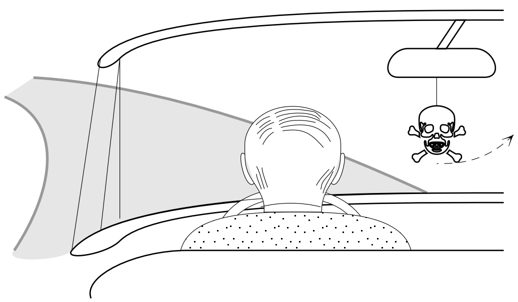
PHYSNET

MISN-0-412

MOTION RELATIVE TO DIFFERENT REFERENCE FRAMES



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MOTION RELATIVE TO DIFFERENT REFERENCE FRAMES

by

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Title: **Motion Relative to Different Reference Frames**

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Input Skills:

1. Vocabulary: inertial frame (MISN-0-408).
2. State the equation of motion for a particle (MISN-0-408).

Output Skills (Knowledge):

K1. Write the relations between the corresponding position vectors, velocities, or accelerations of the same particle described relative to two different reference frames.

K2. State the Galilean relativity principle.

K3. State the equivalence principle.

Output Skills (Problem Solving):

S1. Given S', a reference frame near the earth, and S, an inertial reference frame, apply the Galilean relativity principle to relate descriptions of the same phenomena observed in S' when: (a) when S' is at rest relative to S; (b) S' is moving with constant velocity relative to S.

S2. Apply the equivalence principle to relate the apparent gravitational acceleration $\vec{g}\,\prime$ relative to S', the real gravitational acceleration $\vec{g}$, and the acceleration $\vec{A}$ of S' relative to S.

S3. Use the direction of $\vec{g}\,\prime$ to qualitatively relate descriptions of the same phenomenon when S' is moving relative to S and when S' is at rest relative to S.

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MOTION RELATIVE TO DIFFERENT REFERENCE FRAMES

Abstract:
Although we can describe the motion of particles relative to *any* reference frame, the description may be simplest relative to certain special frames. For example, we have seen that the theory of motion is expressed most conveniently relative to some *inertial* frame. On the other hand, an observer may often find it easiest to describe motion relative to the reference frame in which he is at rest, even when this frame is not an inertial frame. (For example, suppose that a man sitting in a train observes another passenger walking toward him. Then the man would find it easier to describe the motion of the other passenger relative to the train, rather than relative to the surrounding countryside whizzing by at great speed.)

In order to simplify the discussion of many practical situations we should like to choose whatever reference frame seems most convenient. Hence we must be able to answer these questions: (1) What is the relationship between descriptions of motion relative to different reference frames? (2) How can one express the theory of motion relative to a reference frame which is not necessarily inertial?

By examining these questions in the present unit, we shall reach general conclusions which we can use to discuss many important applications (such as the physiological effects of space travel or the utility of the centrifuge in biochemical investigations).

SECT.

A DESCRIPTION OF MOTION RELATIVE TO DIFFERENT FRAMES

We can describe the motion of particles relative to any reference frame with an associated coordinate system. For example, we might choose some reference frame S (such as the ground) with a coordinate system having an origin O and directions specified by unit vectors $\hat{x}$, $\hat{y}$, and $\hat{z}$. Alternatively, we might choose some other reference frame S' (such as a train) with a coordinate system having an origin O' and directions specified by unit vectors $\hat{x}'$, $\hat{y}'$, and $\hat{z}'$. (For simplicity, we shall choose $\hat{x}'$, $\hat{y}'$, and $\hat{z}'$ so that they remain respectively parallel to $\hat{x}$, $\hat{y}$, and $\hat{z}$.) As indicated in Fig. A-1, we can then describe the position of a particle P either by its position vector $\vec{r}$ relative to the origin O of the reference frame S, or by its position vector $\vec{r}\,'$ relative to the origin O' of the reference frame S'. Furthermore, we can describe the position of O' by its position vector $\vec{R}$ relative to the origin O of the frame S. If the reference frames move relative to each other, $\vec{R}$ changes with time.

Any situation may then be described either from the point of view of an observer sitting in the reference frame S, or from the point of view of an observer sitting in the reference frame S'. In making measurements, both observers use identical clocks and identical meter sticks. We shall assume that the observers can synchronize all their clocks and meter sticks so that they indicate the same times and lengths whenever they are read by either observer. *

> * This assumption is open to question since we have never explicitly defined a procedure for synchronizing clocks located at different points. Indeed, Einstein's special theory of relativity (to be discussed in Unit 438) indicates that this assumption is not correct when reference frames move relative to each other with very large speeds comparable to the speed of light (3×10^8 meter/second).

Hence the observers agree about the times at which events occur and thus agree also about the time elapsed between any two events. Furthermore, the observers agree about the distance between any two points.

On the other hand, each observer specifies the position of the same particle by its position vector relative to *his* reference frame. The length and direction of this position vector depend on the origin and the di-

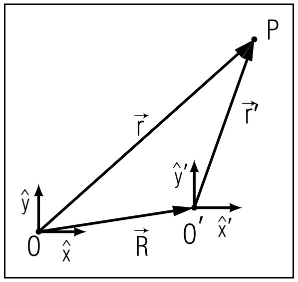


Fig. A-1: Description of the position of a particle P relative to two different reference frames S and S' with associated coordinate systems having origins O and O'.

rections of the coordinate system associated with his reference frame. Hence the same particle is described by different position vectors $\vec{r}$ and $\vec{r}\,'$ relative to two different reference frames S and S'. (See Fig. A-1.) Correspondingly, the particle is also described by different velocities and accelerations relative to these reference frames. [To keep these different descriptions clearly in mind, we shall use unprimed symbols to denote quantities (such as $\vec{r}$) which are measured relative to S, and shall use primed symbols to denote quantities (such as $\vec{r}\,'$) which are measured relative to S'.]

Thus we should like to answer this question: What is the relation between the corresponding position vectors, velocities, or accelerations of the same particle described relative to two different reference frames?

RELATION BETWEEN POSITIONS, VELOCITIES, AND ACCELERATIONS

Figure A-1 indicates at any instant the relationship between a particle's position vector $\vec{r}$ relative to the frame S, its position vector $\vec{r}\,'$ relative to the frame S', and the position vector $\vec{R}$ of the origin O' relative to S. From Fig. A-1 it is apparent that $\vec{r} = \vec{R} + \vec{r}\,'$, or equivalently that:

$$\boxed{\vec{r} = \vec{r}\,' + \vec{R}} \qquad \text{(A-1)}$$

For example, the position of a passenger P on a train can be described either relative to the ground (reference frame S) or relative to the train (reference frame S'). Then Eq. (A-1) says simply that the passenger's position vector $\vec{r}$ relative to the ground is equal to his position vector $\vec{r}\,'$ relative to the train plus the position vector $\vec{R}$ of the point O' on the train relative to the ground.

The relationship Eq. (A-1) is valid at any time t. Since the rate of a sum is equal to the sum of the rates,* Eq. (A-1) then implies that

* This statement is true for vectors as well as for numbers, as can be shown by the same argument as that leading to the result shown in statement (C-11) of Unit 404.

$$\frac{d\vec{r}}{dt} = \frac{d\vec{r}\,'}{dt} + \frac{d\vec{R}}{dt}$$

or

$$\boxed{\vec{v} = \vec{v}\,' + \vec{V}} \qquad \text{(A-2)}$$

Here $\vec{v} = d\vec{r}/dt$ is the velocity of the particle relative to S, $\vec{v}\,' = d\vec{r}\,'/dt$ is its velocity relative to S', and $\vec{V} = d\vec{R}/dt$ is the velocity of S' relative to S. For example, if a man walks along a train, his velocity $\vec{v}$ relative to the ground is equal to his velocity $\vec{v}\,'$ relative to the train plus the velocity $\vec{V}$ of the train relative to the ground.

The relationship Eq. (A-2) is valid at any time t. Since the rate of a sum is equal to the sum of the rates, Eq. (A-2) then implies that

$$\frac{d\vec{v}}{dt} = \frac{d\vec{v}\,'}{dt} + \frac{d\vec{V}}{dt}$$

or

$$\boxed{\vec{a} = \vec{a}\,' + \vec{A}} \qquad \text{(A-3)}$$

Here $\vec{a} = d\vec{v}/dt$ is the acceleration of the particle relative to S, $\vec{a}\,' = d\vec{v}\,'/dt$ is its acceleration relative to S', and $\vec{A} = d\vec{V}/dt$ is the acceleration of S' relative to S. For example, if a man walks along a train, his acceleration $\vec{a}$ relative to the ground is equal to his acceleration $\vec{a}\,'$ relative to the train plus the acceleration $\vec{A}$ of the train relative to the ground.

Example A-1: Air and ground velocity of a plane

A small plane can fly relative to the surrounding air (or relative to the clouds in the air) with an "air velocity" having a magnitude of 240 km/hour. The velocity of the air relative to the ground (i.e., the "wind velocity") is 100 km/hour east. What then is the velocity of the plane relative to the ground (i.e., its "ground velocity") when the plane flies east or south relative to the air?

The relation $\vec{v} = \vec{v}\,' + \vec{V}$ states that the velocity $\vec{v}$ of the plane relative to the ground is equal to the velocity $\vec{v}\,'$ of the plane relative to the air

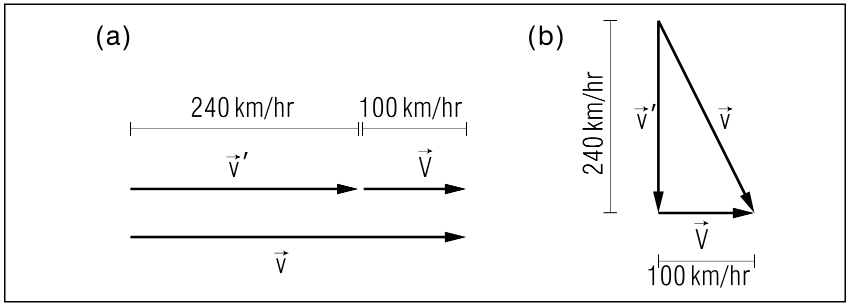


Fig. A-2: Relation between the velocity $\vec{v}\prime$ of a plane relative to the air and its velocity $\vec{v}$ relative to the ground when the air moves with an easterly velocity $\vec{V}$ relative to the ground. (a) $\vec{v}\prime$ directed east. (b) $\vec{v}\prime$ directed south.

plus the velocity $\vec{V}$ of the air relative to the ground. Here we know that $\vec{v}\prime$ has a magnitude $|\vec{v}\prime| = 240$ km/hour and that $\vec{V} = 100$ km/hour east. Hence we can calculate the velocity $\vec{v}$ for any direction of $\vec{v}\prime$.

Suppose that the velocity $\vec{v}\prime$ of the plane relative to the air is east, i.e., in the same direction as the wind velocity $\vec{V}$. (See Fig. A-2a). Then the velocity $\vec{v}$ of the plane relative to the ground is $\vec{v} = \vec{v}\prime + \vec{V} = 340$ km/hour east.

Suppose that the velocity $\vec{v}\prime$ of the plane relative to the air is south, i.e., perpendicular to the wind velocity $\vec{V}$. (See Fig. A-2b.) Then the velocity $\vec{v} = \vec{v}\prime + \vec{V}$ of the plane relative to the ground is indicated in the vector diagram of Fig. A-2b.

Hence the *magnitude* of $\vec{v}$ is such that $v^2 = v'^2 + V^2 = (240\,\text{km/hour})^2 + (100\,\text{km/hour})^2$. Thus $v = 260$ km/hour. Furthermore, the angle θ between the direction of $\vec{v}$ and the southern direction is such that $\tan\theta = 100/240 = 0.417$. Thus $\theta = 23°$.

Illustration

A-1 *Wind velocity relative to a sailboat:* The velocity of the air (or the average velocity of air molecules) is what we commonly call the wind velocity. Suppose that, relative to the reference frame S of the water in a bay, the wind velocity $\vec{v}$ of the air is 8 m/s south. People in a sailboat, however, experience a different wind velocity, the velocity $\vec{v}\prime$ of the air relative to their reference frame S' of the boat. (a) For example,

suppose the boat is sailing "across the wind" with a velocity $\vec{V} = 8$ m/s east relative to the water. What is the wind velocity $\vec{v}\prime$ of the air relative to the boat? (b) The boat then turns and sails "with the wind" with a velocity $\vec{V} = 6$ m/s south relative to the water. An inexperienced sailor remarks that the wind has dropped suddenly. To explain why, find the wind velocity $\vec{v}\prime$ of the air relative to the boat. *(Answer: 106) (Suggestion: [s-11])*

SECT. B FRAMES MOVING WITH CONSTANT RELATIVE VELOCITY

We can use our theory of motion to predict the motion of particles relative to any inertial frame S (such as the ground). How then can we predict the motion of particles relative to another reference frame (such as a vehicle) which moves relative to S with some velocity $\vec{V}$? Let us begin by answering this question for the simple case where the velocity $\vec{V}$ is constant. In the next section we shall then consider the general case where the velocity $\vec{V}$ may be changing.

If the frame S' moves relative to the inertial frame S with a *constant* velocity, we know from our discussion in text section A of Unit 408 that S' is also an inertial frame. Hence we expect that the motion of a particle relative to S' should be described by the same principles as the motion of this particle relative to S. Let us show explicitly that this expectation is correct and explore some of its implications.

Consider any particle P of mass m. Its equation of motion relative to the inertial frame S is then

$$m\vec{a} = \vec{F} \qquad \text{(B-1)}$$

where $\vec{a}$ is the acceleration of the particle relative to S and where $\vec{F}$ is the total force acting on the particle due to all other particles. Suppose now that we wish to describe the motion of the particle P relative to some other reference frame S' which moves relative to S with a *constant* velocity $\vec{V}$. Then the acceleration $\vec{A} = d\vec{V}/dt$ of S' relative to S is zero. Hence the relationship Eq. (A-3) implies that $\vec{a} = \vec{a}\prime + \vec{A} = \vec{a}\prime + 0$ so that

$$\vec{a} = \vec{a}\prime \qquad \text{(B-2)}$$

In other words, the acceleration $\vec{a}$ of the particle relative to S is exactly the same as its acceleration $\vec{a}\prime$ relative to S'. The force on the particle P due to any other particle depends on the distance between these particles. But, as pointed out in Sec. A, all observers agree on the value of the distance between two points. Hence the total force $\vec{F}$ (which is merely the vector sum of the forces on the particle due to all other particles) has the same value relative to any reference frame.[1] By combining the result $\vec{a}\prime = \vec{a}$ with the equation of motion $m\vec{a} = \vec{F}$, we then obtain simply:

[1] For speeds small compared to the speed of light: see the "advice" in A.

$$\boxed{m\vec{a}\prime = \vec{F}} \qquad \text{(B-3)}$$

DISCUSSION

Since the equation of motion $m\vec{a}\prime = \vec{F}$ of *every* particle relative to S' has exactly the same form as its equation of motion $m\vec{a} = \vec{F}$ relative to S, we arrive at this general conclusion:

> Galilean relativity principle: Suppose that a reference frame S' moves relative to some inertial frame S with a constant velocity. Then the motion of particles relative to S' proceeds in exactly the same way as if S' were at rest relative to S. (B-4)

This result is called the "Galilean relativity principle" because its validity was first recognized by Galileo (1564-1642). *

> * The principle is also valid if the original frame S is not an inertial frame.

Consider then any "mechanical" phenomenon, i.e., any phenomenon involving the motion of particles. Then the relativity principle, Rule (B-4), implies that any such phenomenon observed in S' occurs in exactly the same way as if S' were at rest relative to S. Hence no observations of particles relative to some reference frame can ever decide whether this frame is at rest or moving relative to some other reference frame. For example, a ball, initially at rest on the horizontal floor of a train,will remain at rest relative to the train, irrespective of whether this train is stationary or is moving relative to the ground with constant velocity (i.e., without jerks, bumps, or any other forms of acceleration). Similarly, a ball, dropped after being initially at rest relative to the train, will simply fall vertically downward relative to the train, irrespective of whether the train is stationary or is moving relative to the ground with constant velocity. *

> * If the train is moving, the motion of the ball relative to the *ground* is then not vertical, but along some curved path.

Indeed, suppose that a passenger looks out the window and sees an adjacent train moving by him. Then none of the passenger's observations

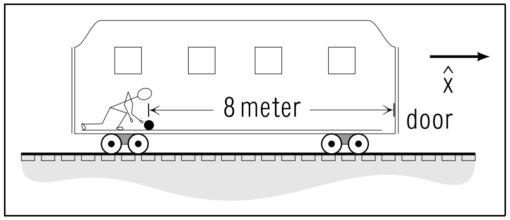


Fig. B-1.

within his train would allow him to decide whether his train is at rest relative to the ground while the other train is moving relative to the ground, or whether his train is moving relative to the ground while the other train is at rest relative to the ground. The relativity principle, Rule (B-4), also implies that all the physiological processes of a biological organism occur in precisely the same manner when the organism is moving with constant velocity relative to some inertial frame as when the organism is at rest relative to this frame. For example, consider a passenger sitting in a jet airplane which moves relative to the ground with some *constant* velocity $\vec{v}$. Then the passenger experiences no physiological effects as a result of his motion (and would, in fact, be unaware of his motion if he closed his eyes) although $\vec{v}$ may be as large as 1000 km/hour. Similarly, astronauts often travel relative to the stars with an extremely large velocity, but experience no physiological effects as long as this velocity is constant.

Applying the Galilean Relativity Principle (Cap. 1)

B-1 While his train is at rest in a station, a boy rolls a ball along the train car to hit the door at the far end, 8 meter away (Fig. B-1). The ball rolls to the right with a constant velocity of $(4\,\text{m/s})\hat{x}$ relative to the train, and thus hits the door 2 second after it leaves the boy's hand. Later, when the train is moving with a constant velocity of $(3\,\text{m/s})\hat{x}$ relative to the ground, the boy rolls the ball from the same spot with the same velocity relative to the train. The ball then has a velocity of $(7\,\text{m/s})\hat{x}$ relative to the ground. How long does it take the ball to arrive at the door this time? *(Answer: 102)(Practice: [p-1])*

Illustration

B-2 *Demonstrating a prediction of Galilean relativity:* To demonstrate that the prediction in problem B-1 "makes sense," let us describe the motion of both the ball and the train car relative to the familiar

reference frame of the ground. (a) When the train is moving relative to the ground, how far and in what direction does the door of the car move in 2 second? (b) During this period of time, how far and in what direction does the ball move relative to the ground? (c) Draw a diagram like Fig. B-1 showing the initial positions of the ball and the door relative to the ground. Then draw on the same diagram the positions of these objects relative to the ground 2 second later. Are the ball and the door in the same position relative to each other after 2 second, as predicted? *(Answer: 108) (Suggestion: [s-8])*

Knowing About Galilean Relativity

B-3 A juggler practicing in a vehicle will have no trouble if the Galilean relativity principle applies (i.e., if the motion of the juggled objects relative to the vehicle proceeds precisely as if the vehicle and juggler were at rest relative to the ground). For which of the following vehicles is this the case? The motion of each vehicle is described relative to the ground. (a) An elevator moving downward with constant speed. (b) A train moving with constant speed around a curve. (c) A train slowing down as it travels west into a station, (d) A helicopter moving with constant speed along a straight path at an angle to the ground. *(Answer: 104) (Suggestion: [s-6])*

SECT. C FRAMES MOVING WITH RELATIVE ACCELERATION

Let us now consider the general case where a reference frame S' moves relative to some inertial frame S with some velocity $\vec{V}$ which is not necessarily constant. Then the equation of motion of any particle P relative to the inertial frame S is, as usual,

$$m\vec{a} = \vec{F} \qquad \text{(C-1)}$$

But the acceleration $\vec{a}$ of the particle relative to S is related to its acceleration $\vec{a}\prime$ relative to S' by Eq. (A-3) so that

$$\vec{a} = \vec{a}\prime + \vec{A} \qquad \text{(C-2)}$$

where $\vec{A} = d\vec{V}/dt$ is the acceleration of S' relative to S. Furthermore we already pointed out in the preceding section that the total force $\vec{F}$ on the particle has the same value in any reference frame. By combining Eq. (C-2) with the equation of motion $m\vec{a} = \vec{F}$ we then obtain

$$m\vec{a}\prime + m\vec{A} = \vec{F}$$

or

$$m\vec{a}\prime = \vec{F} - m\vec{A} \qquad \text{(C-3)}$$

DISCUSSION

Because of the extra term $-m\vec{A}$, the Eq. (C-3) describing the motion of a particle relative to S' is *different* in form from the equation of motion $m\vec{a} = \vec{F}$ describing the particle relative to an inertial frame. But Eq. (C-3) can be made to look like the familiar equation of motion if we write it in the form:

$$\boxed{m\vec{a}\prime = \vec{F}\prime} \qquad \text{(C-4)}$$

where we have introduced the abbreviation:

$$\boxed{\vec{F}\prime = \vec{F} - m\vec{A}} \qquad \text{(C-5)}$$

We shall call $\vec{F}\prime$ the "apparent total force" acting on the particle in S'.

We can express the definition of $\vec{F}'$, Def. (C-5), in a more suggestive way by introducing the symbol $\vec{\mathcal{F}}$ and writing

$$\vec{F}' = \vec{F} + \vec{\mathcal{F}}, \text{ where } \vec{\mathcal{F}} = -m\vec{A} \qquad \text{(C-6)}$$

We can interpret Eq. (C-6) by saying that the apparent total force $\vec{F}'$ consists of the real total force $\vec{F}$ plus an additional "fictitious force" $\vec{\mathcal{F}}$. We call $\vec{\mathcal{F}}$ a fictitious force because (unlike the real total force $\vec{F}$) it does *not* describe any interactions between particles, but describes merely the effects of the acceleration $\vec{A}$ of the frame S'.

We can summarize the preceding comments by this conclusion:

> Suppose that a reference frame S' moves relative to some inertial frame S with an acceleration $\vec{A}$. Then the motion of particles relative to S' proceeds in the same way *as if* S' were an inertial frame within which every particle of mass m is acted on by a fictitious force $\vec{\mathcal{F}} = -m\vec{A}$ in addition to all the real forces acting on the particle. (C-7)

Example C-1: Effect of a sudden acceleration

When a person of mass m is standing in a stationary bus, the total force $\vec{F}$ on him is zero. Suppose that the bus suddenly starts moving to the right so that its acceleration $\vec{A}$ relative to the ground is large and toward the right. (See Fig. C-1a.) The person, viewing himself relative to the bus, then feels an *apparent* total force $\vec{F}'$ which is not zero since it also includes the fictitious force $\vec{\mathcal{F}} = -m\vec{A}$ which is opposite in direction to the acceleration $\vec{A}$ of the bus, i.e., which is horizontal and to the left. (See Fig. C-1b.)

The result is that the person inside the bus gets thrown to the left, opposite to the direction of the acceleration $\vec{A}$ of the bus.

The fictitious force $\vec{\mathcal{F}} = -m\vec{A}$ is analogous to a gravitational force since it is also proportional to the mass m of the particle. To make the analogy explicit, we note that the real total force $\vec{F}$ on the particle consists usually of a gravitational force $\vec{F}_g = m\vec{g}$ and of the total sum $\vec{F}_0$ of all *other* forces. Accordingly, we can write $\vec{F} = \vec{F}_0 + \vec{F}_g$. Hence the *apparent* total force $\vec{F}'$ of Eq. (C-6) is

$$\vec{F}' = \vec{F}_0 + \vec{F}_g + \vec{\mathcal{F}} = \vec{F}_0 + \vec{F}'_g \qquad \text{(C-8)}$$

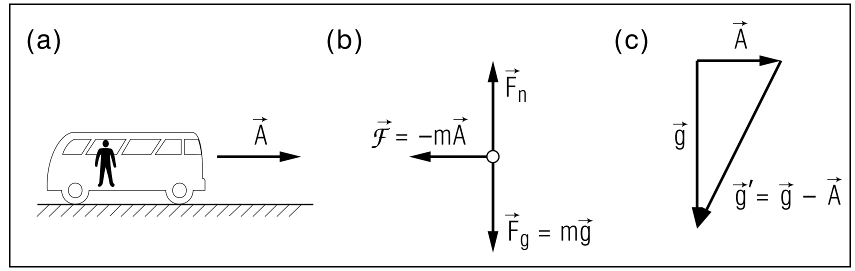


Fig. C-1: Acceleration $\vec{A}$ of a bus relative to the ground. (a) Person in the bus. (b) Forces on the person described relative to the bus, including fictitious force $\vec{\mathcal{F}}$, gravitational force $\vec{F}_g$, and normal force $\vec{F}_n$ exerted by the floor of the bus. (c) Apparent gravitational acceleration $\vec{g}'$ relative to the bus.

where

$$\vec{F}'_g = \vec{F}_g + \vec{\mathcal{F}} = m\vec{g} - m\vec{A} = m(\vec{g} - \vec{A}) = m\vec{g}' \qquad \text{(C-9)}$$

The quantity $\vec{F}'_g$ (which is just the sum of the real gravitational force $\vec{F}_g$ plus the fictitious force $\vec{\mathcal{F}}$ looks like an *apparent* gravitational force corresponding to an apparent gravitational acceleration $\vec{g}' = \vec{g} - \vec{A}$. Hence we can summarize our entire discussion in this section by this useful statement:

> Equivalence principle: Suppose that a reference frame S' moves relative to some inertial frame S with an acceleration $\vec{A}$. Then the motion of particles relative to S' proceeds in the same way *as if* S' were an inertial frame within which the gravitational acceleration is $\vec{g}' = \vec{g} - \vec{A}$ instead of $\vec{g}$. (C-10)

This principle implies that any mechanical phenomenon observed in S' occurs in exactly the same way as if S' were at rest relative to S, but the value of the gravitational acceleration were $\vec{g}' = \vec{g} - \vec{A}$ instead of $\vec{g}$.

Example C-2: Effect of a sudden acceleration

Let us apply the equivalence principle to discuss our previous example C-1 of the person in the bus. Suppose that the bus suddenly starts moving relative to the ground with an acceleration $\vec{A}$ to the right. Then the person, viewing himself relative to the bus, experiences an apparent gravitational acceleration $\vec{g}' = \vec{g} - \vec{A}$, as indicated by the vector diagram in Fig. C-1c. Since this apparent gravitational acceleration $\vec{g}'$ has a component vector to the left, the person is then thrown to the left relative to the bus. (This conclusion agrees, of course, with the one reached in Example C-1 by considering the fictitious force.)

In the special case where S' moves relative to S with a *constant* velocity $\vec{V}$ the acceleration $\vec{A}$ of S' relative to S is zero. The equivalence principle, Rule (C-10), then asserts that any motion relative to S' is exactly the same as it would be when S' is at rest relative to S. In other words, the equivalence principle, Rule (C-10), then reduces to the Galilean relativity principle, Rule (B-4). The equivalence principle is a consequence of the remarkable fact that the gravitational force on a particle is proportional to its mass. This principle is also the cornerstone of Einstein's general theory of relativity which provides a more comprehensive and accurate description of gravitational effects than the Newtonian theory.

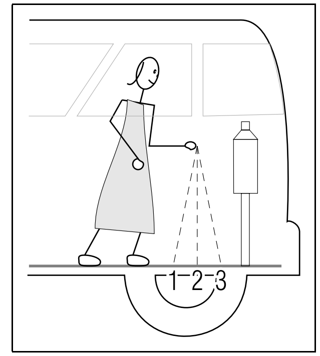


Fig. C-2.

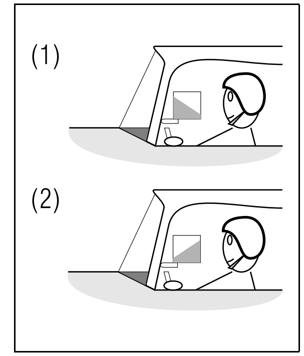


Fig. C-3.

Applying the Equivalence Principle Qualitatively (Cap. 2)

C-1 A woman standing in a bus drops a coin when the bus is at rest relative to the ground. The coin falls along the vertical path (2) in Fig. C-2, i.e., parallel to the gravitational acceleration $\vec{g}$. If the bus is moving relative to the ground, a coin dropped in the same way may fall relative to the bus along any of the paths in Fig. C-2. Which of these paths best indicates how the coin will fall if the bus is moving to the right relative to the ground and (a) speeding up with an acceleration $\vec{A} = 2\,\text{m/s}^2$ to the right, (b) traveling with constant speed, and (c) slowing down with an acceleration $\vec{A} = 2\,\text{m/s}^2$ to the left? *(Answer: 110) (Suggestion: [s-13])*

C-2 The acceleration of a car along a level test track can be indicated by a container of colored liquid mounted on the dashboard. When the car is at rest relative to the ground, the surface of the liquid is horizontal, i.e., perpendicular to the gravitational acceleration $\vec{g}$. Which of the drawings in Fig. C-3 shows the orientation of the liquid surface when, relative to the ground, the car is (a) moving to the left and speeding up, and (b) moving to the left and slowing down? *(Answer: 101) (Suggestion: [s-10])*

C-3 A young man with a flair for decoration hangs a plastic skull from the rear-view mirror of his car. The skull hangs vertically (i.e., with its string parallel to $\vec{g}$ when the car is at rest relative to the ground (Fig. C-4). When the car is moving with constant speed around a level circular curve to the left, does the skull hang in the position shown, to the left of this position, or to the right of this position? *(Answer: 107) (Suggestion: [s-4])*

Fig. C-4.

Knowing About Fictitious Forces

C-4 A car driver often experiences a fictitious force due to the motion of his reference frame, the car, relative to the ground. For each of the following motions of a car, give the direction of this fictitious force (e.g., upward, downward, to the driver's right or left, toward the front or back of the car): (a) moving with constant speed around a level circular curve toward the right, (b) moving with constant speed along a straight level street, and (c) slowing down while approaching a stop sign on a straight level street. *(Answer: 103)*

SECT.

D APPLICATIONS

Suppose that we want to describe the motion of particles within a vehicle moving relative to an inertial frame S (such as the surface of the earth, an approximately inertial frame). Then we can usually describe this motion most simply relative to the reference frame S' consisting of the vehicle and can use the equivalence principle to predict the effects of the vehicle's motion. Let us discuss several practical applications.

WEIGHTLESSNESS

Consider a vehicle S' moving under the sole influence of gravitational forces. Then the vehicle moves relative to some inertial frame S with an acceleration A equal to the gravitational acceleration $\vec{g}$ at the position of the vehicle. (For example, the vehicle might be an elevator which is freely falling after its supporting cable has broken, or it might be a spacecraft orbiting around the earth.)

Since $\vec{A} = \vec{g}$ for such a vehicle, the equivalence principle implies that the apparent gravitational acceleration relative to the vehicle is $\vec{g}\prime = \vec{g} - \vec{A} = \vec{g} - \vec{g} = 0$. Hence a passenger in the vehicle (call him Harry) would observe precisely the same phenomena as if his vehicle were at rest relative to an inertial frame where all gravitational forces had disappeared so that $\vec{g}\prime = 0$. In short, Harry would believe himself to be in a "weightless" environment. For example, suppose that Harry releases a ball after holding it in his hand. Then the unsupported ball would *not* fall relative to the vehicle, but would remain stationary next to Harry's hand.

The phenomenon of weightlessness is commonly observed by astronauts in their spacecrafts. The physiological processes of an astronaut, like all other motions of particles, proceed also as if no gravitational forces existed within the spacecraft. Thus the astronaut may find it difficult to swallow food because the usual gravitational force does not help the passage of food down his throat. Furthermore, the circulation of the astronaut's blood is affected because there is no gravitational force tending to make his blood descend to his feet. Consequently, prolonged periods of weightlessness tend to weaken the muscle tone of the body's cardiovascular system. An astronaut must learn to adapt himself to this novel weightless environment. (A new branch of medicine, called "space medicine," now deals specifically with such physiological problems encountered in space travel.)

WEIGHTLESSNESS VIEWED BY DIFFERENT OBSERVERS

Since the phenomenon of weightlessness appears strange, let us look in greater detail at our previous example of Harry in the falling elevator. Why is it that the unsupported ball does not fall relative to the elevator (so that Harry thinks he is in a weightless environment)?

Let us first look at the situation from Harry's point of view relative to the falling elevator S'. As we have seen, the equivalence principle states that the apparent gravitational acceleration perceived by Harry is $\vec{g}\prime = \vec{g} - \vec{A} = 0$ since the elevator is falling with an acceleration $\vec{A} = \vec{g}$ relative to the ground. Alternatively, Harry might describe the situation by saying that the apparent gravitational force on the ball of mass m is $\vec{F}\prime_g = \vec{F}_g + \vec{\mathcal{F}}$, where $\vec{F}_g = m\vec{g}$ is the real downward gravitational force on the ball and $\vec{\mathcal{F}} = -m\vec{A} = -m\vec{g}$ is the fictitious force on the ball. (See Fig. D-1a.) But since this fictitious force $\vec{\mathcal{F}}$ is upward and equal in magnitude to $\vec{F}_g$, the apparent gravitational force $\vec{F}\prime_g$ on the ball is simply zero.

For comparison, let us now look at the same situation from the point of view of George who is standing on the ground. George describes his observations relative to the ground S which is an *inertial* frame. Knowing that the elevator and the ball interact only with the earth, George concludes that the elevator and the ball both fall downward with the *same* acceleration $\vec{g}$ (since the gravitational acceleration does not depend on the properties of objects). (See Fig. D-1b.) Hence George expects that the ball should, after its release, move in exactly the same way as the elevator, i.e., that it should remain next to Harry's hand. Thus George, too, predicts that Harry should believe himself to be in a weightless environment.

VERTICALLY ACCELERATED VEHICLES

Suppose that a vehicle, such as an elevator or a rocket, moves relative to the ground with some vertical acceleration $\vec{A}$. According to the equivalence principle, a passenger in the vehicle then observes that phenomena described relative to the vehicle proceed as if the gravitational acceleration in the vehicle had the apparent value $\vec{g}\prime = \vec{g} - \vec{A}$. This ap-

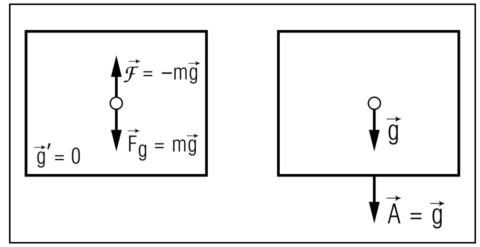


Fig. D-1: Motion of an unsupported ball in a falling elevator. (a) Description relative to the elevator. (b) Description relative to the ground.

parent gravitational acceleration $\vec{g}\prime$ affects the physiological processes of the passenger and must be considered in designing a vehicle acceptable and safe for passengers.

Suppose that the acceleration $\vec{A}$ of the vehicle relative to the ground is in the upward direction, i.e., *opposite* to $\vec{g}$. The apparent gravitational acceleration $\vec{g}\prime = \vec{g} - \vec{A}$ perceived by a passenger is then downward and has a magnitude $(g + A)$ larger than g.* (See Fig. D-2a.)

> The fictitious force $\vec{\mathcal{F}} = -m\vec{A}$ on any particle of mass m is then in the downward direction. (See Fig. D-2b.) Hence the addition of this fictitious force to the real downward gravitational force $\vec{F}_g = m\vec{g}$ leads to an apparent downward gravitational force $\vec{F}\prime_g$ larger than the real one.

Thus the apparent weight of a passenger in the vehicle is larger than his real one. (For example, if a passenger is standing on bathroom scales in an elevator which is accelerating upward relative to the ground, these

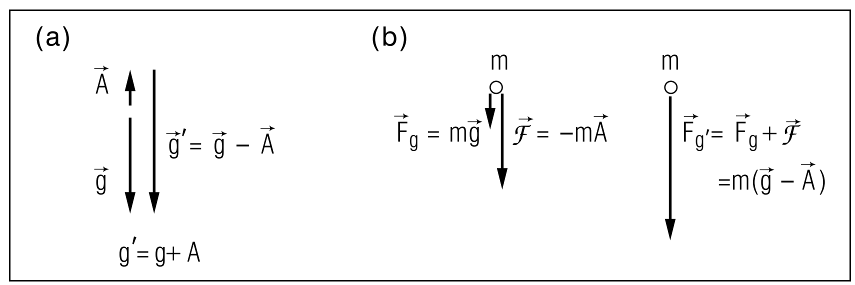


Fig. D-2: Apparent gravity in a vehicle moving with an upward acceleration $\vec{A}$. (a) Apparent gravitational acceleration $\vec{g}\prime = \vec{g} - \vec{A}$. (b) Apparent gravitational force $\vec{F}\prime_g = \vec{F}_g + \vec{\mathcal{F}}$ on a particle of mass m.

scales indicate a larger weight than if the elevator were not accelerating.) Furthermore, the apparently larger downward gravitational force on the passenger's stomach causes the stomach to hang lower down from its supporting muscles. This is why a passenger feels his stomach sinking toward his feet when he is standing in an elevator which is moving with a large upward acceleration.

Example D-1: Apparent gravity in an accelerating elevator

Suppose that an elevator travels relative to the ground with an acceleration of $2\,\mathrm{m/s^2}$ in the upward direction. What then is the apparent gravitational acceleration $\vec{g}\prime$ perceived by a passenger?

If $\hat{x}$ denotes a unit vector in the *downward* direction, the real gravitational acceleration is $g = (10\,\mathrm{m/s^2})\,\hat{x}$ and the upward acceleration of the elevator is $\vec{A} = (-2\,\mathrm{m/s^2})\,\hat{x}$. Hence the apparent gravitational acceleration is $\vec{g}\prime = \vec{g} - \vec{A} = (10\,\mathrm{m/s^2})\,\hat{x} - (-2\,\mathrm{m/s^2})\,\hat{x} = 12\,\mathrm{m/s^2})\,\hat{x}$. In other words, from the point of view of the passenger, all gravitational forces have been increased by 20 percent.

A rocket ship leaving the earth during blast-off is similar to an elevator, except that the upward acceleration $\vec{A}$ of the rocket ship has a magnitude several times larger than g. Indeed, the apparent downward acceleration $\vec{g}\prime = \vec{g} - \vec{A}$ perceived by an astronaut inside the rocket ship may attain a magnitude as large as $7g$. This very large apparent gravitational acceleration imposes severe physiological stresses on the astronaut. For example, it is difficult (or impossible) for the astronaut to move his limbs because they appear to him so heavy. Hence the astronaut cannot be relied upon to manipulate the controls of the space vehicle and all such controls must be automated. Even more important, the large apparent gravitational force tends to force the astronaut's blood downward and makes it difficult for the heart to maintain proper circulation of the blood. Hence the heart rate increases drastically to oppose undue slowing down of the circulation. Even so, the heart cannot pump a standing astronaut's blood to his head if the upward acceleration $\vec{A}$ of the vehicle has a magnitude larger than about $4g$. Such a standing astronaut would thus lose consciousness. To avoid this situation, the astronaut lies horizontally in his rocket ship during the blast-off.

CENTRIFUGES

Very large apparent gravitational forces can be produced in a vehicle if it moves relative to the ground with a very large acceleration $\vec{A}$. Such

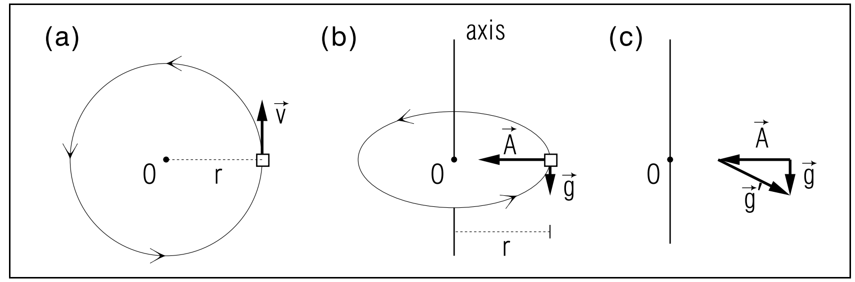


Fig. D-3: Apparent gravity in a centrifuge. (a) Top view looking down upon the horizontal circular path of the rotating vehicle. (b) Perspective front view of the vehicle at some instant. (c) Apparent gravitational acceleration g' relative to the vehicle.

a large acceleration can be achieved most readily in a "centrifuge," i.e., in an apparatus where the vehicle (which might be merely a test tube) is rotated rapidly around a vertical axis. (See Fig. D-3.) If this vehicle is at a distance r from this axis, it moves around a horizontal circle of radius r with a speed $v = 2\pi r\nu$ where ν is the rotational frequency of the centrifuge. [See Eq. (6) of *MISN-0-376*.] When the centrifuge rotates at a constant rate, the acceleration $\vec{A}$ of the vehicle relative to the ground is then (as discussed in Sec. 3 of *MISN-0-376*) directed toward the center of the circle and has a magnitude $A = v^2/r$. For example, the "ultracentrifuge," commonly used in biochemical investigations, can rotate at 1000 revolutions per second while supporting a tiny test tube at a distance of about 7 centimeter from the axis of rotation. The acceleration of this test tube has then a magnitude as large as $3 \times 10^6\,\mathrm{m/s^2}$, i.e., 300,000 times as large as the magnitude g of the gravitational acceleration!

Consider a solution contained in the test tube of a rotating centrifuge. The motion of the molecules in the solution is then described much more conveniently relative to the test tube S' rather than relative to the ground S. By the equivalence principle, all these molecules move relative to the tube as if they were subject to an apparent gravitational acceleration $\vec{g}\prime = \vec{g} - \vec{A}$. *

> * Strictly speaking, this situation is more complex than that contemplated in Sec. A, since the directions of a coordinate system fixed relative to the test tube S' do not remain parallel to those of a coordinate system fixed relative to the ground S. But the distinction is negligible as long as the average velocities of particles relative to S' is small.

As indicated in Fig. D-3c, $\vec{g}\,'$ is not vertically downward, but has a component vector pointing in the *outward* direction. *

> * The fictitious force $\vec{\mathcal{F}} = -\Updownarrow\vec{A}$ on any particle of mass m in the test tube has a direction opposite to $\vec{A}$ and points thus radially outward away from the center of the circular path of the particle. (This fictitious force is commonly called the "centrifugal" force on the particle.) The apparent gravitational force $\vec{F}\,'_g$ on the particle is then the sum of the real downward gravitational force $m\vec{g}$ plus the (usually much larger) outward centrifugal force $\vec{\mathcal{F}}$.

When the centrifuge rotates very rapidly, the acceleration $\vec{A}$ has such a large magnitude that $\vec{g}$ is negligibly small compared to it. In this case the apparent gravitational acceleration $\vec{g}\,' = \vec{g} - \vec{A} \approx -\vec{A}$ so that $\vec{g}\,'$ is nearly opposite to $\vec{A}$, i.e., so that $\vec{g}\,'$ points nearly in the outward direction. Hence all molecules in the test tube behave as if they were in the presence of a very large gravitational acceleration $\vec{g}\,'$ in the outward direction (away from the axis of rotation of the centrifuge).

A centrifuge is useful whenever a large gravitational force is desirable. For example, suppose that we wish to separate particles from a liquid in which they are dispersed. (For example, we might want to separate chemical precipitates from the solution in which they are formed, or to separate blood cells from plasma.) If the particles are small, it takes a very long time before the real gravitational force causes the particles to settle to the bottom of the liquid where they can be separated from the clear liquid above. But if the liquid is placed in the test tube of a rapidly rotating centrifuge, the large *apparent* gravitational force causes the particles to settle rapidly to the bottom of the test tube. (The test tube in the centrifuge is horizontal, with its bottom farthest from the axis of rotation, so that the apparent outward gravitational force drives particles toward the bottom of the tube.)

The rate with which particles descend through a surrounding liquid as a result of the gravitational force (the so-called "sedimentation

rate") provides information about the masses or shapes of the particles. If the gravitational force is sufficiently large, this method can be used to measure the masses of large *molecules*, such as proteins or nucleic acids. Hence the very large apparent gravitational force produced in the ultracentrifuge provides usually the most reliable measurements of the molecular weights of large macromolecules of biological interest. In addition, large macromolecules of slightly different masses can be readily separated in the ultracentrifuge,since the large apparent gravitational acceleration results in an appreciable difference in the apparent weights of such molecules. For these reasons, the ultracentrifuge has become a very important instrument for the study of fundamental biological processes.

Applying the Equivalence Principle Quantitatively (Cap. 2)

D-1 People with cardiac insufficiency can lose consciousness in elevators, because their weakened hearts cannot pump adequate blood to the brain against an apparent gravitational force that is more than a few percent larger than normal. Elevator travel may thus be hazardous for these persons whenever the apparent gravitational acceleration $\vec{g}\,'$ is larger than $\vec{g}$. For each of the following motions an elevator relative to a building, first find the value of $\vec{g}\,'$ and then state whether the motion may be hazardous for a passenger with cardiac insufficiency: (a) The elevator starts moving upward with increasing speed and an acceleration of magnitude $2\,\mathrm{m/s^2}$. (b) The elevator then continues upward with constant speed. (c) As it approaches a floor, the elevator moves upward with decreasing speed and an acceleration of magnitude $2\,\mathrm{m/s^2}$. *(Answer: 109) (Suggestion: [s-7])*

D-2 A rocket is designed so that the astronauts in the rocket experience an apparent gravitational acceleration $\vec{g}\,'$ of $60\,\mathrm{m/s^2}$ downward at blast-off. (Since the astronauts lie horizontally, they can tolerate this value of $\vec{g}\,'$ without loss of consciousness.) What is the acceleration $\vec{A}$ of the rocket relative to the ground in this situation? *(Answer: 105) (Suggestion: [s-2])*

D-3 Drivers of cars which collide unexpectedly may sustain head and neck injuries even if they are protected by lap safety belts. For example, suppose a car 1, initially moving along a direction $\hat{x}$,suddenly rams a similar car 2 at a stoplight (Fig. D-4). During the impact, car 1 has an acceleration $\vec{A}_1 = -10\,\mathrm{m/s^2}\,\hat{x}$ and car 2 has an acceleration $\vec{A}_2 = +10\,\mathrm{m/s^2}\,\hat{x}$ relative to the ground. (a) During the impact, what is

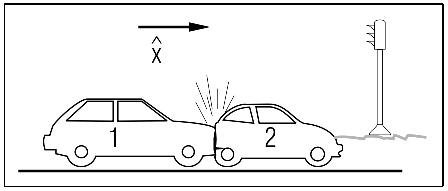


Fig. D-4.

the magnitude of the apparent gravitational acceleration $\vec{g}\prime_1$ relative to car 1? (b) During the impact, the driver of car 1 experiences an unexpected apparent gravitational force in the direction of $\vec{g}\prime_1$. Will this force cause the driver's upper body to pivot forward (causing his head to hit the windshield) or backward (causing a "whiplash" injury to his neck)? (c) Answer the preceding questions for car 2 and its driver. *(Answer: 115) (Suggestion: [s-5])*

D-4 In a standard centrifuge used in medical laboratories, the sample rotates in a horizontal circle of radius 15 cm at a constant rate of about 3000 revolutions per minute, thus moving with a speed of about 50 m/s relative to the ground. What is the magnitude of the apparent gravitational acceleration $\vec{g}\prime$ produced by this rotation? (You can neglect $\vec{g}$ in this calculation, since its magnitude is small compared to that of $\vec{g}\prime$.) Compare the apparent weight $w' = mg'$ of a cell in this sample to its normal weight $w = mg$ by finding the value of the ratio w'/w. *(Answer: 112) (Practice: [p-2])*

SECT.

E SUMMARY

IMPORTANT RESULTS

Descriptions relative to different reference frames: Eq. (A-1), Eq. (A-2), Eq. (A-3)

$$\vec{r} = \vec{r}\prime + \vec{R},\ \vec{v} = \vec{v}\prime + \vec{V},\ \vec{a} = \vec{a}\prime + \vec{A}$$

Galilean relativity principle: Rule (B-4)

If S' moves relative to an inertial frame S with *constant* velocity, motion relative to S' proceeds exactly as if S' were at rest relative to S.

Equivalence principle: Rule (C-10)

If S' moves relative to an inertial frame S with an acceleration $\vec{A}$, motion relative to S' proceeds as if S' were an inertial frame in which the gravitational acceleration is $\vec{g}\prime = \vec{g} - \vec{A}$.

NEW CAPABILITIES

Suppose S' is a reference frame near the earth and S is an inertial reference frame. You should have acquired the ability to:

(1) Apply the Galilean relativity principle to relate descriptions of the same phenomenon observed in S' when S' is at rest relative to S and when S' is moving with constant velocity relative to S. (Sec. B; [p-1])

(2) (a) Apply the equivalence principle to relate the apparent gravitational acceleration $\vec{g}\prime$ relative to S', the real gravitational acceleration $\vec{g}$, and the acceleration $\vec{A}$ of S' relative to S.
(b) Use the direction of $\vec{g}\prime$ to qualitatively relate descriptions of the same phenomenon when S' is moving relative to S and when S' is at rest relative to S. (Sects. C and D, [p-2])

SECT.

F PROBLEMS

F-1 *Behavior of a balloon on a bus:* A child on a bus holds a helium-filled balloon by its string. When the bus is at rest at a stop the string is vertical (i.e., parallel to $\vec{g}$). When the bus moves forward with increasing speed, is the balloon's string vertical, inclined upward toward the front of the bus, or inclined upward toward the back of the bus? *(Answer: 117) ([s-9], [p-3])*

F-2 *Avoiding "blackout" in high-speed aircraft:* At the bottom of a high-speed dive, the pilot of a jet plane experiences an apparent gravitational acceleration $\vec{g}\prime$ which can easily be large enough to cause "blackout" (loss of vision) or even unconsciousness. To examine the limits of safety, suppose the plane is momentarily moving with a constant speed of 300 m/s when it reaches the bottom of the circular arc in Fig. F-1. For what radius R of the plane's circular path will the apparent gravitational acceleration have its maximum safe value $\vec{g}\prime = 4g$? To increase the margin of safety by reducing $\vec{g}\prime$, should the pilot fly the plane along an arc with larger or smaller radius than the preceding value? *(Answer: 114) ([s-12], [p-4])*

F-3 *Liquid accelerometer:* In text problem C-2, we considered a container of liquid mounted on a car's dashboard and observed that the surface of the liquid is inclined at some angle θ to the horizontal when the car has an acceleration $\vec{A}$ relative to the ground (Fig. F-2). To show how this device can be used to measure the car's acceleration, let us find the relation between the magnitude A of this acceleration and the angle θ (which can be measured using a scale on the container). (a) Since the liquid surface is perpendicular to the apparent gravitational acceleration $\vec{g}\prime$, the angle θ is just the angle between $\vec{g}\prime$ and the vertical. Using this observation, and a diagram relating $\vec{g}\prime$ to $\vec{g}$ and $\vec{A}$, express $\tan\theta$ in terms of g and A. (b) What is the value of A corresponding to the typical values $\theta = 0°, 10°$, and $20°$? *(Answer: 111) ([s-3], [p-5])*

F-4 *Effect of the earth's rotation on the gravitational acceleration:* The rotation of the earth causes the apparent gravitational acceleration $\vec{g}\prime$ measured on the earth's surface to differ from the value g due to the real gravitational force exerted by the earth. For example, consider the apparent gravitational acceleration $\vec{g}\prime$ measured relative to a laboratory on the earth's equator. Is $\vec{g}\prime$ larger or smaller than $\vec{g}$? To estimate the size of this effect, find the magnitude $|\Delta\vec{g}|$ of the difference $\Delta\vec{g} = \vec{g}\prime - \vec{g}$. Use the values $R = 6.4 \times 10^6$ meter and $T = 8.6 \times 10^4$ second for the earth's radius and period of rotation. Compare $|\Delta\vec{g}|$ to the approximate magnitude $g = 10\,\text{m/s}^2$ of the real gravitational acceleration by finding the ratio $|\Delta\vec{g}|/g$. *(Answer: 116)*

F-5 *Apparent gravitation and plant growth:* Plants grown on a turntable rotating at a constant rate look like those in Fig. F-3. Use the equivalence principle to give a plausible explanation for (a) the general tilt of the plants toward the center of the turntable, and (b) the increasing tilt of the plants with increasing distance from the center. *(Answer: 113)*

F-6 *"Shaking down" a thermometer:* A mercury thermometer is "shaken down" (to reduce the indicated temperature reading) by shaking it with a flick of the wrist so that the bulb moves rapidly along a circular arc (Fig. F-4). Why does the mercury in the thermometer tube flow outward from the hand toward the bulb? Explain in terms of fictitious forces. *(Answer: 118)*

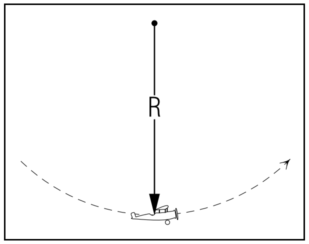


Fig. F-1.

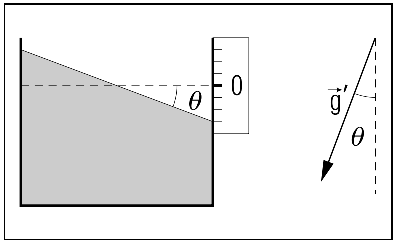


Fig. F-2.

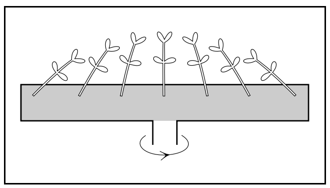


Fig. F-3.

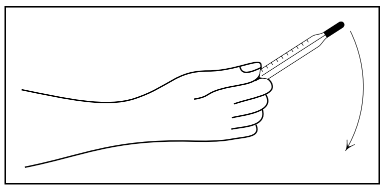


Fig. F-4.

PRACTICE PROBLEMS

p-1 *THE GALILEAN RELATIVITY PRINCIPLE (CAP. 1):* A girl tests her new ping-pong ball by dropping it from a height of 1 meter above an elevator floor when the elevator is at rest relative to the ground. The ball takes 1.4 second to hit the floor, and it rebounds to a height of 0.7 meter above the floor. She repeats this experiment later when the elevator is moving upward with a constant speed of 2.0 m/s relative to the ground. (a) Does the ball take 1.4 second, less time, or more time to hit the upwardly-moving elevator floor? (b) After hitting the moving floor, does the ball rebound to a height of 0.7 meter above the floor, to a smaller height, or to a larger height? *(Answer: 4) (Suggestion: review text problem B-1.)*

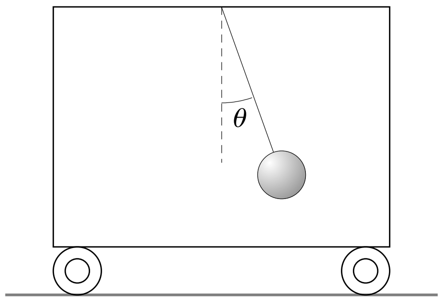
p-2 *APPLYING THE EQUIVALENCE PRINCIPLE QUANTITATIVELY (CAP. 2):* After reading this unit, a curious student of mass $m = 50$ kg weighs herself on a bathroom scale in her apartment elevator. At one time during an upward trip, she finds that her apparent weight $mg' = 450$ N, corresponding to an apparent gravitational acceleration $\vec{g}\,'$ of $9.0\,\text{m/s}^2$ downward. At this time, what is the elevator's acceleration $\vec{A}$ relative to the ground? *(Answer: 2) (Suggestion: review text problems D-1 and D-2.)*

More Difficult Practice Problems (Text Section F)

p-3 *FINDING WHICH WAY IS "UP" ON A MERRY-GO-ROUND:* The flame of a candle at rest (relative to the ground) points up, or opposite to the direction of $\vec{g}$. Suppose a person holds a candle, which is protected from wind by a glass chimney, while standing on a merry-go-round rotating at a constant rate. To find which direction is "up" relative to the rotating merry-go-round, decide whether the candle flame will point roughly (a) along the direction of the person's motion, (b) opposite to the direction of the person's motion, (c) outward from the center of the merry-go-round, or (d) inward toward the center of the merry-go-round. *(Answer: 5) (Suggestion: review text problem F-1.)*

p-4 *APPARENT GRAVITATIONAL EFFECTS ON A FERRIS WHEEL:* Passengers seated on a rotating ferris wheel should experience an apparent gravitational acceleration $\vec{g}\,'$ different from $\vec{g}$. To estimate how appreciable this difference is, consider a typical ferris wheel which has a radius of 6 meter and which takes about 15 second to complete a revolution. (a) When the wheel is rotating, what is the magnitude of the apparent gravitational acceleration $\vec{g}\,'$ relative to a seat located momentarily at the top of the wheel? (b) Is the apparent weight $w' = mg'$ of a passenger in this seat larger or smaller than his real weight $w = mg$? Compare these weights quantitatively by finding the value of the ratio w'/w. *(Answer: 1) (Suggestion: [s-1], review text problem F-2.)*

p-5 *PENDULUM ACCELEROMETER:* The acceleration of a vehicle moving horizontally relative to the ground can be measured using a pendulum suspended within the vehicle. When the vehicle accelerates, the pendulum will hang at some measurable angle θ from the vertical, as shown in the following diagram. (a) Is the acceleration of the vehicle in the diagram directed to the right or to the left relative to the ground? (b) Write an equation for the magnitude A of the vehicle's acceleration in terms of the angle θ. (c) Find the value of A corresponding to the angle $\theta = 30°$ shown in the diagram. *(Answer: 3) (Suggestion: review text problem F-3.)*



SUGGESTIONS

s-1 *(Practice problem [p-4]):* Relative to the ground, a chair on the ferris wheel moves with constant speed v in a circle of radius R. If T is the time required for one revolution of the wheel, recall that $v = 2\pi R/T$.

s-2 *(Text problem D-2):* Since the apparent gravitational acceleration $\vec{g}\prime = \vec{g} - \vec{A}$, solve this equation for the unknown acceleration $\vec{A}$ of the rocket relative to the ground, and use a *vector* diagram to find its value.

s-3 *(Text problem F-3):* In the vector diagram constructing $\vec{g}\prime = \vec{g} - \vec{A}$, the three vectors form a right triangle. The angle θ is one of the acute angles in this triangle.

s-4 *(Text problem C-3):* The car is moving relative to the ground along a circular path at constant speed. Therefore, its acceleration $\vec{A}$ relative to the ground is directed toward the center of the circular path, or toward the left in Fig. C-4.

s-5 *(Text problem D-3):* Since the acceleration of each car relative to the ground determines the value of its apparent gravitational acceleration, $\vec{g}\prime_1 = \vec{g} - \vec{A}_1$ (for car 1) and $\vec{g}\prime_2 = \vec{g} - \vec{A}_2$ (for car 2). Use vector diagrams to construct $\vec{g}\prime_1$ and $\vec{g}\prime_2$. The vectors in each diagram form a right triangle which you can use to find the magnitude and direction of the apparent gravitational acceleration.

s-6 *(Text problem B-3):* The Galilean relativity principle applies only to reference frames (e.g., vehicles) moving with *constant velocity* relative to an inertial reference frame such as the ground. Since the vehicle's velocity is a vector, both its magnitude (the vehicle's speed) and its direction (the vehicle's direction of motion) must be constant.

s-7 *(Text problem D-1):* Use a diagram to find the *vector* difference $\vec{g}\prime = \vec{g} - \vec{A}$ in each case, where $\vec{A}$ is the elevator's acceleration relative to the building. This acceleration $\vec{A}$ is directed *along* the elevator's direction of motion when the elevator's speed is increasing, and opposite to this to this direction when its speed is decreasing.

s-8 *(Text problem B-2):* The door moves relative to the ground with the same constant velocity as the train, 3 m/s to the right. Hence it moves 6 meter to the right in 2 second. Since the ball moves with a constant

velocity of 4 m/s to the right relative to the train, its velocity relative to the *ground* is (4 m/s + 3 m/s) to the right, or 7 m/s to the right.

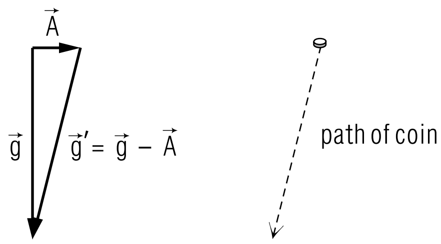
s-9 *(Text problem F-1):* The equivalence principle predicts that the balloon will move toward the *front* of the bus so that its string is parallel to $\vec{g}\prime$ (which points downward toward the *back* of the bus). Another way of seeing this apparently paradoxical result is to think of the balloon floating on layers of denser air. When the bus accelerates forward, the air tends to pile up toward the back of the bus, tilting these layers like the surface of the liquid in text problem C-2.

s-10 *(Text problem C-2):* The liquid surface will be oriented *perpendicular* to the apparent gravitational acceleration $\vec{g}\prime = \vec{g} - \vec{A}$. Thus it is useful to draw a rough vector diagram constructing $\vec{g}\prime$ from $\vec{g}$ and the car's acceleration $\vec{A}$ relative to the ground. Remember that $\vec{A}$ is directed along the car's direction of motion when the car is speeding up, and opposite to the car's direction of motion when the car is slowing down.

s-11 *(Text problem A-1):* The result $\vec{v} = \vec{v}\prime + \vec{V}$ (text equation A-2) states that the velocity $\vec{v}$ of the air relative to the water is just its velocity $\vec{v}\prime$ relative to the boat plus the velocity $\vec{V}$ of the boat relative to the water. You can use this *vector* relation directly to find the wind velocity $\vec{v}\prime = \vec{v} - \vec{V}$ of the air relative to the sailboat in each case. A diagram is essential for part (a).

s-12 *(Text problem F-2):* First use the relation $\vec{g}\prime = \vec{g} - \vec{A}$ to find the acceleration $\vec{A}$ of the plane relative to the ground. Since the plane is moving relative to the ground with constant speed v at the bottom of a circular arc of radius R, its acceleration relative to the ground is also $\vec{A} = v^2/R$ upward. Combine these results to find R. Finally, note that since $\vec{g}$ and $\vec{A}$ have opposite directions, the magnitude of $\vec{g}\prime$ is given by $g' = g + A$. Thus to reduce g', A must be reduced.

s-13 *(Text problem C-1):* When the woman drops a coin on a bus at rest relative to the ground, the coin falls along a path parallel to the real gravitational acceleration $\vec{g}$. Therefore, according to the equivalence principle, when the bus has some acceleration $\vec{A}$ relative to the ground, a coin dropped in the same way will fall (relative to the bus) along a path parallel to the *apparent* gravitational acceleration $\vec{g}\prime = \vec{g} - \vec{A}$. For example, in part (a) the bus has an acceleration $\vec{A}$ of 2 m/s^2 to the right relative to the ground. The apparent gravitational acceleration $\vec{g}\prime$ is thus directed downward and to the left, as shown in the following diagram showing the difference $\vec{g}\prime = \vec{g} - \vec{A}$. Therefore, the coin will fall relative to the bus along a path downward toward the left, rather than vertically downward.



As this example illustrates, the core of the equivalence principle is the *vector* equation $\vec{g}\prime = \vec{g} - \vec{A}$. Vector diagrams using this equation are thus a useful tool in applying the equivalence principle.

ANSWERS TO PROBLEMS

1. a. $g' = g - v^2/R = 9\,\text{m/s}^2$
   b. smaller; $w'/w = 0.9$
2. $\vec{A} = 1.0\,\text{m/s}^2$ downward
3. a. to the left
   b. $A = g \tan\theta$
   c. $A = 5.8\,\text{m/s}^2$
4. a. 1.4 second
   b. 0.7 meter
5. (d)

101. a. drawing 2
     b. drawing 1
102. 2 second
103. a. to driver's left
     b. force is zero
     c. toward front of car
104. (a) and (d)
105. $\vec{A} = 50\,\text{m/s}^2$ upward
106. a. $\vec{v}\prime = 11\,\text{m/s}$ southwest
     b. $\vec{v}\prime = 2\,\text{m/s}$ south
107. to the right
108. a. 6 meter right
     b. 14 meter right
     c. yes
109. a. $\vec{g}\prime = 12\,\text{m/s}^2$ downward; yes
     b. $\vec{g}\prime = \vec{g}$; no
     c. $\vec{g}\prime = 8\,\text{m/s}^2$ downward; no
110. a. path 1

b. path 2

c. path 3

111. a. $\tan\theta = A/g$

b. for $0°$, $A = 0$; for $10°$, $A = 1.8\,\text{m/s}^2$; for $20°$, $A = 3.6\,\text{m/s}^2$

112. $g' = v^2/r = 1.7 \times 10^4\,\text{m/s}^2$; $w'/w = 1.7 \times 10^3$

113. a. $\vec{g}\prime$ is inclined outward

b. the centripetal acceleration $\vec{A}$ increases with the radius

114. $R = v^2/3g = 3 \times 10^3$ meter $= 3\,\text{km}$; larger. ($R \neq v^2/4g$; see [s-12])

115. a. $g_1' = 14\,\text{m/s}^2$

b. forward

c. $g_2' = 14\,\text{m/s}^2$; backward

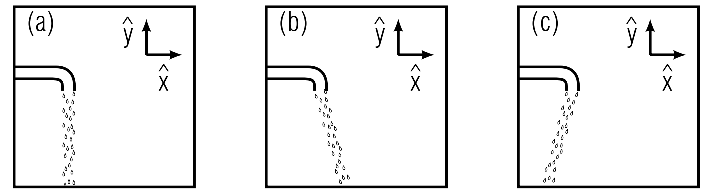
116. smaller; $|\Delta\vec{g}| = v^2/R = 3.4 \times 10^{-2}\,\text{m/s}^2$; $|\Delta\vec{g}|/g = 3.4 \times 10^{-3}$

117. upward toward the *front* of the bus

118. Any part of the tube has an acceleration $\vec{A}$ directed inward toward the hand, so the fictitious force $-m\vec{A}$ is directed outward toward the bulb.

MODEL EXAM

1. **Observations on a moving train.** A boy on a train watches water flowing from a faucet, as illustrated in the following drawing. The train is moving to the right (i.e., along the direction $\hat{x}$ in the drawings) with decreasing speed relative to the ground.



a. Which of the drawings correctly indicates the path of the water flowing from the faucet? (The water flows vertically downward when the train is at rest relative to the ground.)

Later, the train has an acceleration relative to the ground of $5\,\text{m/s}^2$ toward the left (i.e., opposite to $\hat{x}$).

b. What is the magnitude of the apparent gravitational acceleration relative to the train?

Brief Answers:

1. a. (b)

b. $11\,\text{m/s}^2$