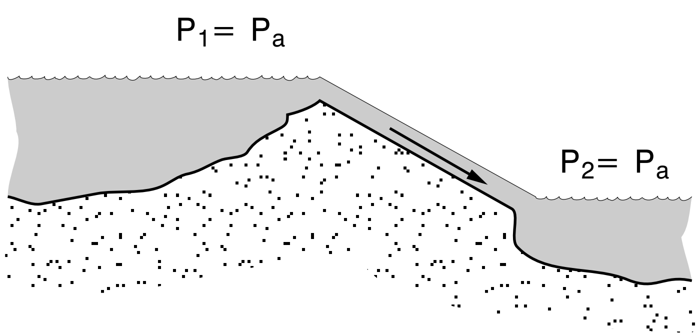
MISN-0-418

FLUIDS IN MOTION



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

FLUIDS IN MOTION

by

F. Reif, G. Brackett and J. Larkin

CONTENTS

Title: **Fluids in Motion**

Author: F. Reif, G. Brackett, and J. Larkin, Department of Physics, University of California, Berkeley.

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Input Skills:

1. Vocabulary: density, pressure (MISN-0-417).
2. State the conservation of energy principle for macroscopic and internal energy (MISN-0-416).

Output Skills (Knowledge):

K1. Vocabulary: mass current, steady state.

K2. State the equation relating mass current and flow velocity.

K3. State the steady-state condition for mass flow.

K4. State the equation for work done on a fluid.

K5. Write an equation to describe dissipative flow in a uniform horizontal tube.

K6. State Bernoulli's equation.

Output Skills (Problem Solving):

S1. Solve problems using: (a) the definition of of mass current; (b) $I = \rho A v$; (c) the steady-state condition; and (d) $I = (p_1 - p_2)/R$.

S2. For an incompressible fluid, entering and leaving a closed region by several channels, apply the steady-state condition to relate the average fluid speeds in these channels and quantities describing the cross-sectional areas of these channels.

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FLUIDS IN MOTION

Abstract:
Now that we have discussed fluids at rest, we can turn our attention to situations where fluids are in motion, e.g., to situations where water flows through pipes or blood flows through arteries.

SECT.

A FLUID CURRENT

DEFINITION OF CURRENT

Consider a fluid which is moving, e.g., water flowing through a pipe. Then fluid passes from one region to another through the surface separating these regions. (See Fig. A-1.) To distinguish between the two regions, let us label one side of the surface by an arrow pointing into one of the two regions. Suppose that, during some small enough time interval dt, a small amount of fluid of mass m passes through this surface. If the fluid flows *into* the region labeled by the arrow, the resulting change in the mass of the fluid in this region is $dM = +m$. If the fluid flows *out* of the region designated by the arrow, the resulting change in the mass of the fluid in this region is $dM = -m$. The *rate* at which the fluid passes through the surface toward the side labeled by the arrow can then be described by the "mass flow rate" or "mass current" I defined as follows:

Def. **Mass current**: $$I = \frac{dM}{dt} \qquad \text{(A-1)}$$

Thus the magnitude $|I| = |dM/dt|$ of the mass current describes the mass of the fluid passing through the surface per unit time. The sign of the current into the region labeled by the arrow is positive if fluid flows through the surface into this region, but is negative if fluid flows out of this region (i.e., if fluid flows opposite to the direction of the arrow).

By its definition, Def. (A-1), the current is a *number*. The "flow direction" of the current does not specify the direction of a vector, but specifies merely whether the current is flowing into one or the other of the two regions separated by the surface.

Sometimes it is convenient to define the "volume current" $I' = dV/dt$ which describes the volume of fluid passing through a surface per unit time. Since the mass of the fluid is related to its volume by its density ρ, $dM = \rho dV$ so that $I = \rho I'$. (However, it is usually easier to consider the mass of the fluid rather than its volume, since a given number of molecules of fluid always corresponds to a fixed mass of fluid, but corresponds to a fixed volume only if the fluid is incompressible.)

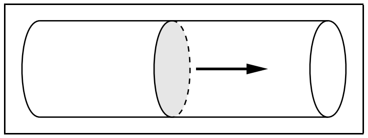

Fig. A-1: Passage of a fluid through a surface separating two regions.

RELATION BETWEEN CURRENT AND FLOW VELOCITY

The flow velocity of a fluid at a point is the average velocity of the particles of the fluid at this point. Suppose that all the particles of the fluid move through a flat surface of area A with the *same* average velocity $\vec{v}$ perpendicular to this surface. What then is the mass current I flowing through this surface?

During a small time interval dt, every particle of the fluid moves then through a distance vdt in the direction of the velocity perpendicular to the surface. Hence any particle in the fluid, which is at a distance less than vdt behind the surface, moves through this surface; but any particle, which is at a distance greater than vdt behind the surface, does not reach the surface and thus does not pass through it. (See Fig. A-2.) Hence the particles which pass through the surface in the time interval dt are all those contained in a length vdt behind the surface of area A, i.e., all those contained in the cylinder of length vdt and area A. The volume of this cylinder is $A(vdt)$. The mass dM of fluid contained in this cylinder (i.e., the mass of fluid passing through the surface in the time dt) is then obtained by multiplying the density ρ of the fluid by the volume of this cylinder. In other words, $dM = \rho(Avdt)$. The mass current $I = dM/dt$ passing through this surface is then equal to $I = (\rho Avdt)/dt$ or

$$I = \rho A v \qquad \text{(A-2)}$$

The flow direction of the current is, of course, into the region toward which the fluid particles move through the surface as a result of their

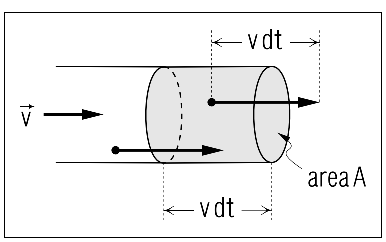

Fig. A-2: Displacement of fluid particles moving during a time dt with a velocity $\vec{v}$ perpendicular to a flat surface of area A.

velocity.

The result Eq. (A-2) is very reasonable since it states that the magnitude of the mass current I through a surface is larger if the area A of this surface is larger and if the speed v of the moving fluid is larger.

Example A-1: Speed of blood flow in the human aorta

About 5 liter of blood, having a mass of about 5 kg, is pumped from the human heart through the aorta every minute. This means that the average mass current I of blood passing through a cross-sectional area of the aorta is

$$I = 5\,\text{kg}/60\,\text{s} = 8 \times 10^{-2}\,\text{kg/s}\,.*$$

* This is the *average* value of the current, since the magnitude of the current fluctuates during the heart cycle.

The radius r of the interior of the aorta is about 1 cm and the density of blood is approximately $10^3\,\text{kg/m}^3$. What then is the average speed v of blood passing through the aorta?

The cross-sectional area A of the aorta is:

$$A = \pi r^2 = 3(1\,\text{cm}^3) = 3(10^{-2}\,\text{m})^2 = 3 \times 10^{-4}\,\text{m}^2\,.$$

Hence we can solve the relation (A-2) for the speed v of the blood in the aorta. Thus we find

$$v = \frac{I}{\rho A} = \frac{8 \times 10^{-2}\,\text{kg/s}}{(10^3\,\text{kg/m}^3)(3 \times 10^{-4}\,\text{m}^2)} = 0.3\,\text{m/s}$$

Understanding the Definition of Mass Current (Cap. 1a)

A-1 *Example:* When the left ventricle of the heart expels blood into the aorta, about 80 gram $= 8 \times 10^{-2}$ kg of blood passes through the aortic valve into the aorta during the small enough time of 0.2 second. What is the corresponding mass current of blood through the aortic valve into the aorta? This value is the maximum current $I_{\max}$ of blood passing through the aorta. *(Answer: 105)*

A-2 *Meaning of dM:* Consider a water-filled region between two imaginary surfaces 1 and 2 which cut across the interior of a pipe carrying steadily-flowing water (Fig. A-3). During a small enough time of

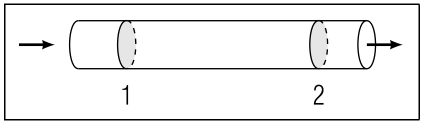

Fig. A-3.

2 second, 4 kg of water flows through surface 1 *into* this region, and 4 kg of water flows through surface 2 *out of* this region. (a) During this time, what are the changes in the mass of water contained in this region due to water flowing through surface 1, through surface 2, and through *both* surfaces? (b) What are the mass currents of water into this region through surface 1, through surface 2, and through *both* surfaces? *(Answer: 108) (Suggestion: [s-4])*

A-3 *Relating quantities:* A typical gas pump delivers gasoline through the hose nozzle with a constant mass current of 0.20 kg/s. (a) What is the mass of the gasoline passing through the nozzle in 1.0 minute? (b) A typical automobile gas tank can hold about 60 kg of gasoline. What is the time required for this mass of gasoline to pass through the nozzle? *(Answer: 101)*

Many problems in this unit concern blood circulation and water flow. Unless stated otherwise, use the value $1.0 \times 10^3\,\text{kg/m}^3$ for the densities of both blood and water.

Knowing About the Relation between Mass and Volume Currents

A-4 (a) What is the average *volume* current I' of blood in the aorta? Use the average mass current $I = 8 \times 10^{-2}\,\text{kg/s}$ found in text example A-1, and express your answer in terms of cm^3/s. (b) When a person inhales, the volume current of air into the lungs is about $5.0 \times 10^{-4}\,\text{m}^3/\text{s}$. What is the mass current of air into the person's lungs? The density of the air is $1.2\,\text{kg/m}^3$. *(Answer: 104)*

Understanding the Relation $I = \rho A v$ (Cap. 1b)

A-5 *Example:* The radius of a capillary is about 4×10^{-6} meter (4 micron), and the average speed of the blood in a capillary is about $0.4\,\text{mm/s} = 4 \times 10^{-4}\,\text{m/s}$. (a) What is the area of the circular cross-section of a capillary? What is the magnitude of the mass current of blood in a capillary (i.e., through a cross-section of a capillary)? (b)

Review: Assume that this mass current is roughly constant. What is the time required for $1\,\text{cm}^3$ of blood having a mass of $1\,\text{gram} = 1 \times 10^{-3}\,\text{kg}$ to pass through a cross-section of a capillary? *(Answer: 110)*

A-6 *Relating quantities:* The average mass current of blood in the vena cava (the large vein leading into the heart) is $8.0 \times 10^{-2}\,\text{kg/s}$, or about equal to that in the aorta. The radius of the inside of the vena cava is 1.2 cm, or about 20 percent larger than that of the aorta. What is the average speed of the blood in the vena cava? Is this speed larger or smaller than the average speed of 0.3 m/s of the blood in the aorta? *(Answer: 102)*

A-7 *Dependence:* The mass current of oil in a certain pipeline #1 is 50 kg/s. Determine the mass current of oil in each of these pipelines: (a) A pipeline having the same cross-sectional area as pipeline #1, but which carries oil having twice the average speed of the oil in pipeline #1. (b) A pipeline having twice the cross-sectional area of pipeline #1, but which carries oil having the same average speed as the oil in pipeline #1. (c) A pipeline having half the cross-sectional area of pipeline #1, but which carries oil having twice the average speed of the oil in pipeline #1. *(Answer: 107) (Practice: [p-1])*

SECT.

B STEADY-STATE FLOW

Consider the flow of water in a garden hose. When the water is first turned on, the situation is quite complicated as the water gradually fills the hose and then emerges from it. But after a while, a much simpler situation is reached where the amount of water in any part of the hose remains unchanged and the current of water flowing through any cross-sectional area of the pipe remains unchanged. Such a simple situation is called a "steady state" according to this definition:

Def. **Steady state**: A situation where the macroscopic properties of all parts of a system remain unchanged. (B-1)

Suppose that a moving fluid is in a steady state. Then the mass of fluid contained in any region remains unchanged. Hence the mass of fluid entering this region during any small time dt must be equal to the mass of fluid leaving the region during this time. *

* In a steady state, the number of fluid particles in in any region remains unchanged. Hence the number of such particles entering the region must be equal to the number of such particles leaving this region. But since this number of particles has an unchanging mass, the mass of fluid entering the region must correspondingly be equal to the mass of fluid leaving the region.

Correspondingly, the current $I_{\rm in}$ flowing into this region must be equal to the current $I_{\rm out}$ flowing out of the region. Thus we arrive at this conclusion, applicable to any region:

$$\boxed{\text{Steady state condition: } I_{\rm in} = I_{\rm out}} \qquad \text{(B-2)}$$

Current may, of course, pass through several portions of the surface enclosing the region of interest. Then $I_{\rm in}$ is the sum of all the currents flowing into the region through several portions of the surface and $I_{\rm out}$ is the sum of the currents flowing out of the region through several other portions of the surface.

As an example, consider a fluid flowing through a tube whose cross-sectional surfaces (perpendicular to the tube) may have different

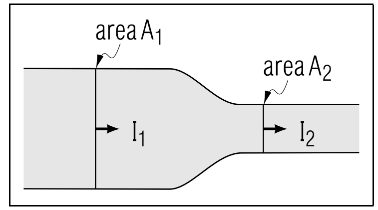

Fig. B-1: Flow of a fluid in a tube of varying cross-sectional area.

areas A_1 and A_2 at two different places. (See Fig. B-1.) In a steady state the current I_1 flowing through the first surface into the region between the two surfaces must then be equal to the current I_2 flowing through the second surface out of this region. Thus

$$I_1 = I_2 \qquad \text{(B-3)}$$

Suppose that the fluid at the first surface of area A_1 has a density ρ_1 and a flow velocity $\vec{v}_1$ (perpendicular to this surface and the same at all of its points). Similarly, suppose that the fluid at the second surface of area A_2 has a density ρ_2 and a flow velocity $\vec{v}_2$. Then we can use the relation (A-2) to express the steady-state condition, Rule (B-2), in the form

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \qquad \text{(B-4)}$$

In the special case where the fluid is an incompressible liquid, $\rho_1 = \rho_2$ and Eq. (B-4) becomes simply $A_1 v_1 = A_2 v_2$. This means that the product Av of the cross-sectional area A multiplied by the speed v of the liquid is everywhere the same. In particular, if the cross-sectional area of the tube is everywhere the same, the speed of the fluid in the tube must be everywhere the same. On the other hand, if the cross-sectional area A of the tube is smaller at some place, then the speed v of the liquid in the tube must there be correspondingly larger (thus assuring that the same amount of fluid passes through the smaller area).

For example, when water emerges from a narrow nozzle at the end of a garden hose, the speed of the water emerging through the nozzle is much larger than the speed of the water inside the hose of larger cross-sectional area. As another familiar example, the speed of water flowing in a mountain river is larger where the cross-sectional area of the river channel is smaller (e.g., in a narrow channel between rocks or at places where the river is shallow).

Consider a junction between a tube 1 and two other tubes 2 and 3. (See Fig. B-2.) In a steady-state the mass of fluid in the junction

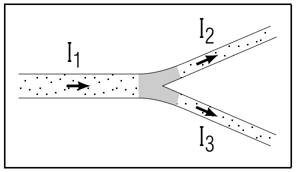

Fig. B-2: Flow of fluid through a junction connecting several tubes.

region (indicated shaded in Fig. B-2) must remain unchanged. Hence the current I_1 flowing into this region from tube 1 must be equal to the current flowing out of this region, i.e., to the sum of the currents I_1 and I_2 flowing out of this region through the tubes 2 and 3. Thus the steady-state condition, Rule (B-2), implies simply that

$$I_1 = I_2 + I_3 \tag{B-5}$$

Example B-1: Speed of blood flowing in capillaries

The main artery (aorta) finally branches out into a large number N of narrow capillaries (where $N \approx 10^9$). Suppose that $I_{\rm c}$ is the mass current of blood in each capillary. In the steady state the mass current $I_{\rm a}$ flowing into the capillaries from the aorta must then be equal to the mass current $NI_{\rm c}$ flowing out of *all* the N capillaries. Hence

$$I_{\rm a} = NI_{\rm c}$$

or

$$I_c = \frac{I_{\rm a}}{N} \approx 10^{-9}\, I_{\rm a}$$

Thus the current of blood in a capillary is very much smaller than that in the aorta.

We know from Example A-1 that the mass current of blood in the aorta is $I_{\rm a} = 8 \times 10^{-2}\,{\rm kg/s}$. Hence the mass current of blood in a capillary is about $I_c = 8 \times 10^{-11}\,{\rm kg/s}$. But by Eq. (A-2), $I_c = \rho A_c v_c$ where $\rho = 10^3\,{\rm kg/m^3}$ is the density of blood, A_c is the cross-sectional area of a capillary, and v_c is the speed of blood in a capillary. Since the inner diameter d_c of a capillary is $d_c \approx 10^{-5}\,{\rm m}$, its cross-sectional area is $A_c = \pi(d_c/2)^2 = 8 \times 10^{-11}\,{\rm m^2}$. Hence we can use this information to find the speed v_c of blood flowing in a capillary. Thus we find $v_c = I_c/(\rho A_c) \approx 10^{-3}\,{\rm m/s}$ or 1 millimeter/second. This is much smaller than the speed $v_a = 0.3\,{\rm m/s}$ of blood of in the aorta as found in Example A-1).

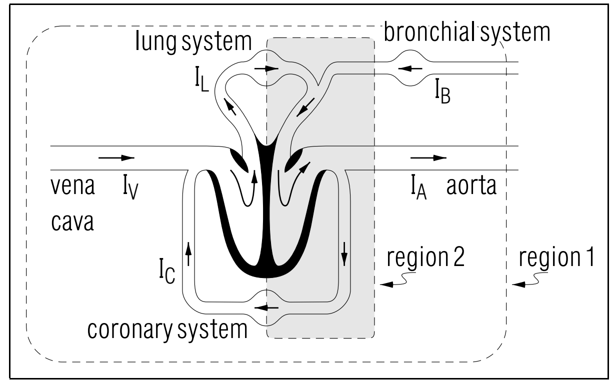

Fig. B-3.

Understanding the Steady State Condition (Cap. 1c)

In blood circulation, there is no net accumulation of blood in any part of the circulatory system over long time periods (i.e., over many heart cycles). Thus we can apply the steady state condition to relate *average* currents in blood vessels.

B-1 *Statement:* Figure B-3 shows schematically the arrangement of the blood circulation channels near the human heart. Each channel is labeled with a symbol for the magnitude of the average current in the channel, and the arrows indicate the direction of blood flow in each channel. (The globular regions represent capillary networks or "beds.") (a) Using the symbols provided, write the steady state condition for each of the two regions outlined by dotted lines in the figure. (b) *Example:* The current I_V through the vena cava is $8.0 \times 10^{-2}\,{\rm kg/s}$, while the current I_L through the lung circulatory system is $8.4 \times 10^{-2}\,{\rm kg/s}$. By relating these currents to the current I_C in the coronary circulatory system, find the value of I_C. *(Answer: 103) (Suggestion: [s-7])*

B-2 *Properties:* What are the possible signs of the quantities appearing in a statement of the steady-state condition, such as text Eq. (B-5)? *(Answer: 106)*

B-3 Figure B-4 shows the blood circulation channels between an artery A and a vein V in an extremity (e.g., a finger). Blood flows from the artery to the vein through the capillary bed C and also through an alternate channel called an "arteriovenous anastomosis" or AVA. Let us call $I_{\rm A}$, $I_{\rm V}$, $I_{\rm C}$, and $I_{\rm AVA}$ the magnitudes of the average currents in these channels. (a) *Relating quantities:* Express the current

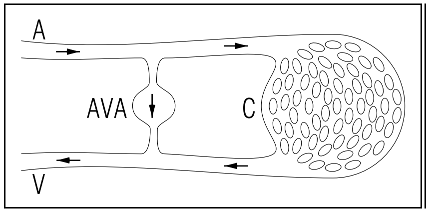

Fig. B-4.

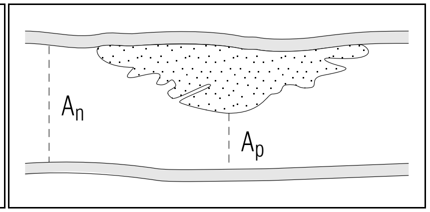

Fig. B-5.

I_C in terms of the currents I_A and I_{AVA}. (b) *Dependence:* Some current normally flows in the AVA. But when the extremity becomes cold enough, the AVA closes abruptly, reducing the current I_{AVA} to zero. If the current I_A in the artery remains constant, what happens to the current I_C of heat-carrying blood in the capillary bed? (In fact, I_A decreases slightly but the result is the same.) *(Answer: 112) (Practice: [p-2])*

Applying the Steady State Condition (Cap. 2)

B-4 Lesions called "atherosclerotic plaques" sometimes develop on the interior of an artery, thus narrowing the blood flow channel. The cross-sectional area A_p of the artery near the plaque is thus smaller than the cross-sectional area A_n of a normal part of the artery (see Fig. B-5). Let us call I_p and I_n the magnitudes of the average blood currents through these areas, and v_p and v_n the average speeds of the blood at these areas. (a) Write an equation relating the currents I_p and I_n. (b) Write an equation relating the average speeds v_p and v_n. Is the blood speed v_p near the plaque larger than, equal to, or smaller than the normal blood speed v_n? (c) Suppose the plaque reduces the cross-sectional area of the artery to one-fifth its normal value, so that $A_p = A_n/5$. For this situation, express v_p as a number times v_n. *(Answer: 109) (Suggestion: [s-3])*

B-5 A cylindrical fire hose has an inside diameter of 10 cm, while the hole at the end of the fire hose nozzle has a diameter of 2.0 cm. In the steady state, water flows in the hose with an average speed of 1.0 m/s. What is the average speed of the water emerging from the nozzle? *(Answer: 114) (Suggestion: [s-5])*

B-6 A garden hose having an inside radius of 1.0 cm is connected to a simple sprinkler, which consists of a hollow metal enclosure with 20 identical holes drilled in the top. In the steady state, the average speed of the water in the hose is 0.40 m/s. What is the radius of the

holes in the sprinkler if the water emerges from each hole with an average speed of 8.0 m/s? *(Answer: 111) (Suggestion: [s-2]) More practice for this Capability: [p-3], [p-4]*

SECT.

C WORK DONE BY PRESSURE

The pressure forces acting on a fluid can do work on the fluid when it moves. To find this work, consider a fluid separated from another system by a flat boundary surface of area A. (For example, Fig. C-1 shows such a fluid enclosed in a cylinder and in contact with the flat surface of a movable piston.) If the pressure at the boundary surface is p, the pressure force exerted *on* the fluid by this boundary has a magnitude $F = pA$ and is in the direction perpendicular to the boundary toward the inside of the fluid. Suppose then that the boundary moves by a distance L in a direction perpendicular to the boundary while the pressure force on the boundary remains constant. The work W done *on* the fluid by the system on the other side of the boundary has the magnitude $FL = (pA)L = pV_s$, where $V_s = AL$ is the volume swept out by the boundary as it moves through the distance L. Thus the work W done *on* the fluid is,

$$\boxed{\text{if } p \text{ is constant, } W = \pm pV_s} \qquad \text{(C-1)}$$

where the plus sign is applicable if the boundary moves toward the inside the fluid (i.e., along the direction of the pressure force on the fluid), and where the minus sign is applicable if the boundary moves toward the outside of the fluid (i.e., opposite to the direction of the pressure force on the fluid).

REMARK

If the entire surface bounding the fluid remains fixed except for the motion of the flat boundary surface considered in Eq. (C-1), the volume V_s swept out by this boundary is related to the change ΔV of the volume of the fluid so that $\Delta V = V_s$ (with a minus sign if the boundary moves toward the inside of the fluid, and a plus sign if it moves toward the outside of the fluid). Hence Eq. (C-1) is then equivalent to the statement that the work W done *on* the fluid is:

$$\boxed{\text{if } p \text{ is constant, } W = -p\Delta V} \qquad \text{(C-2)}$$

Example C-1: Work done on the air inside a lung

The pressure on the air inside a lung has the *same* value p everywhere along the walls of the lung and remains nearly constant when the lung

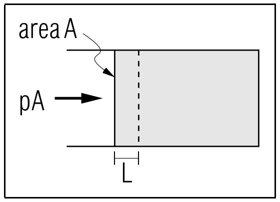

Fig. C-1: Work done by the pressure force acting on a moving boundary of a fluid.

contracts by a small amount. The work done on the air when a *small* part of the wall of the lung moves is then p multiplied by the volume swept out by this small part of the wall. Hence the *total* work done on the air when *all* parts of the wall move is just p multiplied by the *total* volume swept out by the entire wall of the lung (i.e., p multiplied by the magnitude of the volume change of the contracting lung).

WORK DONE ON A FLOWING LIQUID

Consider a tube (or some more complicated system) separated from an outside system by two surfaces S_1 and S_2. (See Fig. C-2.) An incompressible liquid of density ρ flows from the outside system into the tube through the surface S_1 at a pressure p_1; it then leaves the other side of the tube to enter the outside system through the surface S_2 at a pressure p_2. (For example, the outside system might consist of two water reservoirs connected by a pipe.) When the liquid flowing through the tube is in a steady state, the mass of the liquid in the tube remains constant. Hence, when some mass M of liquid enters the tube from the outside system through S_1, a corresponding mass M of liquid must leave the other side of the tube to enter the outside system through S_2. In this process, what is the work done by the outside system on the liquid originally in the tube?

Suppose that some mass M of liquid enters the tube through the surface S_1. Then the liquid boundary originally located at S_1 moves to

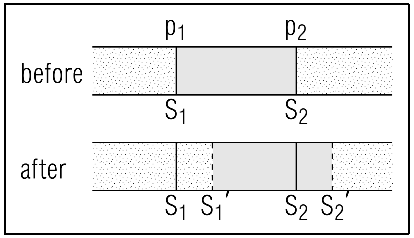

Fig. C-2: Work done on a portion of moving fluid.

some new surface S_1, sweeping out a volume $V_1 = M/\rho$. According to Eq. (C-1), the work done on the liquid by the outside system is then $p_1V_1 = p_1(M/\rho)$. In the preceding process, an equal mass M of liquid must leave the other end of the tube through the surface S_2. Then the liquid boundary originally located at S_2 moves to some new surface S_2', sweeping out a volume $V_2 = M/\rho$. The work done on the liquid by the outside system is then negative (since the liquid boundary moves in a direction *opposite* to the pressure force exerted on it by the outside system on the other side of S_2) and is equal to $-p_2V_2 = -p_2(M/\rho)$. Hence the total work W done on the liquid by the outside system is

$$W = p_1V_1 - p_2V_2 = p_1\frac{M}{\rho} - p_2\frac{M}{\rho} = (p_1 - p_2)\frac{M}{\rho} \qquad \text{(C-3)}$$

Example C-2: Work done on the blood pumped by the heart

Blood leaves the ventricle of the heart at some pressure p_1, flows into the circulatory system (consisting of the aorta, capillaries, and the main vein called the "vena cava"), and finally enters the auricle of the heart at some lower pressure p_2. The pressure difference $p_1 - p_2 = 100\,\text{mm-Hg} = 1.3\times10^4\,\text{N/m}^2$. We know from Example A-1 that every second about $8\times10^{-2}\,\text{kg}$ of blood leaves the ventricle of the heart. (Hence an equal mass of blood enters the auricle of the heart.) What is the work done by the heart on this mass of blood?

Because the density ρ of blood is approximately $1.0\times10^3\,\text{kg/m}^3$, the volume V of blood leaving the heart every second is $V = M/\rho = (8\times10^{-2}\,\text{kg})/(1.0\times10^3\,\text{kg/m}^3) = 8\times10^{-5}\,\text{m}^3$. Hence the relation (C-3) implies that the work done by the heart on this amount of blood is

$$W = (1.3\times10^4\,\text{N/m}^2)(8\times10^{-5}\,\text{m}^3) = 1\,\text{joule}$$

Since this work is done in one second, the average power delivered to the blood by the heart is then (1 joule)/(1 second) = 1 watt. *

> * The delivery of this power requires approximately 10 watt of power consumed by the heart muscle (i.e., the efficiency of the heart is only about 10 percent). These numbers can be compared with the approximately 100 watt of power supplied to a person by his daily food intake.

Knowing About Work Done by Pressure

C-1 The drug in a hypodermic syringe has a constant pressure of $2\times10^5\,\text{N/m}^2$ as the drug is injected. If the end of the syringe's plunger has an area of $0.8\,\text{cm}^2$ and moves a distance of 4 cm, what is the volume V_s swept out by the plunger? What is the work W done on the drug by the plunger? *(Answer: 118)*

C-2 Let us use the relation $W = -p\Delta V$ to find the work done on the air by the lungs during the breathing cycle. During inhalation and exhalation, the volume of air in the lungs increases and then decreases by an amount V_t called the "tidal volume." To estimate the work done, let us assume that the air in the lungs has a constant pressure p_i during inhalation and a constant pressure p_e during exhalation. (a) What is the *change* ΔV in the volume of air in the lungs during inhalation and during exhalation? Express your answers in terms of V_t. (b) What are the works W_i and W_e done on the air by the lungs during inhalation and exhalation? What then is the total work $W = W_i + W_e$ done during the entire breathing cycle? (c) Use the values $V_t = 5\times10^{-4}\,m^3$ and $(p_e - p_i) = 4\times10^2\,\text{N/m}^2$ to find the total work W. (d) The duration of the breathing cycle is about $T = 2$ second. What is the average power $P = W/T$ delivered to the air by the lungs during the breathing cycle? *(Answer: 116)*

C-3 Let us use the relation $W = (p_1 - p_2)(M/\rho)$ to find the work done on the water flowing steadily in a garden hose. The water flows from the faucet end of the hose where the water pressure is $1.2\times10^5\,\text{N/m}^2$ to the other end of the hose where the water pressure equals the atmospheric pressure of $1.0\times10^5\,\text{N/m}^2$. The magnitude of the constant water current in the hose is 0.5 kg/s. (a) *Review:* What is the mass of the water entering the hose at the faucet (or leaving the hose at the other end) in one minute? (b) What is the work done on the flowing water in one minute? *(Answer: 113)*

SECT.

D SIMPLE DISSIPATIVE FLOW

Consider the simple situation where an incompressible fluid (such as water) flows through a straight horizontal tube of constant cross-section. (See Fig. D-1.) Suppose that there were no dissipation of the macroscopic energy of the fluid in the tube into random internal energy. Then the macroscopic energy of the fluid in the tube would remain constant without requiring energy to be supplied from any outside system to which the tube is connected. Since the macroscopic gravitational potential energy of the fluid in a *horizontal* tube does not change, the macroscopic kinetic energy of the fluid would then remain constant and the fluid would simply keep moving along the tube with constant speed.

But in most situations the dissipation of macroscopic energy of a flowing fluid is appreciable. Unless macroscopic energy is supplied to the fluid in the tube from some outside system, the macroscopic energy of the fluid then gradually decreases while its random internal energy increases. The macroscopic kinetic energy of the fluid thus gradually approaches zero so that the fluid comes to rest.

Suppose, however, that the fluid in the tube is connected to an outside system from which it can gain energy (e.g., that it is connected by pipes to a pump). Then the random internal energy of the fluid in the tube can increase at the expense of the macroscopic energy received from the outside system, rather than at the expense of the macroscopic kinetic energy of the fluid in the tube. Thus the fluid in the tube can be kept flowing with constant speed despite the dissipation of macroscopic energy.

To maintain such a steady flow, the fluid in the tube must be acted on by some total force $\vec{F}$ due to the outside system. Suppose that the pressures at the two ends of the tube are p_1 and p_2. Then this force $\vec{F}$ (which is the vector sum of the oppositely directed pressure forces acting on the fluid at the two ends of the tube) is in the direction from the end at the higher pressure to the end at the lower pressure, and has a magnitude proportional to the pressure difference $p_1 - p_2$. Thus we expect that the direction of the fluid current I in the tube should be in the direction of the force $\vec{F}$, i.e., that the fluid should flow from the high-pressure end toward the low-pressure end of the tube. Furthermore the magnitude of the current I should be zero when the force $\vec{F} = 0$, and should increase with increasing magnitude of this force. In other words, $I = 0$ when $p_1 - p_2 = 0$

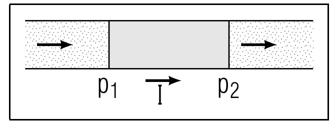

Fig. D-1: Steady flow of a fluid through a horizontal tube of constant cross-section.

and the magnitude of I should increase with increasing magnitude of the pressure difference. Thus the current I should depend on the pressure difference in the manner indicated qualitatively by the graph in Fig. D-2. This graph indicates properly that the direction of I (i.e., its sign) reverses when the pressure difference $p_1 - p_2$ has the opposite sign.]

ENERGY ARGUMENTS

The preceding statements can be justified by considering the energy transformations in the entire system consisting of the fluid in the tube plus the outside system connected to it. (The fluid current through the tube should depend only on the properties of the fluid in the tube, but not on the properties of the outside system. For simplicity, we may then assume that the outside system is such that its random internal energy remains unchanged.)

The conservation of energy implies that the increase of the random internal energy of the entire system is equal to the decrease in the total *macroscopic* energy of this system. But, if the fluid flows with *constant* speed in a *horizontal* tube, neither the macroscopic kinetic energy nor the macroscopic potential energy of the fluid in the tube changes. Hence the decrease in the macroscopic energy of the entire system is just due to the decrease in the macroscopic energy of the outside system, i.e., to the macroscopic work W done by this outside system on the fluid in the tube (since the random internal energy of the outside system is assumed to remain unchanged.) Suppose then that a mass M of fluid enters one end of the tube at a pressure p_1 and a corresponding mass M of fluid leaves the other end of the tube at a pressure p_2. Then the work W done

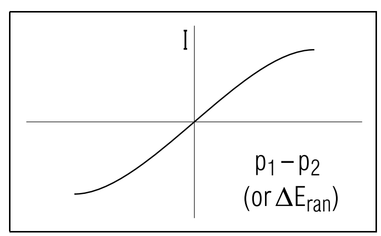

Fig. D-2: Qualitative graph showing how the fluid current in a tube varies with the pressure difference $p_1 - p_2$.

in this process is given by Eq. (C-3), so that the random internal energy of the entire system changes by the amount

$$\Delta E_{\rm ran} = (p_1 - p_2)\frac{M}{\rho} \qquad \text{(D-1)}$$

We know from Unit 416 that the random internal energy $E_{\rm ran}$ of the entire isolated system always tends to increase. Hence any small mass M of fluid tends to pass through the tube in such a direction that the change $\Delta E_{\rm ran}$ is positive, i.e., so that *positive* work W is done on the mass M of fluid. Thus this mass of fluid must pass through the tube from its high-pressure end to its low-pressure end, i.e., the mass current I of the fluid must flow in this direction. This will happen unless the entire system has attained the equilibrium situation where there is no net flow of fluid (so that $I = 0$), and where there is no change in the random internal energy (so that $\Delta E_{\rm ran} = 0$) if some small mass M of fluid passes through the tube. The preceding comments show that the magnitude of the current I is related to the change in random internal energy so that $I \neq 0$ when $\Delta E_{\rm ran} \neq 0$ and that $I = 0$ when $\Delta E_{\rm ran} = 0$. But according to Eq. (D-1), $E_{\rm ran}$ is proportional to the pressure difference $(p_1 - p_2)$ between the ends of the tube. Hence our preceding comments imply that $I \neq 0$ when $(p_1 - p_2) \neq 0$ and that $I = 0$ when $(p_1 - p_2) = 0$. Thus we arrive again at the conclusion that the current I must depend on the pressure difference $(p_1 - p_2)$ in the manner illustrated qualitatively by the graph in Fig. D-2.

Any smooth continuous curve is straight in a small enough region. Near the point where $p_1 - p_2 = 0$ and $I = 0$, the graph in Fig. D-2 must thus be nearly straight in the region where the magnitude of the pressure difference $p_1 - p_2$ is small enough. Accordingly, the current I must then be related to the pressure difference so that

$$\boxed{I = \left(\frac{1}{R}\right)(p_1 - p_2) = \frac{p_1 - p_2}{R}} \qquad \text{(D-4)}$$

where $(1/R)$ is a constant independent of I or $(p_1 - p_2)$. (Equivalently, this means that I is then simply proportional to the pressure difference.) The constant R is conventionally called the "resistance" of the tube and its reciprocal $(1/R)$ the "conductance" of the tube. The value of R depends on the properties of the tube and fluid under consideration. In practice, the relationship Eq. (D-4) is valid for a sufficiently large range of pressure differences to be of considerable practical utility.

The relation (D-4) implies properly that $I = 0$ if the pressure difference $p_{-}p_2 = 0$ Furthermore, Eq. (D-4) has been written with the understanding that the direction of positive current is from the end at the pressure p_1 to the end at the pressure p_2. Thus the current is properly in this direction if p_1 is larger than p_2, and is of opposite direction (i.e., of opposite sign) if p_1 is smaller than p_2 so that $p_1 - p_2$ is negative. The resistance R of the tube describes how large a current is produced by a pressure difference existing between the two ends of the tube. If the resistance R of the tube is large (i.e., if its conductance $1/R$ is small), a given pressure difference produces a small current through the tube. But if the resistance R of the tube is small (i.e., if its conductance $1/R$ is large), the same pressure difference produces a large current through the tube.

The resistance R of a tube depends on the dissipative properties of the fluid and on the dimensions of the tube. For example, for a tube of given dimensions, the resistance is larger if the fluid is a highly viscous (or "sticky") liquid such as molasses, than if the fluid is a fairly nonviscous liquid such as water. For a given fluid, the resistance of a longer tube is larger than that of a shorter tube. (Indeed, the resistance R is proportional to the length L of the tube. But the resistance R of a wider tube is smaller than that of a narrower tube. (Indeed, for a slowly flowing viscous fluid, R is related to the diameter D of the tube so that $R \propto 1/D^4$. Thus a tube having a diameter 2 times as large has a resistance smaller by a factor $1/2^4 = 1/16$).

Example D-1: Flow of water through a hose

Consider a horizontal garden hose connected at one end to a water faucet and open at the other end to the atmosphere. Then the pressure at one end of the hose is the fixed pressure of the water supply, while the pressure at its other end is the atmospheric pressure. Thus a fixed pressure difference $p_1 - p_2$ is maintained between the two ends of the hose. If the hose is long, its resistance is large and the fluid current (or water emerging from the hose per second) is smaller than if the hose is short. But if the hose has a large internal diameter, its resistance is small and the fluid current through the hose is considerably larger than if the hose has a small diameter.

Knowing About Resistance to Fluid Flow

D-1 (a) To obtain the largest possible water current in a fire hose, should one make the resistance of the hose as large or as small as

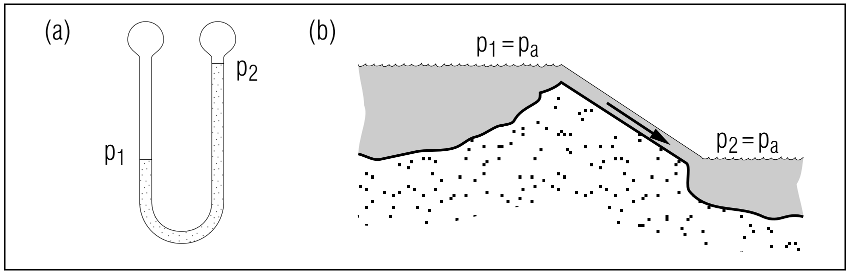

Fig. D-3.

possible? (b) Which of the following steps would *decrease* the resistance to water flow in fire hoses? 1. Making hoses larger in diameter. 2. Making hoses longer. 3. Chemically treating water used in fire-fighting to lower its viscosity. *(Answer: 120)*

Understanding ($I = (P_1 - P_2)/R$) (Cap. 1d)

D-2 *Statement and example:* Two water tanks A and B are connected by a horizontal pipe of length 10 meter and inside diameter 2 cm. The resistance of the pipe to water flow is $R = 2.5 \times 10^3\,\mathrm{m^{-1}\,s^{-1}}$. Let us call p_A and p_B the water pressures in tanks A and B at the two ends of the pipe. (a) Write an expression for the mass current I of water through the pipe in the direction from tank B to tank A. (b) Suppose that $p_A = 3 \times 10^5\,\mathrm{N/m^2}$. What is the magnitude of the current I and the direction of water flow if $p_B = 2 \times 10^5\,\mathrm{N/m^2}$, if $p_B = p_A$, and if $p_B = 4 \times 10^5\,\mathrm{N/m^2}$? *(Answer: 117)*

D-3 *Applicability:* Why does the relation $I = (p_1 - p_2)/R$ not apply to each of the following situations? (a) The U-tube manometer in Fig. D-3a contains a liquid at rest. Thus there is no current in the tube, although the liquid pressures p_1 and p_2 are different so that the pressure difference $(p_1 - p_2)$ is not zero. (b) The pipe in Fig. D-3b carries water between the surfaces of two large reservoirs. Thus there is a current in the pipe, although the water pressures at the end of the pipe are both equal to atmospheric pressure so that the pressure difference $(p_1 - p_2) = 0$. *(Answer: 122)*

D-4 *Relating quantities:* (a) Consider a 10 cm section of the horizontal aorta in a person who is lying down. The resistance of this section is about $80\,\mathrm{m^{-1}\,s^{-1}}$, and the average current of blood flowing away from the heart through this section is 8×10^{-2} kg/s. What is the magnitude of

the difference between the blood pressures at the ends of this section of aorta? Is the pressure at the end near the heart larger or smaller than the pressure at the other "down-stream" end? (b) During a blood transfusion, the horizontal needle inserted into the patient's vein carries a steady mass current of 1×10^{-4} kg/s (which corresponds to a volume current of about $0.1\,\mathrm{cm^3/s}$). The blood pressure in the supply tube at the entrance to the needle is $1.1 \times 10^5\,\mathrm{N/m^2}$, while the blood pressure in the vein at the end of the needle is $1.0 \times 10^5\,\mathrm{N/m^2}$. What is the needle's resistance to blood flow? *(Answer: 115)*

D-5 *Dependence:* A gardener wants to increase the current through the horizontal hose carrying water from his house to his garden. Which of these alternatives would increase the current, and which would decrease it? (The water pressure at the garden end of the hose remains equal to atmospheric pressure in all of these alternatives.) (a) Turning off the dishwasher in the house, which increases the water pressure at the house end of the hose. (b) Attaching a nozzle to the hose, which increases the resistance of the hose. (c) Replacing the hose with a shorter one. (d) Replacing the hose with one of smaller diameter. *(Answer: 119)* *(Practice: [p-5])*

SECT.

E GENERAL STEADY FLOW

Consider the general steady flow of an incompressible fluid flowing through a tube which is not necessarily horizontal and which is not necessarily of the same diameter along its length. (See Fig. E-1.) Then we can readily generalize the energy argument of the preceding section in order to discuss the dissipative steady flow of the fluid through such a tube connected to some outside system.

Consider again the situation where a mass M of incompressible fluid enters the tube at one end and an equal mass of fluid leaves the tube at the other end. The random internal energy $E_{\rm ran}$ of the entire isolated system changes then for several reasons. Part of the increase of this random energy is again due to the work $W = (p_1 - p_2)M/\rho$ done by the outside system on the fluid originally between the two ends of the tube (i.e., due to the decrease in potential energy of interaction between this fluid and the outside system connected to the tube). But, in addition, the random internal energy can now also increase because of the decrease in the macroscopic gravitational potential energy and the decrease in the macroscopic kinetic energy of the fluid originally in the tube. Indeed, when the mass M of fluid enters the tube at one height y_1 and an equal mass M of fluid leaves the tube at another height y_2, the macroscopic gravitational potential energy of the entire fluid decreases by an amount $Mgy_1 - Mgy_2 = Mg(y_1 - y_2)$.*

* If this quantity is negative, this negative decrease corresponds to an actual *increase* in gravitational potential energy.

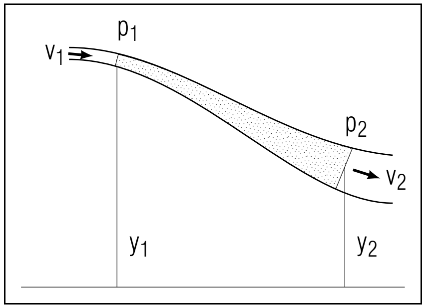

Fig. E-1: Steady flow of a fluid through a tube.

Furthermore, we know from Sec. B that, if the diameter of the tube is different at its two ends, the speed of the fluid is also different at the two ends. Thus, when a mass M of fluid enters the tube at one end with a speed v_1 and an equal mass M of fluid leaves the tube at its other end with a speed v_2, the kinetic energy of the entire fluid decreases by an amount $1/2Mv_1^2 - 1/2Mv_2^2 = 1/2M(v_1^2 - v_2^2)$. The increase in $\Delta E_{\rm ran}$ in the random internal energy of the entire isolated system is then, by conservation of energy, equal to the sum of the decreases in all these other forms of energy. Accordingly we can write:

$$\Delta E_{\rm ran} = (p_1 - p_2)\frac{M}{\rho} + Mg(y_1 - y_2) + \frac{1}{2}M(v_1^2 - v_2^2)$$

or

$$\Delta E_{\rm ran} = \frac{M}{\rho}\left[(p_1 - p_2) + \rho g(y_1 - y_2) + \frac{1}{2}\rho(v_1^2 - v_2^2)\right] \qquad \text{(E-1)}$$

Our energy argument of the preceding section then still holds with this more complicated form of the change of random internal energy. Thus the fluid current $I \neq 0$ if $\Delta E_{\rm ran} \neq 0$ and $I = 0$ if $\Delta E_{\rm ran} = 0$. If $\Delta E_{\rm ran}$ is not too large, the current I is then again proportional to $\Delta E_{\rm ran}$. Thus we can use the result Eq. (E-1) to write our previous result Eq. (D-4) in this more general form:

$$\boxed{I = \frac{1}{R}\left[(p_1 - p_2) + \rho g(y_1 - y_2) + \frac{1}{2}\rho(v_1^2 - v_2^2)\right]} \qquad \text{(E-2)}$$

Note that the fluid current through the tube depends now not only on the pressures at the two ends of the tube, but also on the heights of these two ends and on the speed of the fluid at these two ends.

STEADY FLOW WITH NEGLIGIBLE DISSIPATION

In the special case where the dissipation of the macroscopic energy of the fluid is negligible, its random internal energy remains constant so that $\Delta E_{\rm ran} = 0$. Hence Eq. (E-1) implies then that

$$(p_1 - p_2) + \rho g(y_1 - y_2) + \frac{1}{2}\rho(v_1^2 - v_2^2) = 0$$

Thus

$$p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

or

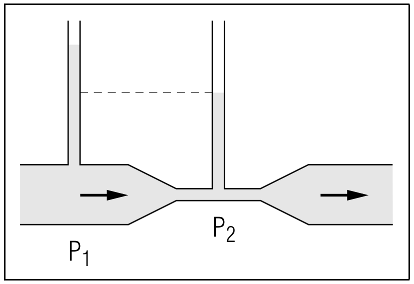

Fig. E-2: Measurement of the flow speed of a liquid from the difference in pressures in the pipe and in a narrow constriction.

$$\boxed{p + \rho g y + \frac{1}{2}\rho v^2 = \text{ constant}} \qquad \text{(E-3)}$$

This last statement (which is also true even if the fluid is compressible) is called "Bernoulli's principle." *

> * When there is no dissipation of macroscopic energy, I can be non-zero despite the fact that $\Delta E_{\text{ran}} = 0$. This result is compatible with Eq. (E-2) since then the resistance $R = 0$.

As a simple application of the relation (E-3), consider a fluid flowing in a steady state with negligible dissipation through a *horizontal* tube. Then the height y of the fluid remains constant and Eq. (E-3) implies simply that $p+(1/2)\rho v^2 = \text{constant}$. This means that, in a narrow section of the tube, where the speed of the fluid must be larger, the pressure in the fluid must be smaller. For example, suppose that an artery becomes narrowed somewhere as a result of fatty deposits on its inside wall. At this place the pressure inside the artery becomes then smaller, so that the artery can be more readily collapsed by the external forces on it. Thus, the narrowing of an artery tends to produce still further narrowing, a self-aggravating process which can ultimately lead to fatal consequences (such as a heart attack).

Example E-1: Measurement of flow speed of a liquid

To measure the speed v_1 of an incompressible liquid moving with negligible dissipation in a horizontal pipe, of cross-sectional area A_1, it is only necessary to connect to the pipe a small section of smaller cross-sectional area A_2. (This arrangement is called a "Venturi meter.")(See Fig. E-2.) In the steady state, the speed v_2 of the fluid in the narrow section is then such that $A_1 v_1 = A_2 v_2$, so that $v_2 = (A_1/A_2)v_1$ is larger than v_1. According to Eq. (E-3), we then find that

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

or

$$p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) \qquad \text{(E-4)}$$

so that p_1 is larger than p_2. The pressure difference $p_1 - p_2$ can readily be measured by noting the heights of the liquid in vertical tubes connected to the wide and narrow sections of the pipe (as shown in Fig. E-2.) From this measured pressure difference one can then use Eq. (E-4) to find the speed v_1 of the liquid in the pipe.

REMARK ON APPLICABILITY

All the arguments in this section apply to any tube, provided that no fluid flows through its sides. This tube might thus be a real tube (such as a copper tube) whose sides consist of solid material, or it might be an imaginary tube chosen within a fluid so that its sides are everywhere parallel to the velocity of the fluid (i.e., so that no fluid flows through the sides of this imaginary tube). By applying our arguments to such imaginary tubes in a fluid not necessarily enclosed in a real tube, all our results, including Eq. (E-3), can be applied to *any* fluid in a steady state.

Applying Bernoulli'S Principle In the Steady State

In the following problems, assume that the fluid described flows steadily with negligible dissipation so that both Bernoulli's principle and the steady state condition apply.

E-1 Water flows from the city mains through a fire hose whose nozzle directs the stream of water vertically upward. What water pressure p_m is required in the mains if the water is to reach a height of 30 meter above the mains? The speed of the water in the mains is negligible, and the water pressure at the top of the stream (where the water is momentarily at rest) is equal to the atmospheric pressure of $1.0 \times 10^5\,\text{N/m}^2$. *(Answer: 124) (Suggestion: [s-6])*

E-2 Consider an atherosclerotic plaque in a horizontal artery so that all points in this artery have about the same height. Suppose that the plaque reduces the diameter of the circular cross-section of the bloodstream to one-third the normal value. (a) If the average speed of the blood in the normal part of the artery is 0.30 m/s, what is the average speed of the blood near the plaque? (b) What is the difference between the blood pressure in the normal part of the artery and the blood pressure near the plaque? *(Answer: 121)*

E-3 Consider the "Venturi meter" described in text example E-1. Let us show how the speed v_1 of the liquid in the main pipe can be found from the known cross-sectional areas A_1 and A_2 and the known heights h_1 and h_2 of the columns of stationary liquid in the left and right vertical tubes. (These heights are measured from the bottom of each tube, where the liquid pressures are p_1 and p_2.) (a) Express the pressure difference $(p_1 - p_2)$ in terms of v_1, ρ, A_1, and A_2 alone. Then express this pressure difference in terms of ρ, g, h_1, and h_2 alone. (b) By combining these results, write an equation for the speed v_1 in terms of known quantities. (c) Suppose the flowing liquid is water and that $A_1 = 2A_2$. If $h_1 = 40\,\text{cm}$ and $h_2 = 25\,\text{cm}$, what is the speed v_1 of the water in the main pipe? *(Answer: 126) (Suggestion: [s-1])*

SECT.

F SUMMARY

DEFINITIONS

mass current; Def. (A-1)

steady state; Def. (B-1)

IMPORTANT RESULTS

Relation between mass current and flow velocity: Eq. (A-2)

$I = \rho A v$ (if $\vec{v}$ is perpendicular to surface)

Steady-state condition:

$I_{\text{in}} = I_{\text{out}}$

Work done on a fluid: Eq. (C-1)

$W = \pm p V_s$ (if p is constant)

Dissipative flow in a horizontal uniform tube: Eq. (D-4)

$I = (p_1 - p_2)/R$ (for incompressible fluid)

Steady dissipationless flow (Bernoulli's principle): Eq. (B-4)

$p + \rho g y + (1/2)\rho v^2 = \text{constant}$

NEW CAPABILITIES

You should have acquired the ability to:

(1) Understand these relations:

(a) the definition $I = dM/dt$ of mass current (Sec. A),

(b) the relation $I = \rho A v$ (Sec. A, [p-1]),

(c) the steady state condition $I_{\text{in}} = I_{\text{out}}$ (Sec. B, [p-2]),

(d) the relation $I = (p_1 - p_2)/R$ (Sec. D, [p-5]).

(2) For an incompressible liquid entering and leaving a closed region by several channels, apply the steady state condition to relate the average liquid speeds in these channels and quantities describing the cross-sectional areas of these channels. (Sec. B, [p-3], [p-4])

Applying Relations Describing Fluid Flow (Cap. 1, 2)

F-1 A horizontal hose of inside diameter 1.0 cm has a resistance to water flow of $4.0 \times 10^4\,\mathrm{m^{-1}\,s^{-1}}$. The water pressure at the end of the hose attached to a faucet has a constant value of $1.20 \times 10^5\,\mathrm{N/m^2}$, and the pressure of the water emerging from the open end of the hose is equal to the atmospheric pressure of $1.00 \times 10^5\,\mathrm{N/m^2}$. The water is in steady flow. (a) What are the magnitudes of the mass current I and the volume current I' of the water in the hose? What is the average speed v of the water in the hose? (b) Suppose that a person now puts a thumb partially over the end of the hose, reducing the cross-sectional area of the emerging water stream to 1/5 of the cross-sectional area of the stream in the hose. If the average speed of the steadily-flowing water in the hose is now $v = 6.2\,\mathrm{m/s}$, what is the magnitude of the mass current I in the hose? What is the average speed V of the water in the narrow opening where the water emerges? If the water pressure at the faucet is the same as before, what is the water pressure p_e in the end of the hose, just behind the person's thumb? *(Answer: 123)*

SECT.
G PROBLEMS

G-1 *Peripheral resistance of the circulatory system:* In analogy to the resistance of a horizontal tube, physiologists define the "total peripheral resistance" R of the circulatory system by the relation $I = (p_A - p_V)/R$, where I is the average current flowing through the circulatory system from the aorta (where the average blood pressure is p_A) to the vena cava (where the average blood pressure is p_V). In contrast with the resistance of a horizontal tube, however, the value of R is not a constant independent of pressure and current. Indeed, R can change dramatically, largely because of changes in the diameters of blood vessels due to both chemical stimuli and the variable distension of the vessels under pressure. (a) In a resting person, the average pressure difference $(p_A - p_V)$ is 100 mm-Hg or $1.3 \times 10^4\,\mathrm{N/m^2}$, and the average current I is $8.0 \times 10^{-2}\,\mathrm{kg/s}$. What is the total peripheral resistance R in a resting person? (b) In moderate exercise, the average current I is about 3 times that in rest, while the average pressure difference $(p_A - p_V)$ is only about 50 percent larger, or 150 mm-Hg $= 2.0 \times 10^4\,\mathrm{N/m^2}$. What is the value of R in moderate exercise? This change is probably due to an increase in the diameter of the arterioles (small arteries leading into the capillary beds), since these vessels contribute about 40 percent of the total resistance R. *(Answer: 128)*

G-2 *Stress on the heart due to a bullet wound in a limb:* Suppose that a bullet passes through a limb, opening an alternate channel between an artery and a vein (Fig. G-1). Let us investigate the blood flow after *external* bleeding has been stopped, so that the blood flows only in the channels shown in the figure. To do so, we shall treat the circulatory system in the limb as a horizontal tube of resistance R_L, so that the average current I_L through this system is $I_L = (p_A - p_V)/R_L$, where p_A and p_V are the average blood pressures in the artery and vein. Suppose that the channel opened by the bullet is a horizontal tube having a typical resistance $R_B = R_L/4$. (a) Write an expression for the current I_B through this channel. Using this result, express I_B as a number times I_C. (b) Because of the body's regulative mechanisms, the pressure difference $(p_A - p_V)$ has about the same value before and after the passage of the bullet, so that the current I_C through the limb remains roughly the same. Let us call I_A and I'_A the currents flowing in the artery before and after the passage of the bullet. Is I'_A larger than, equal to, or smaller than I_A?

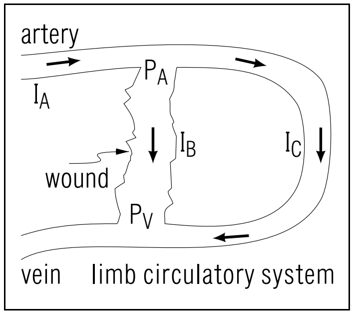

Fig. G-1.

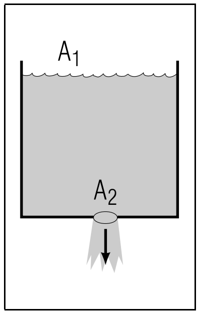

Fig. G-2.

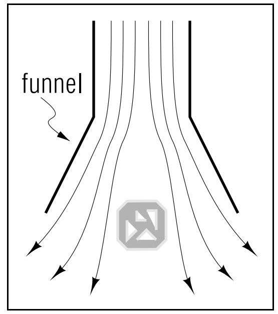

Fig. G-3.

Express I_A and I'_A in terms of I_C, using your previous results, and then express I'_A as a number times I_A. This change in arterial current causes a serious stress on the heart, since the arterial flow becomes similar to that in a person doing exercise. *(Answer: 125)*

G-3 *Speed of a liquid emerging from a tank:* Consider a leaky cylindrical tank of cross-sectional area A_1, which contains a liquid of density ρ (Fig. G-2). The liquid is emerging from a hole in the bottom with an average speed v_2 in a stream of cross-sectional area A_2. The liquid at the surface in the tank, a distance h above the bottom, is thus flowing downward with some smaller speed v_1. The pressures in the emerging stream and at the surface in the tank are both equal to atmospheric pressure. (a) Write an equation for the average speed v_2 of the emerging liquid in terms of ρ, g, h, A_1, and A_2. (b) Suppose that the hole is small, so that A_2 is much smaller than A_1. (Correspondingly, the liquid speed v_1 at the surface is negligible.) Show that $v_2 = \sqrt{2gh}$ in this situation. How does this speed compare with the speed of an object which has fallen a distance h from rest? This result is called "Torricelli's theorem." *(Answer: 129)*

G-4 *Picking up an object without touching it:* Figure G-3 shows an ingenious method for picking up a small object (e.g., a crystal) without touching and contaminating it. Air flows steadily *out* of a funnel and around the object along the "streamlines" shown, so that we can consider the shaded region to be a "tube" of varying cross-section filled with air in steady flow. Let us see how this air can exert an *upward* force on the object. (a) Is the average air speed v_a above the object larger or smaller than the average air speed v_b below it? (b) The air flows with negligible dissipation, and the quantity $\rho g y$ has about the same value above and below the object (since the density ρ of air and the size of the

object are both small? Is the air pressure p_a above the object larger or smaller than the air pressure p_b below it? (c) Is the magnitude of the air pressure force $\vec{F}_a$ on the top surface of the object larger or smaller than the magnitude of the air pressure force $\vec{F}_b$ on the bottom? What is the direction of the sum of these forces? *(Answer: 127)*

PRACTICE PROBLEMS

p-1 *UNDERSTANDING THE RELATION $I = \rho AV$ (CAP. 1B):* The stream of water flowing over the top of a dam has a rectangular cross-section 2 cm high and 5 meter wide. (a) If the average speed of the water in this stream is 2 m/s, what is the mass current of water flowing over the dam? (b) The mass current of water in the river below the dam is the same as that at the top of the dam. If the cross-sectional area of the stream of water in the river is 100 times that at the top of the dam, what is the average speed of the water in the river? *(Answer: 2) (Suggestion: review text problems A-5 through A-7.)*

p-2 *UNDERSTANDING THE STEADY STATE CONDITION (CAP. 1C):* A recirculating fountain in a park has 5 jets from which water shoots upward and falls into a large pool. Water from the pool flows through a drain to a pump, and from the pump to the jets. Suppose that the mass current through each jet is the same, and that the entire system consisting of this fountain is in the steady state. If the magnitude of the mass current of water through the pump is $I_p = 10$ kg/s, what is the magnitude I_j of the mass current through each jet? What is the magnitude I_d of the mass current through the drain? *(Answer: 4) (Suggestion: review text problems B-1 and B-3.)*

p-3 *APPLYING THE STEADY STATE CONDITION (CAP. 2):* A large pipe of diameter D is connected to two smaller pipes, each of diameter d. Water flows steadily from the large pipe into the small pipes. (a) Suppose that the average water speed V in the large pipe is equal to the average water speed v in each of the small pipes. Express the diameter D of the large pipe as a number times the diameter d of the small pipes. (b) Alternatively, suppose that the small pipes are each half the diameter of the large one, so that $d = 1/2D$. Express the average water speed v in each small pipe as a number times the average water speed V in the large one. *(Answer: 1) (Suggestion: review text problems B-4 through B-6.)*

p-4 *APPLYING THE STEADY STATE CONDITION (CAP. 2):* A city water main having an inside radius of 10 cm ultimately branches out into 50 pipes of inside radius 1.0 cm, each of which supplies water to a house. To find the maximum water speed in the main, suppose that water is in steady flow with its maximum average speed of 10 m/s in each pipe supplying a house. What is the average water speed in the main

under these conditions? What is the corresponding mass current in the main? *(Answer: 5) (Suggestion: review text problems B-4 through B-6.)*

p-5 *UNDERSTANDING THE RELATION $I = (P_1 - P_2)/R$ (CAP. 1D):* A 1 km section of horizontal water pipeline has a resistance to flowing water of $25\,\mathrm{m^{-1}\,s^{-1}}$. (a) What is the magnitude of the difference between the water pressures at the ends of this section when water flows steadily through it with a mass current of magnitude 160 kg/s? Is the water pressure larger at the upstream or downstream end of this section? (b) Suppose the resistance of this section were *smaller*. Would the same pressure difference between its ends produce a smaller or a larger mass current through the section of pipeline? *(Answer: 3) (Suggestion: review text problems D-4 and D-5.)*

SUGGESTIONS

s-1 *(Text problem E-3):* Part (a): To obtain the first relation, use the steady state condition to eliminate the speed v_2 from Eq. (E-4) in the text. To obtain the second relation, note that the liquid in each vertical tube is at rest. Thus the liquid pressure at the bottom of each tube is larger than atmospheric pressure by an amount $\rho g h$, where h is the height of the liquid column in the tube.

s-2 *(Text example B-6):* Since each hole in the sprinkler has the same size and the water emerging from the hole has the same average speed, the magnitude I_s of the water current through each sprinkler hole is the *same*. If I_h is the magnitude of the water current in the hose, the steady state condition is then $I_h = 20 I_s$ (since there are 20 holes). By expressing each of the currents I_h and I_s in terms of the corresponding average water speed and radius of the water stream, you can write an equation for the radius of the sprinkler holes in terms of known quantities. (If you need more help, review text example B-1.)

s-3 *(Text problem B-4):* Consider the region between the two areas A_n and A_p. The steady state condition for this region states simply that the magnitudes of the currents through these areas must be equal, or $I_n = I_p$. By expressing each of these currents in terms of the corresponding cross-sectional area and average speed of the blood, you can obtain an equation relating v_n and v_p. (Note that in this situation, as in the others we shall consider, the density of the flowing liquid always has the same value. Thus it may be divided out of equations expressing the steady state condition.)

s-4 *(Text problem A-2):* The mass current through a surface into a region is $I = dM/dt$, where the quantity dM is interpreted as equal in magnitude to the mass m of fluid flowing through the surface (in either direction) during the time interval dt. If fluid is flowing *into* the region through the surface, $dM = +m$; if fluid is flowing *out of* the region through the surface, $dM = -m$. In either case, the sign of dM indicates whether the fluid flowing through the surface tends to increase ($dM = +m$) or decrease ($dM = -m$) the mass M of fluid in the region.

s-5 *(Text problem B-5):* In applying the steady state condition, it is useful to follow the general approach outlined by the questions in text problem B-4. In this problem, you might begin by writing an equation relating the magnitude of the water current through a cross-section of the hose to the magnitude of the water current through the hole in the nozzle. Each of these current magnitudes can then be expressed in terms of symbols for the average water speed and the diameter of the circular cross-sectional area of the water stream, using the result that the area of a circle having a diameter d is $A = \pi(d/2)^2 = \pi d^2/4$. The resulting equation relates the unknown water speed at the nozzle to known quantities. Note that the density of the water is the same in the hose as it is in the nozzle, so that this quantity can be eliminated from the equation.

s-6 *(Text problem E-1):* Since Bernoulli's principle applies, the quantity $p + (1/2)\rho v^2 + \rho g y$ must have the *same* value everywhere along the path of the flowing water. In particular, it must have the same value in the mains as at the top of the stream from the fire hose. If we choose to measure height upward from the mains, this quantity has the value $p_m + 0 + 0 = p_m$ in the mains, since the water speed in the mains is negligible. Similarly, this quantity has the value $p_t + 0 + \rho g y_t$ at the top of the stream, since the water speed at this point is also zero. Therefore, we have the relation

$$p_m = p_t + \rho g y_t$$

Using the known values for the pressure p_t and the height y_t at the top of the stream and for the water density ρ, you can find the value of the pressure p_m in the mains.

Note that applying Bernoulli's principle is very similar to applying a conservation principle such as the principle of conservation of energy. We need only express the value of the conserved quantity at different points in terms of symbols for known and desired quantities, equate these expressions, and solve for the desired quantity.

s-7 *(Text problem B-1):* Part (a): The steady state condition $I_{\text{in}} = I_{\text{out}}$ equates the sum of all currents flowing into a region to the sum of all currents flowing out of a region. (Thus currents flowing in channels which do not cross the region's boundaries do *not* appear in the steady state condition for the region.) To write the steady state condition, first locate all channels in which fluid is flowing *into* the region. The value of I_{in} is then the sum of the *magnitudes* of the currents in these channels. (For example, $I_{\text{in}} = I_B + I_V$ for region 1 in Fig. B-3.) Then locate all channels in which fluid is flowing *out of* the region; the value of $I_{\rm out}$ is the sum of the magnitudes of the currents in these channels. (For example, $I_{\rm out} = I_A$ for region 1.) Then use your results to write the relation $I_{\rm in} = I_{\rm out}$ in terms of the magnitudes of the currents in the channels crossing the boundaries of the region. (For example, $I_B + I_V = I_A$ for region 1.)

Part (b): To relate the magnitudes of currents flowing in several connected channels using the steady state condition, identify (perhaps by drawing a dotted line) the junction region where these channels meet. Then write the steady state condition for this region.

ANSWERS TO PROBLEMS

1. a. $D = \sqrt{2}d = 1.4d$
   b. $v = 2V$
2. a. 2×10^2 kg/s
   b. 2×10^{-2} m/s
3. a. 4.0×10^3 N/m^2, upstream end
   b. larger
4. $I_j = 2$ kg/s, $I_d = 10$ kg/s
5. 5.0 m/s, 1.6×10^2 kg/s

101. a. 12 kg
   b. 3.0×10^2 second (5 minute)
102. 0.18 m/s, smaller (about 2/3 of the speed in the aorta)
103. a. Region 1: $I_B + I_V = I_A$. Region 2: $I_L + I_B = I_A + I_C$.
   b. $I_C = I_L - I_V = 0.4 \times 10^{-2}$ kg/s
104. a. 8.0×10^{-5} m^3/s = 80 cm^3/s
   b. 6.0×10^{-4} kg/s
105. 4×10^{-1} kg/s (or about 5 times the average current)
106. +, 0
107. a. 100 kg/s
   b. 100 kg/s
   c. 50 kg/s
108. a. Surface 1: +4 kg. Surface 2: -4 kg. Both: 0 kg.
   b. Surface 1: +2 kg/s. Surface 2: -2 kg/s. Both: 0 kg/s.
109. a. $I_p = I_n$.
   b. $A_p v_p = A_n v_n$ or equivalent; larger than.
   c. $v_p = 5 v_n$
110. a. Area: 5×10^{-11} m^2. Current: 2×10^{-11} kg/s
   b. 5×10^7 second (about 1 year and 8 months!)

111. 5.0×10^{-4} meter(0.5 mm)

112. a. $I_C = I_A - I_{AVA}$

b. I_C increases.

113. a. 30 kg

b. 6×10^2 J

114. 25 m/s

115. a. 6 N/m^2 (0.05 mm-Hg); larger than.

b. 1×10^8 m^{-1} s^{-1}

116. a. Inhalation: $\Delta V = +V_t$. Exhalation: $\Delta V = -V_t$.

b. $W_i = -p_i V_t$, $W_e = +p_e V_t$, $W = (p_e - p_i)V_t$

c. $W = 0.2$ J

d. $P = 0.1$ watt (or 10 percent of the average power delivered by the heart to the blood)

117. a. $I = (p_B - p_A)/R$

b. If $p_B = 2 \times 10^5$ N/m^2: 40 kg/s, from tank A to tank B. If $p_B = p_A$: current is zero. If $p_B = 4 \times 10^5$ N/m^2: 40 kg/s, from tank B to tank A.

118. $V_s = AL = 3\,\text{cm}^3 = 3 \times 10^{-6}\,\text{m}^3$, $W = +pV_s = 0.6$ J

119. Increase: (a) and (c). Decrease: (b) and (d).

120. a. As small as possible.

b. Steps 1 and 3.

121. a. 2.7 m/s

b. 3.6×10^3 N/m^2 (about 30 mm-Hg)

122. The relation applies only to *horizontal* tubes. Neither of the tubes described is horizontal.

123. a. $I = 0.50$ kg/s, $I' = 5.0 \times 10^{-4}$ m^3/s, $v = 6.4$ m/s

b. $I = 0.49$ kg/s, $V = 31$ m/s, $p_e = (1.01 \text{ or } 1.00) \times 10^5$ N/m^2

124. 4.0×10^5 N/m^2

125. a. $I_B = (p_A - p_B)/R_B$, $I_B = 4I_L$

b. Larger. $I_A = I_L$. $I'_A = I_B + I_L = 5I_L$. $I'_A = 5I_A$.

126. a. $(p_1 - p_2) = (1/2)\rho v_1^2[(A_1/A_2)^2 - 1]$

b. $(p_1 - p_2) = \rho g(h_1 - h_2)$

c. $v_1 = \sqrt{2g(h_1 - h_2)/[(A_1/A_2)^2 - 1]}$

d. $v_1 = 1.0$ m/s

127. a. larger than

b. smaller than

c. smaller than; upward

128. a. 1.6×10^5 m^{-1} s^{-1}

b. 8×10^4 m^{-1} s^{-1} or about half the resting value.

129. a. $v_2 = \sqrt{2gh/[1 - (A_2/A_1)^2]}$

b. If A_2 is much less than A_1, the term $[1 - (A_2/A_1)^2] = 1$, so that $v_2 = \sqrt{2gh}$. It is the same as the speed of an object which has fallen a distance h from rest.