P$  in space, experiences an upward force of magnitude  $F_1 = 3 \times 10^{-6} \text{ N}$ . Particle 1 is removed, and replaced by a particle 2 with charge  $q_2 = -4 \times 10^{-8} \text{ C}$ .
- a. Is the force  $\vec{F}_2$  experienced by 2 different from  $\vec{F}_1$ ? If so, what is  $\vec{F}_2$ ?
- b. Is the electric field at  $P$  in the second case different in any way from the field at  $P$  in the first case? If so, express  $\vec{E}_2$  as a number times  $\vec{E}_1$ .
2. **Fields due to one and two uniformly charged planes.** At a point 2 cm from a uniformly charged plane, and near its center, the electric field is directed toward the plane and has a magnitude of  $9 \times 10^5 \text{ N/C}$ . The area of the plane is  $0.4 \text{ m}^2$ .
- a. What is the total charge on the plane?
- b. Now a second plane, of the same size and shape as the first and identically charged, is placed 1 cm away from the other plane as shown in the figure below. What is the electric field  $\vec{E}$  at each of the points  $A$ ,  $B$ , and  $C$  indicated in the figure? Use the unit vectors  $\hat{x}$  and  $\hat{y}$  as appropriate.



3. **Force on a dipole near a charged sphere.** The figure below shows a dipole near a uniformly-charged sphere.



Determine which of these answers correctly gives the direction of the force on the dipole:

- (a) toward the center of the sphere
- (b) away from the center of the sphere
- (c) the force is zero

**Brief Answers:**

- 1. a. yes,  $6 \times 10^{-6}$  N downward  
b. no
- 2. a.  $-6 \times 10^{-6}$  C  
b. at  $A$ :  $1.8 \times 10^6$  N/C  $\hat{x}$   
at  $B$ : 0  
at  $C$ :  $-1.8 \times 10^6$  N/C  $\hat{x}$
- 3. (b)

