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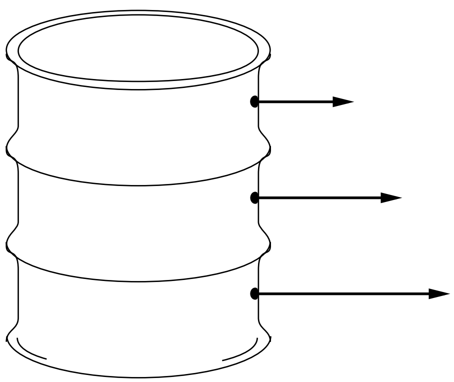
SOLIDS AND FLUIDS AT REST



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

SOLIDS AND FLUIDS AT REST

by

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Title: **Solids and Fluids at Rest**

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Input Skills:

1. Vocabulary: external force, mass of a system (MISN-0-413).

Output Skills (Knowledge):

K1. Vocabulary: density, stress, pressure, gauge pressure.

K2. Describe the pressure force on a fluid.

K3. State the relation between pressures at different levels in a fluid at rest.

K4. State Archimedes' principle.

Output Skills (Problem Solving):

S1. Solve problems using these relations:

S2. For a system of one or more fluids at rest, relate the positions of two points to the pressures or gauge pressure at these points or to the pressure forces exerted by the fluid near these points.

S3. Apply Archimedes' principle and the equation of motion to relate quantities describing the buoyant force on an object at rest in a fluid to quantities describing the remaining forces on the object.

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New authors, reviewers and field testers are welcome.

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SOLIDS AND FLUIDS AT REST

Abstract:
In the preceding units we studied the basic concepts needed to deal with systems consisting of many particles. Hence we can now discuss some of the important properties of common materials (such as solids, liquids, and gases) which consist of enormously many atoms or molecules. For most practical applications, we can consider these substances from a macroscopic (i.e., large-scale) point of view without requiring detailed knowledge about the individual atoms in them. We shall spend most of our time discussing fluids (i.e., liquids and gases) because these have some remarkable properties of great importance for the understanding of physical and biological processes.

SECT.

A MACROSCOPIC PROPERTIES OF MATERIALS

From a macroscopic point of view, any material (such as a solid, a liquid, or a gas) can be described by a few simple properties. Let us examine some of these.

DENSITY

Any small portion of a material has some volume V and some mass M (which is the sum of the masses of all the atoms contained in this portion). If this portion is small enough (although still large enough to contain very many atoms), the number of atoms in it, and thus also its mass M, is proportional to its volume V. (For example, if the volume V of a portion of water would be 3 times as large, the number of atoms in this portion, and thus also the mass of this portion, would be 3 times as large.) Hence the ratio M/V is independent of the volume. This ratio is commonly denoted by ρ (the Greek letter "rho") and is called "density" in accordance with this definition:

Def. **Density**: The density ρ of a material at a point P is the ratio

$$\rho = \frac{M}{V} \qquad \text{(A-1)}$$

where V is a small enough volume enclosing the point P and where M is the mass of the material within this volume.

In short, we can say that the density is the "mass per unit volume" (i.e., the mass divided by the corresponding volume).

A material is said to be "homogeneous" or "uniform" if the intrinsic (i.e., size-independent) properties of every portion of this material are the same. For example, if a material is homogeneous, the density is the same at each point in the material. Then the ratio M/V has the same value for *any* small volume of the material, and has therefore also this value for any large volume. (In other words, *any* volume V of the material is then small enough for finding the density.

The Def. (A-1) implies that the SI unit of density is kg/m^3. The density is also often expressed in terms of the unit gram/cm^3. For example,

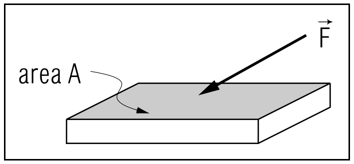


Fig. A-1: Contact force exerted on the surface of a portion of material by its surroundings.

the density of water is 1.0 gram/cm^3 = 1.0×10^3 kg/m^3. Table A-1 lists the densities of a few other homogeneous materials.

ELASTIC PROPERTIES

Any portion of a material (e.g., of a copper block) is acted on by contact forces due to its surroundings. To examine these forces in greater detail, consider any small part of the surface of a portion of the material. (See Fig. A-1.) The atoms adjacent to this surface inside the portion are then acted on by a "contact" force $\vec{F}$ due to the atoms adjacent to this surface outside the portion. If this surface is small enough, the contact force $\vec{F}$ is proportional to the number of atoms adjacent to this surface and is thus proportional to the area A of this surface. (For example, if the area of the surface were 3 times as large, the number of atoms adjacent to the surface would also be 3 times as large.

Table A-1: Densities of some common materials at room temperature and pressure.

material	ρ(kg/m^3)	*material*	ρ(kg/m^3)
wood (maple)	0.7×10^3	alcohol (ethyl)	0.79×10^3
ice	0.92×10^3	oil (olive)	0.92×10^3
bone	1.6×10^3	water	1.00×10^3
glass	2.6×10^3	blood	1.05×10^3
iron	7.7×10^3	glycerin	1.26×10^3
copper	8.5×10^3	mercury	13.6×10^3
lead	11.3×10^3	air	0.0012×10^3

Thus the force $\vec{F}$ exerted on the atoms adjacent to one side of the surface by the atoms adjacent to the other side would also be 3 times as large.) Hence the ratio $\vec{F}/A$ is independent of the size of the area A. This ratio is denoted by $\vec{\sigma}$ (the Greek letter "sigma") and is called "stress."

Def. **Stress**: $\vec{\sigma} = \dfrac{\vec{F}}{A}$ (A small enough) (A-2)

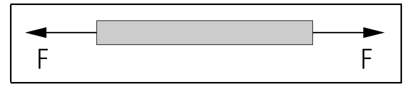


Fig. A-2: A rod pulled on both ends by forces of equal magnitude.

In other words, the stress on a material is the contact force *per unit area* exerted on the material by the material on the other side of this area.

The forces acting on any portion of a material tend to change the spatial arrangement of the atoms relative to each other and thus to produce a deformation of the material. For example, Fig. A-2 illustrates a rod which is being pulled from both ends by contact forces having equal magnitudes F and opposite directions. As a result of these forces, the rod becomes elongated (while its diameter decreases). The magnitude of the elongation depends on the magnitude of the stress, i.e., on the ratio F/A of the magnitude F of the applied force compared to the cross-sectional area A of the rod. (For example, forces of the same magnitude F will stretch a thinner rod more than a thicker one because the magnitude F/A of the stress is larger in the case of the thinner rod which has the smaller area A.) When the magnitude of the stress becomes too large, the rod breaks.

Example A-1: Breaking strength of a steel wire

The tensile strength of a material is the maximum magnitude $\sigma_{\rm max}$ of the stress with which this material can be pulled without breaking. The tensile strength of steel is 5×10^8 N/m^2. What then is the maximum magnitude $F_{\rm max}$ of the force which breaks a steel wire of radius $r = 1$ mm $= 10^{-3}$ m in the arrangement of Fig. A-2?

By the definition of stress, Def. (A-2), $F_{\rm max} = \sigma_{\rm max} A$. The cross-sectional area A of the wire is $A = \pi r^2 = 3 \times 10^{-6}$ m^2. Hence

$$F_{\rm max} = \sigma_{\rm max} A = (5 \times 10^8\,{\rm N/m^2})(3 \times 10^{-6}\,{\rm m^2}) = 1.5 \times 10^3\,{\rm N}$$

(or about 3×10^2 pound).

The "elastic" properties of a material are described by the relation between the contact forces on the material and the resulting deformation of the material. For simplicity, let us consider a portion of material whose center of mass remains at rest so that the total external force on this portion is zero. Let us also assume that the only external forces on this portion are the contact forces due to its surroundings. Then we can examine some especially simple deformations produced by various contact forces. In particular, we may consider a small change of volume without a change of shape (as illustrated in Fig. A-3a), or a small change of shape

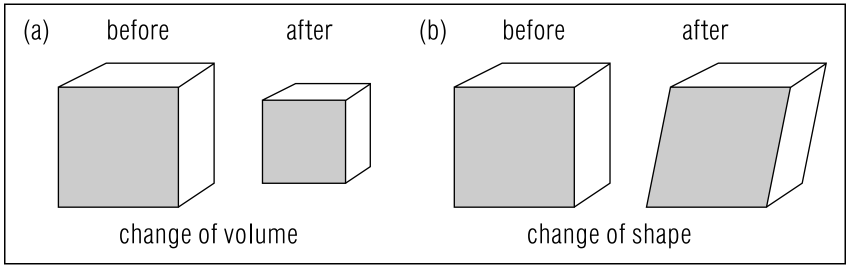


Fig. A-3: Simple small deformations of a material.

without an appreciable change of volume (as illustrated in Fig. A-3b).

Figure A-4 shows a cubical portion of material subjected on all its surfaces to contact forces having equal magnitudes and inward directions perpendicular to these surfaces. The resulting deformation is then a decrease in the volume of the material without a change of shape (as illustrated in Fig. A-3a). If a large change of volume is produced by small forces (e.g., if the material is a sponge), the material is said to be highly "compressible." But if the change of volume is negligible even if the forces are very large (e.g., if the material is a brick), the material is said to be "incompressible." The density of the material remains then unchanged irrespective of the magnitudes of the forces acting on the material. *

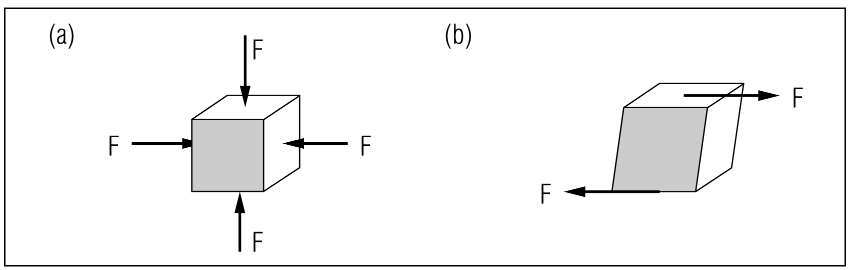


Fig. A-4: Deformations produced by various forces. (a) Change of volume produced by inward compressive forces. (b) Change of shape produced by shear forces.

* If the material of original volume V changes its volume by an amount ΔV, the magnitude $|\Delta V/V|$ of the (relative deformation (or "strain") is related to the magnitude $|\vec{F}/A|$ of the stress by $|\Delta V/V| = K|\vec{F}/A|$, where the quantity K is called the "compressibility" of the material. If the material is highly compressible, K is large. If it is incompressible, $K = 0$.

Figure A-4b shows a portion of material subjected on opposite surfaces to contact forces having equal magnitudes and opposite directions *parallel* to these surfaces. (Such a contact force *parallel* to a surface is called a "shear" force.) The resulting deformation is then a change of shape with negligible change of volume (as illustrated in Fig. A-3b).

FORMS OF MATERIALS

Materials can be classified into various types on the basis of their elastic properties. If *any* kind of deformation of a material can only be produced by large contact forces, the material is called a "solid." But if a sufficiently slow change of shape of a material can be produced by negligibly small shear forces, the material is called a "fluid." (If *fast* changes of shape of a fluid can only be produced by appreciable shear forces, the fluid is said to be "viscous." But if even fast changes of shape can be produced by negligibly small shear forces, the fluid is called "non-viscous" or "inviscous.")

Fluids can be further classified into "liquids" and "gases" on the basis of the contact forces required to produce changes in their volume. A "liquid" is a fluid which is nearly incompressible (so that a change in its volume can only be produced by large contact forces). A "gas" is a fluid which is easily compressed (so that a large change in its volume can be produced even by small forces). Furthermore, the density of a liquid is ordinarily much larger than that of a gas.

Let us look at these different forms of materials from an atomic point of view. In a solid, the atoms are close together and the mutual forces between them are sufficiently strong to keep these atoms locked in nearly fixed positions relative to each other (usually in a highly regular or "crystalline" arrangement). Hence any deformation of the solid can only be produced by large external force applied to the solid. In a liquid, the atoms or molecules are slightly further apart and thus fairly free to move past each other. Thus changes of shape can be produced quite easily by small external shear forces, although large forces are required to compress the liquid and thus to force its atoms closer together. In a gas, the atoms or molecules are far apart and interact only weakly with each other. Hence both changes of shape and changes of volume are readily produced by small external forces.

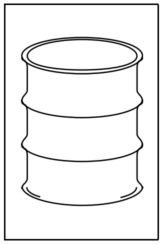


Fig. A-5.

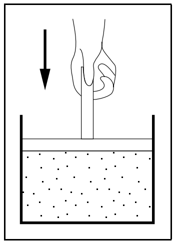


Fig. A-6.

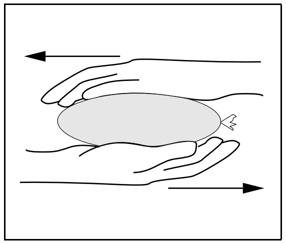


Fig. A-7.

Understanding the Definition of Density (Cap. 1a)

A-1 (a) *Example:* Under standard conditions, one mole of homogeneous hydrogen gas, which has a mass of 2.00×10^{-3} kg, occupies a volume of 22.4 liter $= 2.24 \times 10^{-2}\,\text{m}^3$. What is the density ρ of hydrogen gas under these conditions? (b) *Relating quantities:* The volume of blood in an adult person is about 5 liter $= 5 \times 10^{-3}\,\text{m}^3$. What is the mass of the blood in an adult, assuming the blood is homogeneous? (Use table A-1.) *(Answer: 106)*

A-2 *Applicability:* A hollow 20 kg drum has thin homogeneous steel walls of density $8 \times 10^3\,\text{kg/m}^3$. (Fig. A-5). Either use this information to find each of the following quantities, or explain why the quantity cannot be found. (a) The volume of the entire drum. (b) The volume of the steel walls of the drum. *(Answer: 102)*

A-3 *Dependence:* By measuring the mass of $1\,\text{cm}^3$ of a homogeneous liquid, a student finds the density of the liquid. Suppose the student had used instead one tenth of this volume, or $0.1\,\text{cm}^3$. Would the measured density be the same or one-tenth as large? Would the measured mass be the same or one-tenth as large? *(Answer: 104) (Suggestion: [s-2])*

Knowing About the Properties of Solids, Liquids and Gases

A-4 *Compressibility and density:* Suppose we have four cylindrical containers, each fitted with a piston which can move up or down, and we *completely* fill one container with oxygen gas, one with liquid alcohol, one with liquid molasses, and one with solid ice (as shown in Fig. A-6). (a) Which of these substances is highly compressible, so that we can easily push the piston downward to compress the substance into

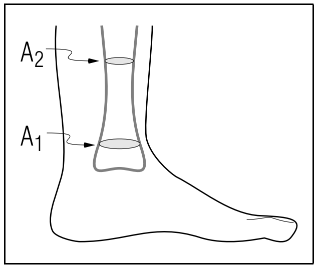


Fig. A-8.

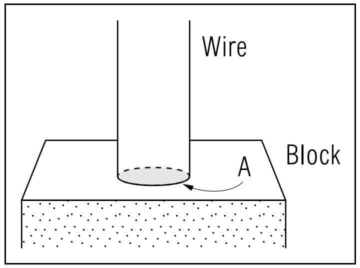


Fig. A-9.

a smaller volume? Which of them is nearly incompressible, so that we can only produce a negligible change in the volume of the substance by pushing downward on the piston? (b) Which of these substances has a density that remains nearly constant when the substance is compressed? *(Answer: 101) (Suggestion: [s-7])*

A-5 *Shear strength and viscosity:* Suppose that we fill four balloons with the four substances described in problem A-4, and we apply shear forces to each substance by pushing horizontally in opposite directions on the top and bottom surfaces of the balloon as shown in Fig. A-7. These forces can change only the shape of the substance in the balloon. (a) Which of these substances require very small shear forces to change shape? Which of them retains its shape even when we apply fairly large shear forces? (b) Molasses requires large shear forces to change shape rapidly, while oxygen requires very small shear forces to do so. Which of these fluids is more viscous? *(Answer: 108)*

Understanding the Definition of Stress (Cap. 1b)

A-6 *Example:* The tibia (the large bone in the lower leg) varies in thickness along its length. Near the ankle joint, the cross-sectional area A_1 of the bone is about $6.0\,\mathrm{cm}^2$, while at the thinnest part of the tibia, one-third of its length above the ankle joint, the cross-sectional area A_2 of the bone is $3.0\,\mathrm{cm}^2$. (See Fig. A-8.) (a) When an 80 kg man lands with stiff legs on the ground after falling about 2 meter, the contact force $\vec{F}$ exerted on the bone above each of these areas by the bone below each area is about $5.0 \times 10^4\,\mathrm{N}$ upward. What are the corresponding stresses $\vec{\sigma}_1$ and $\vec{\sigma}_2$ on the bone at each area, assuming these areas are small enough? (b) Human bone fractures if such "compressive" stresses exceed $1.6 \times 10^8\,\mathrm{N/m}^2$ in magnitude. Is the man's tibia likely to

break when he lands on the ground? If so, will it fracture near the larger area A_1 or near the smaller area A_2? *(Answer: 103)*

A-7 In a traction arrangement, a 15 kg metal block is suspended at rest by a steel wire welded into a hole in the top of the block. Consider a cross-sectional area of the wire just above the block, as shown in Fig. A-9. A contact force equal in magnitude to the block's weight is thus exerted on the wire below this area by the wire above it. (a) *Relating quantities:* To find the thinnest wire that is safe, suppose this contact force produces a stress equal in magnitude to $5 \times 10^8\,\mathrm{N/m}^2$, the tensile strength of steel. What is the wire's cross-sectional area A? What is the radius r of the wire (i.e., of its circular cross-section)? (b) *Dependence:* Suppose instead that the wire has twice the radius and thus four times the cross-sectional area of the one described in part (a). Compare the magnitudes of the contact force and stress for this wire with those for the previous one. *(Answer: 107) (Practice: [p-1])*

SECT.

B PRESSURE IN A FLUID

Consider any fluid which remains at rest (i.e., which is in equilibrium). Then the contact force parallel to the surface of any portion of the fluid (i.e., the shear force) must be zero, since any such non-zero force would gradually produce changes of shape in the fluid and would thus not leave the fluid at rest.

In any fluid at rest, the contact force acting *parallel* to the surface of any portion of the fluid is thus zero. *

> * This conclusion is also true for a *nonviscous* fluid even if it is moving, since the shear force in such a fluid is always equal to zero.

Hence the contact force acting on this surface due to its surroundings must always be *perpendicular* to this surface. Furthermore, this contact force is ordinarily directed *inward* toward the inside of the fluid on which it acts (i.e., this force tends to compress the fluid) since the force required to pull the fluid apart is usually negligibly small. (See Fig. B-1.) Thus we arrive at this conclusion:

In a fluid at rest, the contact force exerted on any small surface of a portion of the fluid by its surroundings is directed *perpendicularly inward* to this surface.	(B-1)

Such a contact force perpendicular to the surface on which it acts is called a "pressure force."

Because of the reciprocal relation between mutual forces, Rule (B-1) implies that the contact force exerted by any portion of a fluid on any small surface of its surroundings must have the same magnitude but the opposite direction as the force in Rule (B-1), i.e., this force must be directed perpendicular to the surface *outward* on the surroundings. This conclusion holds irrespective of the nature of the surroundings, e.g., irrespective of whether the surroundings consist of another portion of the fluid or whether they consist of the walls of a container.

Consider a small surface near a point in the fluid. How does the magnitude of the pressure force exerted on this surface depend on how the surface is oriented in space (e.g., on whether it is horizontal, vertical, or has any other orientation)? As we show in Section H, the equation of motion applied to the fluid then implies this simple result:

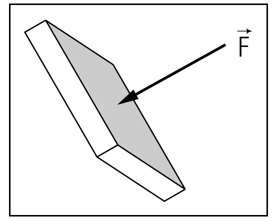


Fig. B-1: Force $\vec{F}$ exerted on the surface of a small portion of fluid by its surroundings.

The magnitude of the pressure force exerted by a fluid on a small surface near a point is *independent* of the orientation of this surface.	(B-2)

Like any other contact force, the pressure force $\vec{F}$ on a small enough surface is proportional to the area A of this surface. Hence the ratio F/A is independent of A. The *magnitude* of this pressure force per unit area (i.e., of this stress) is simply called the "pressure" in accordance with this definition:

Def. **Pressure**: The pressure p at a point in a fluid is the ratio

$$p = \frac{F}{A}$$

where F is the magnitude of the pressure force exerted on the fluid on one side of a small enough surface of area A (at this point) by the material on the other side of this surface. (B-3)

Note that the pressure at any point in a fluid has a unique value since it is independent of both the size and the orientation of the small surface considered at this point.

Example B-1: Force exerted by atmospheric pressure

The pressure p of the air (the "atmospheric" pressure) at sea level is $p = 1 \times 10^5\,\mathrm{N/m^2}$. What is the magnitude F of the force exerted by the surrounding air on the top of a can of soup if this top has a radius $r = 5\,\mathrm{cm} = 5 \times 10^{-2}\,\mathrm{m}$?

The area of the top of the can is $A = \pi r^2 = 8 \times 10^{-3}\,\mathrm{m}^2$. Because of the reciprocal relation between mutual forces, the pressure force exerted on the top of the can by the air is perpendicularly inward on the can and

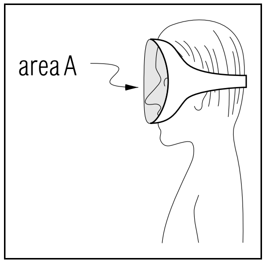


Fig. B-2.

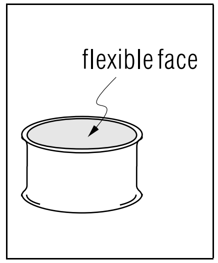


Fig. B-3.

has a magnitude $F = pA$ equal to the magnitude of the outward pressure force of Def. (B-3) exerted on the air by the top of the can. Thus we find that

$$F = pA = (1 \times 10^5\,\mathrm{N/m^2})(8 \times 10^{-3}\,\mathrm{m^2}) = 8 \times 10^2\,\mathrm{N}$$

i.e., about the same as the weight of a man. It is only because the soup and air inside the can exert on the top of the can a compensating outward pressure force that the can does not collapse as a result of the pressure exerted by the surrounding air.

Understanding the Definition of Pressure (Cap. 1c)

B-1 The faceplate of a skin-diver's mask has an area A (which we shall assume is small enough) of $1.5 \times 10^{-2}\,\mathrm{m^2}$. (a) *Example:* When the faceplate is vertical, as shown in Fig. B-2, the force $\vec{F}$ exerted on it by the surrounding water is $3.0 \times 10^3\,\mathrm{N}$ to the right. What is the pressure p of the water near the faceplate? (b) *Dependence:* Suppose the diver now looks downward, so that the faceplate is horizontal but is located at the same position. Is the direction of the force $\vec{F}$ exerted on the faceplate by the water the same or different? Is the magnitude of this force the same or different? Is the water pressure near the faceplate the same or different? *(Answer: 105)*

B-2 *Properties:* (a) List the following properties of the quantities stress and pressure: kind of quantity, possible signs of numerical quantities, SI unit. (b) Which of these properties differ for the two quantities? (c) *Comparisons:* Both stress and pressure are related to the contact force exerted on a small enough area of a surface. Which of them is related only to contact forces *perpendicular* to the surface? *(Answer: 111)*

B-3 *Interpretation and relating quantities:* The pressure-sensing element of an "aneroid barometer" (a mechanical device for measuring atmospheric air pressure) is an evacuated can having circular top and bottom faces of radius 3.0 cm (Fig. B-3). The flexible top face of the can bends slightly inward under the influence of the pressure force exerted by the surrounding air, and this deflection is indicated by a pointer on the barometer's dial. Let us investigate the pressure forces on the can, assuming that the surrounding air has uniform pressure of $1.0 \times 10^5\,\mathrm{N/m^2}$. (a) What are the forces exerted on the top face and on the bottom face of the can by the surrounding air? (b) The sum of these forces is the total vertical force exerted on the can by the surrounding air, since the surrounding air exerts only horizontal forces on areas on the side of the can. What is the total vertical force exerted on the can by the surrounding air? (c) The air pressure inside the can is negligible. What is the total force exerted on the *top face* of the can by the air inside and outside the can? *(Answer: 119)*

B-4 *Dependence:* (a) Suppose another barometer has a can half the radius of the one described in problem B-3, so that the area of the top face of this smaller can is one-fourth that of the larger can. Compare the air pressure forces on the top faces of these cans when both are surrounded by air having the same pressure. (b) As a storm approaches, the atmospheric air pressure decreases. Does the air pressure force on the top face of such cans increase (so that the face flexes inward slightly more) or decrease (so that the face flexes inward slightly less)? *(Answer: 113) (Practice: [p-2])*

SECT.

C RELATION BETWEEN PRESSURES AT VARIOUS POINTS

Consider any fluid at rest while under the influence of gravity near the surface of the earth. Then the total external force on every portion of the fluid must be zero (since the acceleration of the center of mass of every such portion must be zero). But this total external force is the vector sum of the pressure forces, exerted on the fluid portion by its surroundings, and of the gravitational force exerted on the fluid portion by the earth. The condition that the sum of these forces is zero then implies that the pressures at various points in the fluid must be related in some definite way. To examine *how* these pressures are related, we need only examine a few simply chosen portions of the fluid.

Consider first a thin horizontal cylindrical portion of the fluid. As illustrated in Fig. C-1, the long sides of this cylinder of fluid are parallel to the horizontal unit vector $\hat{x}$ and each end surface has a small area A. Since the sum of all external forces on this fluid cylinder must be zero, the sum of the horizontal component vectors of these forces must also be zero. If the pressure at the left surface of the fluid cylinder is p_1, the pressure *force* on this surface of the fluid cylinder is $p_1 A\hat{x}$ (since this force is directed inward perpendicular to the surface, i.e., along $\hat{x}$). If the pressure at the right side of the fluid cylinder is p_2, the pressure *force* on this surface of the fluid cylinder is $-p_2 A\hat{x}$ (since this force is inward perpendicular to this surface, i.e., *opposite* to $\hat{x}$). The pressure forces on the horizontal side surfaces of the fluid cylinder are perpendicular to these surfaces (i.e., perpendicular to $\hat{x}$) so that their component vectors parallel to i are zero. The gravitational force on the fluid cylinder is vertically downward (and thus perpendicular to $\hat{x}$) so that its component vector parallel to $\hat{x}$ is also zero. Since the sum of the component vectors parallel to $\hat{x}$ of all the external forces on the fluid cylinder must be zero, we conclude that

$$p_1 A\hat{x} - p_2 A\hat{x} = 0$$

Hence

$$p_1 = p_2 \qquad \text{(C-1)}$$

This relation must be true irrespective of the length of the horizontal fluid cylinder, i.e., irrespective of the location of the end surfaces of this cylinder. Since these end surfaces are at the same height, Eq. (C-1) implies this conclusion:

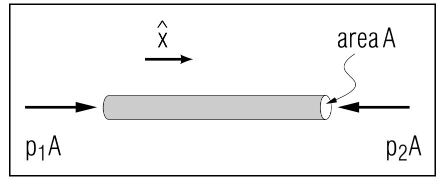


Fig. C-1: A thin horizontal cylindrical portion of fluid at rest.

In a fluid at rest, the pressures at any two points at the same height are equal.	(C-2)

Consider now a thin vertical cylindrical portion of the fluid. As illustrated in Fig. C-2, the long sides of this cylinder of fluid are parallel to the vertical unit vector $\hat{y}$ and have a length h. Each end surface of the cylinder has a small area A. Since the sum of all external forces on this fluid cylinder must be zero, the sum of the vertical component vectors of all these forces must also be zero. If the pressure at the bottom surface of the fluid cylinder is p_1, the pressure force on this surface of the fluid cylinder is $p_1 A\hat{y}$ (since this force is inward perpendicular to the surface, i.e., upward along $\hat{y}$).

If the pressure at the top surface of the fluid cylinder is p_2, the pressure force on this surface of the fluid cylinder is $-p_2 A\hat{y}$ (since this force is inward perpendicular to the surface, i.e., downward opposite to $\hat{y}$). The pressure forces on the vertical side surfaces of the fluid cylinder are perpendicular to these surfaces (i.e., perpendicular to $\hat{y}$) so that their component vectors parallel to $\hat{y}$ are zero. The gravitational force on the

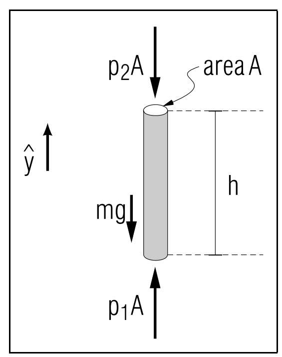


Fig. C-2: A thin vertical cylindrical portion of fluid at rest.

fluid cylinder is $m\vec{g} = -mg\hat{y}$ since it is downward opposite to $\hat{y}$. Here the mass m of the fluid cylinder is related to its volume $V = Ah$ by the density ρ of the fluid so that $m = \rho V = \rho Ah$. *

> * We assume that the height h is sufficiently small so that the density ρ of the fluid is the same at all points within the cylindrical portion of fluid.

Since the sum of the component vectors along $\hat{y}$ of all the external forces on the fluid cylinder must be zero, we conclude that

$$p_1 A\hat{y} - p_2 A\hat{y} - (\rho Ah)g\hat{y} = 0$$

so that

$$p_1 - p_2 - \rho hg = 0$$

Hence

$$\boxed{p_1 - p_2 = \rho gh} \qquad \text{(C-3)}$$

or $p_1 = p_2 + \rho gh$. Thus we arrive at this conclusion: If a point in a fluid at rest is located a vertical distance h directly below another point in the fluid, the pressure p_1 at the lower point is larger than the pressure p_2 at the higher point by an amount ρgh, where ρ is the density of the fluid. [This statement merely expresses the fact that the pressure force on the lower surface of any fluid portion (such as that in Fig. C-2) must be larger than the pressure force on the upper surface of this portion in order to support the weight of this portion of fluid.]

We can use the preceding conclusion to compare the pressures at *any* two points B_1 and B_2 in a fluid at rest, even if one point is not directly below the other. (See Fig. C-3.) Indeed, according to Eq. (C-1), the pressure p_1 at B_1 is the same as the pressure p_3 at the point B_3 which is at the same level directly below B_2. By using Rule (C-3) to compare the pressures at B_3 and B_2, we then obtain $p_1 - p_2 = p_3 - p_2 = \rho gh$ where h is the vertical distance of B_1 (or B_3) below B_2. Thus we can summarize the entire discussion of this section by this conclusion:

> If a point B_1 is a vertical distance h below any other point B_2 in a homogeneous fluid at rest, the pressures at these points are related to the density ρ of the fluid by
>
> $$p_1 - p_2 = \rho gh\,.$$
>
> (C-4)

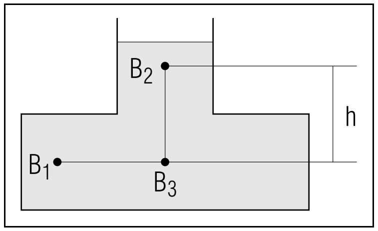


Fig. C-3: Comparison of the pressures at any two points B_1 and B_2 in a fluid at rest.

If the vertical distance h between the points is zero, Rule (C-4) implies $p_1 - p_2 = 0$ or $p_1 = p_2$, in agreement with Rule (C-2).

Relating Pressure, Pressure Force, and Position (Cap. 2)

C-1 Figure C-4 shows two swimming pools, one shallow and one deep, which are connected by a horizontal pipe and filled with water to the same level. List all of the indicated points at which the water pressure is (a) larger than that at the point P, (b) equal to that at the point P, and (c) smaller than that at the point P. *(Answer: 115)*

C-2 (a) If the water filling the pools described in problem C-1 has a uniform density of $1.0 \times 10^3\,\text{kg/m}^3$, what is the pressure difference $p_2 - p_1$ between the water pressures at the points 1 and 2? (b) Suppose instead that the pools are "empty"; i.e., filled with air of uniform density $1.2\,\text{kg/m}^3$. What is the pressure difference $p_2 - p_1$ between the *air* pressures at the points 1 and 2? (c) In both of these situations, the pressure p_1 is equal to the atmospheric air pressure of $1.0 \times 10^5\,\text{N/m}^2$. What is the pressure p_2 in each situation? (d) Consider two points in a fluid, where one point is no more than a few meters higher than the other. For the precision we are using, are the fluid pressures at these points the same if the fluid is a liquid? Are they the same if the fluid is a gas? *(Answer: 112)*

C-3 Consider the two pools filled with water as shown in Fig. C-4. (a) The submerged side of the deep pool is a rectangle 3.0 meter high and 10 meter long, and so the submerged side has an area $A = 30\,\text{m}^2$. By applying the definition of pressure, a student states that the magnitude F of the pressure force exerted on this submerged side by the water is given by $F = pA$, where $p = 1.0 \times 10^5\,\text{N/m}^2$ is the water pressure at the surface. Why has the student *incorrectly* applied this relation? (b) The bottom of the deep pool is a rectangle 4.0 meter wide and 10 meter long. What is the magnitude F of the pressure force exerted on the bottom by

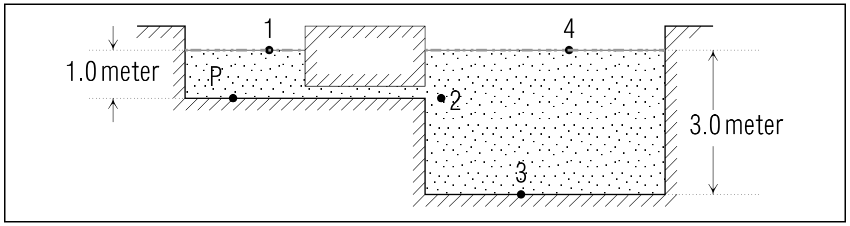


Fig. C-4.

the water? Why can one correctly apply the relation $F = pA$ in this case? *(Answer: 110)*

SECT.

D APPLICATIONS TO FLUIDS AT REST

PRESSURE BELOW THE SURFACE OF A FLUID

Suppose that the pressure at the top surface of a homogeneous fluid at rest is p_a. Then we know from Rule (C-4) that the pressure p at any point at a depth h below the surface of the fluid must be

$$p = p_a + \rho g h \qquad \text{(D-1)}$$

Thus the pressure in a fluid is larger at points which are farther below its surface. (This larger pressure is, of course, due to the larger weight of fluid which must be supported above the lower points.) More specifically, Eq. (D-1) tells us that the pressure at a depth h below the surface of the fluid is larger than that at its surface by an amount $\rho g h$. In the case of a liquid such as water, whose density is fairly large (about $10^3\,\mathrm{kg/m^3}$), the pressure inside the liquid increases quite rapidly with increasing depth. (Thus animals which live at great depths below the surface of the ocean must have physiological characteristics which allow them to survive despite the large pressure forces exerted on them.) In the case of a gas, such as air, the density is typically about a 1000 times smaller. Hence the pressure in gas changes by a negligible amount over a vertical distance of a few meters. On the other hand, the pressure does change by a significant amount if the vertical distance is sufficiently large. For example, the layer of air (the "atmosphere") above the surface of the earth is several hundred kilometers thick. Hence the pressure of the air at the bottom of the atmosphere near the surface of the earth (the so-called "atmospheric pressure") is much larger than the nearly zero pressure at the top of the atmosphere. *

> * The atmospheric pressure cannot be calculated directly from Eq. (D-1) since the air is quite compressible and is thus not homogeneous. (Indeed, the density of the air decreases with increasing height above the surface of the earth.)

Example D-1: Pressure below the surface of the ocean

The atmospheric pressure p_a at sea level (i.e., at the surface of the ocean) is $1 \times 10^5\,\mathrm{N/m^2}$. What then is the pressure p experienced by a fish (or a diver) at a depth 100 meter below the surface of the ocean?

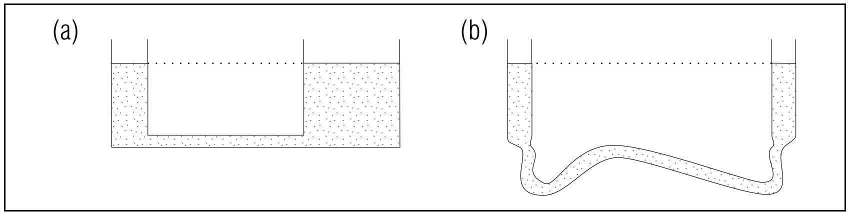


Fig. D-1: Surface levels of a liquid. (a) Container of complicated shape. (b) Device for determining horizontal levels.

Since the density of sea water is approximately $1.0 \times 10^3\,\mathrm{kg/m^3}$, the pressure difference due to this depth of water is

$$p - p_a = \rho g h = (1.0 \times 10^3\,\mathrm{kg/m^3})(10\,\mathrm{m/s^2})(100\,\mathrm{m}) = 10 \times 10^5\,\mathrm{N/m^2}$$

Hence $p = p_a + \rho g h = 11 \times 10^5\,\mathrm{N/m^2}$. Thus this pressure is 11 times larger than the atmospheric pressure experienced at sea level.

SURFACE LEVELS OF LIQUIDS

Consider a liquid at rest in a container and suppose that two parts of the liquid surface are in contact with gas at the same pressure p_0. (For example, the two parts of the liquid surface might be the liquid surfaces in the two parts of the container of Fig. D-1, where both these surfaces are in contact with the air in the surrounding atmosphere.) Then the same pressure difference $p - p_0$ exists between the pressure p at some point B in the liquid and the pressure p_0 at each part of the liquid surface. Hence the relation $p - p_0 = \rho g h$ implies that the vertical distance h between B and each part of the liquid surface must also be the same, i.e., that each part of the liquid surface must be at the same height.

The preceding conclusion must be true no matter how complicated the shape of the container might be. For example, Fig. D-1b illustrates a device consisting of two glass tubes connected by a long garden hose. When filled with water at rest, the surfaces of the water in both glass tubes must then be at the same height (since they are both in contact with air at atmospheric pressure). This device is of practical utility in constructing buildings, since it can be used to determine whether two distant points in the building are at the same level.

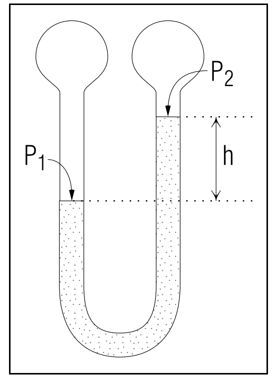


Fig. D-2: Manometer consisting of a liquid contained in a U-shaped tube.

MEASUREMENT OF PRESSURE

Consider a liquid of density ρ at rest in the U-shaped tube shown in Fig. D-2. Then the pressures p_1 and p_2 at the surfaces of the liquid in the two sides of the tube must be related to the vertical distance h between these surfaces by the relation $p_1 - p_2 = \rho g h$, the surface at the higher pressure being below that at the lower pressure. If we know the density of the liquid, we can then measure the vertical distance h between the two liquid surfaces in order to determine the difference in pressure at these surfaces. Thus we can use the liquid-filled U-shaped tube as a "manometer" (i.e., as a device for measuring pressure or pressure differences).

If p_1 is the pressure of the gas above the liquid surface in the left side of the U-shaped tube and p_2 is the pressure of the gas above the liquid surface in the right side of the tube, the measured vertical distance h between the liquid surfaces allows one to find the pressure difference $p_1 - p_2$ between the two gases. If the region above the liquid surface on the right side of the tube is a vacuum, $p_2 = 0$ and the vertical distance h can be used to find the pressure $p_1 = \rho g h$ of the gas above the liquid surface on the left side. For example, if the left side of the tube is open to the surrounding atmosphere, p_1 is simply the atmospheric pressure. (A manometer used for measuring the atmospheric pressure is called a "barometer.")

To measure the atmospheric pressure with the previous arrangement, one commonly uses a tube filled with mercury. Since this liquid has a very large density ($\rho = 13.6 \times 10^3\,\mathrm{kg/m^3}$), the height h between the liquid surfaces can be kept conveniently small. The atmospheric pressure p_a is found to vary slightly depending on weather conditions, but its normal value corresponds to a height of mercury equal to $h = 0.760\,\text{meter}$. The corresponding value of the atmospheric pressure is thus

$$p_a = \rho g h = (13.6 \times 10^3\,\text{m}^3)(9.80\,\text{m/s}^2)(0.760\,\text{m})$$

or

$$p_a = 1.01 \times 10^5\,\text{N/m}^2 \qquad \text{(D-2)}$$

Pressures are often expressed in various common units such as "atmosphere" (indicating the conventionally accepted *standard* value p_a of atmospheric pressure at sea level) or "mm-Hg" (indicating the particular pressure which would support the specified height, in *millimeters*, of a column of *mercury*.) These units are *defined* in terms of the SI unit (newton/meter^2) so that

$$\boxed{p_a = 1\,\text{atmos.} = 760\,\text{mm of Hg} = 1.01325 \times 10^5\,\text{N/m}^2} \qquad \text{(D-3)}$$

GAUGE PRESSURE

Since we usually work in an environment of air at atmospheric pressure, it is often convenient to indicate how much a pressure p differs from the standard atmospheric pressure p_a specified in Eq. (D-3). Accordingly, we introduce this definition of the "gauge pressure" p^* corresponding to the actual pressure p:

Def. | **Gauge pressure**: $p^* = p - p_a$ | (D-4)

For example, a mercury-filled manometer is commonly used to measure the difference between the arterial blood pressure p_b and the atmospheric pressure p_a. *

> * One actually measures what pressure of air in a cuff surrounding the arm is equal to the pressure of the blood in the artery in the arm.

The measured "systolic" pressure (about 120 mm-Hg) is then the gauge pressure $p_b^* = p_b - p_a$ of the blood.

According to Def. (D-4), the gauge pressure is less than the actual pressure and is negative when the actual pressure is less than the standard atmospheric pressure. But the *difference* between any two gauge

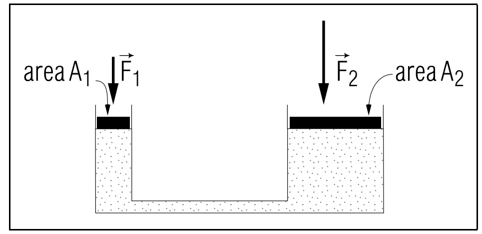


Fig. D-3: A hydraulic press

pressures is simply equal to the difference between the corresponding actual pressures since

$$p_1^* - p_2^* = (p_1 - p_a) - (p_2 - p_a) = p_1 - p_2 \qquad \text{(D-5)}$$

HYDRAULIC PRESS

Consider the arrangement of Fig. D-3 where the two surfaces of the liquid at rest in the container are in contact with movable pistons. For simplicity, assume that the two surfaces of the liquid are at the same height so that the pressures p_1 and p_2 at the liquid surfaces are equal. The pressure p at each surface is related to the magnitude F of the force on this surface and to the area A of this surface (and of the piston on top of it) by the definition $p = F/A$. Hence the equality $p_1 = p_2$ of the pressures at the pistons implies that the magnitudes of the forces on the pistons are related to that

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \qquad \text{(D-6)}$$

Thus the magnitudes of the forces exerted on the pistons are *not* equal unless the areas of the pistons are equal. Indeed, if A_2 is much larger than A_1, the magnitude F_2 of the force exerted by the liquid on the larger piston is much larger than the magnitude F_1 of the force exerted by the liquid on the smaller piston (or of the magnitude F_1 of the force exerted *on* the liquid by the smaller piston). The device illustrated in Fig. D-3, and called a "hydraulic press," can thus be used to produce a large force on a large piston by applying a much smaller force on a small piston. For example, a person can exert a relatively small force on such a hydraulic press in order to support a very heavy object, such as a car.

In these and later problems, assume that atmospheric air has the standard atmospheric air pressure p_a unless stated otherwise. For convenience, we shall use the approximate value $p_a = 1.00 \times 10^5\,\mathrm{N/m^2}$.

Knowing About the Properties of Fluid Surfaces

D-1 Consider a horizontal surface between two fluids at rest (e.g., the surface between the water in a glass and the air). If we call the upper fluid A and the lower fluid B, is the pressure in fluid A just above this surface larger than, equal to, or smaller than the pressure in fluid B just below this surface? *(Answer: 127)*

D-2 Consider this statement: "All parts of the surface of a liquid in a container near the earth's surface have the same height." (a) Using this statement, a student decides that Fig. D-2 cannot be correct, since it shows two parts of a liquid surface at different heights. What important condition on this statement has the student overlooked? (b) Another student uses this statement to assert that the surface of the water contained in a river must be level, so that he cannot understand why the surface of a large river slopes downward toward the sea. What important condition on this statement has the student overlooked? *(Answer: 117)*

Knowing About Gauge Pressure and Units of Pressure

D-3 The gauge pressure of the air in an automobile tire is usually about $p^* = 2$ atmosphere. (a) Express this gauge pressure in terms of $\mathrm{N/m^2}$. (b) What is the pressure p of the air in such a tire? Express your answer in terms of $\mathrm{N/m^2}$ and in terms of atmosphere. (c) What is the *gauge* pressure of air having the standard atmospheric pressure p_a? *(Answer: 114)*

D-4 The minimum or "diastolic" blood pressure in a person's brachial artery is about $1.10 \times 10^5\,\mathrm{N/m^2}$. (a) What is the corresponding diastolic *gauge* pressure in $\mathrm{N/m^2}$? (b) Express this gauge pressure in terms of the unit mm-Hg. *(Answer: 121)*

D-5 Figure D-4 shows a "sphygmomanometer" used to measure blood pressure. The "manometer" consists of a vertical tube, which is open to the atmosphere at the top, immersed in a pool of mercury in a reservoir. The closed space above the mercury in the reservoir is connected by a tube to a bulb and an inflatable cuff which is placed around the patient's upper arm. The bulb is used to pump air into the cuff, thus increasing the air pressure in the cuff and constricting the brachial

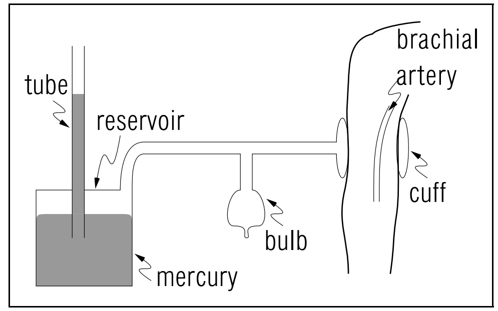


Fig. D-4.

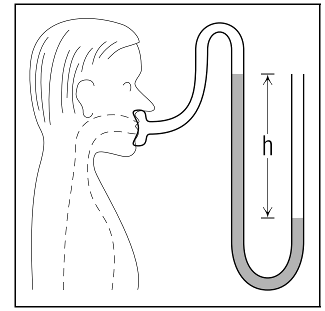


Fig. D-5.

artery until the artery barely opens when the blood in the artery has its maximum or "systolic" pressure. The resulting spurt of blood through the constriction can be heard in a stethoscope. The air pressure in the cuff is then equal to the patient's systolic blood pressure.

(a) If the systolic blood pressure is $1.17 \times 10^5\,\mathrm{N/m^2}$, what is the pressure in the mercury at its surface in the reservoir and at its surface in the tube?

(b) Using the value $1.4 \times 10^4\,\mathrm{kg/m^3}$ for the density of mercury, find the height of the mercury column in the tube (i.e., find the height of the mercury surface in the tube above the mercury surface in the reservoir). Express your answer in terms of $\mathrm{mm} = 10^{-3}$ meter.

(c) Does the height of the mercury column indicate directly (in mm-Hg) the patient's systolic blood *pressure* or *gauge pressure*?

(Answer: 118) (Suggestion: [s-10])

D-6 The "U-tube" manometer shown in Fig. D-5 is used to measure the lung air pressure a patient produces by inhaling or exhaling. One side of the tube is open to the atmosphere, while the other is connected by a tube to the patient's mouth and thus to his lungs. The patient's nose is clamped to prevent air from passing through it. The liquid in the manometer tube is water, of density $1.0 \times 10^3\,\mathrm{kg/m^3}$.

(a) At the time illustrated in Fig. D-5, is the gauge pressure of the stationary air in the patient's lungs larger than, equal to, or smaller than the zero gauge pressure of the atmospheric air?

(b) If the vertical distance $h = 1.0$ meter, what is the gauge pressure of the air in the patient's lungs?

(c) If the manometer tube contained mercury instead of water, what would be the vertical distance h between the liquid surfaces for the same gauge pressure in the patient's lungs? Use the value $1.4 \times 10^4\,\mathrm{kg/m^3}$ for the density of mercury.

(d) In measuring small gauge pressures with a liquid-filled manometer, which of the following should you use to obtain a large and easily-measured vertical distance h between the liquid surfaces: a liquid with large density (e.g., mercury) or a liquid with small density (e.g., oil)?

(Answer: 116) (Suggestion: [s-5])

D-7 A closed cylindrical container has a bottom of area A and is filled with a liquid of density ρ to a height h above the bottom. The region above the liquid has been evacuated, so that the pressure of the gas above the liquid is negligible.

(a) Write an expression for the magnitude F of the pressure force exerted by the liquid on the container bottom.

(b) *Review:* The volume of the liquid in the container is $V = Ah$. Using this result, express the mass m and weight w of the liquid in terms of h and A. Is the magnitude of the pressure force exerted by the liquid on the container bottom equal to the weight of the liquid?

(Answer: 133) More practice for this Capability: [p-3], [p-4]

SECT.

E BUOYANT FORCE

Suppose that an object is surrounded by a fluid at rest. (This fluid may consist of several different fluids, e.g., it might consist of water and air, as indicated in Fig. E-1a.) Then the total force exerted on the object by the surrounding fluid is called the "buoyant force" $\vec{F}_b$ exerted on the object by the surrounding fluid. How can we find this buoyant force?

We can use the following simple argument to find the buoyant force without the need for detailed calculation. Let us compare the situation of Fig. E-1a, showing the object surrounded by fluid, with the situation of Fig. E-1b where the object is absent. Consider, in this Fig. E-1b, the portion of fluid occupied by the object in the original situation of Fig. E-1a. (This portion of fluid is called the "fluid *displaced* by the object.") The buoyant force $\vec{F}_b$ exerted on this fluid portion by the surrounding fluid must then be the same as the buoyant force exerted on the object in Fig. E-1a. (The reason is that the pressure force exerted by the surrounding fluid on *any* small surface of the object is the same as the pressure force on the corresponding small surface of the fluid portion, because both these small surfaces are at the same depth.) But since the fluid in Fig. E-1b is at rest, the *total* external force on the fluid portion in Fig. E-1b must be zero. This total external force is the sum of the buoyant force $\vec{F}_b$ and of the gravitational force $m_d\vec{g}$ on the fluid portion of mass m_d. (In other words, m_d is the mass of the fluid *displaced* by the object.) Hence

$$\vec{F}_b + m_d\vec{g} = 0$$

or

$$\vec{F}_b = -m_d\vec{g} \qquad \text{(E-1)}$$

This result shows that the direction of the buoyant force $\vec{F}_b$ is opposite to that of the gravitational force. Hence the direction of the buoyant force is vertically *upward*. Furthermore the magnitude of the buoyant force is equal to $m_d\vec{g}$, the weight of the fluid portion displaced by the object in Fig. E-1a. Thus we arrive at the conclusion discovered by

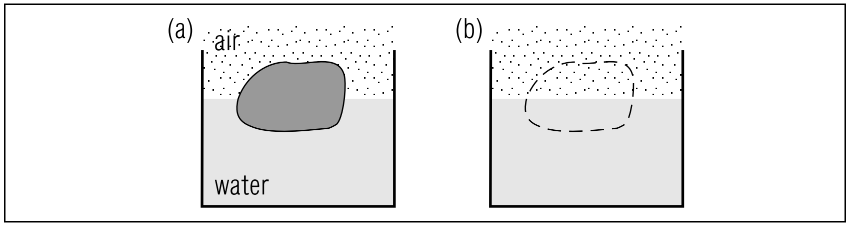


Fig. E-1: Buoyant force on an object. (a) Object submerged in a fluid (e.g., water and air). (b) Fluid without the object.

Archimedes (287-227 B.C.):

Archimedes' principle: The buoyant force on an object is directed upward and has a magnitude equal to the total weight of the fluid displaced by this object.	(E-2)

We can summarize this principle by writing

$$\boxed{F_b = w_d \text{ upward}} \qquad \text{(E-3)}$$

where w_d is the total weight of the displaced fluid, i.e., the sum of the weights of all fluids displaced by the object. Thus,

$$\text{for each fluid, } w_d = m_d g = \rho V_d g \qquad \text{(E-4)}$$

where V_d is the volume of displaced homogeneous fluid of density ρ and corresponding mass $m_d = \rho V_d$.

Since the density of air is very small, the weight of air displaced by an object is usually negligibly small. For example, in a situation such as that of Fig. E-1a, the magnitude of the buoyant force is essentially equal to the weight of the water displaced by the object, since the weight of the air displaced by the object is very much smaller and can thus be neglected.

DIRECT CALCULATION OF THE BUOYANT FORCE

To understand better how the buoyant force arises, let us calculate the total force exerted by the surrounding fluid on a simple object completely immersed in a homogeneous fluid. To be specific, we shall consider an object which has rectangular sides, the bottom and top sides each having a small area A and the other sides being vertical. (See Fig. E-2.) If the pressure at the bottom of this object is p_1, the pressure force exerted on this side by the surrounding fluid is $p_1 A\hat{y}$, where $\hat{y}$ is a unit vector in

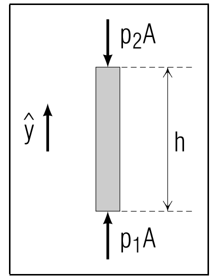


Fig. E-2: Buoyant force on a thin vertical object.

the upward direction. If the pressure at the top side of the object is p_2, the pressure force exerted on this side by the surrounding fluid is $-p_2 A\hat{y}$ (since it is downward, opposite to $\hat{y}$). The pressure forces on opposite vertical sides have equal magnitudes and opposite directions. Thus the sum of the pressure forces on these vertical sides is zero. Hence the total buoyant force exerted on the object by the surrounding fluid is merely the sum of the pressure forces on the bottom and top sides of the object. Accordingly

$$\vec{F}_b = p_1 A\hat{y} - p_2 A\hat{y} = (p_1 - p_2)A\hat{y} \qquad \text{(E-5)}$$

If the object has a vertical length h, the pressures at the bottom and top surfaces of the object are related so that $p_1 - p_2 = \rho g h$ where ρ is the density of the fluid. Thus.Eq. (E-4) becomes

$$\vec{F}_b = \rho g h A\hat{y} = \rho g V\hat{y} = m_d g\hat{y} \qquad \text{(E-6)}$$

where $V = Ah$ is the volume of the object (and thus the volume of the fluid displaced by the completely immersed object) and where $m_d = \rho V$ is the mass of this displaced fluid. Hence Eq. (E-6) shows that the buoyant force is upward and has a magnitude equal to the weight $m_d g$ of the fluid displaced by the object.

Thus we arrive again at Archimedes' principle stated in Rule (E-2).
*

> * The preceding calculation can readily be extended to the case where the object is immersed in an inhomogeneous fluid or to an object of any shape (since any such object can be regarded as consisting of many thin vertical objects of the type considered in our simple example).

DISCUSSION

If an object is completely immersed in a homogeneous fluid, the volume V_d of fluid displaced by the object is simply equal to the volume V_0 of the object. Hence the weight of the displaced fluid (of density ρ) is $\rho V_0 g$. According to Archimedes' principle, the magnitude of the upward buoyant force on the object is then $F_b = \rho V_0 g$. If F_b is *smaller* than the weight of the object (i.e., smaller than the magnitude F_g of the downward gravitational force on the object), the total external force on the object is directed downward and the object sinks to the bottom of the fluid. If F_b is *equal* to the weight of the object, the total external force on the object is zero and the object remains at rest in the middle of the fluid. If F_b is *larger* than the weight of the object, the total external force on the object is directed upward and the object rises to the top of the fluid.

Consider the last situation in the case where the fluid is a liquid with air above its top surface. If the density ρ_0 of the object is less than the density ρ of the liquid, the object rises to the surface of the liquid and emerges through it (so as to remain only partially submerged in the liquid). Since the weight of air displaced by the object is negligibly small, the magnitude of the buoyant force on the object is always equal to the weight of the *liquid* displaced by the object. But, as the object emerges through the surface, the volume of the object submerged below the liquid surface decreases. This decreased volume of liquid displaced by the object then results in a decreased buoyant force on the object. Thus the object finally comes to rest when it is submerged below the liquid to such an extent that the magnitude of the upward buoyant force on the object (i.e., the weight of the liquid actually displaced by the object) is just equal to the weight of the object.

Example E-1: Homogeneous object floating in a liquid

Suppose that a homogeneous object, having a density ρ_0 smaller than the density ρ of a liquid, floats at the surface of this liquid. Then the part of the object submerged below the surface of the liquid has some volume V_d which is the volume of the liquid displaced by the object. (See Fig. E-3.) The buoyant force on the object is then $\vec{F}_b = \rho V_d g \hat{y}$ if $\hat{y}$ denotes a unit vector in the upward direction. If the object has a volume V_0, the mass of the object is $\rho_0 V_0$ so that the gravitational force on the object if $\vec{F}_g = -\rho_0 V_0 g \hat{y}$ (because this force is downward, *opposite* to $\hat{y}$). Since the floating object is at rest, the total force $\vec{F}$ on the object must be zero so that

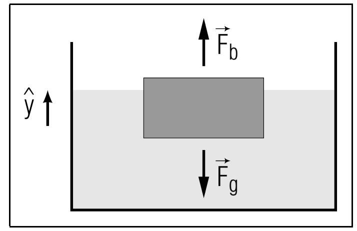


Fig. E-3: Object floating in a liquid.

$$\vec{F} = \vec{F}_g + \vec{F}_b = -\rho_0 V_0 g \hat{y} + \rho V_d g \hat{y} = 0$$

Hence

$$\rho_0 V_0 = \rho V_d$$

Thus the fraction V_d/V_0 of the volume of the object submerged below the level of the liquid is equal to

$$\frac{V_d}{V_0} = \frac{\rho_0}{\rho} \qquad \text{(E-6)}$$

For example, if the object is a block of wood (of density $\rho_0 = 0.7 \times 10^3\ \mathrm{kg/m^3}$) submerged in water (of density $\rho = 1.0 \times 10^3\ \mathrm{kg/m^3}$), $V_d/V_0 = 0.7$.

If a homogeneous object has a density ρ_0 larger than the density ρ of the liquid in which it is immersed, the object sinks since its weight is then larger than the maximum magnitude of the buoyant force on the object (i.e., larger than the weight of the liquid displaced by the object when it is completely immersed in the liquid). But if the object is hollow, the volume of liquid displaced by the object can be much larger than the actual volume of material in the object. Hence the magnitude of the buoyant force on the object can be much larger than the weight of the material in the object. This is why it is possible to build floating ships of steel, despite the fact that the density of steel is much larger than that of water.

> When finding the buoyant force on an object immersed in a liquid, you may neglect the weight of any gas displaced by the object as negligible in comparison with the weight of the liquid displaced by the object.

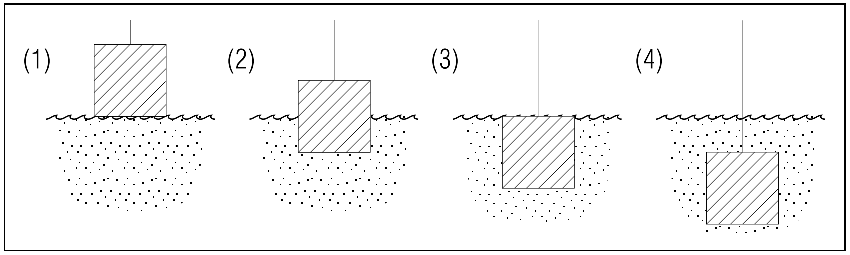


Fig. E-4.

Understanding Archimedes' Principle (Cap. 1d)

E-1 *Statement and example:* Suppose a homogeneous solid cube of volume $V_0 = 1\,\text{cm}^3 = 1 \times 10^{-6}\,\text{m}^3$ and density ρ_0 is held rigidly on a thin rod and immersed by different amounts in a liquid of density ρ_l which has gas of density ρ_g above it. (a) For each of the situations shown in Fig. E-4, use the symbols provided to write an expression for the magnitude F_b of the buoyant force exerted on the cube by the surrounding fluids. (Half of the cube is submerged in situation 2.) (b) Suppose that the liquid is water of density $1 \times 10^3\,\text{kg/m}^3$, the gas is air of density $1\,\text{kg/m}^3$, and the cube is made of wood of density $7 \times 10^2\,\text{kg/m}^3$. What is the buoyant force F_b on the cube in situations 1 and 2 of Fig. E-4? (c) *Dependence:* Consider a cube made of lead having a density 16 times that of the wood cube. Compare the buoyant force on this cube in situation 2 with the buoyant force on the wood cube in the same situation. *(Answer: 125) (Suggestion: [s-11])*

E-2 *Interpretation:* Use Table E-1 to answer these questions: (a) A rectangular barge 10 meter long and 5.0 meter wide has vertical sides 2.0 meter high and a flat bottom. When this barge is empty, it floats at rest with its bottom 0.20 meter below the water surface. What is the buoyant force exerted on the barge by the surrounding air and water? (b) A child's helium-filled balloon is a sphere of radius 10 cm. What is the buoyant force on this balloon due to the surrounding air, which has a density of $1.2\,\text{kg/m}^3$? *(Answer: 120) (Suggestion: [s-9])*

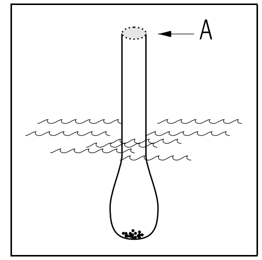


Fig. E-5.

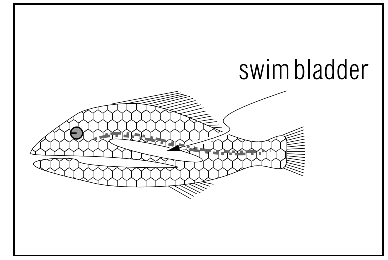


Fig. E-6.

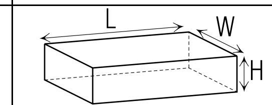
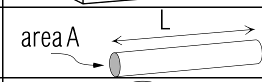
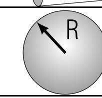
Solid	*Dimensions*	*Volume*
Rectangular Parallelopiped		$V = LWH$
Cylinder		$V = AL$
Sphere		$V = (4/3)\pi R^3$

Table E-1: Volumes of selected solids.

E-3 *Relating quantities:* A "hydrometer" used to measure the density of liquids such as urine consists of a circular glass cylinder, of cross-sectional area $A = 2.0\,\text{cm}^2$ and height 6.0 cm, which is weighted at the bottom so that it will float upright (Fig. E-5). The mass of this hydrometer is 8.0 gram. When the hydrometer floats at rest in a urine sample, the magnitude of the buoyant force exerted on the hydrometer is equal to the hydrometer's weight. Suppose that the bottom of the hydrometer is 3.6 cm below the urine surface. What is the density of the urine? *(Answer: 124)*

E-4 *Dependence:* Suppose the hydrometer described in problem E-3 is placed in urine of *smaller* density, where it again floats at rest. (a) Is the buoyant force exerted on the hydrometer larger, smaller, or the same in this situation? (b) Is the submerged volume of the hydrometer (or the depth of its bottom below the urine surface) larger, smaller, or the same in this situation? *(Answer: 128) (Suggestion: [s-8])*

Now: Go to tutorial section E.

Applying Archimedes' Principle And $\vec{F} = m\vec{a}$ (Cap. 3)

E-5 Consider the empty barge described in part a of problem E-2 (a) What is the mass of the barge? (b) To illustrate the huge carrying capacity of such a barge, find the mass of ore this barge supports when it floats at rest with its bottom 1.5 meter below the water surface. (c) Suppose that this barge is loaded in the fresh water of a river and is then towed to sea. The density of sea water is slightly larger than that of fresh water. When the barge is at rest, is the depth of the barge bottom below the surface larger, smaller, or the same in sea water as it is in fresh water? *(Answer: 122) (Suggestion: [s-12])*

E-6 Fish have a "swim bladder," a thin-walled sac filled with air which the fish extracts from its bloodstream (Fig. E-6). By inflating or deflating this bladder, the fish can increase or decrease its volume V without changing its mass m. Consider a fish of mass $m = 5.0\,\text{kg}$ which is initially floating at rest while completely submerged in water of density $1.0 \times 10^3\,\text{kg/m}^3$. (a) What is the buoyant force $\vec{F}_b$ on the fish? What is the volume V of the fish? Is the *average* density $\rho_f = m/V$ of the fish larger than, equal to, or smaller than the density of the water? (b) Suppose the fish now inflates its swim bladder slightly, thus increasing the volume V of the fish. What happens to the buoyant force $\vec{F}_b$ on the fish and the average density of of the fish? Does the fish sink, rise, or remain at rest? (c) Suppose instead that the fish deflates its swim bladder slightly. Answer the questions in part (b) for this situation. *(Answer: 126) (Practice: [p-5])*

SECT.
F SUMMARY

DEFINITIONS

density; Def. (A-1)

stress; Def. (A-2)

pressure; Def. (B-3)

gauge pressure; Eq. (D-3)

IMPORTANT RESULTS

Pressure force on a fluid: Rule (B-1), Rule (B-2)

Pressure force on any small surface of fluid portion is directed perpendicularly inward and independent of orientation of surface.

Definition of pressure: Def. (B-3)

$p = F/A$ where F is magnitude of pressure force and A is area of surface.

Relation between pressures in a fluid at rest: Rule (C-4)

$p_1 - p_2 = \rho g h$

Buoyant force on an object: Rule (E-2). Eq. (E-3), Eq. (E-4)

$F_b = w_d$ upward, where w_d =total weight of displaced fluid
(For each fluid, $w_d = \rho V_d g$)

NEW CAPABILITIES

You should have acquired the ability to:

(1) Understand these relations:

(a) the definition of density $\rho = m/V$ (Sec. A),

(b) the definition of stress $\vec{\sigma} = \vec{F}/A$ (Sec. A, [p-1]),

(c) the definition of pressure $p = F/A$ (Sec. B, [p-2]),

(d) Archimedes' principle $\vec{F}_b = w_d$ upward, where $w_d = \rho V_d g$ for each fluid (Sec. E).

(2) For a system of one or more fluids at rest, relate the positions of two points to the pressures or gauge pressures at these points or to the pressure forces exerted by the fluid near these points. (Sects. C and D, [p-3], [p-4].

(3) Apply Archimedes' principle and the equation of motion to relate quantities describing the buoyant force on an object at rest in a fluid to quantities describing the remaining forces on the object. (Sec. E, [p-5]).

Applying Relations for Solids and Fluids At Rest (Cap. 1, 2, 3)

F-1 What would be the magnitude of the pressure force exerted by the sea water on a $1.0\,\text{cm}^2$ area on the surface of an animal living at the bottom of the Java Trench in the Indian Ocean? The depth of this trench is 7.5 kilometer. (Assume that sea water has a uniform density of $1.03 \times 10^3\,\text{kg/m}^3$.) *(Answer: 131)*

F-2 An iceberg of uniform density ρ_i and volume V_i f1oats at rest in sea water of density ρ_w. (a) Write an expression for the mass M_i of the iceberg. (b) Write an expression for the submerged volume V_s of the iceberg (i.e., the volume of the iceberg below the water surface). (c) What is the fraction V_s/V_i of the iceberg's volume that is below the water surface? Express your answer in terms of ρ_i and ρ_w (d) Use the values $\rho_i = 0.93 \times 10^3\,\text{kg/m}^3$ and $\rho_w = 1.03 \times 10^3\,\text{kg/m}^3$ to find the fraction of the volume of the iceberg that is below the water surface. *(Answer: 129)*

F-3 A healthy person cannot expand the chest to breathe if the pressure outside the chest is more than about $5 \times 10^3\,\text{N/m}^2$ larger than the pressure of the air in the lungs. Suppose such a person is using a "snorkel" tube to breathe atmospheric air while swimming underwater. If the air pressure in this person's lungs is equal to the atmospheric pressure, what is the depth of the person's chest below the water surface when the water pressure outside the chest exceeds the lung air pressure by $5 \times 10^3\,\text{N/m}^2$? (This is the maximum possible depth for snorkeling.) *(Answer: 132)*

F-4 Sharpening a dull knife greatly decreases the thickness of the cutting edge and thus the area of the knife in contact with the material being cut. (a) Suppose we hold first a dull knife, and then the same knife after sharpening, so that it exerts the same force on some material. Is the stress produced on the material larger, smaller or the same for the dull knife as it is for the sharp one? (b) To barely cut any material, a knife must produce a certain minimum stress on the material. To produce this stress, must a dull knife exert on the material the same force as a sharp knife, a smaller force, or a larger force? Does this result agree with your experience that sharp knives cut more easily? *(Answer: 134)*

F-5 To "suck" liquid up a straw, a person expands the mouth cavity or lungs, thus decreasing the air pressure in these cavities and above the liquid in the straw. (a) If the person's lips are 20 cm above the surface of the lemonade (of density $1.0 \times 10^3\,\text{kg/m}^3$) in a glass, what air pressure must the person produce in the mouth so that the lemonade surface in the straw just reaches the lips? (b) The minimum gauge pressure a person can produce in the air in the mouth is about -76 mm-Hg. What is the corresponding maximum height to which a person can raise the lemonade surface in a straw? (This is the height of the tallest practical straw for sipping lemonade!) (c) Answer the preceding question if the liquid is not lemonade but gin, which has a smaller density of $0.92 \times 10^3\,\text{kg/m}^3$. *(Answer: 109)*

Note: Further applications are provided in tutorial section F.

SECT.

G PROBLEMS

G-1 *Using a hydrometer to measure the specific gravity of a liquid:* The "specific gravity" of a substance is defined as the ratio ρ_s/ρ_w, where ρ_s is the density of the substance and ρ_w is the density of water. A "hydrometer" is often used to measure the specific gravity of liquids (e.g., battery acid specific gravity, an indicator of the "charge" of the battery). Consider the hydrometer of Fig. G-1, which has a glass bulb containing lead shot and a vertical glass stem of cross-sectional area A. When this hydrometer floats at rest in water, its entire volume V is submerged. Suppose that when this hydrometer floats at rest in a liquid, the top of its stem is a distance h above the liquid surface. (a) Is the density ρ_s of the liquid larger or smaller than the density ρ_w of water? (b) Express the specific gravity ρ_s/ρ_w of the liquid in terms of V, h, and A. *(Answer: 123) (Suggestion: [s-3])*

G-2 *Measuring the specific gravity of a solid:* Consider a sample of some homogeneous solid having a density ρ_s and a weight w. The specific gravity of the substance (see problem

G-1 *F:* G2. (a) Find an expression for the specific gravity ρ_s/ρ_w of the substance in terms of F and w. (b) For a sample of glass, $F = (2/3)w$ (i.e., the measured weight of the sample when immersed in water is two-thirds of its actual weight). What is the specific gravity of the glass in this sample? *(Answer: 138) (Suggestion: [s-1])*

G-3 *Sensitivity of the ear:* A person with good hearing can barely hear a sound which causes the air pressure outside the eardrum to differ from the air pressure behind the eardrum by about $3\times10^{-5}\,\text{N/m}^2$. What is the magnitude of the total force exerted by the air on the eardrum under these conditions? The diameter of the eardrum is about 0.6 cm. *(Answer: 136) (Suggestion: [s-6])*

G-4 *Ear pain due to changes in height:* Because of blockage of the Eustachian tube, the air pressure behind the eardrum of a person with a cold remains roughly constant over short time periods. Suppose such a person rides downward in an elevator. What distance can this person travel before the difference in pressure across the eardrum reaches $300\,\text{N/m}^2$? This value marks the threshold of pain in human hearing. (b) Answer the preceding question if the person dives downward in a swimming pool. *(Answer: 130)*

G-5 *Change in water level due to melting ice:* A homogeneous ice cube floats at rest in a glass of water. When the ice cube has melted completely, is the water level in the glass higher, lower, or the same as before? *(Answer: 135)*

G-6 *Change in water level due to discarding a load of scrap:* A barge loaded with scrap iron floats at rest in a canal lock. The lock gates and sluices are closed, so that the volume of water in the lock remains constant. If the scrap iron is thrown overboard and sinks to the bottom of the lock, does the level of the water in the lock rise, fall, or remain the same? *(Answer: 137)*

Note: Additional problems are provided in tutorial section G.

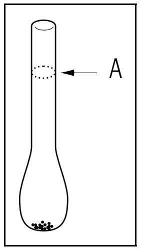


Fig. G-1.

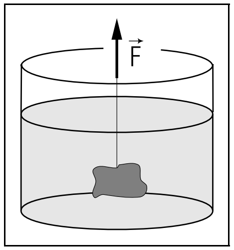


Fig. G-2.

SECT.

H DEPENDENCE OF PRESSURE FORCE ON ORIENTATION OF A SURFACE

To examine how the pressure force on a small surface near a point B depends on the orientation of this surface, consider a small wedge-shaped portion of fluid enclosing the point B and having two surfaces 1 and 2 which have different orientations but the same area A. (See Fig. H-1.) Let us then apply the equation of motion $m\vec{a} = \vec{F}$ to this small wedge of fluid of mass m. The total external force on this wedge is the sum of the gravitational force $m\vec{g}$ and of the pressure forces perpendicular to all the surfaces of the wedge. Hence the equation of motion of the wedge is

$$\boxed{m\vec{a} = m\vec{g} + (\text{sum of all pressure forces})} \qquad \text{(H-1)}$$

But if the wedge is so small that all its surfaces are almost at the point B, the volume V of the wedge, and thus also its mass $m = \rho V$ (where ρ is the density of the fluid), is negligibly small. Hence (1) implies that the pressure forces on very small surfaces near a point must be related so that

$$\boxed{\text{sum of pressure forces} = 0} \qquad \text{(H-2)}$$

In particular, the sum of the component vectors of these pressure forces along the direction $\hat{x}$ parallel to the surface 3 of the wedge in Fig. H-1 must then also be zero. But this component vector of the pressure force $\vec{F}_1$ on the surface 1 is $F_1 \cos\theta \hat{x}$; this component vector of the pressure force $\vec{F}_2$ on surface 2 is $-F_2 \cos\theta \hat{x}$; and this component vector of the pressure force $\vec{F}_3$ is zero since this force is perpendicular to $\hat{x}$. Hence (2) implies that

$$F_1 \cos\theta \hat{x} - F_2 \cos\theta \hat{x} = 0$$

so that

$$F_1 = F_2 \qquad \text{(H-3)}$$

In other words, no matter what the relative orientation of the surfaces 1 and 2 may be, the magnitudes of the pressure forces on these surfaces are always the same. Thus we arrive at Rule (B-2).

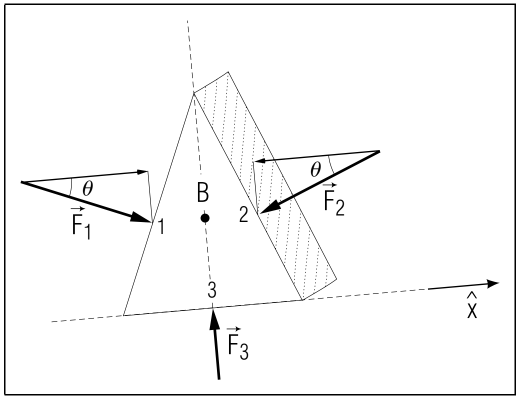


Fig. H-1: A small wedge-shape portion of fluid surrounding the point B.

TUTORIAL FOR E

APPLYING ARCHIMEDES' PRINCIPLE AND $\vec{F} = m\vec{a}$

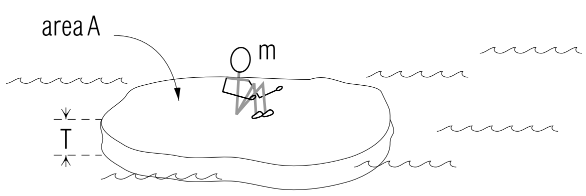
e-1 *PURPOSE:* When an object is at rest in a gas or a liquid, the equation of motion states that the total force on the object (i.e., the vector sum of the buoyant force on the object and the other forces on the object) must be zero. The purpose of the following frame is to illustrate how we can use this statement to relate a variety of quantities describing the forces acting on the object. Such quantities include the dimensions of the fluid volume displaced by the object, the density and volume (or the mass) of the object, and the density of the fluid. As usual, we shall follow the useful problem-solving strategy outlined in text section D of Unit 409.

e-2 *A METHOD FOR APPLYING ARCHIMEDES' PRINCIPLE AND $\vec{F} = m\vec{a}$:* Let us systematically solve this problem:

What is the surface area of a 20 cm thick ice floe that can barely support an 80 kg man while floating at rest in sea water of density $1.03 \times 10^3\,\text{kg/m}^3$? When the man sits on such a floe, it floats at rest with its top surface level with the water surface. Assume that the floe has vertical sides and a flat top, and that it is formed of homogeneous ice having a density of $0.93 \times 10^3\,\text{kg/m}^3$. (The volume of this floe is given by its area multiplied by its thickness.)

DESCRIPTION:

Diagram:



Known information:

Thickness of floe: $T = 0.20$ meter.

Density of ice forming floe: $\rho_1 = 0.93 \times 10^3\,\text{kg/m}^3$

Volume of floe: $V = AT$, where A is its surface area.

Mass of man: $m = 80\,\text{kg}$.

Density of water: $\rho_w = 1.03 \times 10^3\,\text{kg/m}^3$.

Desired information:

Surface area A of floe.

PLANNING:

(1) *Choose the particle to be considered, and write its equation of motion in terms of symbols for the forces acting on it.*
Since the water supports both the man and the floe, let us choose the two together as a composite particle. The forces on this particle are the buoyant force $\vec{F}_b$ due to the surrounding water and air and the gravitational force $\vec{F}_g$ due to the earth. Since this composite particle is at rest, its equation of motion is

$$\vec{F}_b + \vec{F}_g = 0$$

(2) *Express each force in this equation in terms of symbols for known and desired information.*
The buoyant force is given by Archimedes' principle, $\vec{F}_b = w_d\hat{x}$, where $\hat{x}$ is a unit vector directed upward. Since the weight of the air displaced is negligible, the weight w_d of fluid displaced is just the weight of the water displaced. Thus $w_d = \rho_w V_d g$. The volume of water displaced is equal to the volume of the floe, so that $V_d = AT$. Thus

$$\vec{F}_b = \rho_w AT g\hat{x}$$

The gravitational force on this composite particle is given by $\vec{F}_g = -(m + M)g\hat{x}$, where M is the mass of the floe. Since the floe is homogeneous, its density $\rho_i = M/V = M/AT$, so that $M = \rho_i AT$. Thus

$$\vec{F}_g = -(m + \rho_i AT)g\hat{x}$$

Our equation of motion then becomes

$$\rho_w A T g \hat{x} - (m + \rho_i A T) g \hat{x} = \vec{0}$$

We know all the quantities in this equation except the desired quantity A, so we can find the value of A.

IMPLEMENTATION:

(1) *Solve the equation algebraically for the desired quantity.*
Writing both sides of our equation as multiples of $\hat{x}$:

$$[\rho_w A T g - (m + \rho_i A T) g]\hat{x} = 0\hat{x}$$

Thus we have the algebraic equation

$$\rho_w A T g - (m + \rho_i A T) g = 0$$

Dividing both sides of this equation by g and then solving for A:

$$\rho_w A T - (m + \rho_i A T) = 0$$

$$A(\rho_w T - \rho_i T) - m = 0$$

$$A = m/(\rho_w T - \rho_i T) = m/(\rho_w - \rho_i)T$$

(2) *Substitute known values, and find the desired quantity.*
Since $\rho_w - \rho_i = (1.03 - 0.93) \times 10^3\,\mathrm{kg/m^3} = 0.10 \times 10^3\,\mathrm{kg/m^3}$,

$$\begin{aligned} A &= (80\,\mathrm{kg})/(1.0 \times 10^2\,\mathrm{kg/m^3})(2.0 \times 10^{-1}\,\mathrm{meter}) \\ &= \frac{8.0 \times 10^1}{2.0 \times 10^1\,\mathrm{kg}} \left(\frac{m^3}{\mathrm{kg}}\right) \left(\frac{1}{\mathrm{meter}}\right) \\ &= -4.0\,\mathrm{m^2} \end{aligned}$$

CHECKING:

The algebra and arithmetic are correct, and the area of the floe has the correct positive sign and a reasonable magnitude. In addition, our equation for the required area A shows that A would be larger if the supported man's mass m were larger, and smaller if the thickness T of the floe were larger; both of these results make sense.

Using an approach similar to the one illustrated in this problem, you should be able to solve systematically other problems requiring application of Archimedes' principle and $\vec{F} = m\vec{a}$.

Now: Go to text problem E-5.

TUTORIAL FOR F

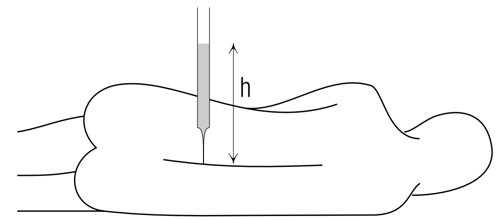
APPLYING RELATIONS FOR SOLIDS AND FLUIDS AT REST

The problems in this section illustrate further applications of the important relations discussed in this unit, and offer further practice and review for Capabilities 1, 2, and 3.

f-1 *THE HIGH BLOOD PRESSURE OF THE GIRAFFE:* What would be high blood pressure for a man is normal for a giraffe, even though both animals have the same average blood pressure in the brain. The reason is the giraffe's extraordinary height. To illustrate, suppose for simplicity that the blood in both an erect man and an erect giraffe is at rest, and that the arterial blood in the brains of both animals has the normal average gauge pressure of 60 mm-Hg $= 7.8 \times 10^3\,\mathrm{N/m^2}$. The man's heart is 0.50 meter below the brain, while The giraffe's heart is 3.0 meter below the brain. What is the gauge pressure (in mm-Hg) of the blood in the heart of each animal? Take the blood of both animals to have a density of $1.1 \times 10^3\,\mathrm{kg/m^3}$. *(Answer: 2) (Suggestion: Review Text Sects. C and D. Reference: James V. Warren, "The Physiology of the Giraffe," Scientific American, November 1974.)*

f-2 *DANGER OF PRESSURIZED AIRCRAFT CABINS:* Suppose that an airplane maintains its interior or cabin air pressure at the airport air pressure of $1.0 \times 10^5\,\mathrm{N/m^2}$ so that the cabin air is "pressurized." The airplane leaves the airport and ascends to a height of 4.0 kilometer (about 13,000 foot) above the airport. One of the airplane's windows is a plexiglass square 20 cm on a side. (a) What are the magnitudes and directions (outward or inward) of the pressure forces exerted on this window by the air inside and outside the airplane? Use the value $1.0\,\mathrm{kg/m^3}$ for the density of air. (b) If the window is not securely fastened to the airplane's fuselage, what is likely to happen? To avoid this result, cabin air pressure is usually adjusted to a value less than the original airport air pressure, although still larger than the exterior air pressure. *(Answer: 5) (Suggestion: review Text Sec. D.)*

f-3 *HEIGHT OF FLUID IN A CEREBROSPINAL TAP:* The cerebrospinal fluid in the spinal and cranial cavities has a gauge pressure of about 11 mm-Hg and a density about equal to that of water. To tap this fluid, a needle attached to a vertical tube is inserted into the spinal cavity as shown in the following drawing. The fluid rises up the tube, which is open to the atmosphere at the top. (a) What is the height h of the column of fluid in the tube when the fluid is at rest?

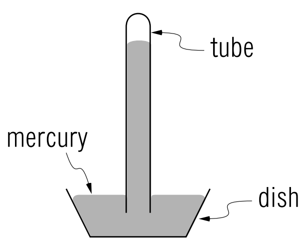


(b) In the "Queckensted test," the veins in the neck are then compressed, increasing the venous blood pressure in the brain. If the patient is normal, this action also increases the pressure through the cerebrospinal fluid. What should happen to the height of the fluid column in the tube if the patient is normal? *(Answer: 12) (Suggestion: review text section D.)*

f-4 *BUOYANCY CORRECTION IN WEIGHING:* When we weigh a homogeneous object of density ρ_0 and volume V_0, the measured weight of the object differs from its actual weight because of the object's buoyancy due to the surrounding air of density $\rho_a = 1\,\mathrm{kg/m^3}$. Although this difference is small, it must be corrected for in making accurate weight measurements using a chemical balance. Let us find the size of this difference (the error due to buoyancy) relative to the object's actual weight. (a) Express the object's actual weight w in terms of ρ_0, ρ_a, and V_0. (b) When the object is weighed on a scale, the scale indicates the magnitude F of the upward force exerted on the object by the scale, so that F is the measured weight of the object. Express the measured weight F in terms of ρ_0, ρ_a, and V_0, assuming that the object is surrounded by air. Is the measured weight larger or smaller than the actual weight? (c) Express the ratio $(w - F)/w$ in terms of ρ_0, ρ_a, and V_0. This ratio is the *relative* error in weighing due to buoyancy, since the difference $(w - F)$ is the error. Is the relative error larger for small-density or large-density objects? (d) Find the relative error if the object is a piece of wood of density $7 \times 10^2\,\mathrm{kg/m^3}$ and if the object is a chunk of lead of density $1 \times 10^4\,\mathrm{kg/m^3}$. *(Answer: 9) (Suggestion: review text section E and the method outlined in tutorial section E.)*

f-5 *USING AIR PRESSURE TO MEASURE ALTITUDE:* The density of air has a roughly constant value of $1\,\mathrm{kg/m^3}$ up to a height of about 5 kilometer above the earth's surface. Thus the atmospheric air pressure changes uniformly with height within this range, and a sensitive barometer can be easily used as an "altimeter," a device for measuring altitude or height. (a) To illustrate, suppose the atmospheric air pressure at a sea-level airport is $1.0 \times 10^5\,\mathrm{N/m^2}$. What is the air pressure at the position of an airplane which is 1000 meter (about 3300 feet) above the airport? (b) A sensitive aircraft altimeter can detect a change in air pressure of only $20\,\mathrm{N/m^2}$. What is the corresponding change in altitude this altimeter can detect? (In other words, what is the vertical distance between two points at which the air pressure differs by $20\,\mathrm{N/m^2}$?) *(Answer: 7) (Suggestion: review text section D.)*

f-6 *HEIGHT OF A WATER BAROMETER:* The following drawing shows a mercury-filled barometer like the one constructed by Torricelli in the mid 1600's. The mercury in the dish is in contact with air at atmospheric pressure, while the space above the mercury in the closed tube contains air at negligible pressure (i.e., a vacuum). The height of the mercury column in the tube (about 760 mm or 0.76 meter) thus indicates the atmospheric air pressure. (a) Shortly after this barometer was made, Pascal made a similar one containing water instead of mercury. If the atmospheric pressure is $1.0 \times 10^5\,\mathrm{N/m^2}$, what is the height of the water column in the tube of such a barometer? This is the minimum height of the tube needed to make this barometer. (b) Suppose we wish to make a similar barometer using an oil having a smaller density than that of water. Will the necessary tube height be larger or smaller than your previous answer? *(Answer: 11) (Suggestion: review text section D.)*

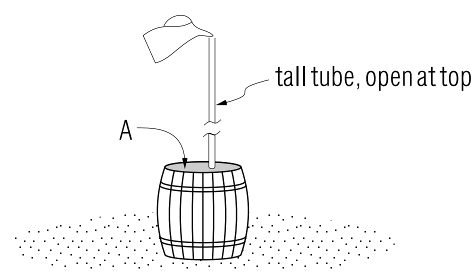


TUTORIAL FOR G

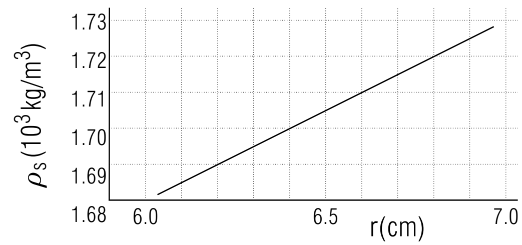
PROBLEMS

The problems in this section illustrate more complex applications of the important relations discussed in this unit.

g-1 *BURSTING A BARREL WITH TWO POUNDS OF WATER:* The following drawing shows a barrel like the one Pascal used in 1646 to convince his contemporaries that the pressure force exerted by a liquid at a given level can have a magnitude much *larger* than the weight of the liquid above this level. The barrel's thin tight-fitting lid, of area $A = 0.20\,\mathrm{m^2}$, has a tall tube of negligible cross-sectional area inserted through it. At the start of Pascal's demonstration, water is poured into this tube until the water surface in the barrel just reaches the bottom surface of the barrel lid. (a) In this situation, what are the forces $\vec{F}_a$ and $\vec{F}_w$ exerted on the barrel lid by the air above it and the water below it? If the mass of this lid is negligible, what is the vertical force $\vec{F}_b$ the barrel walls must exert on the lid to keep it stationary? (b) What are the forces $\vec{F}_a$, $\vec{F}_w$, and $\vec{F}_b$ if 1 liter of water, having a weight of about 10 N or 2 lb, is poured into the tube? This action raises the surface of the water in the tube to a height of 10 meter above the bottom of the lid. (c) Suppose the maximum magnitude of the vertical force $\vec{F}_b$ the barrel walls can exert on the lid is $2.0 \times 10^4\,\mathrm{N}$ (about 2 ton). What will happen when a little more water is poured into the tube? *(Answer: 15)*



g-2 *DNA SEPARATION USING DENSITY-GRADIENT ULTRACENTRIFUGATION:* When a concentrated solution of cesium chloride (CsCl) is placed in the sample cell of an ultracentrifuge spinning at 60,000 revolutions per minute, the large apparent gravitational acceleration $\vec{g}\prime$ relative to the cell causes the CsCl molecules to concentrate toward the outer part of the cell. The density ρ_s of the solution thus increases with distance r from the axis of the centrifuge in the manner shown by this approximate graph:



When particles such as DNA molecules are placed in this solution, they move under the influence of the apparent gravitational force and the buoyant force due to the solution until they reach a certain distance r from the axis at which they remain at rest relative to the cell. Particles with different properties collect at different distances from the axis, and can thus be separated. (For example, this result has been used to separate ordinary DNA molecules from those grown in an environment containing heavy ^{15}N atoms instead of ordinary ^{14}N atoms. This technique was crucial to experiments exploring DNA replication mechanisms.) (a) Let us call ρ_m the density and V the volume of a DNA molecule in the cell. Write an expression for the total force $\vec{F}$ on the molecule (relative to the cell) in terms of ρ_m, V, g', and the density ρ_s of the solution at the molecule's position. Use a unit vector $\hat{x}$ to indicate the direction along $\vec{g}\prime$. (b) Using this result, find an expression for the density ρ_s of the solution where the molecule remains at rest relative to the cell. (c) Ordinary DNA molecules containing only ^{14}N atoms have a density of $1.700 \times 10^3\,\text{kg/m}^3$, while "heavy" DNA molecules containing only ^{15}N atoms have a density of $1.714 \times 10^3\,\text{kg/m}^3$. Using this information and the preceding graph, find the separation between these two types of molecules when both have come to rest relative to the cell. *(Answer: 8)*

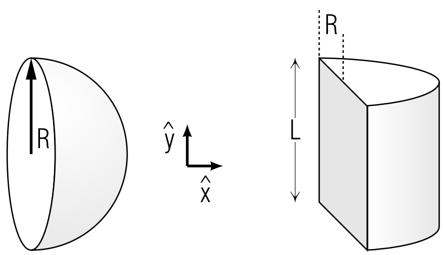
g-3 *USING WORK TO RELATE FORCES ON A HYDRAULIC PRESS:* We can use work instead of pressure to relate the forces $\vec{F}_1$ and $\vec{F}_2$ acting on the pistons of the hydraulic press shown in Fig. D-3. Since the press is in macroscopic equilibrium, we know from text section G of Unit 416 that the total work δW done on this system is *zero* for any small displacement. Since internal forces do no work, the total work δW is done only by the external forces $\vec{F}_1$ and $\vec{F}_2$. Suppose that piston 1

moves downward by a small distance L_1, so that piston 2 moves upward by a small distance L_2. (a) Express the total work δW done on the press in terms of L_1, L_2, and the magnitudes F_1 and F_2 of the forces on the pistons. Then use the result $\delta W = 0$ to express the ratio F_1/F_2 in terms of L_1 and L_2. (b) Since piston 1 moved downward a distance L_1, a volume of fluid $V_1 = L_1 A_1$ left cylinder 1. Similarly, a volume of fluid $V_2 = L_2 A_2$ was added to cylinder 2. Using the fact that the volume of the fluid in the press remained constant as the pistons moved, express the ratio L_1/L_2 in terms of the areas A_1 and A_2 of the pistons. (c) Use your results to express the ratio F_1/F_2 in terms of A_1 and A_2. Does your result agree with Eq. (D-4)? *(Answer: 17) (Suggestion: [s-4])*

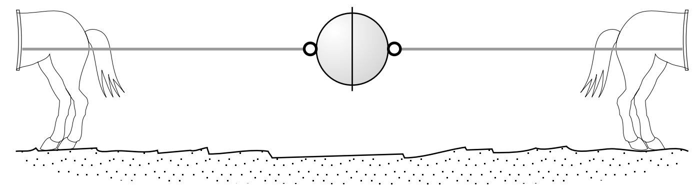
g-4 *SCALING OF BONE THICKNESS WITH BODY SIZE:* In *Gulliver's Travels*, Jonathan Swift pointed out that animals of similar construction but different size cannot have bones of the same shape. This problem illustrates why.

Consider two animals of similar shape (e.g., a man and a Lilliputian), one animal being twice the size of the other. Since the larger animal is twice as tall, twice as wide, and twice as thick as the smaller one, the larger animal's volume V' is $2 \times 2 \times 2 = 8$ times as large as the smaller animal's volume V, or $V' = 8V$. (a) Let us assume that both animals have the same average density (i.e., the animal's mass divided by its volume). Express the mass M' of the larger animal as a number times the mass M of the smaller one. (b) Since the leg bones of each animal must support the animal's weight, the contact forces $\vec{F}\prime$ and $\vec{F}$ exerted at the cross-sectional areas A' and A of these bones must be related in the same way as the masses of the animals. Thus $F'/F = M'/M$. Since the breakage of bones depends on stress, however, the stress produced by these forces should be the same for each animal. Using these observations, express A' as a number times A. (c) If we assume the bones of both animals are solid and have circular cross-sections with diameters d' and d, then $A' = (\pi/4)d'^2$ and $A = (\pi/4)d^2$. Using your previous result, express d' as a number times d. (d) The larger animal's leg bone is twice as long as the smaller animal's leg bone. If these bones have the same shape, d' should be twice as large as d. Is it? If not, is the larger animal's bone thicker or less thick in comparison to its length than the smaller animal's leg bone? *(Answer: 10)*

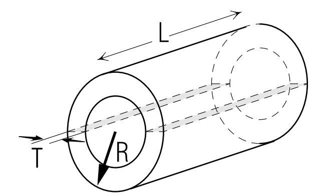
g-5 *PRESSURE FORCES ON SPECIAL CURVED SURFACES:* The following drawing shows a solid hemisphere and a solid half-cylinder which are both floating at rest in a fluid having a *uniform* pressure p, and which both interact only with the fluid (e.g., this fluid might be in a tank located in space far from everything else). (a) What is the magnitude and the direction of the pressure force $\vec{F}_f$ exerted by the fluid on the *flat* surface shown in gray on each object? (b) What is the pressure force $\vec{F}_c$ exerted by the fluid on the curved surface of each object? These results are excellent approximations for the pressure forces exerted by a gas near the earth's surface, and for the pressure forces exerted by a liquid near the earth's surface if the dimensions of the curved surfaces are less than a few centimeters. *(Answer: 16)*



g-6 *FORCE REQUIRED TO PART THE MAGDEBURG HEMISPHERES:* In Magdeburg in 1654, Otto von Guericke staged a dramatic demonstration of atmospheric pressure, as illustrated in the following drawing. He placed the rims of two hollow hemispherical shells together and used an air pump of his invention to evacuate the air inside the resulting sphere. Two teams of eight horses each were unable to pull the hemispheres apart. The radius of the hemispheres was 0.30 meter. If the air pressure inside the sphere was 0.10 atmosphere, what is the magnitude of the tension force exerted by each rope that would just overcome the pressure force on each hemisphere and thus pull them apart? Use your results from tutorial frame [g-5], and assume that the interior and exterior radii of the spheres are the same. *(Answer: 14)*



g-7 *STRESS IN THE WALL OF A BLOOD VESSEL:* Consider a cylindrical portion of blood vessel having a length L, a radius R, and a *thin* wall of thickness T, as shown in the following drawing. (The thickness of the vessel in the drawing is greatly exaggerated for clarity.)



Suppose the blood in this vessel has a uniform pressure p_i while the fluid in the tissues outside the vessel has a smaller uniform pressure p_0. The wall of the vessel remains at rest despite the pressure force tending to expand it because different parts of the wall material exert forces on each other. Let us find the stress in the wall. (a) Consider the upper and lower pieces of the vessel shown in the figure. The upper piece exerts on the lower one a total contact force $\vec{F}$ equal to the sum of the contact forces exerted at the *two* areas of contact shown in gray. Express the magnitude F of this force in terms of p_i, p_0, and the dimensions of the vessel, using your result in tutorial problem [g-5]. (b) What is the magnitude σ of the stress produced by this contact force? (c) Find the value of σ for the aorta, using the values $(p_i - p_0) = 100\,\text{mm Hg} = 1.3 \times 10^4\,\text{N/m}^2$, $R = 1\,\text{cm}$, and $T = 2\,\text{mm}$. Compare this value with the tensile strength of $10^6\,\text{N/m}^2$ for the elastin fibers which support most of this stress. (d) The wall of a capillary is only $1\,\text{micron} = 10^{-6}$ meter thick (1/2000 of the aorta wall thickness), and yet the stress required to keep the wall in equilibrium for a given "transmural pressure" $(p_i - p_0)$ is about the *same* for a capillary as it is for the aorta. Use your result to explain why the thin wall of the capillary can support a given pressure difference as easily as the thick wall of the aorta. (In fact, the pressure difference across a capillary wall is about one-fourth that across the aorta wall.) *(Answer: 18)*

PRACTICE PROBLEMS

p-1 *UNDERSTANDING THE DEFINITION OF STRESS (CAP. 1B):* Women wearing spike-heeled shoes used to dent (and even punch through) the thin aluminum floors of commercial aircraft. To see why, suppose a 50 kg woman happens to have only the spike heel of one shoe in contact with the floor at some instant, so that the magnitude of the downward contact force exerted by the heel on the floor is equal to the woman's weight. (a) If the bottom of the heel is a square having a side of 0.50 cm, what is the stress on the floor just under the heel? (b) For comparison, suppose instead that the woman is wearing shoes with low heels having a bottom area 200 times that of the spike heels. In the same situation, what is the stress on the floor under the heel? *(Answer: 4) (Suggestion: review text problems A-6 and A-7.)*

p-2 *UNDERSTANDING THE DEFINITION OF PRESSURE (CAP. 1C):* Consider a region on the wall of an artery that is a square 3.0 mm on a side. Suppose the pressure of the blood in the artery is $1.1 \times 10^5\,\mathrm{N/m^2}$, a typical value. (a) What is the magnitude of the pressure force exerted on this small enough square region by the blood? (b) Answer the preceding question for a square region of arterial wall that is 1.0 mm on a side. (c) Suppose that the pressure of the blood in the artery decreases. Does the pressure force exerted by the blood on the two small regions of the arterial wall increase, decrease, or remain the same? *(Answer: 1) (Suggestion: review text problem B-4.)*

p-3 *RELATING PRESSURE, PRESSURE FORCE, AND POSITION (CAP. 2):* Deep-sea divers work inside a flexible rubber suit into which compressed air is fed from a hose leading to the surface. To keep the suit from shriveling inward (which would make it difficult for the diver to breathe) or ballooning outward (which would make it difficult for the diver to move and which might even rupture the suit), the force exerted by the air on any portion of the suit must be equal in magnitude to the force exerted by the surrounding water on this portion. (a) If the diver is 200 meter below the ocean surface, what is the magnitude of the pressure force exerted by the water on a flat portion of the suit having an area of $1.0\,\mathrm{in}^2 = 6.5\,\mathrm{cm}^2$? Assume that sea-water has a density of $1.0 \times 10^3\,\mathrm{kg/m^3}$, and use the relation $10\,\mathrm{N} = 2.2\,\mathrm{lb}$ to express your answer in terms of the unit lb (pound). (b) If the air inside the suit is to exert a force of equal magnitude on this portion, what must be the air pressure in the suit?

Express your answer in terms of lb/in^2. *(Answer: 3) (Suggestion: review text problem D-7.)*

p-4 *RELATING PRESSURE, PRESSURE FORCE, AND POSITION (CAP. 2):* In a simple intravenous feeding arrangement, a solution of density $1.0 \times 10^3\,\text{kg/m}^3$ is contained in a bottle from which a tube leads downward to a needle inserted in the patient's arm. The air pressure above the solution in the bottle is equal to the atmospheric air pressure. The solution will barely flow into the vein if the gauge pressure of the solution in the needle barely exceeds the gauge pressure of the blood in the vein. Suppose the gauge pressure of the blood in the vein is $10\,\text{mm-Hg} = 1.3 \times 10^3\,\text{N/m}^2$. What height must the surface of the solution be above the needle to ensure that the gauge pressure of the solution in the needle is just equal to this venous gauge pressure? This is the minimum height required for intravenous feeding. *(Answer: 13) (Suggestion: review text problem D-6.)*

p-5 *APPLYING ARCHIMEDES' PRINCIPLE AND $\vec{F} = M\vec{A}$ (CAP. 3):* A skin diver can send an object to the ocean surface by suspending it from a plastic bag which the diver inflates from his air supply. When released, the bag and object rise to the surface because of the buoyancy of the bag. To estimate the mass of an object which can be lifted to the surface by a typical bag, find the mass of a dense object which can be suspended *at rest* from an inflated bag having a volume of $400\,\text{cm}^3$ when both are located at a depth of 10 meter. At this depth, the density of the water is $1.0 \times 10^3\,\text{kg/m}^3$ and the density of the air in the bag is $2.4\,\text{kg/m}^3$. The buoyant force on the dense object and the weight of the plastic forming the bag are both negligible. *(Answer: 6) (Suggestion: review text problem E-5 and the method outlined in tutorial frame [e-2].)*

SUGGESTIONS

s-1 *(Text problem G-2):* Note that the weight of the sample is $w = \rho V g$, where V is the volume of the sample. Use this relation to eliminate V from the equation relating the forces acting on the sample when it is immersed in water. (If you need more help, review tutorial section E.)

s-2 *(Text problem A-3):* The mass m of any amount of a homogeneous substance is proportional to the volume V of this amount so that, for example, twice the volume of the substance has twice the mass. Consequently, the density of the substance, which is the ratio of these two quantities, has the same value when measured with any amount of the substance.

s-3 *(Text problem G-1):* The buoyant forces on the hydrometer in the water and the liquid must be equal, since both are equal in magnitude to the weight of the hydrometer. Note that the submerged volume of the hydrometer in the liquid is $(V - hA)$, since the quantity hA is the volume of the stem above the liquid surface.

s-4 *(Tutorial frame [g-3]):* Since piston 1 moves downward a distance L_1 along the direction of the force $\vec{F}_1$, the work done on the press by the force $\vec{F}_1$ is $\delta W_1 = +F_1 L_1$. Since piston 2 moves upward a distance L_2 in a direction opposite to that of the force $\vec{F}_2$, the work done on the press by the force $\vec{F}_2$ is $\delta W_2 = -F_2 L_2$.

s-5 *(Text problem D-6):* The *gauge* pressures at any two points in a fluid are related in the same way as the *pressures* at these points are related. For example, the water pressure at the surface on the left side of the manometer tube shown in Fig. D-5 is less than the water pressure at the surface on the right side by the amount $\rho g h$, where ρ is the density of water. The same is true of the gauge pressures of the water at these surfaces.

s-6 *(Text problem G-3):* Write each pressure force on the eardrum in terms of the pressure of the air exerting the force and the area A of the eardrum, using a unit vector to indicate direction (either outward from the eardrum or inward toward it). Then find the sum of these forces. Since the forces have opposite directions, their sum will be proportional to the *difference* between the air pressures outside and inside the eardrum.

s-7 *(Text problem A-4):* Part (b): An amount of homogeneous matter having a mass m and a volume V has a density $\rho = m/V$. The mass m of the substance in the container remains constant when the substance is compressed. Therefore, the density of the substance changes appreciably only if the *volume* of the substance changes appreciably. Thus the density of an incompressible substance remains constant under compression, while the density of a highly compressible substance increases greatly under compression.

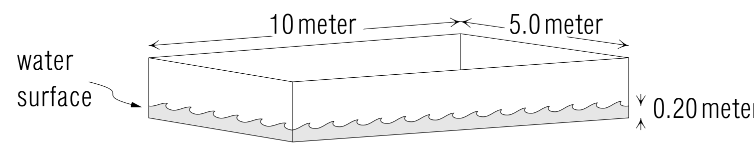
s-8 *(Text problem E-4):* Part (a): The only forces acting on the hydrometer are the gravitational force and the buoyant force due to the surrounding liquid and air. If the hydrometer is at rest, the magnitudes of these forces must be equal. Thus the buoyant force is the same in this situation as it was in the preceding one.

Part (b): Since the weight of air displaced is negligible, the buoyant force on the hydrometer is equal to $\rho V_d g$ upward, where ρ is the urine density and V_d is the volume of urine displaced. The buoyant force is the same, so that the displaced volume V_d (and thus the submerged volume of the hydrometer) must be *larger* in order to compensate for the smaller urine density. The hydrometer's bottom will thus be at a larger depth below the urine surface.

s-9 *(Text problem E-2):* Part (a): Since the weight of the air displaced by the barge is negligible in comparison to the weight of the water displaced by the barge, you need only find the weight of the displaced water.

To find the volume V_d of liquid displaced by an object, it is usually helpful to sketch the object and locate the liquid surface on the sketch. The displaced volume V_d is then the volume of only that part of the object that is *below* the liquid surface. For example, the following sketch shows that the part of the barge below the water surface is a block-shaped region 10 meter long, 5.0 meter wide, and 0.20 meter high. Using table E-1:

$$V_d = (10\,\text{meter})(5.0\,\text{meter})(0.20\,\text{meter})$$



Part (b): Since the balloon is completely surrounded by air, the volume of air displaced by the balloon is equal to the entire volume of the balloon, and the buoyant force on the balloon is equal to the weight of this volume of air.

s-10 *(Text problem D-5):* Part (a): Since the pressure in a gas varies little between points differing in height by a few meter, the pressure of the air in the cuff is the same as the pressure of the air above the mercury in the reservoir. Consequently, the pressure of the mercury at its surface in the reservoir is equal to the cuff air pressure and thus to the patient's systolic blood pressure, while the pressure of the mercury at its surface in the tube is equal to the atmospheric pressure of the air above this surface.

Part (b): Since the mercury surface in the reservoir is a depth h below the mercury surface in the tube (where h is the height of the mercury column in the tube), the pressure of the mercury at its surface in the reservoir must be *larger* than the pressure of the mercury at its surface in the tube by an amount $\rho g h$, where ρ is the density of mercury. Using this relation and the pressures found in part (a), you can find the value of h.

s-11 *(Text problem E-1):* Part (a): In each case, the magnitude F_b of the buoyant force equals the weight w_d of fluid displaced by the object. The weight of each kind of fluid displaced by the object is given by $\rho V_d g$, where ρ is the density of the *fluid* and V_d is the volume of fluid displaced.

For example, in situation 1, the object displaces a volume of gas equal to its own volume V_0 but it displaces no liquid at all. Thus the weight of fluid displaced is just the weight of the gas displaced, or

$$w_d = \rho_g V_0 g$$

In situation 2, the object displaces a volume of gas equal to half its own volume V_0, and it displaces a volume of liquid equal to half its own volume V_0. The weight of fluid displaced is thus the *sum* of the weight of the gas displaced and the weight of the liquid displaced, or

$$w_d = \rho_g \left(\frac{1}{2}V_0\right) g + \rho_l \left(\frac{1}{2}V_0\right) g$$

Since the density ρ_l of a liquid is typically about 1000 times the density ρ_g of a gas, the weight of gas displaced is negligible in comparison with the weight of liquid displaced. Thus the following approximate relation is also correct:

$$w_d \approx \rho_l \left(\frac{1}{2}V_0\right) g$$

s-12 *(Text problem E-5):* Follow the method outlined in tutorial frame [e-2].

Part (a): Note that only the buoyant and gravitational forces act on the barge, and that you have already found the value of the buoyant force in text problem E-2.

Part (b): Note that this is a *different* situation from that described in part (a), because the barge now supports an amount of ore having some mass M. Choose the barge and ore together as a composite particle of mass $m + M$, where m is the mass of the barge. By expressing the forces on the barge in terms the length L and width W of the barge, the depth D of the barge bottom below the surface, and the density ρ_w of the water, you should arrive at this equation for the mass M of the ore:

$$M = \rho_w(LWD) - m$$

Part (c): The equation found in part (b) applies also in this part; all that is necessary is to rearrange this relation to express the depth D in terms of the water density ρ_w so that you can see how D will change if ρ_w becomes larger. Alternatively, you might want to recall how the hydrometer behaved when it was placed in liquids of different density (text problems E-4 and E-5).

ANSWERS TO PROBLEMS

1. a. 0.99 N
   b. 0.11 N
   c. decrease
2. Man: 100 mm-Hg, Giraffe: 310 mm-Hg !
3. a. 3.1×10^2 lb
   b. 3.1×10^2 lb/in^2
4. a. 2.0×10^7 N/m^2 downward
   b. 1.0×10^5 N/m^2 downward
5. a. Inside air: 4.0×10^3 N outward. Outside air: 2.4×10^3 N inward.
   b. The window may blow out, because of the net force of 1600 N (350 lb) outward exerted on it by the air.
6. 0.40 kg
7. a. 9×10^4 N/m^2
   b. 2 meter
8. a. $\vec{F} = (\rho_s - \rho_0)Vg'\hat{x}$
   b. $\rho_s = \rho_0$, because $\vec{F} = \vec{0}$
   c. 0.3 cm = 3 mm, an easily-measurable separation.
9. a. $w = \rho_0 V_0 g$
   b. $F = (\rho_0 - \rho_a)V_0 g$, smaller
   c. $(w - F)/w = \rho_a/\rho_0$, small density objects
   d. Wood: $1 \times 10^{-3} = 0.1$ percent. Lead: $1 \times 10^{-4} = .01$ percent.
10. a. $M' = 8M$
    b. $A' = 8A$
    c. $d' = \sqrt{8}d = 2.8d$
    d. No, the larger animal's bone is thicker in comparison to its length.
11. a. 10 meter = 33 foot !
    b. larger
12. a. 14 or 15 cm (either is acceptable)

b. It should increase.

13. 0.13 meter (13 cm). To obtain a satisfactory flow, the height should be several times larger than this value.

14. 2.5×10^4 N (or 5500 lb!)

15. a. $\vec{F}_a = 2.0 \times 10^4$ N downward, $\vec{F}_w = 2.0 \times 10^4$ N upward. $\vec{F}_b = -(\vec{F}_a + \vec{F}_w) = \vec{0}$

b. $\vec{F}_a = 2.0 \times 10^4$ N downward, $\vec{F}_w = 4.0 \times 10^4$ N upward. $\vec{F}_b = 2.0 \times 10^4$ N downward.

c. The barrel will burst!

16. a. Hemisphere: $\vec{F}_f = \pi R^2 p\,\hat{x}$. Cylinder: $\vec{F}_f = 2RLp\,\hat{x}$.

b. Hemisphere: $\vec{F}_c = -\pi R^2 p\,\hat{x}$. Cylinder: $\vec{F}_c = -2RLp\,\hat{x}$.

17. a. $\delta W = \delta W_1 + \delta W_2 = F_1 L_1 - F_2 L_2$. $F_1/F_2 = L_2/L_1$.

b. $L_1/L_2 = A_2/A_1$

c. $F_1/F_2 = A_1/A_2$, yes

18. a. $F = (p_i - p_0)2RL$

b. $\sigma = (p_i - p_0)R/T$

c. $\sigma = 7 \times 10^4\,\text{N/m}^2$. It is about 15 times smaller.

d. The stress is the same because the *radius* of a capillary is also about 1/2000 of the radius of the aorta.

101. a. Oxygen gas is highly compressible. The remaining liquids and solid are nearly incompressible.

b. The liquids alcohol and molasses and the solid ice.

102. a. Cannot be found, because the drum is hollow and thus not homogeneous.

b. $2.5 \times 10^{-3}\,\text{m}^3$

103. a. $\vec{\sigma}_1 = 8.3 \times 10^7\,\text{N/m}^2$ upward, $\vec{\sigma}_2 = 1.7 \times 10^8\,\text{N/m}^2$ upward

b. Yes, it is likely to break near the smaller area A_2.

104. Density: same. Mass: one-tenth as large.

105. a. $p = 2.0 \times 10^5\,\text{N/m}^2$ (note that p is not a vector)

b. The direction of $\vec{F}$ is different (i.e., upward), the magnitude of $\vec{F}$ is the same, the pressure p is the same.

106. a. $8.93 \times 10^{-2}\,\text{kg/m}^3$

b. 5 kg

107. a. $A = 3 \times 10^{-7}\,\text{m}^2$, $r = 3 \times 10^{-4}$ meter = 0.03 cm

b. Contact force is the same, stress is one-fourth as large.

108. a. Small shear forces are required for the oxygen gas and for the liquid alcohol and molasses. The solid ice retains its shape.

b. Molasses is more viscous.

109. a. $-(1.00 - 0.02) \times 10^5\,\text{N/m}^2 = 0.98 \times 10^5\,\text{N/m}^2$

b. 1.0 meter

c. 1.1 meter

110. a. Because this relation applies only if the area A is small enough, i.e., only if the pressure p is the same near all parts of the area. This is not true for the side of the pool.

b. $F = p_3 A = (1.3 \times 10^5\,\text{N/m}^2)(40\,\text{m}^2) = 5.2 \times 10^6$ N. Because the pressure p is the same everywhere on the bottom of the pool, since all points on the bottom are at the same depth.

111. a.

	Stress	*Pressure*
Kind	vector	number
Signs		+, 0
SI unit	N/m^2	N/m^2

b. Kind of quantity (and signs)

c. pressure

112. a. $p_2 - p_1 = \rho g h = 1.0 \times 10^4\,\text{N/m}^2$

b. $p_2 - p_1 = 12\,\text{N/m}^2$

c. Water: $p_2 = 1.1 \times 10^5\,\text{N/m}^2$. Air: $p_2 = 1.0 \times 10^5\,\text{N/m}^2$.

d. Liquid: No (unless the height difference is less than 0.5 meter). Gas: yes.

113. a. The pressure force on the face of the smaller can is one-forth that on the face of the larger can.

b. Decreases.

114. a. $p^* = 2 \times 10^5\,\mathrm{N/m^2}$
   b. $p = 3 \times 10^5\,\mathrm{N/m^2} = 3$ atm
   c. Zero.
115. a. Point 3
   b. Point 2
   c. Points 1 and 4
116. a. smaller than
   b. $-1.0 \times 10^4\,\mathrm{N/m^2}$
   c. $h = 0.071$ meter
   d. a liquid with small density
117. a. The liquid surfaces must be in contact with a gas at the same pressure, unlike the situation in Fig. D-2.
   b. The liquid must be at rest, unlike the water in a river.
118. a. Reservoir: $1.17 \times 10^5\,\mathrm{N/m^2}$. Tube: $1.00 \times 10^5\,\mathrm{N/m^2}$.
   b. 0.12 meter = 120 mm
   c. Gauge pressure
119. a. Top: 2.8×10^2 N downward. Bottom: 2.8×10^2 N upward.
   b. Zero. Similarly, the total horizontal force exerted on the can by the surrounding air is also zero. Thus we have been justified in neglecting the force due to the air in writing the equation of motion for stationary objects. The air pressure near a moving object is *not* uniform, however, so that the sum of the air pressure forces on the object can be appreciable. This total force is commonly called the "force due to air resistance."
   c. 2.8×10^2 N downward.
120. a. 1.0×10^5 N upward.
   b. 5.0×10^{-2} N upward
121. a. $1.0 \times 10^4\,\mathrm{N/m^2}$
   b. 76 mm-Hg
122. a. 1.0×10^4 kg
   b. 6.5×10^4 kg
   c. smaller

123. a. larger
   b. $\rho_s/\rho_w = V/(V - hA)$
124. $1.1 \times 10^3\,\mathrm{kg/m^3}$
125. a. Situation 1: $F_b = \rho_g V_0 g$.
   Situation 2: $F_b = \rho_l(1/2V_0)g + \rho_g(1/2V_0)g \approx \rho_l(1/2V_0)g$.
   Situations 3 and 4: $F_b = \rho_l V_0 g$.
   b. Situation 1: $F_b = 1 \times 10^{-5}$ N upward.
   Situation 2: $F_b = 5 \times 10^{-3}$ N upward.
   c. They are the *same*.
126. a. $\vec{F}_b = 50$ N upward, $V = 5.0 \times 10^{-3}\,\mathrm{m^3}$, equal to
   b. $\vec{F}_b$ increases, ρ_f decreases (because V increases). It rises.
   c. $\vec{F}_b$ decreases, ρ_f increases. It sinks.
127. Equal to.
128. a. the same
   b. larger
129. a. $M_i = \rho_1 V_i$
   b. $V_s = M_i/\rho_w = \rho_i V_i/\rho_w$
   c. $V_s/V_i = \rho_i/\rho_w$
   d. $V_s/V_i = 0.90 = 90$ percent
130. a. 25 meter
   b. 3.0 cm, or about an inch
131. 7.7×10^3 N (or 1700 lb!)
132. 0.5 meter
133. a. $F = \rho g h A$
   b. $m = \rho h A$, $w = \rho g h A$, yes
134. a. smaller
   b. larger force, yes
135. The same. Try the experiment!
136. 8×10^{-10} N !
137. The level falls.
138. a. $\rho_s/\rho_w = w/(w - F)$
   b. $\rho_s/\rho_w = 3$, or the glass is three times as dense as water.

MODEL EXAM

USEFUL INFORMATION

: Density of water: $1.0 \times 10^3\,\mathrm{kg/m^3}$

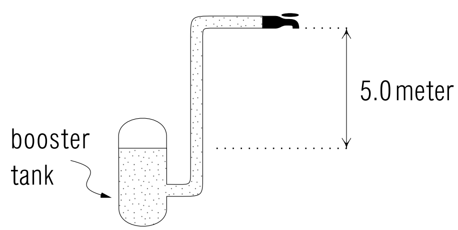
1. **Buoyant force on a block of wood.** A boy holds a block of wood of density $7 \times 10^2\,\mathrm{kg/m^3}$ so that it is completely submerged in the water of a lake. The volume of the block is $80\,\mathrm{cm^3}$.

   a. What is the buoyant force exerted on the block by the water?

   The boy then releases the block so that it floats at rest on the water with about 1/3 of its volume above the water surface.

   b. Is the buoyant force on the block in this situation larger than, equal to, or smaller than that in the previous situation?

2. **Improving the water pressure in a farm house.** To overcome the poor water pressure in a farm house, a "booster tank" is installed in the basement, as shown in this drawing: When the tank is nearly filled by water pumped from the farm well, an air compressor is used to pressurize the air above the water surface in the tank.



   If the air pressure in the tank is $2.0 \times 10^5\,\mathrm{N/m^2}$, what is the water pressure of the stationary water in the second-floor pipes, which are 5.0 meter above the water surface in the tank?

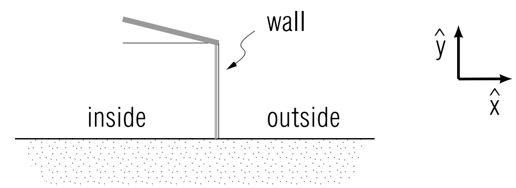
3. **Stress in a nail head.** The area of a circular nail head is $2 \times 10^{-5}\,\mathrm{m^2}$. When a hammer drives the nail downward into a board, the contact force exerted on the nail by the hammer is equal to $8 \times 10^3\,\mathrm{N}$ downward. If the hammer strikes the nail squarely, the contact force is exerted across the entire area of the nail head.

   a. If this area of contact is small enough, what is the stress in the nail just under this area?

   If the hammer strikes the nail off-center, the area of contact is smaller than the area of the nail head, although the contact force is the same.

   b. Is the stress in the nail just under the area of contact larger than, equal to, or smaller than that in the previous situation?

4. **Pressure forces on a wall during a tornado.** When a tornado passes near the wall of a building shown in the following drawing, the air pressure p_0 just outside the wall drops suddenly to a value of $8 \times 10^4\,\mathrm{N/m^2}$, while the air pressure p_1 just inside the wall remains equal to the normal atmospheric pressure of $1 \times 10^5\,\mathrm{N/m^2}$.



   If the wall is a rectangle 5 meter high and 10 meter wide, what are the pressure forces $\vec{F}_0$ and $\vec{F}_i$ exerted on the wall by the air outside and inside the wall? Indicate both the magnitudes and the directions of these forces.

Brief Answers:

1. a. $0.8\,\mathrm{N}$ upward
   b. smaller than
2. $1.5 \times 10^5\,\mathrm{N/m^2}$
3. a. $4 \times 10^8\,\mathrm{N/m^2}$ downward
   b. larger than
4. $\vec{F}_0 = -4 \times 10^6\,\mathrm{N}\hat{x}$ or $4 \times 10^6\,\mathrm{N}$ to left

   $\vec{F}_i = 5 \times 10^6\,\mathrm{N}\hat{x}$ or $5 \times 10^6\,\mathrm{N}$ to right