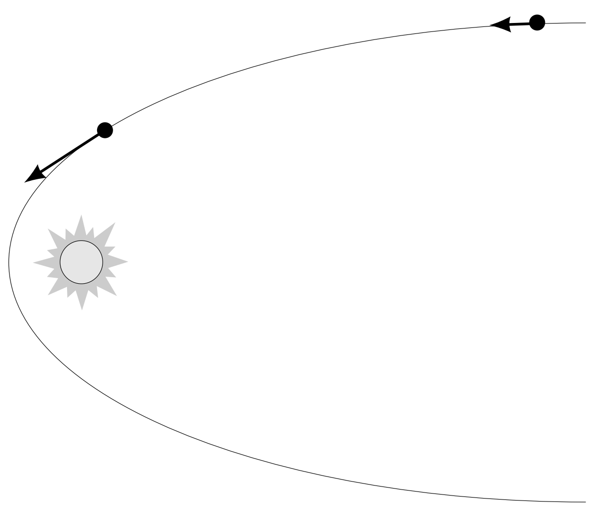
PHYSNET

MISN-0-408

THEORY OF MOTION



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

THEORY OF MOTION

by

F. Reif, G. Brackett and J. Larkin

CONTENTS

Title: **Theory Of Motion**

Author: F. Reif, G. Brackett, and J. Larkin, Department of Physics, University of California, Berkeley.

Version: 4/30/2002 Evaluation: Stage 0

Length: 1 hr; 52 pages

Input Skills:

1. Vocabulary: reference frame, (MISN-0-405); velocity, acceleration (MISN-0-406).
2. Given a vector equation, solve it for any quantity in the equation (MISN-0-405).

Output Skills (Knowledge):

K1. Vocabulary: system of particles, isolated system, inertial frame, mass, kilogram, force, newton, equilibrium.

K2. State the principle of inertia.

K3. State two properties of two-particle forces.

K4. State the superposition principle for the forces on a particle interacting with two or more other particles.

K5. State the "equation of motion" for a particle.

Output Skills (Problem Solving):

S1. Solve problems using these relations: (a) equal but opposite forces acting on each of two particles due to their mutual interaction; (b) the (vector) equation relating the force acting on a particle to the mass of the body and its acceleration; (c) the superposition principle for forces acting on a particle.

S2. Recognize an inertial reference frame and describe the motion of an isolated particle relative to such a frame.

S3. Given two parallel forces acting on a particle, quantitatively relate these forces to the particle's acceleration.

THIS IS A DEVELOPMENTAL-STAGE PUBLICATION
OF PROJECT PHYSNET

The goal of our project is to assist a network of educators and scientists in transferring physics from one person to another. We support manuscript processing and distribution, along with communication and information systems. We also work with employers to identify basic scientific skills as well as physics topics that are needed in science and technology. A number of our publications are aimed at assisting users in acquiring such skills.

Our publications are designed: (i) to be updated quickly in response to field tests and new scientific developments; (ii) to be used in both classroom and professional settings; (iii) to show the prerequisite dependencies existing among the various chunks of physics knowledge and skill, as a guide both to mental organization and to use of the materials; and (iv) to be adapted quickly to specific user needs ranging from single-skill instruction to complete custom textbooks.

New authors, reviewers and field testers are welcome.

MISN-0-408

THEORY OF MOTION

Abstract:
Up to now we have been mostly concerned with the *description* of motion. Hence we are now prepared to undertake the more ambitious task of formulating a theory capable of *prediction* so that information about particles at one time can be used to predict their motion at *all other* times. In accordance with the comments in text section B of Unit 402, we shall try to construct such a predictive theory of motion by making a few basic assumptions (or "hypotheses") suggested by some experimental observations or plausible arguments. If predictions based on these assumptions are extensively verified by observations, we shall then adopt these assumptions as the basic premises of our theory. To facilitate the task of formulating a theory of motion, we shall pursue the useful strategy of starting with simple situations and gradually proceeding to more complex ones. Accordingly, we shall discuss first the motion of a single particle, then the motion of two particles, and finally the motion of three or more particles. At that point we shall have obtained a completely general theory essentially identical to that first formulated by Isaac Newton (1642-1727).

SECT.

A MOTION OF A SINGLE ISOLATED PARTICLE

Our ultimate aim is to predict the motion of some "system" of particles, by which we mean:

Def. | **System of particles**: Any set of particles upon which we wish to focus attention. | (A-1)

Such a system is especially simple if it is "isolated".

Def. | **Isolated system**: A system which is uninfluenced by anything outside the system. | (A-2)

In the present section we shall consider the simplest isolated system, i.e., one consisting of a *single* isolated particle. In practice, we can expect a particle to be isolated if it is sufficiently far from all other particles (e.g., if it is in outer space millions of miles away from the nearest star).

A convenient reference frame for describing the motion of an isolated particle consists of the sun and some distant stars. For example, the coordinate system associated with such a "solar frame" might have its origin at the sun and its coordinate directions pointing from the sun toward some distant stars (since these directions remain essentially unchanged in time).

How does an isolated particle move relative to this solar reference frame? The observed motion of comets suggests a simple answer. When a comet moves in outer space very far from the sun (so that it is effectively isolated), it travels with a nearly constant velocity, i.e., with a nearly constant speed in a nearly fixed direction. It is only when the comet comes near to the sun that its velocity starts changing appreciably, as indicated in Fig. A-1. Many such observations show that any isolated particle moves very simply relative to some special reference frame (such as the solar frame) and the observations suggest this general assumption:

Principle of inertia: There exists a reference frame relative to which every isolated particle moves with constant velocity. (A-3)

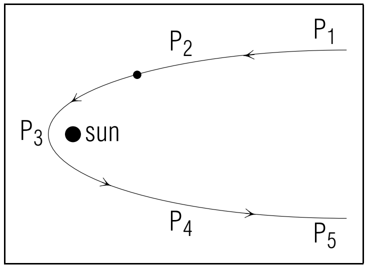


Fig. A-1: Motion of a comet around the sun. The velocity of the comet is nearly constant when the comet is very far from the sun (between P_1 and P_2, and also between P_4 and P_5.)

Such a reference frame is called an "inertial frame" in accordance with this definition:

Def. | **Inertial frame**: A reference frame relative to which every *isolated* particle moves with constant velocity. | (A-4)

The solar frame is thus an inertial frame.

OTHER INERTIAL FRAMES

What would happen if we described the motion of an isolated particle (e.g., of a distant comet) relative to some other reference frame, e.g., relative to a spaceship moving relative to the solar frame with some velocity $\vec{V}$? If the particle moves relative to the solar frame with a velocity $\vec{v}$, its velocity $\vec{v}\prime$ relative to the spaceship is then

$$\vec{v}\prime = \vec{v} - \vec{V} \qquad \text{(A-5)}$$

[For example, in Fig. A-2 the particle moves relative to the solar frame with a velocity $\vec{v} = 5\,\text{km/sec}$ along $\hat{x}$, and the spaceship moves relative to the solar frame with a velocity $\vec{V} = 2\,\text{km/sec}$ along $\hat{x}$. Hence the particle

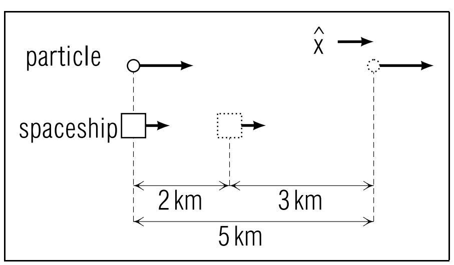


Fig. A-2: Motion of a particle and of a spaceship relative to the solar frame. The dotted outlines indicate the positions of these objects after 1 second.

moves relative to the spaceship with a velocity $\vec{v}\prime = 3\,\text{km/sec}$ along $\hat{x}$ (i.e., with a velocity $\vec{v} - \vec{V}$).] *

* As we shall see in text section A of Unit 412, the relation $\vec{v}\prime = \vec{v} - \vec{V}$ is generally valid, even if $\vec{v}$ and $\vec{V}$ do not have the same direction.

According to Eq. (A-5), the velocity $\vec{v}\prime$ of an *isolated* particle relative to the spaceship differs from its *constant* velocity $\vec{v}$ relative to the inertial solar frame merely by the velocity $\vec{V}$ of the spaceship. If $\vec{V}$ is constant, $\vec{v}\prime = \vec{v} - \vec{V}$ is then also constant. On the other hand, if $\vec{V}$ is not constant, $\vec{v}\prime = \vec{v} - \vec{V}$ is also not constant. Hence an isolated particle moves with constant velocity $\vec{v}\prime$ relative to a spaceship if (and only if) this spaceship moves with *constant* velocity $\vec{V}$ relative to the solar frame. According to Def. (A-4), the spaceship (or other such reference frame) would then also be an inertial frame. Hence we reach this conclusion:

| Any reference frame is an inertial frame if (and only if) it moves with *constant* velocity relative to some other inertial frame (such as the solar frame). | (A-6) |

Thus there exists a whole class of inertial reference frames in addition to the solar frame. The description of motion relative to any such inertial frame is especially simple since every isolated particle moves relative to this frame with constant velocity. Thus we shall henceforth describe the motion of all particles relative to an inertial frame (unless we specifically state that we are using some other reference frame).

APPLICATIONS TO MOTION NEAR THE SURFACE OF THE EARTH

The motion of objects observed from the surface of the earth can usually be described most conveniently relative to a reference frame (such as the walls of a room) fixed with respect to the surface of the earth. Such a "terrestrial reference frame" moves (just like our preceding spaceship) with some velocity $\vec{V}$ relative to the solar frame because the earth rotates around its axis and also moves around the sun. This velocity $\vec{V}$ is not constant since its direction changes. Hence a terrestrial frame is *not* an inertia1 frame.

But the velocity $\vec{V}$ of a terrestrial frame changes its direction very slowly (since the earth rotates about its axis only once every day). Suppose then that one is interested in describing the motion of particles during a time much less than 1 day. In this case the velocity $\vec{V}$ changes so little that it remains nearly constant. Accordingly, a terrestrial frame can then be considered *approximately* an inertial frame. Indeed, it is usually quite adequate to regard a terrestrial frame as an inertial frame when one is dealing with the motion of particles close to the surface of the earth. *

> * Observations which are sufficiently precise, or which extend over sufficiently long periods of time, do show that a terrestrial frame is not inertial and thus demonstrate the rotation of the earth. (For example, isolated stars, photographed from the earth with long-time exposure, are observed to move along curved paths so that their velocity relative to the earth is *not* constant.)

No particle near the surface of the earth is ever isolated since it interacts by gravity with the earth. But this gravitational interaction affects only motion along the vertical direction. Hence such a particle may still act like an isolated particle in its motion along any horizontal direction. For example, as discussed in text section E of Unit 407, the horizontal velocity of a particle projected near the earth remains constant if effects of the surrounding air are negligible. Similarly, if a particle moves along a horizontal surface which is quite smooth (like a sheet of ice), the horizontal motion of the particle is not affected appreciably by the surface and the particle moves with constant horizontal velocity. In short, projectiles moving with negligible air resistance and particles moving along smooth horizontal surfaces behave, in their horizontal motion, approximately like isolated particles.

Using Inertial Frames to Describe Isolated Particles (Cap. 2)

A-1 Far from all other particles, a space probe moves with constant velocity relative to the solar reference frame. (a) Is this probe an inertial reference frame? (b) A bolt breaks loose and moves within the probe without interacting with any other particles. Relative to the frame of the probe, is the bolt's path straight or curved? Is its speed constant or changing? Note: Here we use the earth's surface as our reference frame. For the precision desired this is an inertial reference frame. *(Answer:*

103)

A-2 *Particles isolated for horizontal motion:* (a) A heavy box on a dolly (a platform supported by well-oiled wheels) is isolated for horizontal motion as it rolls along a warehouse floor. Describe the path and speed of the box. (b) Which of the following particles are isolated for horizontal motion? An ice puck moving with constant velocity along a horizontal ice surface. A tether-ball on a rope swinging with constant speed along a horizontal circular path. *(Answer: 108)*

SECT.

B INTERACTION OF TWO PARTICLES

Let us now turn our attention to the case of an isolated system consisting of *two* interacting particles (e.g., two billiard balls). If these particles are sufficiently far from each other (so that each is isolated from the other as well as from the rest of the world), each moves with *constant* velocity relative to an inertial frame. But if the particles are sufficiently close to each other, the interaction between the particles becomes apparent because their velocities are ordinarily no longer constant, i.e., because each particle moves with some acceleration. (See Fig. B-1.) A theory dealing with the motion of two interacting particles must then focus attention on the *acceleration* of each of the particles.

The particles may interact in various ways. For example, the particles may interact when they are at appreciable distances from each other (like the moon interacting with the earth, or like one small magnet interacting with another). Alternatively, the particles may interact when they are in apparent contact with each other (either directly like one billiard ball colliding with another, or indirectly like one ball tied to another ball by a rubber band.)

Let us then consider any two particle system specified interacting particles, such as 1 and 2 in Fig. B-1, and ask this question: How does the acceleration $\vec{a}_1$ of particle 1 (or the acceleration $\vec{a}_2$ of particle 2) depend on the spatial relationship between these particles? The simplest assumption we can make is:

At any instant, the acceleration of one particle interacting with another depends only on its position relative to the other particle (valid for many cases).	(B-1)

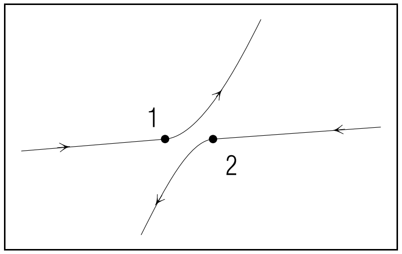


Fig. B-1: Paths of two interacting particles.

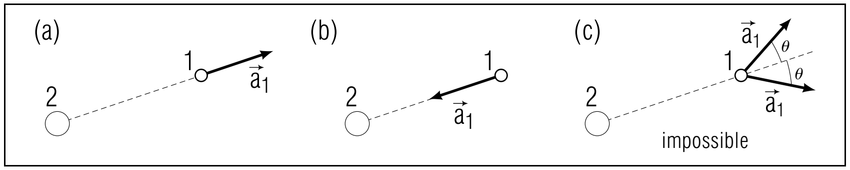


Fig. B-2: Possible directions of the acceleration $\vec{a}_1$ of the particle 1 interacting with another particle 2.

Of course, it would be possible to suggest more complicated assumptions. For example, the acceleration of each particle might also depend on the velocities of the particles, or it might depend on their positions at some time in the past. However, the simple assumption in Rule (B-1) leads to correct predictions in many cases. [The exceptions can be handled by later refinements of the theory through suitable modifications of Rule (B-1).]

According to Rule (B-1), the acceleration $\vec{a}_1$ of particle 1 at any instant can depend only on its position relative to particle 2, i.e., only on the displacement vector $\vec{R}$ from particle 2 to particle 1. (See Fig. B-2.) Hence the acceleration $\vec{a}_1$ can depend only on the direction of $\vec{R}$ and on the magnitude of $\vec{R}$ (i.e., on the distance between the particles).

To examine more closely the possible directions of $\vec{a}_1$, suppose the particles are separated by a given distance. Then, for cases that obey Rule (B-1), the direction of $\vec{a}_1$ can depend only on the direction of $\vec{R}$. For example, the direction of $\vec{a}_1$ may certainly be parallel to $\vec{R}$. In other words, if the arrow representing $\vec{A}_1$ is drawn from particle 1, the direction of $\vec{a}_1$ may be along the line joining the particles so that it points either away from, or toward, the other particle, particle 2 (as illustrated in Fig. B-2a and Fig. B-2b). On the other hand, under Rule (B-1) $\vec{a}_1$ can *not* have any other direction, such as one making some angle θ with the line joining the particles. For then *several* possible directions of $\vec{a}_1$ (such as the two directions illustrated in Fig. B-2c) would be consistent with the same direction of $\vec{R}$ and the direction of $\vec{a}_1$ would thus *not* depend solely on the direction of $\vec{R}$.

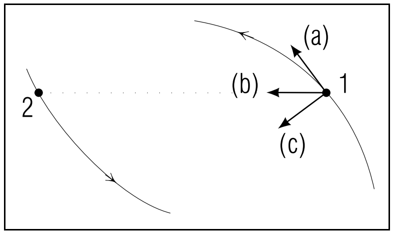


Fig. B-3.

The preceding comments show that Rule (B-1) implies this conclusion for any two specified particles:

At any instant, the acceleration of one particle interacting with another has a direction along the line joining the particles and depends only on the distance between them [for cases that obey Rule (B-1)].	(B-2)

Knowing About Two-Particle Interactions

B-1 The two ions of gas shown in Fig. B-3 are isolated from other particles and move along the indicated paths. Which of the arrows indicates the direction of the acceleration of atom 1? *(Answer: 106)*

SECT. C RELATION BETWEEN ACCELERATIONS AND MASSES

Now that we have discussed the acceleration of each of interacting particles interacting particles, let us examine the relationship existing between the accelerations of these particles.

SPECIAL CASE OF IDENTICAL PARTICLES

Consider first the special simple case where the two particles are completely identical and thus indistinguishable (so that they are of the same size, consist of the same substance, etc.). Then each particle must have an acceleration of the same magnitude so that $a_1 = a_2$.

Furthermore, if the particles are completely identical, the direction of the acceleration of each must be specified relative to the other particle in the same way. If the acceleration $\vec{a}_1$ of particle 1 is directed *away* from particle 2 (as shown in Fig. C-1a), then the acceleration $\vec{a}_2$ of particle 2 must be similarly directed *away* from particle 1. Alternatively, if the acceleration $\vec{a}_1$ of particle 1 is directed *toward* particle 2 (as shown in Fig. C-1b), the acceleration $\vec{a}_2$ of particle 2 must be similarly directed *toward* particle 1. As is seen from Fig. C-1, the preceding statements thus imply that the accelerations of the particles always have *opposite* directions.

GENERAL CASE

When the two interacting particles are not identical, it is plausible to assume that the preceding conclusion about the directions of their accelerations remains valid. In other words,

In an isolated system of two particles, the accelerations of the particles have at any instant opposite directions.	(C-1)

On the other hand, the accelerations of different particles do not ordinarily have the same magnitude. What then is the relationship between the magnitudes a_1 and a_2 of these accelerations?

In the special case where the particles are identical, $a_1 = a_2$ so that the ratio $a_1/a_2 = 1$ at all times, irrespective of the positions of

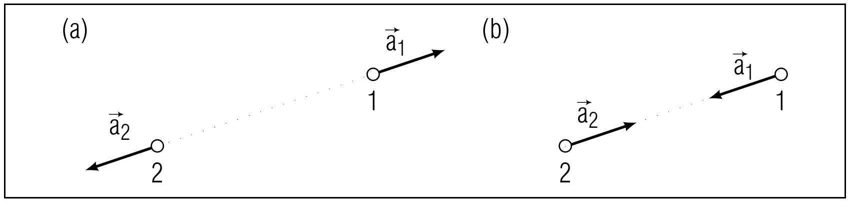


Fig. C-1: Possible relations between the accelerations of two identical interacting particles.

the particles or of the nature of the interaction between them. In the general case of different particles, it is then simplest to assume (and verified by observations) that the ratio a_1/a_2 has some constant positive value which is ordinarily different from 1, but which depends *only* on the properties of the particles (i.e., which again does not depend on the positions of the particles or on the nature of the interaction between them). This assumption suggests that each particle can be described by a property, called its "mass" and denoted by m, such that the ratio of the accelerations of the particles is related to the ratio of their masses. Conventionally one writes this relationship in the form

$$\frac{a_1}{a_2} = \frac{m_2}{m_1} \qquad \text{(C-2)}$$

This can be expressed more simply as

$$m_1 a_1 = m_2 a_2 \qquad \text{(C-3)}$$

If the particles are identical, their masses are the same so that $m_1 = m_2$. Then the relation (C-3) implies properly that $a_1 = a_2$. But if $m_1 > m_2$, the relation (C-3) implies that $a_1 < a_2$. In other words, the particle with the larger mass has the smaller acceleration, i.e., its velocity changes more slowly. Thus the mass of a particle describes quantitatively the qualitative notion of "inertia," the tendency of a particle to resist changes in its velocity. [For example, when a child throws a ball against the front end of a stationary baby carriage, the ball bounces back (so that its velocity changes drastically) while the carriage barely starts moving (so that its velocity changes only slightly). Thus the mass, or inertia, of the carriage is much larger than that of the ball.]

Thus we have been led to the fundamental assumption that every particle can be characterized by a property called "mass" and defined as

follows:

Def. **Mass**: The mass of a particle is a positive numerical quantity specifying the relative magnitude of the acceleration of this particle compared to that of any other particle with which it interacts (so that $m_1\vec{a}_1 = -m_2\vec{a}_2$). (C-4)

Our entire discussion of this section can then be summarized by combining the relations (C-1) and (C-3) into the single vector relation

$$m_1\vec{a}_1 = -m_2\vec{a}_2 \qquad \text{(C-5)}$$

where the minus sign indicates merely that the accelerations have opposite directions.

PRACTICAL APPLICATIONS

Suppose that a bullet of mass m_1 is fired horizontally from a gun of mass m_2 (under conditions where the gun is on a smooth surface so that the gun and bullet are free to move horizontally like an isolated system of particles). Then the explosive interaction between the bullet and the gun results not only in an acceleration $\vec{a}_1$ of the bullet, but also in an opposite acceleration $\vec{a}_2$ of the gun (i.e., the gun "recoils"). But the acceleration $\vec{a}_1$ of the bullet has a much larger magnitude than that of the acceleration $\vec{a}_2$ of the gun since the mass of the bullet is much smaller than that of the gun. Correspondingly, the velocity $\vec{v}_1$ acquired by the bullet has a much larger magnitude than the recoil velocity $\vec{v}_2$ acquired by the gun. The same principle Eq. (C-5) explains the propulsion of rockets. Here a chemical reaction is used to accelerate a mass of gas propelled out of the rear of the rocket. Hence the rocket itself is accelerated in the opposite direction, i.e., its velocity is increased in the forward direction.

As a last example, consider a person lying on a table supported so as to be free to move in the horizontal direction. When the action of the heart accelerates some mass of blood in one direction (e.g., toward the feet) the body of the person must then, by Eq. (C-5), be accelerated in the opposite direction. Although the magnitude of the acceleration of the body is quite small (since the mass of the body is much larger than that of the accelerated blood), it can be observed with sensitive instrumentation. In this way one can obtain information about the motion of the blood to study heart action and diagnose heart disease. This method is used in medicine and is called "ballistocardiography."

The objects discussed in this unit interact like particles, and thus are adequately described by the principles of this unit.

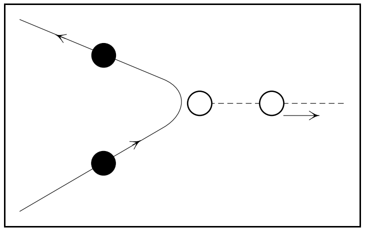


Fig. C-2.

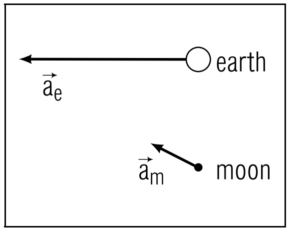


Fig. C-3.

Understanding the Relation of Masses and Accelerations (Cap. 1a)

C-1 *Statement and example:* A black marble in a children's game strikes a white marble on a playground surface sufficiently smooth and flat that the marbles are isolated for horizontal motion. During the interaction shown in Fig. C-2, the black marble's velocity changes by a large amount, so that its acceleration $\vec{a}_b$ is large. The white marble, initially at rest, acquires only a small velocity, so that its acceleration $\vec{a}_w$ is small. (a) If the white and black marbles have masses m_w and m_b, use the symbols provided to state the relation between their masses and accelerations. (b) Which marble has the larger mass? (c) If the accelerations are related by $a_w = (1/3)a_b$, express the mass m_w in terms of the mass m_b. *(Answer: 101) (Suggestion: [s-1])*

C-2 *Relating quantities:* Each of the following pairs of objects collides on a surface sufficiently smooth that the pair is isolated for horizontal motion: a basketball and a bowling ball, a ping-pong ball and a bowling ball, a ping-pong ball and a basket ball. (a) Which object in each pair has the larger acceleration? Which has the larger mass? (b) List these balls in order of increasing mass. (c) As we shall see in Unit 409, an object with larger mass "feels heavier" than an object with smaller mass. Check your answers. Are the balls arranged in order of increasing "heaviness"? *(Answer: 104)*

C-3 *Applicability:* Figure C-3 shows the accelerations of the earth and moon relative to the inertial solar reference frame. Since these accelerations do not have opposite directions, the relation $m_e\vec{a}_e = -m_m\vec{a}_m$ does *not* relate their masses and accelerations. Why is this relation not applicable to this situation? *(Answer: 110)*

More practice for this Capability: [p-1], [p-2]

SECT.

D MASS AND THE DEFINITION OF FORCE

The preceding two sections have discussed all the basic principles describing the interaction between two particles isolated from the rest of the universe. We shall now merely introduce some conventional definitions which are used to express these principles in more convenient form.

STANDARD OF MASS

According to our definition of mass, $m_1 a_1 = m_2 a_2$ or

$$\frac{m_1}{m_2} = \frac{a_2}{a_1} \tag{D-1}$$

Hence the ratio m_1/m_2 of the masses of any two particles can be simply determined by letting them interact (when they are isolated from the rest of the universe) and measuring the ratio of the magnitudes of their accelerations at any instant. For example, the masses of a book and a brick might be compared by letting them collide (when both are free to slide along a smooth horizontal surface) and measuring the magnitudes of their resulting accelerations. Although such a collision between particles is not the most convenient or precise method of comparing the masses of objects of everyday size, it is a method commonly used to compare the masses of atomic particles.

As usual, such a comparison procedure determines only the ratio of the masses of any two particles, but does not assign a value to the mass of any one particle. We can remove this ambiguity (as we did in the case of the quantities length and time) by agreeing always to compare the mass of any particle with the mass m_S of one specified particle S chosen as the "standard of mass." By international agreement, this standard is a chunk of metal carefully stored in a vault near Paris. This standard is called the "standard kilogram" and the value of its mass m_S is denoted by the algebraic symbol (or unit) "kilogram" or the corresponding abbreviation "kg." Comparison of any particle 1 with the standard kilogram then leads to the result

$$\frac{m_1}{m_S} = \frac{a_S}{a_1} \text{ or } m_1 = \frac{a_S}{a_1}\,\text{kilogram} \tag{D-2}$$

since $m_S =$ kilogram. For example, if the magnitude a_1 of the acceleration of a book interacting with the standard kilogram is 2 times as large as the magnitude a_S of the acceleration of the standard, the mass m_1 of the book is 1/2 kilogram. (The mass of an adult person is between 50 kg and 100 kg.)

DEFINITION OF FORCE

The fundamental relation $m_1\vec{a}_1 = -m_2\vec{a}_2$ of Eq. (C-5) involves the product of the mass m and the acceleration $\vec{a}$ of each particle. Hence it is convenient to introduce this abbreviation:

$$\boxed{\vec{F} = m\vec{a}} \tag{D-3}$$

The quantity $\vec{F}$ is called "force" in accordance with this definition:

Def. | **Force**: The force $\vec{F}$ on a particle due to specified other particles is the vector $\vec{F} = m\vec{a}$, where m is the mass of the particle and $\vec{a}$ is its acceleration due to its interaction with these other particles. | (D-4)

For example, when the particle interacts with only one other particle, $\vec{F} = m\vec{a}$ is the force on the particle due to this other particle. When the particle interacts with several other particles, $\vec{F} = m\vec{a}$ is the force on the particle due to all these other particles. [Note that our precise of force, Def. (D-4), resembles the everyday notion of force as a "push" or a "pull" since such a push or pull can accelerate a particle.]

The Def. (D-3) implies that the force $\vec{F}$ is a *vector* having the same direction as the acceleration $\vec{a}$ (since $\vec{F}$ is the vector $\vec{a}$ multiplied by the positive number m). Furthermore, the unit of force is the same as that of $m\vec{a}$ so that

$$\text{unit of force} = \text{kg}\frac{\text{meter}}{\text{sec}^2} = \text{newton} \tag{D-5}$$

where the new unit "newton" (abbreviated "N") is merely a convenient abbreviation for the combination of units kg meter/sec^2. For example, since an object near the surface of the earth falls with a gravitational acceleration $\vec{g} = 10\,\text{meter/sec}^2$ downward, the gravitational force on an object having a mass $m = 1\,\text{kg}$ is $m\vec{g} = (1\,\text{kg})(10\,\text{meter/sec}^2)$ downward $=$ 10 newton downward. (The relation between the unit "newton" and the English unit of force called the "pound" is approximately 1 newton = 0.2 pound.)

Now: Go to tutorial section D.

Understanding the Definition of Force (Cap. 1b)

D-1 *Statement and example:* (a) State the definition of force as an equation. (b) A trailer, towed by a car, has a mass of 250 kg, and moves with an acceleration of $0.30\,\mathrm{m/s^2}$ east. What is the force on the trailer (due to its interaction with the particles in the road, in the car, and in the earth)? *(Answer: 107)*

D-2 *Properties:* (a) Answer the following questions about the quantities force, mass, and acceleration. Is this quantity a vector or a number? What single SI unit is associated with it? (b) Which of the values, 0.2 kg, 2 kg, 60 kg, 600 kg, is the best estimate of the mass of a large book? of your mass? (c) Use your estimates of masses to estimate typical magnitudes for the following forces: the force on a large book as it falls vertically to the floor; the force on you as you jump upward with an acceleration of magnitude $5\,\mathrm{m/s^2}$. *(Answer: 105)*

D-3 *Meaning of force:* Suppose the trailer described in problem D-1 is at rest, uncoupled from the car. What then is the force on the trailer due to all other particles? Why is this answer different from the answer to problem D-1? *(Answer: 102)*

SECT.

E PROPERTIES OF TWO-PARTICLE FORCES

The Def. (D-3) of force allows us to summarize very compactly the general properties of the interaction between two specified particles constituting an isolated system. Thus Rule (B-2) implies that the force is "central" (i.e., along the centers of the particles) in this sense:

Central-force property: At any instant, the force on one particle due to another has a direction along the line joining the particles and depends only on the distance between them.	(E-1)

Furthermore, the conclusion $m_1\vec{a}_1 = -m_2\vec{a}_2$ of Eq. (C-5) relates the accelerations of the two particles. Since $\vec{a}_1$ is the acceleration of particle 1 due to its interaction with particle 2, we can write $m_1\vec{a}_1 = \vec{F}_{1,2}$ as the "force on particle 1 due to particle 2." Similarly we can write $m_2\vec{a}_2 = \vec{F}_{2,1}$ as the "force on particle 2 due to particle 1." With these abbreviations, the relation (C-5) between the accelerations is equivalent to this "reciprocal relation" between the mutual forces:

Reciprocal relation: At any instant, $\vec{F}_{1,2} = -\vec{F}_{2,1}$	(E-2)

In other words, at any instant the mutual forces on two interacting particles have equal magnitudes but opposite directions. Since the mutual forces are so closely related, a knowledge of one of the forces allows one immediately to find the other. Thus a complete specification of both forces requires only the following information provided by a "force law":

Def.	**Force law**: Specification of how the force on either of two interacting particles depends on the properties of these particles and on the distance between them.	(E-3)

The information about the direction of the force can be provided very simply since the central-force property permits only the following two possibilities: (1) Each force is directed *away* from the other particle (as shown in Fig. E-1a), in which case the force is said to be "*repulsive*." (2) Each force is directed *toward* the other particle (as shown in Fig. E-

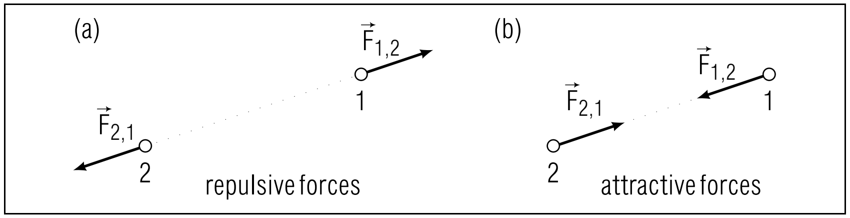


Fig. E-1: Mutual forces on two interacting particles.

1b), in which case the force is said to be "*attractive*."

FUNDAMENTAL FORCES IN NATURE

It is remarkable that all forces in nature, and thus the observed interactions between all objects from atoms to stars, can be explained in terms of only four fundamental forces each of which can be characterized by a different force law. *

> * Recent theoretical work suggests that some of these forces may be related so that the number of fundamental forces might be even less than four.

One of these fundamental forces is called the "gravitational force." This force is always attractive. Its magnitude depends on the masses of the interacting particles and is only appreciable if the mass of at least one of these particles is of astronomical size. Thus this force is very important in astronomical problems or in discussing the interaction of objects with the earth, but is negligibly small otherwise.

Another fundamental force is the "electric force" which can be either repulsive or attractive. It is responsible for the interaction between atoms or molecules and is thus ultimately responsible for the interactions studied in most of physics, all of chemistry, and all of biology! This force is thus of enormous practical importance. (The "magnetic" force is closely related to this electric force.)

The remaining two fundamental forces are only appreciable if the distance between the two interacting particles is very small (i.e., less than about 10^{-15} meter). These forces are responsible for the interaction between particles in the atomic nucleus. Hence they are important for understanding the structure of the atomic nucleus, radioactivity, nuclear energy, and other nuclear processes. These fundamental forces and their

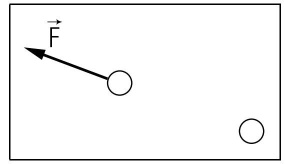


Fig. E-2.

many important consequences will be studied at length in later units of this book.

Understanding the Relation between Mutual Forces (Cap. 1c)

E-1 Art (who has a mass $m_A = 20\,\text{kg}$) and Betty (who has a mass $m_B = 30\,\text{kg}$) push each other while sliding on a frozen puddle, sufficiently smooth that the children are isolated for horizontal motion. (a) *Statement*: State the reciprocal relation between the force $\vec{F}_{A,B}$ on Art due to Betty and the force $\vec{F}_{B,A}$ on Betty due to Art. (b) *Example*: If Art pushes Betty with a force of 60 newton west, what is the force on Art due to Betty? (c) *Comparison*: Use the definition of force to find the acceleration of each child. *(Answer: 115)*

E-2 *Relating quantities:* Imagine that the earth and one ball are isolated from all other objects, and that we measure the motion of these two particles relative to an inertial reference frame. The ball (of mass 0.6 kg) falls towards the earth's surface with an acceleration of magnitude $10\,\text{m/s}^2$. (a) Compare the magnitudes of the force on the ball due to the earth and the force on the earth due to the ball. Which is larger, or are both equal? (b) What is the magnitude of the force on the earth due to the ball? (c) The earth has a mass of 6.0×10^{24} kg. What is the magnitude of the earth's acceleration? *(Answer: 118)*

More practice for this Capability: [p-4]

Knowing About Forces

E-3 *Attractive and repulsive forces:* Figure E-2 shows two particles isolated from other particles, and the direction of the force $\vec{F}$ on one particle due to the other. Is this force an attractive or a repulsive force? Suppose the particles were much farther apart. Would the magnitude of $\vec{F}$ be larger or smaller? *(Answer: 113)*

SECT.

F INTERACTION OF SEVERAL PARTICLES

Our understanding of the interaction between two particles can be readily generalized to discuss the interaction between three or more particles. Let us first consider an isolated system consisting of threeparticles 1, 2, and 3. (See Fig. F-1a.) Suppose that we focus attention on particle 1. Then we would like to know its acceleration $\vec{a}_1$ in the presence of the other two particles or, equivalently, the force $\vec{F}_1 = m_1\vec{a}_1$ on particle 1 due to its interaction with both other particles. If particle 3 were not there, the force on particle 1 would be simply the force $\vec{F}_{1,2}$ on 1 due to particle 2. Similarly, if particle 2 were not there, the force on particle 1 would be simply the force $\vec{F}_{1,3}$ on 1 due to particle 3. When both particles 2 and 3 are present simultaneously, particle 1 interacts with *both* of them. Then it is simplest to assume, and verified by observations, that the force $\vec{F}_1$ on particle 1 due to its interaction with both other particles is just the vector sum of the individual forces $\vec{F}_{1,2}$ and $\vec{F}_{1,3}$. (See Fig. F-1b.) Thus

$$\vec{F}_1 = \vec{F}_{1,2} + \vec{F}_{1,3} \qquad \text{(F-1)}$$

This relationship, called the "superposition principle," is similarly applicable to a system consisting of more than three particles and can be stated this way:

Superposition principle: The force interacting with several other particles is the vector sum of the individual forces due to these other particles separately.	(F-2)

The force on a particle due to its interaction with *all* other particles can thus be called the "total force" on the particle, i.e., the vector sum of the individual forces due to all other particles separately. Note that each individual force on a particle due to any other particle is understood to have the same value as if the remaining particles were absent. Thus the individual mutual forces on two such particles have the familiar properties discussed in Sec. E and satisfy the reciprocal relation $\vec{F}_{1,2} = -\vec{F}_{2,1}$.

As a simple example of the superposition principle, consider the earth interacting with the moon and with the sun by gravitational forces. Then the total force on the earth due to *both* the moon and the sun is just the

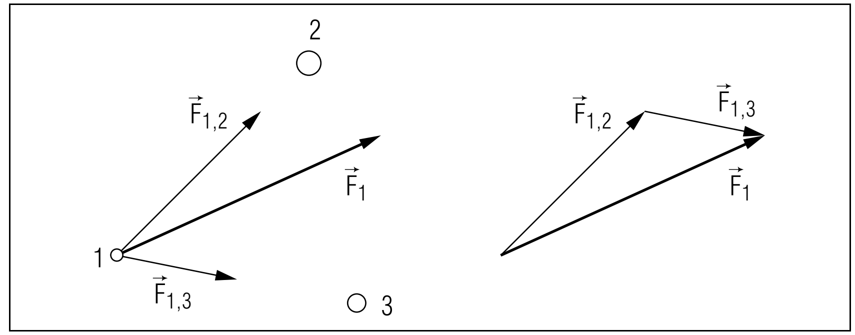


Fig. F-1: Isolated system of three interacting particles (a) Individual forces and total force on particle 1. (b) Superposition principle relating these forces.

vector sum of the force on the earth due to the moon alone, plus the force on the earth due to the sun alone. (This total force is responsible for the ocean tides observed on the earth.)

The superposition principle, Rule (F-2), is extremely useful since it allows us to discuss the motion of *any* number of particles once we know the interaction between any two of these particles separately. Hence the superposition principle is the last basic assumption necessary for a complete theory of motion for any number of particles.

Understanding the Superposition Principle (Cap. 1d)

F-1 *Statement and example:* A particle A interacts with two particles B and C, and so is acted on by the two forces $\vec{F}_{A,B}$ and $\vec{F}_{A,C}$ shown in Fig. F-2. (a) Use the symbols provided to state the superposition principle. (b) What is the value of the total force on particle A? *(Answer: 116)*

F-2 *Meaning of total force:* For the precision desired here, the sun, earth, and moon interact only with each other. The table in Fig. F-3 gives values for the individual forces on each particle due to each of the others. What is the force on the earth due to all other particles? What is the force on the moon? *(Answer: 119) (Suggestion: [s-2])*

F-3 *Comparison of total and individual forces:* Two particles, 1 and 2, interact but are isolated from other particles. (a) Particle 1 has a mass of 2.0 kg, and an acceleration of $(3.0\,\mathrm{m/s^2})\hat{x}$. What is the total

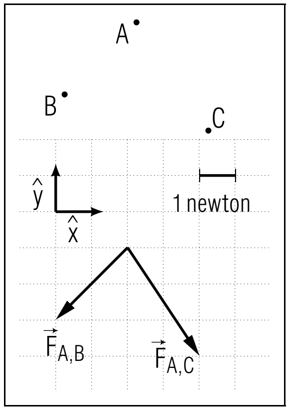


Fig. F-2.

Force	Value
On sun due to earth	3.55×10^{22} newton to right
On sun due to moon	4.36×10^{20} newton to right
On earth due to moon	1.98×10^{20} newton to left
On earth due to sun	3.55×10^{22} newton to left
On moon due to earth	1.98×10^{20} newton to right
On moon due to sun	4.36×10^{20} newton to left

Fig. F-3.

force on 1? What is the individual force on 1 due to 2? (b) Particles 1 and 2 remain as in part (a), but a third particle 3 now interacts with them. The individual force on 1 due to 3 is $(-6.0\,\text{newton})\hat{x}$. Is the individual force on 1 due to 2 different than it was in part (a)? If so, find it. Is the total force on 1 different than it was in part (a)? If so, find it. *(Answer: 124) (Suggestion: [s-5])*

F-4 *Dependence of total force on individual forces:* An artificial satellite A is acted on by individual forces $\vec F_{A,E}$ and $\vec F_{A,S}$ due to the earth and sun. Both of these forces always have a magnitude of 100 newton, but for different positions of the satellite, they have different directions. For each pair of individual forces shown in Fig. F-4, what is the magnitude of the total force on the satellite? For which pair is this magnitude largest? *(Answer: 111) (Suggestion: [s-8])*

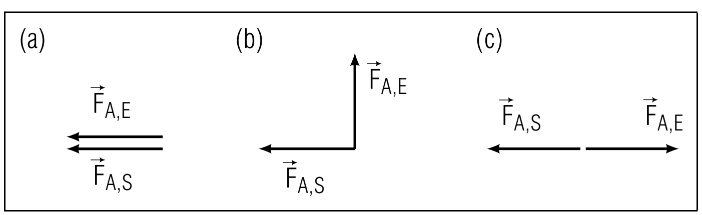


Fig. F-4.

SECT.

G EQUATION OF MOTION

We have now discussed all the basic assumptions of the theory of motion originally formulated by Newton. These basic assumptions are summarized by these three principles:

(1) The principle of inertia, Rule (A-3).

(2) The properties of the two-particle interaction [i.e., the central-force property, Rule (E-1), and reciprocal relation (E-2) between mutual forces.]

(3) The superposition principle Eq. (F-1).

If the superposition principle is combined with the definition $\vec F = m\vec a$ of the force on a particle, it is equivalent to the following statement called the "equation of motion" of a particle:

$$\boxed{\text{Equation of motion: } m\vec a = \vec F \text{ where } \vec F = \text{ total force.}} \qquad \text{(G-1)}$$

This equation must be interpreted with care. Thus the equation applies to any single *particle*. The quantity m then refers to the mass of this particle, $\vec a$ to the acceleration at some instant of this particle relative to some *inertial* reference frame, and $\vec F$ to the *total* force (i.e., to the vector sum of all individual forces) acting on this particle at this instant because of its interaction with all other particles. *

> * Historically, Newton called the basic principles of his theory of motion "laws of motion." His "first law" was the principle of inertia, his "second law" the equation of motion $m\vec a = \vec F$, and his "third law" the reciprocal relation between mutual forces.

DISCUSSION OF THE EQUATION OF MOTION

The equation of motion $m\vec a = \vec F$ relates the acceleration $\vec a$ of a particle to the total force $\vec F$ on this particle. The acceleration $\vec a$ of the particle describes merely the motion of *this* particle, i.e., it describes how the velocity of this particle changes with time. On the other hand, the total force on the particle depends not only on this particle but also on all the *other* particles with which it interacts (e.g., the total force depends on the relative positions of the interacting particles). Hence the equation of motion relates the motion of a particle to its interaction with all other particles.

Accordingly the equation of motion allows one to predict the motion of particles on the basis of a knowledge about the interaction between them (i.e., about their mutual forces). Thus the equation $m\vec{a} = \vec{F}$ is the most important summary of our theory of motion and constitutes the starting point for all arguments used to predict motion. This is true even in the case of complicated objects (such as automobiles or flowing liquids) which have many parts moving relative to each other; for such complicated objects can always be regarded as consisting of many constituent particles, each of which may be analyzed by the equation $m\vec{a} = \vec{F}$. Hence the equation of motion gives rise to an extremely large and varied range of applications which are studied extensively in most of the natural sciences.

Let us now consider the special case where the total force $\vec{F}$ on a particle is zero. (This may happen either because the particle is isolated so that *no* forces act on it, or because the individual forces on the particle are such that their vector sum is zero.) Then the equation $m\vec{a} = \vec{F}$ implies that $\vec{a} = 0$ so that the particle must move with *constant* velocity relative to an inertial frame. An especially simple situation is that where this constant velocity is zero. Then the particle is said to be in "equilibrium" according to the following definition:

Def. | **Equilibrium**: An object is in equilibrium relative to some reference frame if all particles in this object remain at rest relative to this frame | (G-2)

If an object is in equilibrium relative to an inertial frame, the acceleration a of every particle in the object must be zero. Hence the equation of motion $m\vec{a} = \vec{F}$ implies that the *total* force $\vec{F}$ on every particle must be zero, i.e.,

$$\boxed{\text{for equilibrium, } \vec{F} = 0} \qquad \text{(G-3)}$$

Example G-1: Acceleration of an elevator

An elevator of mass $m = 5 \times 10^2$ kg is acted on by a downward gravitational force $\vec{F}_g$ due to the earth and by an upward force $\vec{F}_c$ due to

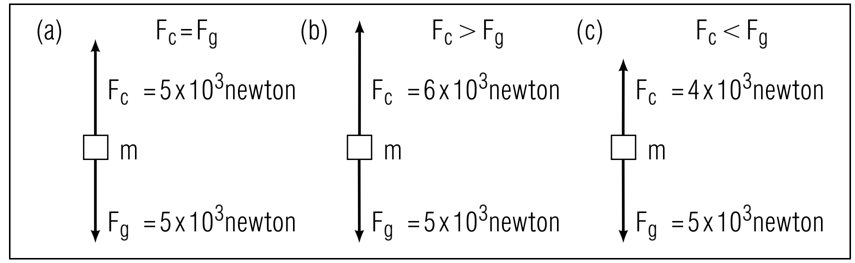


Fig. G-1: Elevator moving under the influence of a downward gravitational force $\vec{F}_g$ and an upward cable force $\vec{F}_c$.

the cable supporting the elevator. The magnitude of $\vec{F}_g$ is 5×10^3 newton. What then is the acceleration $\vec{a}$ of the elevator (relative to the earth) when the cable force $\vec{F}_c$ has a magnitude of 5×10^3 newton, 6×10^3 newton, and 4×10^3 newton?

Consider the elevator as the particle of interest. (See Fig. G-1.) Then the total force $\vec{F}$ on the elevator is $\vec{F} = \vec{F}_g + \vec{F}_c$. Thus the equation of motion of the elevator is

$$m\vec{a} = \vec{F}_g + \vec{F}_c\,. \qquad \text{(G-4)}$$

Hence

$$\vec{a} = \frac{\vec{F}_g + \vec{F}_c}{m} \qquad \text{(G-5)}$$

Now, suppose that $\vec{F}_c = 5 \times 10^3$ newton upward, as illustrated in Fig. G-1a. Then $\vec{F}_g + \vec{F}_c = 0$. Hence Eq. (G-5) implies that $\vec{a} = 0$. In this case the elevator simply moves with constant velocity. In particular, if the elevator is initially at rest, it remains at rest (i.e., it is in equilibrium).

Next, suppose that $\vec{F}_c = 6 \times 10^3$ newton upward, as illustrated in Fig. G-1b. Then $\vec{F}_g + \vec{F}_c = 1 \times 10^3$ newton upward. Hence Eq. (G-5) implies that $\vec{a} = 2\,\text{meter/sec}^2$ upward.

Finally, suppose that $\vec{F}_c = 4 \times 10^3$ newton upward, as illustrated in Fig. G-1c. Then $\vec{F}_g + \vec{F}_c = 1 \times 10^3$ newton downward. Hence Eq. (G-5) implies that $\vec{a} = 2\,\text{meter/sec}^2$ downward.

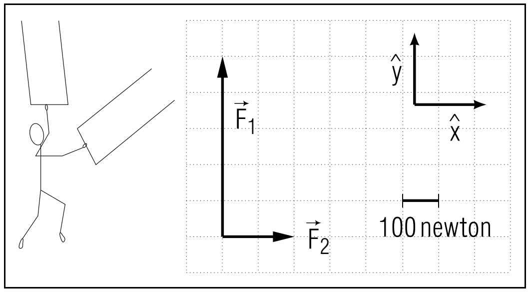


Fig. G-2.

Understanding the Equation of Motion (Cap. 1e)

G-1 *Statement and example:* Tarzan of the apes has a mass $m_T = 80\,\text{kg}$. When he is at the lowest point of his swing on a vine, he has an acceleration $\vec{a}_T$, and he is acted on by these forces: $\vec{F}_{T,V} = 1,000$ newton upward (due to the vine), and $\vec{F}_{T,E} = 800$ newton downward (due to the earth). (a) Use the symbols provided to state Tarzan's equation of motion. (b) What is Tarzan's acceleration at the time described? *(Answer: 114)*

G-2 *Meaning of $\vec{F}$:* The trapeze artist shown in Fig. G-2 has a mass $m = 50\,\text{kg}$, and moves with an acceleration $\vec{a} = (4.0\,\text{m/s}^2)\hat{x}$. He is acted on by the forces $\vec{F}_1$ and $\vec{F}_2$ due to the trapezes, and by the gravitational force of 500 newton downward. Which of the following correctly describes the total force on this trapeze artist? Briefly describe what is wrong with each of the incorrect descriptions. (a) $\vec{F}_1 + \vec{F}_2 =$ 539 newton upward and towards the right. (b) the sum of F_1, F_2 and the gravitational force, 1,200 newton. (c) $\vec{F}_1 + \vec{F}_2 +$ (500 newton downward) $=$ (200 newton)$\hat{x}$. (d) $\vec{F}_1 + \vec{F}_2 +$ (500 newton downward)$+m\vec{a} =$ (400 newton)$\hat{x}$. (e) $m\vec{a} =$ (200 newton)$\hat{x}$. *(Answer: 121) (Suggestion: [s-11])*

G-3 *Relating quantities:* (a) A dropped book having a mass of 1.5 kg moves subject only to gravitational interaction with the earth. What is the total force on the book? What is the force on the book due to the earth? (b) After falling, the book lies at rest so that it interacts with the floor as well as with the earth. What is the total force on the book? What is the force on the book due to the earth? What is the force on the book due to the floor? *(Answer: 109) ([s-3], [p-5])*

Relating Motion to Parallel Forces (Cap. 3)

G-4 A bag of fruit is placed on a grocery spring scale where it oscillates up and down before coming to rest. The bag has a mass M, moves with an acceleration $\vec{a}$, and is acted on by the downward gravitational force $\vec{F}_g$ due to the earth and by the upward force $\vec{F}_s$ due to the scale. (a) Write the equation of motion for the bag in terms of the symbols provided. (b) Which force on the bag has the larger magnitude for each of the following motions of the bag? Acceleration is upward. Acceleration is downward. Bag is at rest. *(Answer: 122) (Suggestion: [s-6])*

G-5 In a sea rescue, a man with a mass of 100 kg is lifted by a rope suspended from a helicopter. The man is acted on by an upward force of magnitude 1,200 newton due to the rope, and by a downward gravitational force of magnitude 1,000 newton. What is the total force on the man? Is his acceleration directed upward or downward? *(Answer: 117)*

SECT.

H SUMMARY

DEFINITIONS

system of particles; Def. (A-1)

isolated system; Def. (A-2)

inertial frame; Rule (A-3)

mass; Def. (C-4)

kilogram; Eq. (D-2)

force; Eq. (D-3), Def. (D-4)

newton; Eq. (D-5)

equilibrium; Def. (G-2)

IMPORTANT RESULTS

Principle of inertia: Rule (A-3)

Every isolated particle moves with constant velocity relative to special reference frames called "inertial frames."

Properties of two-particle forces: Rule (E-1), Rule (E-2)

(a) Central-force property: $\vec{F}$ is along line joining particles and depends only on the distance between them.

(b) Reciprocal relation: $\vec{F}_{1,2} = -\vec{F}_{2,1}$

Superposition principle: Rule (F-2)

The total force on a particle is the vector sum of all separate forces on it.

Equation of motion: Eq. (G-1)

For any particle, $m\vec{a} = \vec{F}$ (where $\vec{F}$ = vector sum of all forces).

NEW CAPABILITIES

You should have acquired the ability to:

(1) Understand these relations:

(a) the relation between masses and accelerations, $m_1\vec{a}_1 = -m_2\vec{a}_2$ (Sec. C [p-1], [p-2])

(b) the definition of force, $\vec{F} = m\vec{a}$ (Sec. D, [p-3])

(c) the reciprocal relation between mutual forces, $\vec{F}_{1,2} = -\vec{F}_{2,1}$ (Sec. E, [p-4])

(d) the superposition principle (Sec. F)

(e) the equation of motion, $m\vec{a} = \vec{F}$ (Sec. G, [p-5])

(2) Recognize an inertial reference frame and describe the motion of an isolated particle relative to such a frame. (Sec. A)

(3) When only two parallel forces act on a particle, qualitatively relate these forces to the particle's acceleration. (Sec. G)

Organization of Relations (Cap. 1)

H-1 For each relation listed in Cap. 1, indicate which of the following phrases best describes this relation. (a) Relates the acceleration of a particle to the vector sum of individual forces due to other particles. (b) Relates the mutual forces on each of two interacting particles due to the other (even if these particles are not isolated from others). (c) Relates the masses and accelerations of two particles, *only* when they are isolated from other particles. (d) Defines the total force on a particle due to all other particles. (e) Relates the total force on a particle to individual forces due to each other particle. *(Answer: 123)*

H-2 For the following questions (a), (b), and (c) first choose the relation listed in Cap. 1 which you will use to answer the question. Then answer the question. (a) A car of mass 2000 kg tows a trailer of mass 500 kg along a level road. If the trailer exerts on the car a force of 750 newton towards the left, what is the force on the trailer due to the car? (b) The trailer is acted on not only by this force due to the car, but also by a gravitational force of 5,000 newton downward, and by a force of 5,000 newton upward due to the road surface. What is the total force on the trailer? (c) What is the trailer's acceleration? (d) Since the car has the same acceleration, $m_c\vec{a}_c = -m_t\vec{a}_t$ does not relate the masses and accelerations of the car and trailer. Briefly explain why this relation does not apply here. *(Answer: 112) (Suggestion: [s-9])*

SECT.

I PROBLEMS

I-1 *Truth in advertising:* A manufacturer advertises the strength of his wire by using this demonstration: A piece of wire is attached to a fixed post and to a tractor. When the tractor is started the wire fails to break, demonstrating its strength. A competitor uses a similar demonstration, but replaces the fixed post with a second tractor, and suggests the superior strength of his wire by demonstrating that it fails to break when both tractors exert opposite forces on the wire. Does the second demonstration really indicate superior strength? Explain by describing the individual forces due to the tractor(s) and post on each piece of wire. (The gravitational force on the wire is negligible. All the tractors exert forces of equal magnitude.) *(Answer: 120)*

TUTORIAL FOR D

UNDERSTANDING THE DEFINITION OF FORCE

d-1 *STATEMENT:*

State an equation summarizing the definition of force.

► ______________

Understanding this relation means being able to answer questions of the type listed in Appendix B, questions like those in the following frames.

(Answer: 7)

d-2 *EXAMPLE:* A wagon of mass 10 kg rolls down a hill with an acceleration $\vec{a} = (0.5\,\mathrm{m/s^2})\hat{x}$, where $\hat{x}$ is a unit vector directed downward along the hill.

What is the total force on the wagon due to all the particles with which it interacts? (These include particles near the surface of the hill, and particles making up the earth.)

► ______________

(Answer: 4) (Practice: text problem D-1.)

d-3 *PROPERTIES:* (1) Complete this table summarizing the properties of force, mass, and acceleration:

►

	force	mass	acceleration
number or vector:			
for numbers, possible signs:			
Single SI unit:			
Common algebraic symbol:			

TYPICAL MAGNITUDES

(2) For each object listed below, indicate which of these is the best estimate of its mass: 0.5 kg, 5 kg, 50 kg, 100
unitkg, 1000 kg.

► a baby: ______

a grown woman: ______

an automobile: ______

(3) For each of the following situations, use your preceding answers to estimate the magnitude F of the force on the specified object due to all other objects.

A baby is tossed in the air and moves with the gravitational acceleration.

► $F =$ ______

A car on a highway moves with an acceleration of magnitude $2\,\mathrm{m/s^2}$.

► $F =$ ______

(Answer: 1)

d-4 *MEANING OF $\vec{F}$, m, and $\vec{a}$:* A girl holding a snowball stands on the icy surface of a frozen puddle, so that she and the snowball are isolated for horizontal motion. The girl has a mass of 30 kg, and she throws the snowball so that it has a horizontal acceleration of $(20\,\mathrm{m/s^2})\hat{x}$.

Briefly describe why the product $(6.0 \times 10^2\,\text{newton})\hat{x}$ of this mass and acceleration is *not* the total force on the girl.

►

Is $(6.0 \times 10^2\,\text{newton})\hat{x}$ the total force on the snowball?

► yes, no

(Answer: 10)

d-5 *MEANING OF $\vec{F}$:* A baseball of mass 0.15 kg travels along a curved path (subject only to gravitational interaction with the earth) and then comes to rest on a rooftop.

As it moves through the air, what is the force on the baseball due to all other particles?

► ____________

As it lies on the roof, what is the force on the baseball due to all other particles?

► ____________

Indicate all of the following sentences which describe your preceding answers:

(a) The forces are different because the ball has different accelerations.

(b) The forces are different, because the ball interacts with the roof in one case but not in the other.

(c) The forces are the same because the gravitational acceleration $\vec{g}$ is the same in both cases.

(d) The forces are the same, because in each case the ball's acceleration is constant.

► (a) (b) (c) (d)

(Answer: 11)

d-6 *COMPARISON OF FORCE AND ACCELERATION:* A particle of mass 1 kg moves with an acceleration of $5\,\mathrm{m/s^2}$ north.

What is the force on this particle due to all other particles?

► ____________

Compare the force on this particle with its acceleration. Do these quantities have the same magnitude? Do they have the same direction? Are the quantities themselves equal?

► magnitudes: same, different

directions: same, different

quantities: equal, not equal

(Answer: 9) (Suggestion: [s-7])

d-7 *DEPENDENCE OF ACCELERATION ON FORCE AND MASS:* A car is stalled on a level road with the brakes released and the gears in neutral. The force on this car (due to all other particles) is approximately equal to the force on the car due to persons pushing on it. A boy pushes such a car so that the force on the car is $\vec{F}_1$ and its acceleration is $\vec{a}_1$. He then gets two friends to help him, and when they all push, the force on the car has a value $\vec{F}_3$ which is three times as large as $\vec{F}_1$.

Compare the acceleration $\vec{a}_3$ of the car when three boys push it with the original acceleration $\vec{a}_1$ by writing an expression for $\vec{a}_3$ as a number times $\vec{a}_1$.

► $\vec{a}_3 =$ ____________

Suppose the first boy now pushes a second car which has a larger mass than the first.

If he pushes so that the force on this second car is again $\vec{F}_1$, will the acceleration of this car have a larger or a smaller magnitude than the acceleration $\vec{a}_1$ of the first car?

► larger, smaller

(Answer: 3) (Suggestion: [s-10])

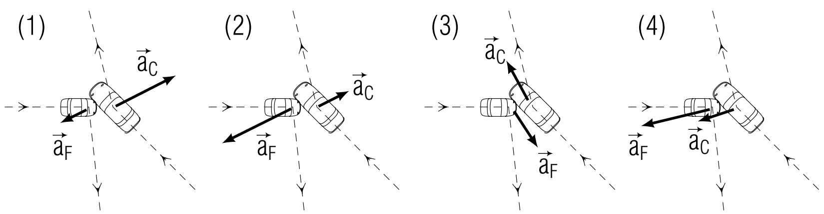
d-8 *SUMMARY:* The definition of force, $\vec{F} = m\vec{a}$, states that the force on a particle (due to all other particles) is the product of the particle's mass and its acceleration. Because the acceleration of a particle depends on its interaction with all other particles, the force on the particle also depends on these interactions.

We have examined this relation with great care, because we shall be using it extensively in this way: By studying the interaction of a particle P with other particles, one can often determine directly the force on P (without measuring its acceleration). Then knowing the force on P and the mass of P, one can use the definition of force to find the acceleration of P, and thus to predict its motion.

Now: Go to text section E.

PRACTICE PROBLEMS

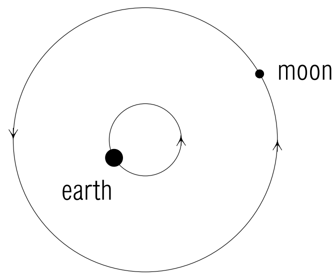
p-1 *UNDERSTANDING THE RELATION OF MASSES AND ACCELERATIONS (CAP. 1A):* A Fiat (of small mass m_F) and a Cadillac (of larger mass m_C) collide in an intersection sufficiently flat and icy that both vehicles are isolated for horizontal motion. (a) Which of the following diagrams correctly indicates the directions and relative magnitudes of the accelerations $\vec{a}_F$ and $\vec{a}_C$ of the Fiat and Cadillac in this collision? (b) When a vehicle has a large acceleration, its occupants are likely to be injured. In which car is injury more likely? (c) If the masses are related by $m_C = (5/2)m_F$ express $\vec{a}_C$ in terms of $\vec{a}_F$. *(Answer: 5) Suggestion: [s-4], or review text problem C-1. (Further practice: [p-2])*



p-2 *UNDERSTANDING THE RELATION OF MASSES AND ACCELERATIONS (CAP. 1A):* Have you ever wondered whether the earth's motion is affected by the motion of objects at its surface? For simplicity, imagine that the earth and one person on it are an isolated system of two particles described relative to the solar reference frame. The person pushes on the earth with his feet so that he jumps upward with an acceleration of magnitude of $5\,\mathrm{m/s^2}$. If the mass of the earth is 10^{23} times as large as the mass of the person, what is the magnitude of the earth's acceleration during this jump? *(Answer: 2) (Suggestion: Review text problem C-1.)*

p-3 *UNDERSTANDING THE DEFINITION OF FORCE (CAP. 1B):* Text problems D-1 through D-3 provide practice for this capability.

p-4 *UNDERSTANDING THE RELATION BETWEEN MUTUAL FORCES (CAP. 1C):* The earth and moon can be considered as particles (isolated from other particles) which move along the circular paths shown in the following drawing (relative to an inertial reference frame). The mass of the earth is 80 times the mass of the moon.



Compare the mutual forces on each particle by expressing the force $\vec{F}_{m,e}$ on the moon due to the earth in terms of the force $\vec{F}_{e,m}$ on the earth (due to the moon). Compare the accelerations of the particles by expressing the acceleration $\vec{a}_m$ of the moon in terms of the acceleration $\vec{a}_e$, of the earth. *(Answer: 8) (Suggestion: Review text problem E-1 and E-2.)*

p-5 *UNDERSTANDING THE EQUATION OF MOTION (CAP. 1E):* (a) A ball of mass 5.0 gram is attached to a rubber band and moves with an acceleration of 15 m/s^2 upward. What is the total force on the ball? (To express your answers in terms of newton, use the relation gram = 10^{-3} kilogram.) (b) The rubber band breaks and the ball moves subject only to gravitational interaction with the earth. What is the ball's acceleration? What is the total force on the ball? *(Answer: 6) (Suggestion: Review text problem G-3.)*

SUGGESTIONS

s-1 *(Text problem C-1):* When two particles interact while isolated from other particles, their masses and accelerations are related by $m_1\vec{a}_1 = -m_2\vec{a}_2$. Thus the particle with the larger acceleration must have a correspondingly smaller mass. For example, if a_1 is four times as large as a_2, then m_1 must be one fourth as large as m_2.

s-2 *(Text problem F-2):* According to the superposition principle, the force on a particle X is the vector sum of all the individual forces *on X* due to all other particles. For example, the force on the earth is the vector sum of the forces 3.55×10^{22} newton left (due to the sun) and 1.98×10^{20} newton left (due to the moon).

s-3 *(Text problem G-3):* Part (a): Since the book interacts only with the earth, the total force on the book *is* the force on the book due to the earth. This total force is equal to $m\vec{a}$ where $\vec{a}$ is the gravitational acceleration.

Part (b): The total force on the book is again $m\vec{a}$, where $\vec{a}$ is the acceleration of the book while at rest. The gravitational force on the book due to the earth is the same whether or not the book interacts with the floor. The force on the book due to the floor must be such that the vector sum of this force and the gravitational force equals the total force on the book.

s-4 *(Practice problem [p-1]):* When two particles interact while isolated from other particles, their accelerations have opposite directions and are parallel to the line joining the particles. The magnitudes of these accelerations are related by $m_1a_1 = m_2a_2$, so that the particle with the smaller mass has correspondingly larger acceleration. Thus, if m_1 is three fourths as large as m_2, then a_1 must be four thirds as large as a_2.

s-5 *(Text problem F-3):* If only two particles interact, then the total force on one particle is just the individual force due to the other particle. The individual force on one particle due to another is the same whether or not other particles are present.

s-6 *(Text problem G-4):* Review text example G-1, noting particularly Fig. G-1. A particle's acceleration has the same direction as the total force on that particle. For example, when the boy's acceleration is upward, the vector sum of the individual forces acting on him must be upward.

s-7 *(Tutorial frame [d-6]):* The magnitude of a vector quantity includes its units. Since force and acceleration have different units, they have different magnitudes, and they cannot be equal.

s-8 *(Text problem F-4):* Remember to find the *vector* sum of the two forces. Draw the forces with the tail of one vector at the tip of the other, construct their vector sum, and use geometry to find the magnitude of this sum.

s-9 *(Text problem H-2):* Part (b): The total force on the trailer is the vector sum of the three individual forces on the trailer. Draw arrows representing each of these individual forces, and then find their vector sum. Notice that two of these individual forces have equal magnitudes but opposite directions.

s-10 *(Tutorial frame [d-7]):* The total force on the car is related to its mass and acceleration by $\vec{F} = m\vec{a}$ or $\vec{a} = \vec{F}/m$. Thus if the total force on each of two particles is the same, but one particle has a larger mass than the other, the particle with the larger mass must have the smaller acceleration.

s-11 *(Text problem G-2):* The individual forces on a particle are each due to its interaction with some other object. The trapeze artist is acted on by a gravitational force due to the earth, and by forces due to each of two trapezes. Thus the total force on him is equal to the *vector* sum of these three individual forces. According to the equation of motion, this total force is also equal to the product of the trapeze artist's mass and acceleration.

ANSWERS TO PROBLEMS

1. (1)

	force	mass	acceleration
Number or vector:	vector	number	vector
Possible signs:		positive	
Single SI unit:	newton (N)	kilogram (kg)	meter/sec^2
Common symbol:	$\vec{F}$	m	$\vec{a}$

(2) baby: 5 kg; woman: 50 kg; automobile: 1000 kg

(3) baby: F = 50 newton; car: $F = 2 \times 10^3$ newton

2. 5×10^{-23} m/s^2, certainly negligible

3. $\vec{a}_3 = 3\vec{a}_1$, smaller

4. (5 kg meter/sec^2) $\hat{x}$ = (5 newton) $\hat{x}$

5. a. 2

   b. Fiat

   c. $\vec{a}_C = -2/5\vec{a}_F$

6. a. 7.5×10^{-2} newton upward

   b. 10 m/s^2 downward, 5.0×10^{-2} newton downward

7. $\vec{F} = m\vec{a}$, or equivalent using other symbols

8. $\vec{F}_{m,e} = -\vec{F}_{e,m}$, $\vec{a}_m = -80\vec{a}_e$

9. 5 newton north; magnitudes: different; directions: same; quantities: not equal

10. In the relation $\vec{F} = m\vec{a}$, the symbols $\vec{F}$, m, and $\vec{a}$ mean quantities all referring to the *same* particle. The product of the mass of the *girl* and the acceleration of the *snowball* is not the force on the girl, nor is it the force on the snowball.

11. 1.5 newton downward, 0 newton, (a), and (b)

101. a. $m_b\vec{a}_b = -m_w\vec{a}_w$

   b. white

   c. $m_w = 3m_b$

102. 0. Since the trailer's acceleration is zero, $\vec{F} = m\vec{a} = 0$. The total force on a particle depends on its interaction with *all* other particles. In D-1 the trailer interacted with the car, and here it does not.

103. a. Yes

 b. Straight path, constant speed

104. a.

larger acceleration	*larger mass*
basket ball	bowling ball
ping-pong ball	bowling ball
ping-pong ball	basket ball

 b. ping-pong ball, basket ball, bowling ball

 c. yes

105. a. force: vector, newton; mass: number, kilogram; acceleration: vector, m/s^2

 b. book: 2 kg; person: 60 kg

 c. 20 newton, 3×10^2 newton

106. (b)

107. a. $\vec{F} = m\vec{a}$, or equivalent using your symbols

 b. 75 newton or 75 kg m/s^2 east (must include unit and direction)

108. a. Straight path, constant speed

 b. ice puck

109. a. 15 newton downward, 15 newton downward

 b. 0 newton, 15 newton downward, 15 newton upward

110. Earth and moon are not isolated from other particles (e.g., from the sun.)

111. a. 200 newton

 b. 141 newton

 c. 0 newton, largest for (a).

112. a. reciprocal relation, 750 newton towards the right

 b. superposition principle, 750 newton towards the right

 c. equation of motion, 1.50 m/s^2 toward the right

 d. car and trailer are not isolated from other particles

113. repulsive, smaller

114. a. $m_T\vec{a}_T = \vec{F}_{T,V} + \vec{F}_{T,E}$

 b. 2.5 m/s^2 upward

115. a. $\vec{F}_{B,A} = -\vec{F}_{A,B}$

 b. 60 newton east

 c. Art: 3 m/s^2 east; Betty: 2 m/s^2 west

116. a. $\vec{F}_A = \vec{F}_{A,B} + \vec{F}_{A,C}$ ($\vec{F}_A$ is total force on A)

 b. $(-5\text{ newton})\hat{y}$

117. 200 newton upward, upward

118. a. equal

 b. 6 newton

 c. 1×10^{-24} m/s^2

119. 3.57×10^{22} newton to left, 2.38×10^{20} newton to left

120. In the second case, suppose the tractors exert forces of magnitude F_t directed toward the right and left. Then in the first case, the one tractor exerts a force of magnitude F_t directed towards the right. But, since the wire remains at rest, the post must exert on it a force of magnitude F_t toward the left. Thus the forces on the two wires are identical.

121. a. gravitational force not included

 b. not the *vector* sum

 c. correct

 d. $m\vec{a}$ is *not* an individual force. It is the total force.

 e. correct

122. a. $M\vec{a} = \vec{F}_g + \vec{F}_s$

 b. $\vec{F}_s$, $\vec{F}_g$, equal magnitudes

123. (The relation between masses and accelerations: c). (Definition of force: d). (Reciprocal relation: b). (Superposition principle: e). (Equation of motion: a).

124. a. $(6.0\text{ newton})\hat{x}$, $(6.0\text{ newton})\hat{x}$

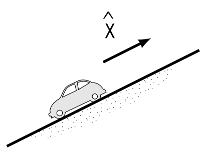
 b. individual force same; total force different, 0 newton

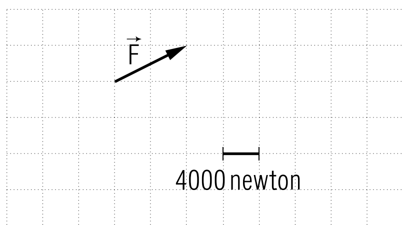
MODEL EXAM

1. **Relating the masses of two stars.** Observations of two interacting stars (which are isolated from all other particles) indicate that star 1 has an acceleration of $5 \times 10^5\,\mathrm{m/s^2}$ *toward* the sun, while star 2 has an acceleration of $2 \times 10^5\,\mathrm{m/s^2}$ *away* from the sun.

   Compare the masses of these stars by writing an expression for the mass m_1 of star 1 as a number times the mass m_2 of star 2.

2. **Forces on a car as it descends a hill.** The 1,600 kg car shown in the following drawing slows down as it descends a steep hill. The car is acted on by a total force $\vec{F} = (9,000\,\text{newton})\hat{x}$ due to the combined effect of its interaction with particles in the road surface and its gravitational interaction with the earth (the gravitational force is 16,000 newton downward).





   a. What is the car's acceleration?

   b. Construct an arrow representing the force on the car due to the road surface alone.

   c. Construct an arrow representing the force on the road surface due to the car.

3. **Relating forces and accelerations.** A student's 2 kg physics book lies at rest on a desk, but the book is acted on by a downward gravitational force of magnitude 20 newton. Considering this book, the student thinks, "The book's acceleration must be given by $m\vec{a} = \vec{F}$. Therefore $\vec{a} = \vec{F}/m = (20\,\text{newton downward})/(2\,\text{kg}) = 10\,\text{m/s downward}$."

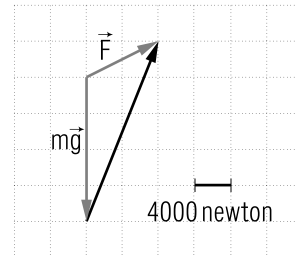
   If this student's reasoning is correct, just write *correct.* If it is not correct, briefly explain why. Then find the book's correct acceleration.

Brief Answers:

1. $m_1 = (2/5)m_2$

2. a. $5.625\,\mathrm{m/s^2}\,\hat{x}$

   b.



   Arrow with same magnitude and direction as the one shown.

   c. Arrow equal in magnitude and opposite in direction to the answer for part (b).

3. Any statement indicating: the gravitational force is not the total force on the book, or the book interacts with the desk as well as with the earth. Correct acceleration is 0.