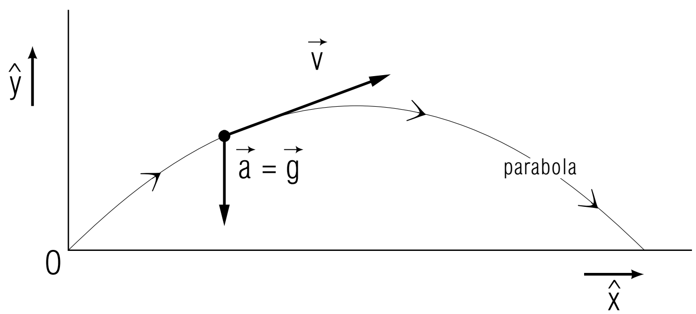
PHYSNET

MISN-0-407

COMPONENT DESCRIPTION OF VECTORS AND MOTION



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

COMPONENT DESCRIPTION OF VECTORS AND MOTION

by

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CONTENTS

Title: **Component Description of Vectors and Motion**

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Input Skills:

1. Vocabulary: unit vector (MISN-0-405).
2. Define addition of two vectors and illustrate with a sketch (MISN-0-405).
3. Calculate the position or velocity of a particle subject only to gravitational interaction with the earth (MISN-0-406).

Output Skills (Knowledge):

K1. Vocabulary: component vector, numerical component of a vector.

K2. Given a vector drawn on a graph which also shows x- and y-axes, write the vector in terms of its components along those axes.

K3. Define the equality of vectors and their components.

K4. Describe projectile motion using vector components.

Output Skills (Problem Solving):

S1. Using an arrow to represent a vector, construct its component vectors parallel and perpendicular to a specified direction.

S2. Given a vector, calculate its components parallel and perpendicular to a specified direction.

S3. Describe any vector in terms of: (a) its magnitude and direction; (b) the sum of its component vectors; (c) its numerical components.

S4. Given two particles moving near the earth's surface, subject only to gravitational interaction and starting with the same height and vertical component of velocity, describe the motion of one particle using information about the motion of the other.

MISN-0-407

COMPONENT DESCRIPTION OF VECTORS AND MOTION

Abstract:
In discussing motion and many other problems, we shall often have to deal with various vectors (such as velocities and accelerations) which have complicated directions relative to each other. Such problems can usually be greatly simplified by decomposing the vectors into convenient parts, handling these parts separately in all necessary calculations, and finally recombining the parts to obtain the desired results. Such methods for decomposing and recombining vectors are of enormous utility in physics and related sciences. Accordingly, we shall use the present unit to introduce these methods and to apply them to some familiar kinds of motion.

SECT.

A COMPONENT VECTORS

In many situations there are certain directions which are specially significant (e.g., the northern direction, the downward direction, or the downstream direction of a river). How then can various vectors be described relative to such a special direction without a cumbersome specification of angles?

An answer to this question is suggested by the familiar way of specifying relative positions on the surface of the earth. For example, although we might say that Boston is 210 miles in a direction 35° east of north from New York, it is easier to specify that Boston is 172 miles north and 120 miles east of New York. This last statement describes the displacement from New York to Boston by specifying two mutually perpendicular displacements, without the need to mention any angles.

Consider then any vector $\vec{A}$ and some special direction specified by the unit vector $\hat{x}$. The preceding example then suggests that $\vec{A}$ can be expressed (as indicated in Fig. A-1) as the vector sum of a vector $\vec{A}_{\parallel}$ parallel to $\hat{x}$ and another vector $\vec{A}_{\perp}$ perpendicular to $\hat{x}$ so that

$$\vec{A} = \vec{A}_{\parallel} + \vec{A}_{\perp} \qquad \text{(A-1)}$$

The vector $\vec{A}_{\parallel}$ is called the "component vector of $\vec{A}$ parallel to $\hat{x}$" and the vector $\vec{A}_{\perp}$ the "component vector of $\vec{A}$ perpendicular to $\hat{x}$." Here we have introduced this definition:

Def. | **Component vectors**: The component vectors of a vector $\vec{A}$ relative to some direction are the vectors, parallel and perpendicular to this direction, whose sum is equal to $\vec{A}$. | (A-2)

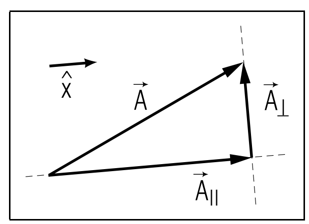


Fig. A-1: Component vectors of a vector $\vec{A}$ parallel and perpendicular to some direction $\hat{x}$.

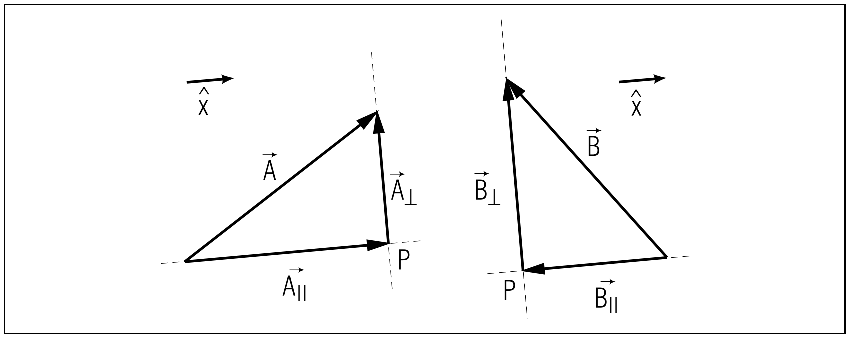


Fig. A-2: Construction of component vectors parallel and perpendicular to $\hat{x}$ for two vectors $\vec{A}$ and $\vec{B}$.

The component vectors of a vector $\vec{A}$ relative to some direction $\hat{x}$ can be found by the following procedure: Draw the arrows representing the vector $\vec{A}$ and the direction specified by $\hat{x}$. (See Fig. A-2.) Through the tail of $\vec{A}$ draw a line parallel to $\hat{x}$ and through the tip of $\vec{A}$ draw a line perpendicular to $\hat{x}$. These lines then intersect at some point P. The arrow drawn from the tail of $\vec{A}$ to P then represents the component vector of $\vec{A}$ parallel to $\hat{x}$; similarly, the arrow drawn from P to the tip of $\vec{A}$ represents the component vector of $\vec{A}$ perpendicular to $\hat{x}$. (Note that the vector $\vec{A}$ itself is always the side opposite to the right angle in the triangle formed by these vectors.)

SPECIAL CASES

By writing $\vec{A} = \vec{A}_{\parallel} + \vec{A}_{\perp}$, we have decomposed the vector $\vec{A}$ into a part $\vec{A}_{\parallel}$ parallel and another part $\vec{A}_{\perp}$ perpendicular to $\hat{x}$. This decomposition becomes especially simple when the vector $\vec{A}$ is itself either parallel or perpendicular to $\hat{x}$. For example, suppose that $\vec{A}$ has a direction along $\hat{x}$, as illustrated in Fig. A-3a. Then $\vec{A}$ itself is entirely along $\hat{x}$ and has no part perpendicular to $\hat{x}$. Thus $\vec{A}_{\parallel} = \vec{A}$ and $\vec{A}_{\perp} = 0$. On the other hand, suppose that $\vec{A}$ is perpendicular to $\hat{x}$, as illustrated in Fig. A-3b. Then $\vec{A}$ itself is entirely perpendicular to $\hat{x}$ and has no part parallel to $\hat{x}$. Thus $\vec{A}_{\perp} = \vec{A}$ and $\vec{A}_{\parallel} = 0$.

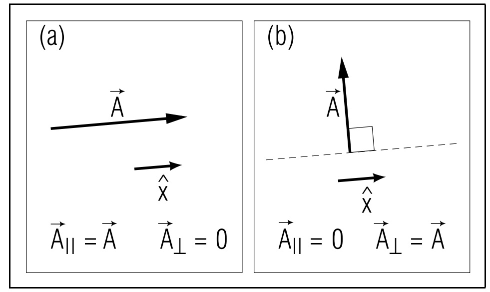


Fig. A-3: Component vectors of $\vec{A}$ in special cases. (a) $\vec{A}$ has a direction along $\hat{x}$. (b) $\vec{A}$ is perpendicular to $\hat{x}$.

Constructing Component Vectors (Cap. 1)

A-1 Draw arrows representing the horizontal vector $\vec{H}$ and the unit vector $\hat{x}$ shown in Fig. A-4. Construct and label arrows representing the component vectors $\vec{H}_{\parallel}$ and $\vec{H}_{\perp}$ of $\vec{H}$ parallel and perpendicular to $\hat{x}$. Is the component vector $\vec{H}_{\parallel}$ parallel to $\vec{H}$, parallel to $\hat{x}$, or perpendicular to $\hat{x}$? In the right triangle formed by $\vec{H}$ and its component vectors, which vector forms the hypotenuse? *(Answer: 101) (Suggestion: [s-13])*

A-2 (a) Suppose a ball is thrown vertically upward with the velocity $\vec{v}$ of magnitude 2 m/s indicated in Fig. A-5. Use the unit vectors $\hat{x}$ and $\hat{y}$ to express the following component vectors. What are the component vectors $\vec{v}_{\parallel}$ and $\vec{v}_{\perp}$ of $\vec{v}$ parallel and perpendicular to $\hat{y}$? (b) If a second ball has a horizontal velocity $\vec{v}\,'$ of magnitude 3 m/s, what is its component $\vec{v}_{\parallel}$ parallel to $\hat{y}$? *(Answer: 104) (Suggestion: [s-1])*

A-3 A boy in a sailboat wishes to travel along the southeast path indicated in Fig. A-6. As he maneuvers the boat in the wind, it travels through the series of displacements $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{D}$. Draw the component vectors of each displacement parallel and perpendicular to the desired direction of travel. (Indicate zero vectors by dots.) *(Answer: 107) (Suggestion: [s-5])*

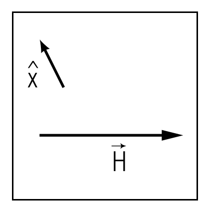


Fig. A-4.

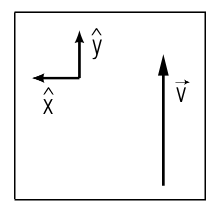


Fig. A-5.

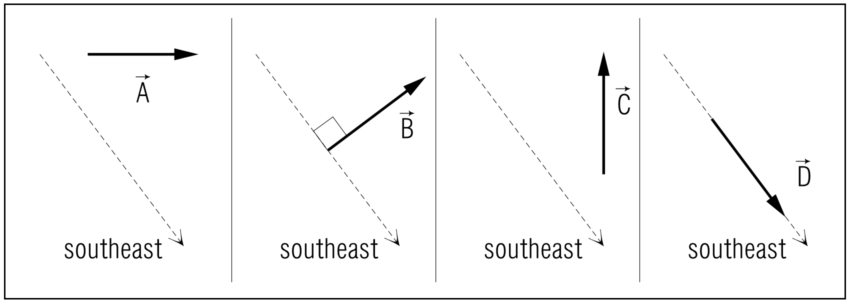


Fig. A-6.

More practice for this Capability: [p-1]

SECT.
B NUMERICAL COMPONENTS OF VECTORS

Consider the component vector $\vec{A}_{\|}$ of $\vec{A}$ parallel to a direction specified by the unit vector $\hat{x}$. As we observed in text section C of Unit 405, any such vector parallel to $\hat{x}$ can always be expressed as a multiple of $\hat{x}$. Thus we can write

$$\vec{A}_{\|} = A_x \hat{x} \qquad \text{(B-1)}$$

where A_x is some *number* which is called the "*numerical* component" (or simply the "component") of $\vec{A}$ along $\hat{x}$. Here we have introduced this definition:

Def. **Numerical component of a vector**: The numerical component of $\vec{A}$ along a direction $\hat{x}$ is the *number* whose magnitude is equal to that of the component vector of $\vec{A}$ parallel to $\hat{x}$, and whose sign is positive or negative depending on whether the direction of this component vector is along or opposite to $\hat{x}$. (B-2)

Example B-1: Numerical component of a velocity

A jumping rabbit leaves the ground with a velocity $\vec{v}$ of 6 meter/sec in a direction 60° from the upward vertical as indicated in Fig. B-1a). What then is the numerical component v_x of this velocity along the unit vector $\hat{x}$ specifying the *downward* direction of the gravitational acceleration? (This value v_x determines the height reached by the jumping rabbit.)

Figure B-1b shows the component vectors of $\vec{v}$ parallel and perpendicular to $\hat{x}$. The *magnitude* $\vec{v}_{\|}$ of the component vector parallel

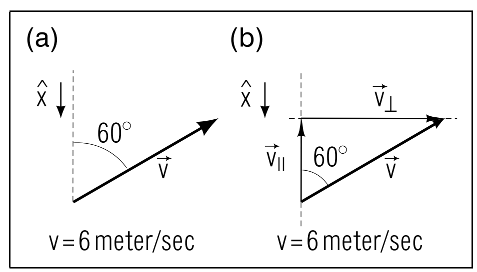


Fig. B-1: Components of a velocity. (a) Velocity $\vec{v}$ of a jumping rabbit. (b) Component vectors of this velocity.

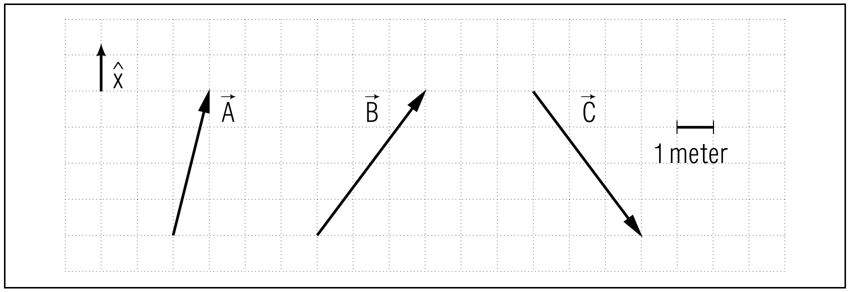


Fig. B-2.

to $\hat{x}$ is then related to the magnitude v of the velocity by the relation $\cos 60^\circ = v_{\|}/v$, so that $v_{\|} = v\cos 60^\circ$. Furthermore, the *direction* of $\vec{v}_{\|}$ is seen to be *opposite* to $\hat{x}$. Hence we can write

$$\vec{v}_{p}arallel = -(v\cos 60^\circ)\hat{x} = -(3\,\text{meter/sec})\hat{x} \qquad \text{(B-3)}$$

because $v = 6\,\text{meter/sec}$ and $\cos 60^\circ = 0.5$. Since $\vec{v}_{\|} = v_x\hat{x}$, the numerical component of $\vec{v}$ along $\hat{x}$ is then just

$$v_x = -v\cos 60^\circ = -3\,\text{meter/sec} \qquad \text{(B-4)}$$

Now: Go to tutorial section B.

Understanding the Definition of a Numerical Component (Cap. 2)

B-1 *Example and comparison:* (a) Carry out the following steps illustrating a general procedure for finding numerical components. Use a grid to draw arrows representing the vector $\vec{A}$ and the unit vector $\hat{x}$ shown in Fig. B-2. Construct arrows representing the component vectors of $\vec{A}$ parallel and perpendicular to $\hat{x}$. Express the component vector $\vec{A}_{\|}$ parallel to $\hat{x}$ as a multiple of $\hat{x}$. Use this expression to find the numerical component A_x of $\vec{A}$ along $\hat{x}$. (b) Repeat the procedure described in part (a) for the vectors $\vec{B}$ and $\vec{C}$. (c) Compare the vectors $\vec{A}$ and $\vec{B}$ by stating whether $\vec{A} = \vec{B}$, and whether $A_x = B_x$. (d) Does $\vec{B} = \vec{C}$? Does $B_x = C_x$? *(Answer: 110)*

B-2 *Interpretation:* The numerical component of a plane's velocity $\vec{v}$ along the unit vector $\hat{y}$ shown in Fig. B-3 indicates how rapidly the plane is receding from or approaching the mountainside below. Each

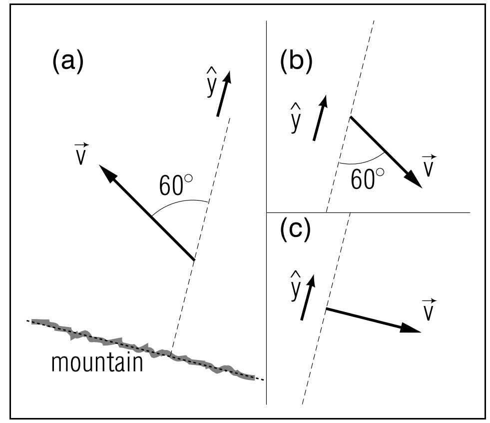


Fig. B-3.

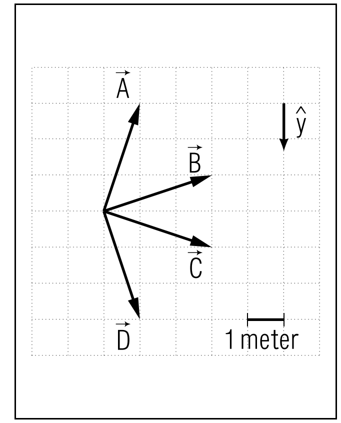


Fig. B-4.

of the velocities shown has a magnitude of 50 meter/sec. For each value of $\vec{v}$, find the numerical component of $\vec{v}$ along $\hat{y}$. *(Answer: 102)*

B-3 *Dependence:* The vectors $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{D}$ in Fig. B-4 all have the same magnitude. Thus their numerical components along $\hat{y}$ depend on their directions relative to $\hat{y}$. (a) What is the numerical component along $\hat{y}$ of each vector? (b) What is the sign of the numerical component along $\hat{y}$ of *any* vector if this vector is directed roughly along $\hat{y}$ (like $\vec{D}$)? If this vector is directed roughly opposite to $\hat{y}$ like $\vec{A}$? (c) Suppose two vectors have the same magnitude, but one is directed roughly along $\hat{y}$ (like $\vec{D}$) and one is roughly perpendicular to $\hat{y}$ (like $\vec{C}$). Which vector has the larger numerical component along $\hat{y}$? *(Answer: 105)*

SECT.

C COMPONENTS RELATIVE TO A COORDINATE SYSTEM

Consider a coordinate system specified by two mutually perpendicular unit vectors $\hat{x}$ and $\hat{y}$. (See Fig. C-1.) Then any vector $\vec{A}$ parallel to the plane containing $\hat{x}$ and $\hat{y}$ can be expressed in terms of its component vectors parallel and perpendicular to $\hat{x}$ (or equivalently, parallel to $\hat{x}$ and parallel to $\hat{y}$). Thus we can write:

$$\boxed{\vec{A} = A_x\hat{x} + A_y\hat{y}} \qquad \text{(C-1)}$$

where we have used the conventional symbols A_x and A_y to denote the numerical components of $\vec{A}$ along the $\hat{x}$ and $\hat{y}$ directions of a coordinate system. *

> In general, a vector $\vec{A}$ in space can similarly be expressed in terms of a three-dimensional coordinate system by writing $\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$.

Example C-1: Description of a vector in terms of its components

The velocity $\vec{v}$ in Fig. C-2 can be expressed as

$$\vec{v} = -(4\,\text{meter/sec})\hat{x} + (3\,\text{meter/sec})\hat{y}$$

so that

$$v_x = -4\,\text{meter/sec and } v_y = 3\,\text{meter/sec}$$

Similarly, the velocity $\vec{v}\prime$ in Fig. C-2 can be expressed as

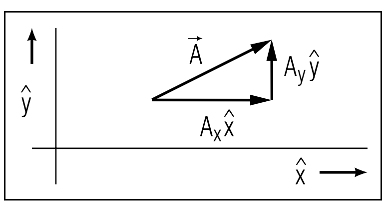


Fig. C-1: Components of a vector $\vec{A}$ relative to a coordinate system.

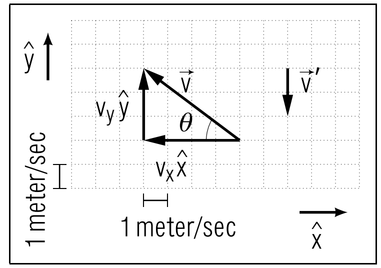


Fig. C-2: Specification of velocities in terms of their components.

$$\vec{v}\prime = -(2\,\text{meter/sec})\hat{y} = 0\hat{x} - (2\,\text{meter/sec})\hat{y}$$

so that

$$v'_x = 0 \text{ and } v'_y = -2\,\text{meter/sec}$$

If we know the magnitude of a vector and the angle specifying its direction, we can easily find its numerical components relative to some coordinate system.

Example C-2: Finding the numerical components of a vector

Consider the velocity $\vec{v}$ having a magnitude $v = 8.0\,\text{meter/sec}$ and a direction making an angle of 30° with the $\hat{x}$ direction, as shown in Fig. C-3. What are the numerical components v_x and v_y of this velocity?

We draw the component vectors of $\vec{v}$, as indicated in Fig. C-3. The magnitude of the component vector $v_x\hat{x}$ parallel to $\hat{x}$ can then be found from the magnitude v of the velocity $\vec{v}$ and the definition of the cosine. Thus $v_x\hat{x} = (v\cos 30°)\hat{x}$. Similarly, the magnitude of the component vector $v_y\hat{x}$ parallel to $\hat{y}$ can be found from v and the definition of the sine. Thus $v_y\hat{y} = -(v\sin 30°)\hat{y}$ where the minus sign indicates that the direction of this component vector is *opposite* to $\hat{y}$.

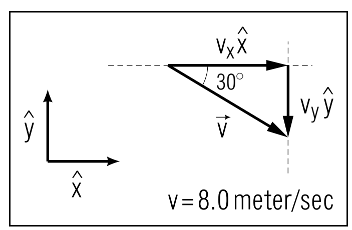


Fig. C-3: Calculation of the components of a velocity $\vec{v}$.

Hence we find

$$v_x = v\cos 30^\circ = (8.0\,\text{meter/sec})(0.866) = 6.9\,\text{meter/sec}$$

$$v_y = -v\sin 30^\circ = -(8.0\,\text{meter/sec})(0.500) = -4.0\,\text{meter/sec}$$

Conversely, if we know the numerical components (or component vectors) of a vector relative to some coordinate system, we can readily find the magnitude of this vector and the angles specifying its direction. For example, the perpendicular sides of the right triangle in Fig. C-1 have magnitudes $|A_x|$ and $|A_y|$. Hence we can find the magnitude A of the side opposite the right angle by the Pythagorean theorem. Thus:

$$\boxed{A^2 = |A_x|^2 + |A_y|^2 = A_x^2 + A_y^2} \qquad \text{(C-2)}$$

The last step is justified even if A_x or A_y is negative since the square of a negative number is equal to the square of its magnitude. [For example, $(-5)^2 = (-5)(-5) = 5^2$.] *

* In three dimensions, where one can write $\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$ a generalization of Eq. (C-2) leads to the result $A^2 = A_x^2 + A_y^2 + A_z^2$.

Example C-3: Finding a vector from its numerical components

Consider a velocity $\vec{v}$ whose numerical components relative to the directions $\hat{x}$ and $\hat{y}$ of a coordinate system are $v_x = -4\,\text{meter/sec}$ and $v_y = 3\,\text{meter/sec}$. What is the magnitude v of this velocity and what is the angle between $\vec{v}$ and the $\hat{x}$ direction?

We can use the numerical components of $\vec{v}$ to draw its component vectors and hence also $\vec{v}$ itself, as shown in Fig. C-2. We can then calculate v from the right triangle in this figure. Thus

$$v^2 = (4\,\text{meter/sec})^2 + (3\,\text{meter/sec})^2 = 25\,\text{meter}^2/\text{sec}^2$$

so that

$$v = 5\,\text{meter/sec}$$

Furthermore, the angle θ in Fig. C-2 is such that

$$\tan\theta = \frac{|v_y|}{|v_x|} = \frac{3\,\text{meter/sec}}{4\,\text{meter/sec}} = 0.75$$

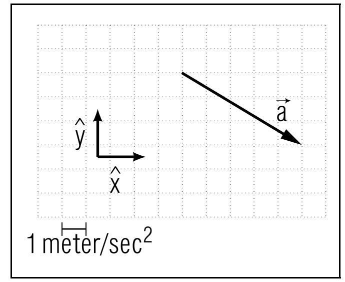


Fig. C-4.

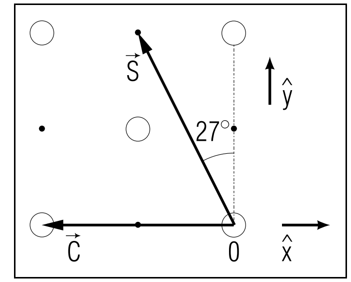


Fig. C-5.

so that

$$\theta = 37^\circ$$

Hence the angle between $\vec{v}$ and the $\hat{x}$ direction is $180^\circ - \theta = 143^\circ$.

The specification of a vector in terms of its components is usually much simpler than its specification in terms of its magnitude and direction. We shall illustrate the simplicity and utility of such a component description in the next two sections.

Finding Component Expressions for Vectors (Cap. 3)

C-1 (a) Using a grid, draw an arrow representing the acceleration $\vec{a}$ shown in Fig. C-4, and construct arrows representing its component vectors parallel to the coordinate directions $\hat{x}$ and $\hat{y}$. (b) Express $\vec{a}$ as a sum of these component vectors (i.e., as a sum of multiples of $\hat{x}$ and $\hat{y}$). (c) What are the numerical components a_x and a_y of $\vec{a}$ relative to these coordinate directions? *(Answer: 108)*

C-2 Figure C-5 shows the arrangement of sodium atoms (dots) and chlorine atoms (open circles) in a sodium chloride crystal. Positions of such atoms are commonly specified by numerical components of their position vectors. (a) If the position vectors $\vec{C}$ and $\vec{S}$ have magnitudes $C = 5.3$ Å, and $S = 6.0$ Å, express each vector as a sum of its component vectors relative to the coordinate system shown. (The symbol Åstands for angstrom = 10^{-10} meter.) (b) What are the numerical components of $\vec{C}$ and $\vec{S}$ relative to this coordinate system? *(Answer: 111) (Suggestion: [s-2])*

More practice for this Capability: [p-3]

Finding Vectors From Their Component Expressions (Cap. 3)

C-3 The displacement $\vec{D}$ from New York to Washington, D.C. has the components $D_x = -160\,\text{km}$ and $D_y = -120\,\text{km}$ relative to coordinate directions east and north (specified by unit vectors $\hat{x}$ and $\hat{y}$ respectively). (a) Express $\vec{D}$ as a sum of its component vectors (i.e., as a sum of multiples of $\hat{x}$ and $\hat{y}$). (b) Draw a diagram showing $\hat{x}$ and $\hat{y}$, and these component vectors. Then add the component vectors to construct an arrow representing $\vec{D}$. (c) What is the magnitude D of the distance from New York to Washington? Is the direction of $\vec{D}$ roughly northeast, northwest, southeast, or southwest? *(Answer: 114)*

C-4 Relative to the coordinate directions east and north (specified by unit vectors $\hat{x}$ and $\hat{y}$), the velocity $\vec{v}$ of a light plane has the numerical components: $v_x = 60\,\text{m/s}$ (due to the planets motion relative to the air) and $v_y = -25\,\text{m/s}$ (due to the motion of the air relative to the ground). (a) Draw arrows representing the component vectors of $\vec{v}$ along these directions, and then construct an arrow representing $\vec{v}$. (b) What is the magnitude of $\vec{v}$? Is the direction of $\vec{v}$ roughly northeast, northwest, southeast, or southwest? *(Answer: 103)*

More practice for this Capability: [p-4]

SECT.

D UTILITY OF COMPONENT DESCRIPTIONS

Problems involving vectors can usually be decomposed into easily handled parts by this procedure: (1) Decompose all vectors by expressing them in terms of their component vectors relative to some conveniently chosen coordinate system. (2) Make all calculations by using these component vectors or their corresponding numerical components. (Such calculations are easy since numerical components are merely numbers). (3) Recombine the component vectors obtained in the calculation to find all vectors of interest. By following this procedure, even complex geometrical problems can be handled by simple arithmetic and algebra, without any need for visualizing geometrical relationships.

Example D-1: Subtracting vectors by using components

Consider two velocities $\vec{v}$ and $\vec{v}\,'$ expressed in terms of their component vectors so that $\vec{v} = v_x\hat{x} + v_y\hat{y}$ and $\vec{v}\,' = v'_x\hat{x} + v'_y\hat{y}$. The vector difference $\vec{v}\,' - \vec{v}$ is then

$$\vec{v}\,' - \vec{v} = (v'_x\hat{x} + v'_y\hat{y}) - (v_x\hat{x} + v_y\hat{y})$$

We can then rearrange the terms on the right side, corresponding to $\hat{x}$ and $\hat{y}$ by writing

$$\vec{v}\,' - \vec{v} = v'_x\hat{x} - v_x\hat{x} + v'_y\hat{y} - v_y\hat{y}$$

Hence

$$\vec{v}\,' - \vec{v} = (v'_x - v_x)\hat{x} + (v'_y - v_y)\hat{y} \qquad \text{(D-1)}$$

a result which expresses the vector difference $(\vec{v}\,' - \vec{v})$ directly in terms of its numerical components. Note that our calculation involved only simple algebra and required neither diagrams nor a concern with angles.

The geometrical significance of Eq. (D-1) is illustrated in Fig. D-1 which shows the two vectors $\vec{v} = (5\hat{x} + 2\hat{y})$ meter/sec and $\vec{v}\,' = (6\hat{x} + 5\hat{y})$ meter/sec. The difference $(\vec{v}\,' - \vec{v})$ of these vectors, indicated by the vector drawn from the tip of $\vec{v}$ to the tip of $\vec{v}\,'$, is then seen to be equal to $(\hat{x} + 3\hat{y})$ meter/sec. This result is indeed the same as that obtained by Eq. (D-1).

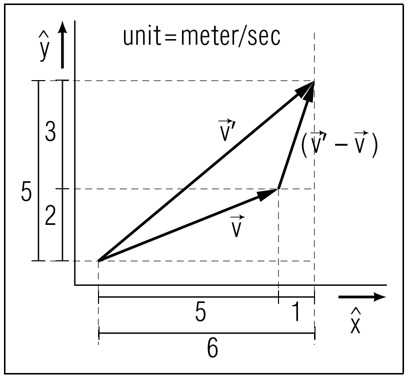


Fig. D-1: Subtraction of two vectors.

CORRESPONDENCE BETWEEN VECTOR AND COMPONENT RELATIONS

The addition of vectors in terms of their numerical components can be carried out in a manner completely analogous to that illustrated in the preceding example for the subtraction of vectors. Hence the result Eq. (D-1) leads to this conclusion:

Every numerical component of the sum (or difference) of two vectors is equal to the sum (or difference) of the corresponding numerical components of these vectors.	(D-2)

Suppose that two vectors are equal so that they have the same magnitude and the same direction. Then each of these vectors must also have the same component vector parallel to any specified direction. Thus we arrive at this useful conclusion:

If two vectors are equal, their component vectors parallel to any direction (and hence also their numerical components along any direction) are equal.	(D-3)

For example,

$$\text{if } \vec{A} = \vec{B}, \quad A_x = B_x \text{ and } A_y = B_y \tag{D-4}$$

Example D-2: Relation between components of velocity and acceleration

Consider an equation such as the definition of the acceleration

$$\vec{a} = \frac{d\vec{v}}{dt} \tag{D-5}$$

Here $d\vec{v}$ denotes the small velocity change, during the small time dt, between some original velocity $\vec{v}$ and some slightly changed velocity $\vec{v}\prime$. But our previous result Eq. (D-1) implies that $d\vec{v} = dv_x\hat{x} + dv_y\hat{y}$, where $dv_x = v'_x - v_x$ and $dv_y = v'_y - v_y$. If both $\vec{a}$ and $\vec{v}$ are expressed in terms of their numerical components, the Eq. (D-5) then becomes

$$a_x\hat{x} + a_y\hat{y} = \frac{dv_x}{dt}\hat{x} + \frac{dv_y}{dt}\hat{y} \tag{D-6}$$

The equality of the corresponding numerical components on both sides of this equation then implies that

$$a_x = \frac{dv_x}{dt} \text{ and } a_y = \frac{dv_y}{dt} \tag{D-7}$$

Using Component Vectors to Solve Problems (Cap. 3)

D-1 Suppose we wish to find a value for the vector $\vec{D} = \vec{B} - \vec{A}$ shown in Fig. D-2. We might make a larger, very accurate drawing of these vectors, using a protractor and ruler to measure $\vec{D}$, or we could use trigonometry. But the following component method is often most useful. (a) Express $\vec{A}$ and $\vec{B}$ as sums of their components vectors parallel to $\hat{x}$ and $\hat{y}$. (b) Subtract these expressions to obtain an expression for $\vec{D}$ as a sum of its component vectors. (c) Use arrows representing the two component vectors of $\vec{D}$ to construct an arrow representing $\vec{D}$. What is the magnitude D? *(Answer: 106)*

D-2 The elasticity of polymer substances (e.g., some plastics, and silicone rubber) results from long molecules (polymers) forming tangled coils which straighten as the substance stretches. For example, the four-atom polymer segments shown in Fig. D-3 can either be "stretched" as in (a) or "coiled" as in (b). We call the distance between atoms 1 and 4 the "length" of a polymer segment. Thus the stretched segment has a "length" equal to the magnitude of $\vec{L} = \vec{P} + \vec{Q} + \vec{R}$, and the coiled segment has a "length" equal to the magnitude of $\vec{L}\prime = \vec{P} + \vec{Q} + \vec{R}\prime$. Use the following procedure to find these "lengths" (a) What are the

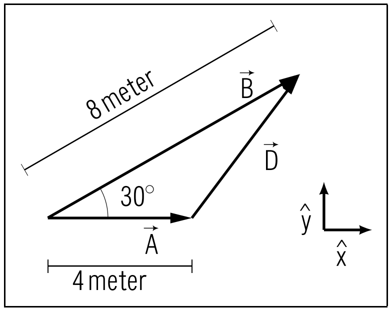


Fig. D-2.

numerical components of $\vec{P}$, $\vec{Q}$, and $\vec{R}$ relative to the coordinate system shown? (b) What are the numerical components L_x and L_y of $\vec{L}$? What is the "length" L of the stretched segment? (c) What are the components R'_x and R'_y of the vector $\vec{R}'$? (d) What are the components L'_x and L'_y of the vector $\vec{L}' = \vec{P} + \vec{Q} + \vec{R}'$? What is the "length" L' of the coiled segment? *(Answer: 109) (Suggestion: [s-6])*

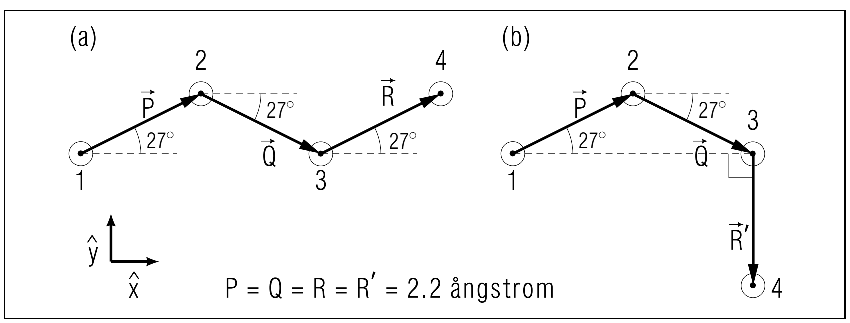


Fig. D-3.

SECT.

E PROJECTILE MOTION

Consider the motion of some particle, such as a baseball or some other projectile, which has been launched near the surface of the earth. (See Fig. E-1.) *After* the initial launching process, the particle is subject only to the gravitational interaction with the earth (if we neglect the interaction of the particle with the surrounding air). Then we know from text section E of Unit 406 that the particle moves with a constant downward acceleration

$$\vec{a} = \vec{g} \tag{E-1}$$

where the gravitational acceleration $\vec{g}$ is independent of all properties of the particle and has a magnitude $g \approx 10\,\text{meter/sec}^2$. How can we use this information to predict how the velocity and position of the particle change with time, or to determine along what kind of curved path (or trajectory) the particle travels during its flight?

At first glance this question seems complicated because the particle moves along a *curved* path so that its velocity $\vec{v}$ continually changes direction relative to its downward acceleration $\vec{g}$. But by decomposing all relevant vectors into convenient components, the problem can be discussed quite easily.

To describe the motion of the particle, we choose a convenient coordinate system, one of whose directions (say the $\hat{y}$ direction) is parallel to the vertical direction of the constant acceleration $\vec{g}$ of the particle. To be specific, let us choose $\hat{y}$ to be vertically upward and $\hat{x}$ to be horizontal, as illustrated in Fig. E-1. The position vector $\vec{r}$ of the particle may then be written as $\vec{r} = x\hat{x} + y\hat{y}$ (where

the numerical components x and y of $\vec{r}$ are called the "position coordinates" of the particle). Similarly, the velocity $\vec{v}$ of the particle can be

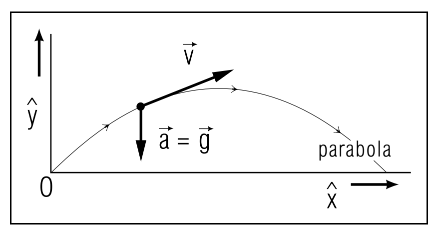


Fig. E-1: Motion of a particle subject solely to the gravitational interaction with the earth.

written as $\vec{v} = v_x\hat{x} + v_y\hat{y}$ and its acceleration as $\vec{a} = a_x\hat{x} + a_y\hat{y}$.

Since $\vec{g}$ is directed downward (i.e., opposite to $\hat{y}$) we can write $\vec{g} = -g\hat{y}$. Hence our basic relation $\vec{a} = \vec{g}$ in Eq. (E-1) is equivalent to

$$a_x\hat{x} + a_y\hat{y} = -g\hat{y} \tag{E-2}$$

But this equality between vectors implies that their corresponding numerical components along $\hat{x}$ and $\hat{y}$ must also be equal, i.e., that

$$\text{along } \hat{x}, \quad a_x = 0 \tag{E-3}$$

$$\text{along } \hat{y}, \quad a_y = -g \tag{E-4}$$

The separation of the original vector equation $\vec{a} = \vec{g}$ into the two equivalent component Eqs. (E-3) and (E-4) simplifies the discussion of the motion enormously since the motion along the $\hat{x}$ and $\hat{y}$ directions can now be discussed separately.

Horizontal motion: Since $a_x = 0$, the horizontal acceleration $a_x\hat{x}$ of the particle is zero. Correspondingly Eq. (D-7) implies that $dv_x/dt = 0$ so that the horizontal velocity $v_x\hat{x}$ of the particle remains unchanged. Hence the horizontal motion of the particle is one with *constant* velocity $v_x\hat{x}$. All these statements are true irrespective of the motion of the particle along the vertical $\hat{y}$ direction.

Vertical motion: Since $a_y = -g$, the particle has a constant acceleration $a_y\hat{y} = -g\hat{y}$ of magnitude g along the downward direction (i.e., opposite to $\hat{y}$). Hence the vertical motion of the particle is exactly the same as that of a particle which moves *entirely* vertically after starting with the same initial vertical velocity from the same height. This statement is true irrespective of the motion of the particle along the horizontal $\hat{x}$ direction.

The preceding comments are illustrated by Fig. E-2 which shows the paths of several particles projected from the same point with different initial *horizontal* velocities $\vec{v}_A$. In each case, the particle moves then with a *constant* horizontal velocity $v_x\hat{x}$ equal to its initial value $\vec{v}_A$.

The vertical motion of each particle is exactly the same since each particle starts with zero initial vertical velocity (because $\vec{v}_A$ is horizontal) and since each moves with the same downward acceleration $-g\hat{y}$. Thus the vertical motion of each particle (i.e., its height and vertical velocity at any time) is the same as that of a particle which starts from rest and falls

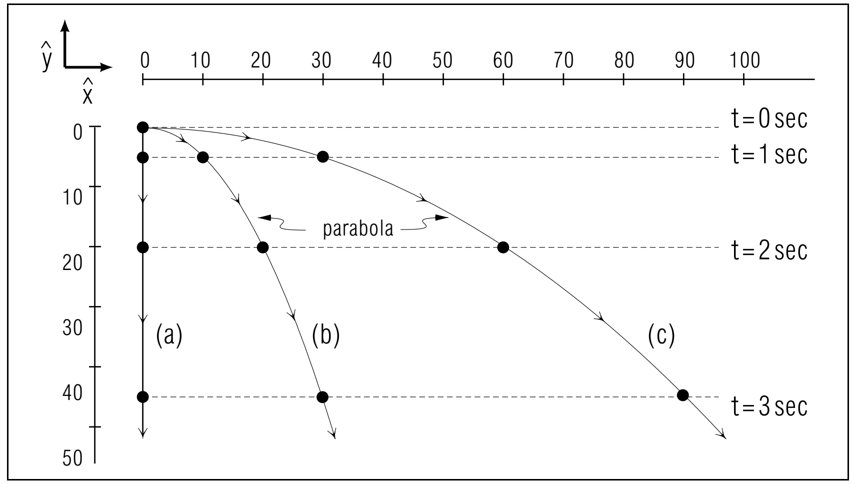


Fig. E-2: Path of a particle moving under the influence of gravity near the surface of the earth after being projected with a horizontal velocity $\vec{v}_A$. The dots indicate successive positions of the particle. (a) $-\vec{v}_A = 0$ (b) $\vec{v}_A = (10\,\text{meter/sec})\hat{x}$ (c) $\vec{v}_A = (30\,\text{meter/sec})\hat{x}$.

purely vertically. The only difference between the particles in Fig. E-2 is that the *horizontal* distance traveled by a particle is proportionately larger if it starts with a larger initial horizontal velocity.

Similar comments can be made about Fig. E-1, which illustrates the motion of a baseball thrown from the ground. The horizontal motion of the ball proceeds with constant velocity, while its vertical motion is exactly the same as that of a ball thrown vertically upward from the ground with the same initial vertical velocity. The combination of these two motions results in the parabolic trajectory illustrated in Fig. E-1.

QUANTITATIVE DISCUSSION

Since Eq. (E-3) and Eq. (E-4) show that the motion of a projected particle can be separated into motions with constant acceleration along each of these directions, the result Relation (D-9) of Unit 406

$$\Delta\vec{r} = \vec{v}_A\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2 \tag{E-5}$$

is applicable to motion along each of these directions. For example, along the $\hat{x}$ direction, $\vec{r} = x\hat{x}$, $\vec{v}_A = v_{Ax}\hat{x}$, and $\vec{a} = a_x\hat{x} = 0$. Hence Eq. (E-5) implies that

$$\Delta x = v_{Ax}\Delta t \tag{E-6}$$

Along the $\hat{y}$ direction, $\vec{r} = y\hat{y}$, $\vec{v}_A = v_{Ay}\hat{y}$, and $\vec{a} = -g\hat{y}$. Hence Eq. (E-5) implies

$$\Delta y = v_A y \Delta t - \frac{1}{2}g(\Delta t)^2 \tag{E-7}$$

The displacement of the particle is then simply $\Delta\vec{r} = \Delta x\hat{x} + \Delta y\hat{y}$. The particle paths shown in Fig. E-2 are consistent with these equations in the case where $v_{Ay} = 0$.

Now: Go to tutorial section E.

Comparing Particle Motion near the Earth's Surface (Cap. 4)

For the precision desired here, you may assume that the objects described in the following problems move subject only to gravitational interaction with the earth.

E-1 A rabbit pursued by hounds jumps off a vertical cliff with an initial velocity of magnitude 5.0 meter/sec. As the rabbit leaves the cliff, it dislodges a pebble which falls vertically downward striking the ground below the cliff 1.4 sec later. For both the rabbit and the pebble, the initial vertical components of velocity are zero. At what time after leaving the cliff does the rabbit land on the horizontal ground below? What is the rabbit's horizontal distance from the base of the cliff at the time it lands? *(Answer: 116)*

E-2 A boy throws three water balloons each with the same initial vertical components of position and velocity, but with differing horizontal components of velocity. The three balloons follow the paths indicated in Fig. E-3, striking the horizontal ground surface below: (a) Which balloon strikes the ground after the smallest time interval (or do all three strike after the same interval)? (b) Which balloon's vertical component of velocity has the largest magnitude at the time it strikes the ground (or is this component equal for all balloons)? (c) Do all balloons strike the ground with the same velocity? *(Answer: 112)*

E-3 A very large man and a very small woman leave the surface of a trampoline with the same upward velocity. Which person reaches the greater height before falling back to the trampoline surface? *(Answer: 117) ([s-10], [p-5])*

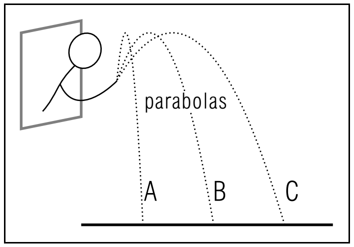


Fig. E-3.

SECT.

F SUMMARY

DEFINITIONS

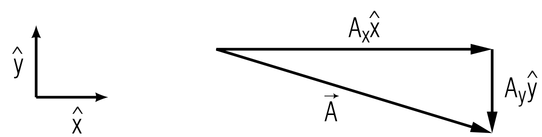
component vectors; Def. (A-2)

numerical component of a vector; Def. (B-2)

IMPORTANT RESULTS

Decomposition of a vector into components: Eq. (C-1), Eq. (C-2)

$\vec{A} = A_x\hat{x} + A_y\hat{y}$, where $A^2 = A_x^2 + A_y^2$



Equality of vectors and of their components: Rule (D-3)

If two vectors are equal, their component vectors parallel to any direction (and hence also their numerical components along any direction) are equal.

Projectile motion: (Sec. E)

The motion along the horizontal and vertical directions can be discussed separately. The horizontal motion proceeds with constant velocity, the vertical motion is the same as that of a particle moving entirely vertically (after starting with the same vertical velocity from the same height.)

NEW CAPABILITIES

(1) Use an arrow representing a vector to construct its component vectors parallel and perpendicular to a specified direction, and to find its numerical component along that direction. (Sec. A, [p-1])

(2) Understand the definition of the numerical component of a vector. (Sec. B, [p-2])

(3) Use interchangeably the following descriptions of a vector: (a) its magnitude and direction, (b) the sum of its component vectors, (c) its numerical components. (Sec. C, [p-3], [p-4])

(4) If two particles move near the earth's surface, subject only to gravitational interaction, and starting with the same height and vertical component of velocity, describe the motion of one particle using information about the motion of the other. (Sec. E, [p-5])

SECT.

G PROBLEMS

G-1 *Using components to describe a river crossing:* A girl heads her rowboat directly across a river of width 0.5 mile and rows (relative to the water) with a constant speed of 1 mile/hour. Her boat is carried downstream by the river current with a velocity (4 mile/hour)$\hat{y}$. The boat then has a velocity (relative to the banks) of $\vec{V}$ = (1 mile/hour)$\hat{x}$ + (4 mile/hour)$\hat{y}$, where $\hat{x}$ and $\hat{y}$ are the unit vectors shown in Fig. G-1. The equation $\vec{V} = \vec{D}/T$ relates the velocity $\vec{V}$ to the time T required to cross the river and the displacement $\vec{D}$ from the boat's initial position on one bank to its final position on the opposite bank. (a) Write two equations relating the numerical components of the boat's displacement and velocity along $\hat{x}$ and $\hat{y}$. (b) Use one of these component relations to find the time T required for crossing the river. (c) Use this value of T to find the distance D_y which the boat travels downstream (along $\hat{y}$) during the crossing. *(Answer: 115) (Suggestion: [s-7])*

Describing Particle Motion near the Earth's Surface

For the precision desired here, the objects described in the following problems are particles moving subject only to gravitational interaction with the earth.

G-2 *The range of a projected particle:* The range of a particle, such as a thrown baseball or a bullet from a gun, is the horizontal distance $|\Delta x|$ it travels from its initial position before striking the horizontal

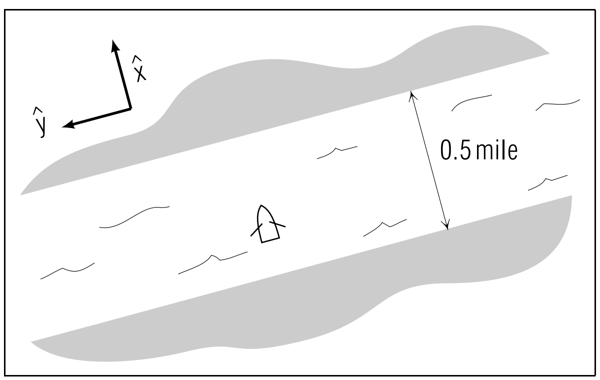


Fig. G-1.

ground surface. Consider such a particle which has an initial position at the ground, and an initial velocity with horizontal and vertical components v_{Ax} and v_{Ay}. After a time Δt the particle strikes the ground a horizontal distance $|\Delta x|$ from its initial position. What is the vertical component Δy of the particle's displacement from its initial position after time Δt? Use the Eqs. (E-6) and (E-7) to find an expression for the range $|\Delta x|$ in terms of the initial components of velocity and known quantities, including the magnitude g of the gravitational acceleration. A golf ball is hit at an angle of 45° from the horizontal so that $v_{Ax} = v_{Ay} = 30\,\text{m/s}$. What is the range of the golf ball? *(Answer: 113)*

G-3 *Athletic performance and the value of g:* In the 1968 Olympics Robert Beaman of the US broke the previous running broad jump record by jumping 8.25 meter at Mexico City where the gravitational acceleration has a magnitude $g = 9.786\,\text{m/s}^2$. Suppose instead these games had been held in Munich where $g' = 9.809\,\text{m/s}^2$. If Beaman had begun his jump in exactly the same way (with the same initial velocity) what would be the ratio $|\Delta x'|/|\Delta x|$ of the distance jumped in Munich to the distance jumped in Mexico city? Express the ratio in terms of g and g'. In which location does he jump farther? *(Answer: 120) (Suggestion: [s-9])*

G-4 *Fire fighting by airplane:* A fire-fighting airplane approaching a spot fire has a constant horizontal velocity of magnitude v, and flies at an altitude of $|\Delta y|$ above the horizontal ground surface. At what horizontal distance $|\Delta x|$ from the fire should the pilot release a canister of fire-extinguishing chemicals so that it will land in the fire? The initial velocity of the canister as it is released is the same as the velocity of the plane. Express your answer in terms of $|\Delta y|$, g, and v_0. If the plane's speed is 40 meter/sec, and its altitude 500 meter, what is this distance $|\Delta x|$? *(Answer: 118) (Suggestion: [s-11])*

Note: Tutorial section G includes problems on biological applications of component descriptions.

TUTORIAL FOR B

UNDERSTANDING THE DEFINITION OF NUMERICAL COMPONENT (Cap. 2)

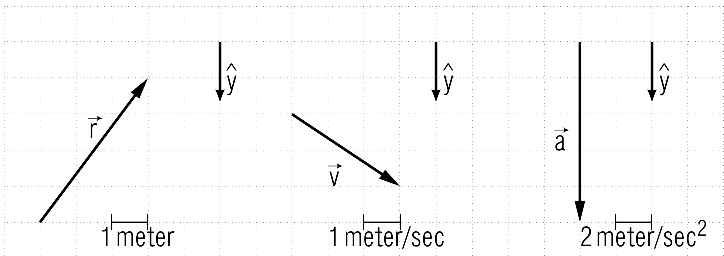
b-1 *EXAMPLE:* As the first step towards acquiring an understanding of the definition of numerical component, we apply it to a simple example. Because this definition describes a procedure, such an example is the most useful summary of the relation.

Consider a ball moving near the earth's surface and influenced only by gravity. The acceleration $\vec{a}$ of the ball is always along the downward unit vector $\hat{y}$, but its velocity $\vec{v}$ and its position vector $\vec{r}$ may have any direction.

Find the numerical component of $\vec{r}$ along $\hat{y}$ by using the following procedure: (1) Draw arrows representing the component vectors of $\vec{r}$ parallel and perpendicular to $\hat{y}$. (Use dots to indicate zero vectors.) (2) Express the component vector $\vec{r}_{\|}$ parallel to $\hat{y}$ as a multiple of $\hat{y}$. (3) Use this expression to find the numerical component r_y.

►

Repeat this procedure to find the numerical components of $\vec{v}$ and $\vec{a}$ along $\hat{y}$.



► $\vec{r}_{\|} = (\_\_\_\_\_\_\_\_\_\_)\hat{y}$

$r_y = (\_\_\_\_\_\_\_\_\_\_)$

$\vec{v}_{\|} = (\_\_\_\_\_\_\_\_\_\_)\hat{y}$

$v_y = (\_\_\_\_\_\_\_\_\_\_)$

$\vec{a}_{\|} = (\_\_\_\_\_\_\_\_\_\_)\hat{y}$

$a_y = (\_\_\_\_\_\_\_\_\_\_)$

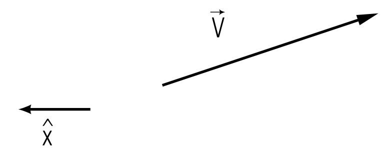
Check your work by making sure that each component vector equals the corresponding numerical component multiplied by $\hat{y}$.

(Answer: 10) (Practice: text problem B-1.)

b-2 *STATEMENT:* Having worked a simple example, let us summarize in words how to find the numerical component of a vector $\vec{V}$ along a direction $\hat{x}$. The first step is to construct component vectors of $\vec{V}$ parallel and perpendicular to $\hat{x}$.

Construct these component vectors:

►



Briefly describe how to use this diagram to find the numerical component $\vec{V}_x$ of $\vec{V}$ along $\hat{x}$.

►

(Answer: 13)

b-3 *PROPERTIES:* Consider a vector $\vec{V}$, its magnitude, its component vector parallel to a direction $\hat{x}$, and its numerical component along $\hat{x}$.

Complete the following chart summarizing the simple properties of these quantities

►

	vector	magnitude	component vector	numerical component
Vector or number?				
Possible signs? (for numbers only)				
Common algebraic symbols?				

Suppose the vector $\vec{V}$ is a velocity with units of meter/sec. What are the units of the other quantities?

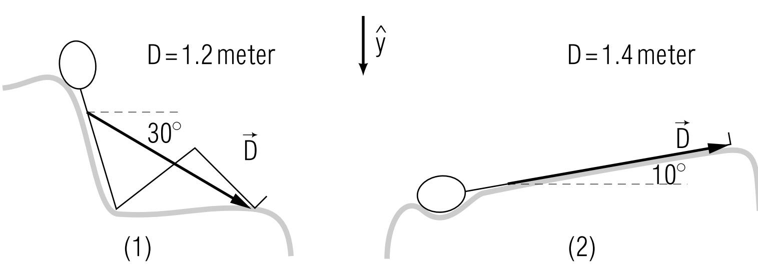
meter/sec			

(Answer: 2)

b-4 *INTERPRETATION:* Applying the definition of numerical component often requires using trigonometry to interpret information provided as angles. The following example illustrates this interpretation.

The work of the heart consists in part of lifting blood from the lower extremities. Thus the strain on a weakened heart can be reduced by reducing the vertical distance of the feet below the heart. If $\vec{D}$ is the displacement from the heart to the feet, this distance is the magnitude of the numerical component D_y along the downward unit vector $\hat{y}$.

For each of these positions of a heart patient, find the numerical component D_y.



► 1. $D_y =$ __________

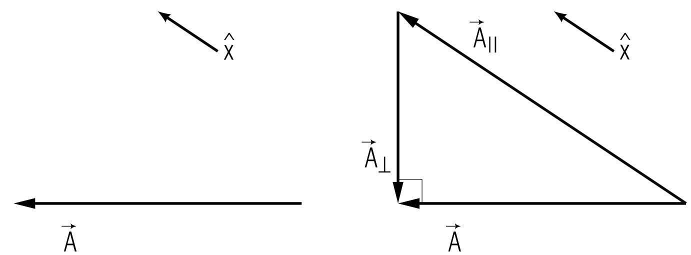
2. $D_y =$ __________

(Answer: 11) (Suggestion: [s-3].) (Practice: text problem B-2.)

b-5 *MEANING OF COMPONENT VECTORS:* Applying the definition of numerical component requires first constructing correct component vectors. To help yourself avoid commonly made mistakes, consider the following *incorrect* constructions which can *not* be used to find numerical components.

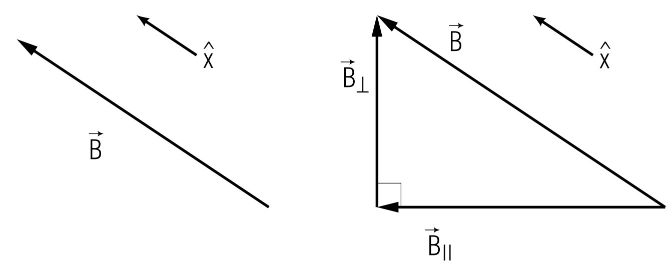
Construct correct component vectors of $\vec{A}$, $\vec{B}$, and $\vec{C}$ parallel and perpendicular to $\hat{x}$. Then briefly describe why each "construction" shown at the right is *incorrect.*

► (1)



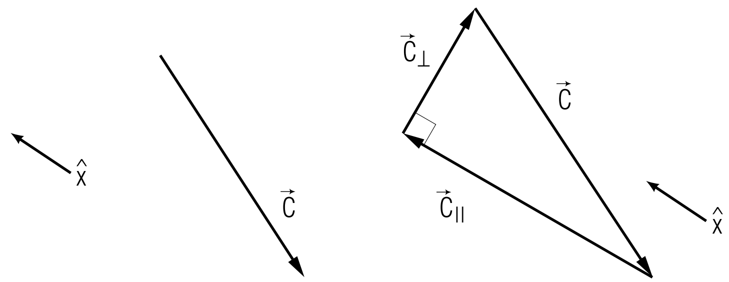
Incorrect because: __________

(2)



Incorrect because: __________

(3)



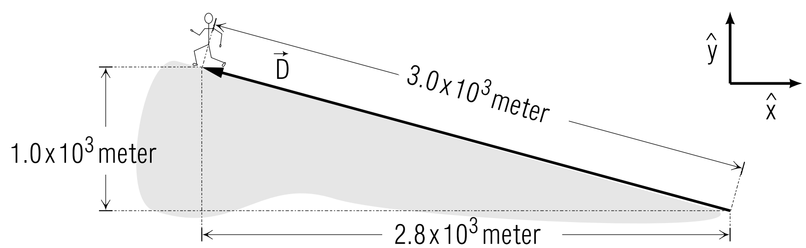
Incorrect because: ______________________

____

(Answer: 5)

b-6 *COMPARISON OF QUANTITIES:* The numerical component of a vector along a direction is closely related to the vector itself, to its magnitude, and to one of its component vectors. Because care must be taken to distinguish between these quantities, we have compared their simple properties in tutorial frame [b-3]. We now consider an example which requires finding all these quantities without confusion.

A hiker climbs up a slope, traveling through the displacement $\vec{D}$ indicated in the following diagram. (The vectors $\hat{x}$ and $\hat{y}$ are unit vectors.)



(1) What is the numerical component of $\vec{D}$ along $\hat{y}$?

► ______________

(2) What is the magnitude of the displacement $\vec{D}$?

► ______________

(3) What is the horizontal component vector of $\vec{D}$?

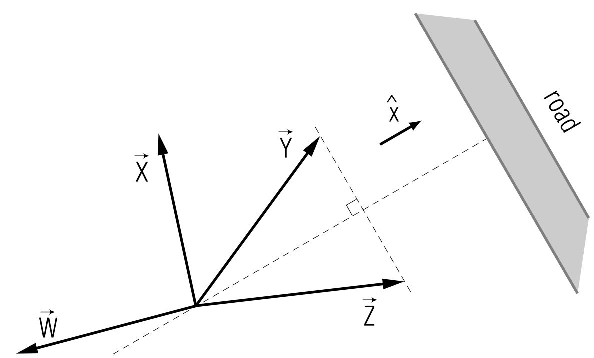
► ______________

Check carefully the signs of these quantities, and make sure that directions are specified for vector quantities.

(Answer: 8)

b-7 *DEPENDENCE:* If two vectors have the same magnitude, their numerical components along a given direction depend on the direction of each vector relative to the given direction.

For example, a hiker wishing to reach a road wants to travel along the direction indicated by $\hat{x}$ in the following drawing. However, due to obstacles in the terrain, he might travel through one of the displacements $\vec{W}$, $\vec{X}$, $\vec{Y}$, or $\vec{Z}$, all of which have the same magnitude. Answer each of these questions by indicating *one or more* of these vectors:



(1) Which of these vectors has the largest numerical component along $\hat{x}$?

► $\vec{W}$, $\vec{X}$, $\vec{Y}$, $\vec{Z}$

(2) Which has the smallest numerical component along $\hat{x}$?

► $\vec{W}$, $\vec{X}$, $\vec{Y}$, $\vec{Z}$

(3) Which vector has the numerical component along $\hat{x}$ which is most nearly equal to zero?

► $\vec{W}$, $\vec{X}$, $\vec{Y}$, $\vec{Z}$

(Answer: 14) (Suggestion: [s-8].)

b-8 *DEPENDENCE QUIZ:* The preceding example illustrates that the numerical component of any vector along a direction $\hat{x}$ depends on the direction of this vector relative to $\hat{x}$ (i.e., whether it is directed *roughly along* $\hat{x}$ (like $\vec{Y}$ and $\vec{Z}$), *roughly opposite to* $\hat{x}$ (like $\vec{W}$), or *roughly perpendicular to* $\hat{x}$ (like $\vec{X}$)).

Summarize this dependence by completing these statements with *positive*, *negative*, or *nearly zero*.

► If a vector $\vec{V}$ is directed roughly perpendicular to a direction $\hat{x}$, then the numerical component V_x is ________________

If $\vec{V}$ is directed roughly along $\hat{x}$, V_x is ________________

If $\vec{V}$ is directed roughly opposite to $\hat{x}$, V_x is ________________

(Answer: 3) (Practice: text problem B-3.)

b-9 *SUMMARY:* To find the numerical component V_x of any vector $\vec{V}$ along a direction $\hat{x}$, first construct the component vector $\vec{V}_{\|}$ and express $\vec{V}_{\|}$ as a number times $\hat{x}$. This number is the numerical component V_x. Thus V_x can be positive, negative, or zero, depending on whether $\vec{V}$ and $\hat{x}$ have roughly the same, roughly opposite, or perpendicular directions.

Now: Go to text problem B-1.

TUTORIAL FOR E

COMPARING PARTICLE MOTION NEAR THE EARTH'S SURFACE (Cap. 4)

e-1 *A METHOD FOR COMPARING MOTION NEAR THE EARTH'S SURFACE:* To help you develop the capability of comparing the motion of two particles influenced only by the earth's gravity, this tutorial section describes and illustrates a systematic method for making such comparisons. This method includes these steps:

Description Describe the problem, listing the known and desired information, and drawing a picture showing the paths of both particles.

Solution On your picture, indicate the position of each particle at each of the times of interest. You may then be able to find the desired information. If not, use one of the following descriptions of motion for a particle influenced only by the earth's gravity.

(1) The horizontal component of such a particle's velocity is constant. Thus the horizontal distance traveled during a time interval is just the magnitude of this component multiplied by the time interval.

(2) If two such particles initially have the same vertical components of velocity and position, then their vertical motion thereafter is identical. Thus, after any given time interval, the two particles have the same height, and the same vertical component of velocity.

e-2 *ILLUSTRATION OF THE METHOD:* We illustrate the method outlined in the preceding frame by applying it to this problem: An inexperienced baseball player hits a ball vertically upward so that it reaches its maximum height of 20 meter at a time 2 sec after being hit, and strikes the horizontal ground surface 4 sec after being hit. He then hits a second ball from the same vertical position and with the same vertical component of velocity. But this ball has an initial horizontal component of velocity with magnitude 15 m/s. Use the description of the first ball to answer the following questions about the second ball. At what time after being hit does this ball reach its maximum height? What is its maximum height? At what horizontal distance from the batter does this ball strike the ground surface?

DESCRIPTION

► *Known*

Ball 1: Maximum height is 20 meter.

Reaches maximum height after 2 sec.

Strikes ground after 4 sec.

Ball 2: Initial vertical components of position and velocity are the same as those of ball 1.

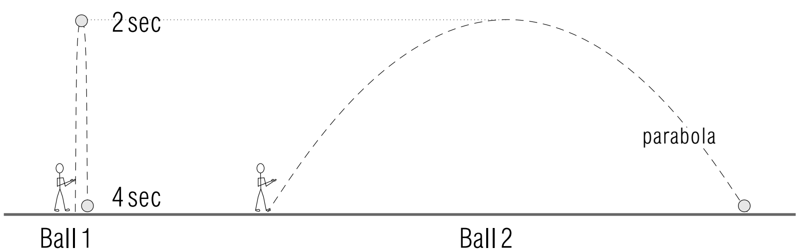
Initial horizontal component of velocity has magnitude 15 m/s.

► *Desired*

Time for ball 2 to reach maximum height

Maximum height of ball 2

Horizontal distance ball 2 travels from the batter



SOLUTION

Use the preceding drawing to indicate the positions of ball 2 2 sec and 4 sec after being hit. Label these positions by the corresponding times.

►

At what time after being hit does ball 2 reach its maximum height? What is its maximum height?

► time of maximum height: ________________

maximum height: ________________

To find the distance from the batter at which ball 2 strikes the ground, remember that the horizontal component of this ball's velocity has a constant magnitude of 15 meter/sec during the 4 sec before it strikes the ground.

What is the horizontal distance from the batter at which ball 2 strikes the ground?

► horizontal distance: ________________

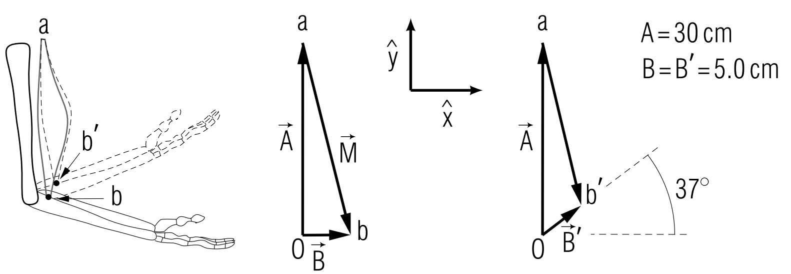
(Answer: 17)

Now: Go to text problem E-1.

TUTORIAL FOR G

BIOLOGICAL APPLICATIONS OF COMPONENT DESCRIPTIONS

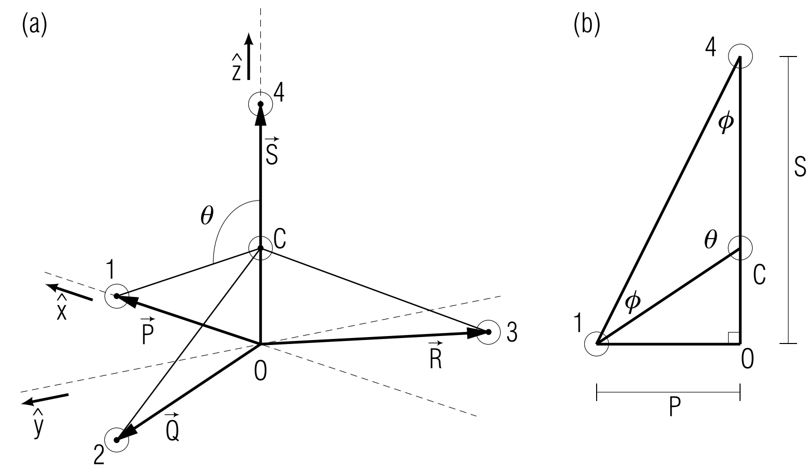
g-1 *RELATING MOTION TO CHANGE IN MUSCLE LENGTH:* Motion of a limb can be produced rapidly if it involves only a small change in muscle length. For example, a small change d in the length of the biceps muscle produces the $37°$ folding of the forearm indicated in the following drawing. The change d is equal to the difference between the initial and final muscle lengths M and M'. To find this difference, consider the displacements $\vec{M}$ and $\vec{M}'$ from the upper to the lower end of the biceps for the two forearm positions. The vectors $\vec{M}$ and $\vec{M}'$ are related known displacements $\vec{A}$, $\vec{B}$, and $\vec{B}'$, from the elbow to the muscle endpoints. (a) Express $\vec{M}$ in terms of $\vec{A}$ and $\vec{B}$, and $\vec{M}'$ in terms of $\vec{A}$ and $\vec{B}'$.



To find the change $d = M - M'$, we need the magnitudes M and M', which can be found using component descriptions. (b) Express $\vec{A}$, $\vec{B}$,and $\vec{B}'$ as sums of their component vectors relative to the coordinate system shown. (c) Use the expressions found in part (b) and the equations found in part (a) to write expressions for $\vec{M}$ and $\vec{M}'$ in terms of their component vectors relative to this coordinate system. (d) What are the magnitudes M and M'? (e) What is the change d in the length of the biceps required to produce the motion shown in the preceding drawing? *(Answer: 15) (Suggestion: [s-4])*

g-2 *THE STRUCTURE OF METHANE:* The structure of methane (shown in the following drawings) is completely symmetric, so that there is an equal distance between any pair of the four hydrogen atoms (1, 2, 3, and 4) surrounding the carbon atom (C). The angle θ between any two of the carbon-hydrogen bonds is useful in determining the structure

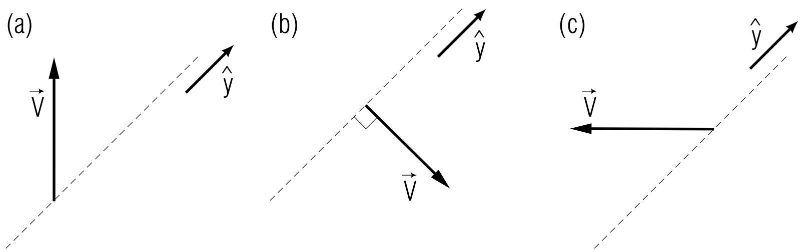
of carbon compounds. To find this angle, we use component descriptions of vectors relative to the coordinate system shown, which has its origin O at the center of the equilateral triangle formed by atoms 1, 2, and 3. The coordinate directions are: $\hat{z}$ upward from O through atom 4, $\hat{x}$ horizontal from O through atom 1, and $\hat{y}$ horizontal and perpendicular to $\hat{x}$ and $\hat{z}$. The vectors $\vec{P}$, $\vec{Q}$, $\vec{R}$, and $\vec{S}$ are the position vectors of the four hydrogen atoms relative to the origin O. Part b of the following drawing shows the triangle with a right angle at the origin O and vertices at the hydrogen atoms 1 and 4. We shall find the angle θ by using the magnitudes S and P to find the angle ϕ in the isosceles triangle formed by atoms 1, 4, and C. Then we can find θ by using $\phi + \theta = 180°$.



Suppose the position vectors $\vec{P}$, $\vec{Q}$, and $\vec{R}$ all have magnitude 2.0 Å. Since the distance between any pair of hydrogen atoms is the same, the distance $|\vec{D}| = |\vec{P} - \vec{Q}|$ between 1 and 2 is equal to the distance $|\vec{D}'| = |\vec{P} - \vec{S}|$ between 1 and 4. (a) Express $\vec{P}$, $\vec{Q}$, and $\vec{S}$ as sums of their component vectors, expressing $\vec{S}$ in terms of its magnitude S. (b) Find the value of $|\vec{D}|$, and express $|\vec{D}'|$ in terms of $\vec{S}$. (Write your answers as square roots.) (c) Use the relation $|\vec{D} = |\vec{D}'|$ to find the magnitude of $\vec{S}$. (d) Use the right triangle in part b of the preceding drawing to find the angle ϕ. (e) What is the angle θ? *(Answer: 16) (Suggestion: [s-12])*

PRACTICE PROBLEMS

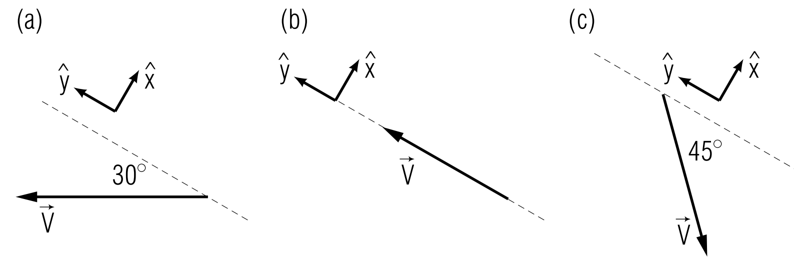
p-1 *CONSTRUCTING COMPONENT VECTORS (CAP. 1):* Navigating a plane in the wind requires calculating the effect of the air's motion relative to the ground. This is most easily done by decomposing the velocity $\vec{V}$ of the air (or wind) relative to the ground into component vectors parallel and perpendicular to the desired direction of travel. Suppose a plane is to fly northeast along the unit vector $\hat{y}$ in these diagrams:



For each of the wind velocities $\vec{V}$ shown in these diagrams, construct the component vectors $\vec{V}_{\parallel}$ and $\vec{V}_{\perp}$ of $\vec{V}$ parallel and perpendicular to $\hat{y}$. *(Answer: 7) Now: Return to text problem A-1 and make sure your work is correct.*

p-2 *UNDERSTANDING THE DEFINITION OF NUMERICAL COMPONENT (CAP. 2):* For practice on this capability, use text problems B-1 through B-3.

p-3 *FINDING COMPONENT EXPRESSIONS FOR VECTORS (CAP. 3):* In order to describe the motion of a sailboat, it is convenient to use coordinate directions $\hat{x}$ and $\hat{y}$ parallel and perpendicular to the wind direction. The sailboat velocities shown in the following diagrams each have the magnitude 10 m/s.

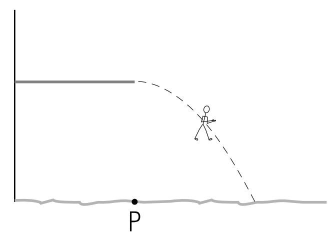


Express each $\vec{v}$ as a sum of its component vectors relative to these coordinate directions, and then state the components v_x and v_y of $\vec{v}$ along $\hat{x}$ and $\hat{y}$. *(Answer: 6) (Suggestion: review your work in text problems C-1 and C-2.)*

p-4 *FINDING VECTORS FROM THEIR COMPONENT DESCRIPTIONS (CAP. 3):* The velocity of a star (relative to the center of our galaxy) is found from numerical components along directions $\hat{x}$ and $\hat{y}$ parallel and perpendicular to a line joining the star and the earth. The numerical component $v_y$along $\hat{y}$ is found by observing a change in the position of the star in photographs taken several years apart. The numerical component v_x along $\hat{x}$ can be inferred from measurements of the light emitted from the star.

Relative to the coordinate directions in this drawing: the star "ξ Herculis" has a velocity $\vec{v}$ with numerical components: $v_x = -18$ km/sec and $v_y = 24$ km/sec. What is the magnitude v of this star's velocity? Draw an arrow roughly indicating the direction of $\vec{v}$ on this drawing: *(Answer: 12) (Suggestion: review your work in text problems C-3 and C-4.)*

p-5 *COMPARING MOTION NEAR THE EARTH'S SURFACE (CAP. 4):* Three boys run off a high diving board, falling into the pool below as shown in this drawing:

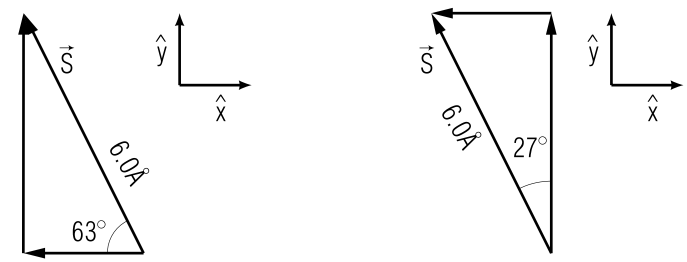


Tom just steps off the board, so that his initial velocity (as he leaves the board) is zero, and he falls vertically downward striking the water after a time of 1 sec. Dick runs off the board so that he has an initial horizontal velocity of magnitude 2.0 meter/sec. Harry runs off the board so that his initial velocity is horizontal, but with a magnitude of 4.0 meter/sec. (a) At what time after leaving the diving board do Dick and Harry strike the water? (b) At what horizontal distance from the point P below the end of the diving board do Dick and Harry enter the water? *(Answer: 1) (Suggestion: review your work in text problems E-1, E-2, and E-3.)*

SUGGESTIONS

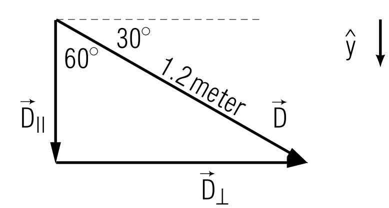
s-1 *(Text problem A-2):* Part (a): We wish to decompose $\vec{v}$ into parts parallel and perpendicular to $\hat{y}$. But since $\vec{v}$ is *parallel* to $\hat{y}$, it has no part perpendicular to $\hat{y}$, i.e., $\vec{v}_\perp = \vec{0}$. Correspondingly, the "part" of $\vec{v}$ parallel to $\hat{y}$ is just $\vec{v}$ itself. Part (b): Draw a diagram showing $\vec{v}\prime$ and $\hat{y}$. Then use the same reasoning applied to part (a).

s-2 *(Text problem C-2):* Both of the following sketched diagrams show the component vectors of the position vector $\vec{S}$ relative to the coordinate system provided.



Using either diagram, express each component vector as a number times $\hat{x}$ or $\hat{y}$. These numbers (which can be found using trigonometry) are just the numerical components of $\vec{S}$ along $\hat{x}$ and $\hat{y}$. (If you need further help, review your work in tutorial frame [b-4].)

s-3 *(Tutorial frame [b-4]):* Let us illustrate with the first example how to use trigonometry to find numerical components. We first sketch arrows representing $\vec{D}$ and its component vectors parallel and perpendicular to $\hat{y}$.



To find the desired magnitude of $|\vec{D}_\parallel|$, use the definition:

$$\cos 60^\circ = \frac{|\vec{D}_{\parallel}|}{|\vec{D}|}$$

Thus: $|\vec{D}_{\parallel}| = |\vec{D}|\cos 60^\circ$. Since $\vec{D}_{\parallel}$ and $\hat{y}$ have the same direction, D_y is positive. If you need further help, review your work in tutorial frame [b-1].

s-4 *(Tutorial frame [g-1]):* Part (c): Use the expressions for $\vec{M}$ and $\vec{M}\prime$ from part (a) and substitute values found in part (b). For example:

$$\vec{M}\prime = \vec{B}\prime - \vec{A} = [(4\,\text{cm})\hat{x} + (3\,\text{cm})\hat{y}] - [(30\,\text{cm})\hat{y}] = (4\,\text{cm})\hat{x} + (-27unitcm)\hat{y}$$

Part (d): Sketch diagrams showing the component vectors of $\vec{M}$ and $\vec{M}\prime$ [e.g., $(4\,\text{cm})\hat{x}$ and $(-27\,\text{cm})\hat{y}$], and add them to construct arrows representing $\vec{M}$ and $\vec{M}\prime$. Use the Pythagorean theorem to find the magnitudes M and M'.

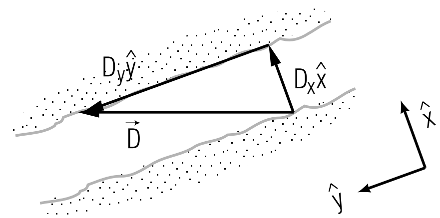
s-5 *(Text problem A-3):* Follow the same procedure you used in problems A-1 and A-2. For example, through the tail of A, draw a line parallel to the direction southeast, and through the tip of $\vec{A}$ draw a line perpendicular to this direction. If you need help with vectors $\vec{B}$ or $\vec{D}$, review your work in problem A-2, reading suggestion [s-1] if necessary.

s-6 *(Text problem D-2):* Part (a): Draw each vector individually and construct its component vectors along the coordinate directions.

Part (b): First add the numerical components found in part (a) to find L_x and L_y. Then make a drawing showing the component vectors $L_x\hat{x}$ and $L_y\hat{y}$, and add these vectors to find $\vec{L}$. Use the Pythagorean theorem to find the magnitude $|\vec{L}|$.

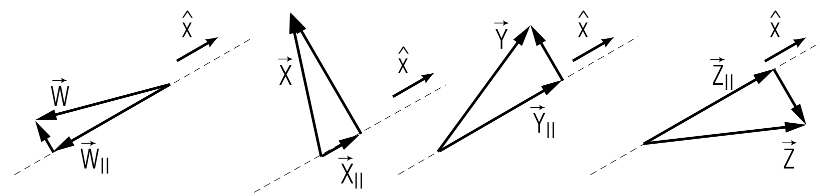
s-7 *(Text problem G-1):* Part (a): As discussed in text section D, if two vectors are equal ($\vec{V} = \vec{D}/T$), then their numerical components are equal: $V_x = D_x/T$ and $V_y = D_y/T$.

Part (b): The following sketch roughly indicates the boat's displacement $\vec{D}$ and component vectors of $\vec{D}$ parallel to $\hat{x}$ and $\hat{y}$.



But according to this drawing, D_x is just the width of the river. Write an equation relating D_x and V_x . Then find T.

s-8 *(Tutorial frame [b-7]):* The following diagrams show the component vector parallel to $\hat{x}$ for each of the vectors described in tutorial frame [b-7].



(1) Determine the sign of each of these numerical components along $\hat{x}$:

► W_x: ____, X_x: ____, Y_x: ____, Z_x: ____

(2) Which of these numerical components has the smallest magnitude?

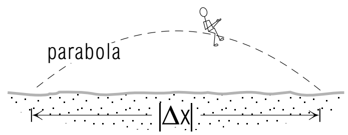
► W_x, X_x, Y_x, Z_x

(3) Which numerical component has the smallest value? (Remember that negative numbers are smaller than positive numbers.)

► W_x, X_x, Y_x, Z_x

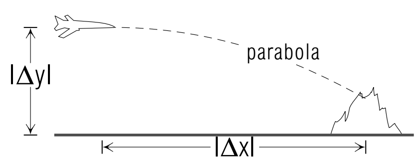
(Answer: 9) Now: Return to tutorial frame [b-7].

s-9 *(Text problem G-3):* We consider the broad jumper as a particle projected from the ground surface. Use the results of text problem G-2 to write an expression for his range $|\Delta x|$ in terms of the numerical components v_{Ax} and v_{Ay} of his initial velocity and the magnitude of the gravitational acceleration.

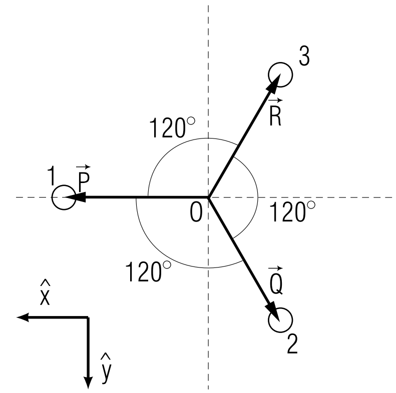


s-10 *(Text problem E-3):* Consider any two particles moving subject only to gravitational interaction with the earth. If these particles have the same initial vertical components of position and velocity, then their subsequent vertical motion is identical. If these particles have the same initial horizontal components of position and velocity, then their subsequent horizontal motion is identical. This is true even if the particles have very different masses, because the gravitational acceleration $\vec{g}$ is independent of all particle properties.

s-11 *(Text problem G-4):* The horizontal component of the canister's velocity has the constant magnitude v_0. Thus the horizontal distance $|\Delta x|$ traveled by the canister during the time Δt before it strikes the ground is just $|\Delta x| = v_0(\Delta t)$. The vertical distance $|\Delta y|$ traveled by the canister during the time Δt is just the same *as if* the canister had been released with an initial velocity of zero. Thus $|\Delta y| = |(g/2)(\Delta t)^2|$. Combine these equations to eliminate the unknown time Δt, and then solve for $|\Delta x|$.



s-12 *(Tutorial frame [g-2]):* The following drawing shows the plane containing the origin O and the three hydrogen atoms 1, 2, and 3. Using this drawing you should be able to express the vectors $\vec{P}$ and $\vec{Q}$ as sums of their component vectors parallel to $\hat{x}$ and $\hat{y}$.

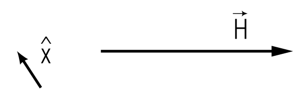


s-13 *(Text problem A-1):* Let us systematically carry out the procedure described in the text for constructing component vectors.

(1) Through the tail of $\vec{H}$ on the following diagram, draw a line *parallel* to $\hat{x}$. Then through the tip of $\vec{H}$ draw a line *perpendicular* to $\hat{x}$. Extend these lines to intersect in a right angle at a point R.

(2) Draw and label the component vector $\vec{H}_{\parallel}$ which begins at the tail of $\vec{H}$ and ends at the intersection R. Draw and label $\vec{H}_{\perp}$ which begins at the tip of $\vec{H}_{\parallel}$ and ends at the tip of $\vec{H}$.

►



Check your work by making sure that:

(1) The original vector $\vec{H}$ is the hypotenuse (longest side) of the right triangle formed by $\vec{H}$ and its component vectors.

(2) The vector $\vec{H}_{\parallel}$ is parallel to $\hat{x}$, and $\vec{H}_{\perp}$ is perpendicular to $\hat{x}$.

(Answer: 4) Now: Now: Go to practice problem [p-1].

ANSWERS TO PROBLEMS

1. a. after 1 sec for both

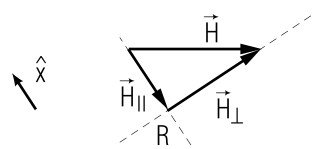
   b. 2.0 m for Dick, 4.0 m for Harry

2.

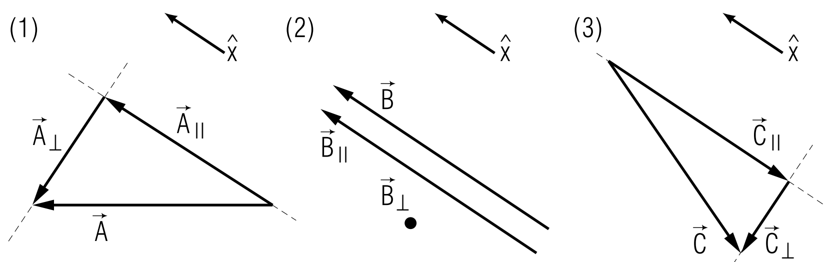
	vector	magnitude	component vector	numerical component
Vector or number?	vector	number	vector	number
Possible signs:		$+, 0$		$+, 0, -$
Algebraic symbol:	$\vec{V}$	$V,\ \lvert\vec{V}\rvert$	$\vec{V}_{\parallel}$	V_x
Unit:	m/s	m/s	m/s	m/s

3. nearly zero, positive, negative.

4.



5. Correct constructions:



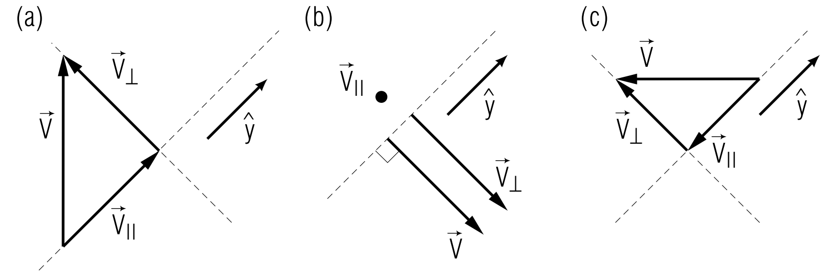
Given constructions are incorrect because: (1) $\vec{A}_\perp$ is not perpendicular to $\hat{x}$; (2) $\vec{B}_\parallel$ and $\vec{B}_\perp$ are not parallel and perpendicular to $\hat{x}$; (3) $\vec{C}$ is not the sum of $\vec{C}_\parallel$ and $\vec{C}_\perp$.

6. a. $\vec{v} = (-5.0\,\text{m/s})\hat{x} + (8.7\,\text{m/s})\hat{y}$, $v_x = -5.0\,\text{m/s}$, $v_y = 8.7\,\text{m/s}$

   b. $\vec{v} = (10\,\text{m/s})\hat{y}$, $v_x = 0$, $v_y = 10\,\text{m/s}$

   c. $\vec{v} = (-7.1\,\text{m/s})\hat{x} + (-7.1\,\text{m/s})\hat{y}$, $v_x = -7.1\,\text{m/s}$, $v_y = -7.1\,\text{m/s}$

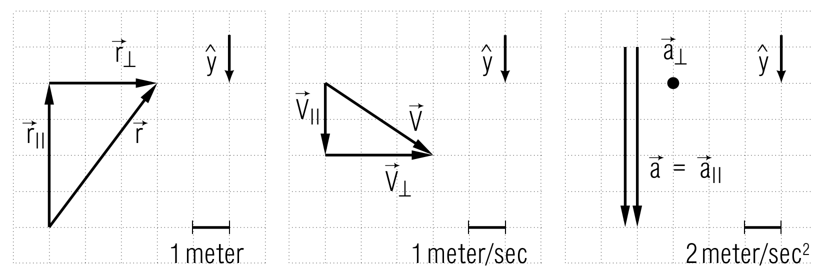
7.



8. (1) 1.0×10^3 meter; (2) 3.0×10^3 meter; (3) $(-2.8 \times 10^3\,\text{meter})\hat{x}$

9. (1) $W_x : -$, $X_x : +$, $Y_x : +$, $Z_x +$ (2) X_x (3) W_x

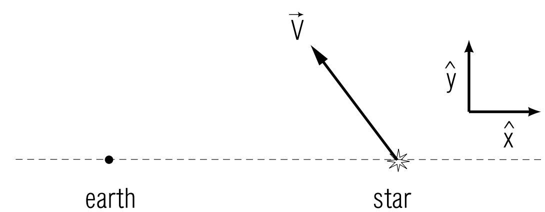
10.



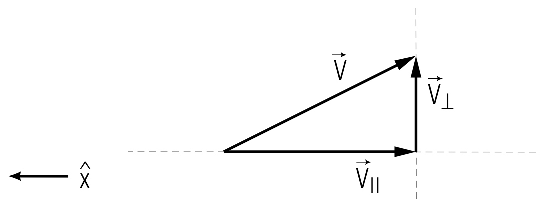
$\vec{r}_\parallel = (-4\,\text{m})\hat{y}$, $r_y = -4\,\text{m}$, $\vec{v}_\parallel = (2\,\text{m/s})\hat{y}$, $v_y = 2\,\text{m/s}$, $\vec{a}_\parallel = (10\,\text{m/s}^2)\hat{y}$, $a_y = 10\,\text{m/s}^2$

11. (1) 0.60 meter (2) -0.24 meter

12. $v = 30$ kilometer/sec



13.



Either of the following: (1) V_x has the same magnitude as $\vec{V}_{||}$, and V_x is positive or negative according to whether $\vec{V}_{||}$ has the same or opposite direction as $\hat{x}$. (2) Express $\vec{V}_{||}$ as a number times $\hat{x}$. Then that number is the numerical component V_x.

14. (1) $\vec{Y}$ and $\vec{Z}$ (2) $\vec{W}$ (3) $\vec{X}$

15. a. $\vec{M} = \vec{B} - \vec{A}$, $\vec{M}\prime = \vec{B}\prime - \vec{A}$

    b. $\vec{A} = (30\,\text{cm})\hat{y}$, $\vec{B} = (5\,\text{cm})\hat{x}$, $\vec{B}\prime = (4\,\text{cm})\hat{x} + (3\,\text{cm})\hat{y}$

    c. $\vec{M} = (5\,\text{cm})\hat{x} + (-30\,\text{cm})\hat{y}$, $\vec{M}\prime = (4\,\text{cm})\hat{x} + (-27\,\text{cm})\hat{y}$

    d. $M = \sqrt{925\,\text{cm})} \approx 30\,\text{cm}$, $M' = \sqrt{745}\,\text{cm}) \approx 27\,\text{cm}$

    e. $d = 3\,\text{cm}$

16. a. $\vec{P} = (2.0\,\text{Å})\hat{x}$, $\vec{Q} = (-1.0\,\text{Å})\hat{x} + (1.7\,\text{Å})\hat{y}$, $\vec{S} = S\hat{z}$

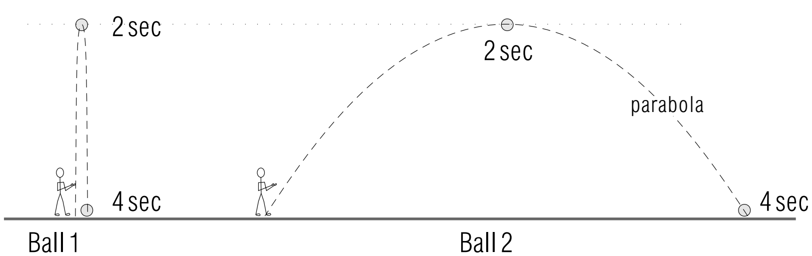
    b. $|\vec{D}| = \sqrt{(3.0\,\text{Å})^2 + (1.7\,\text{Å})^2} = \sqrt{12}\,\text{Å}$, $|\vec{D}\prime| = \sqrt{(2.0\,\text{Å})^2 + S^2} = \sqrt{4.0\,\text{Å} + S^2}$

    c. $S = \sqrt{12 - 4.0}\,\text{Å} = 2.8\,\text{Å}$

    d. $\tan\phi = P/S = 2.0/2.8$, $\phi = 36°$

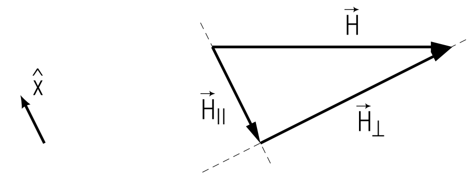
    e. $\theta = 108°$

17.



time of maximum height: 2 s; maximum height: 20 m; horizontal distance: 60 m.

101.



$\vec{H}_{||}$ is parallel to $\hat{x}$. $\vec{H}$ is the hypotenuse.

102. a. 25 m/s

     b. −25 m/s

     c. 0

103. a. $\vec{v}$ has component vectors 60 m/s east and 25 m/s south.

     b. 65 m/s, southeast

104. a. $\vec{v}_{||} = \vec{v} = (2\,\text{m/s})\hat{y}$, $\vec{v}_\perp = 0\hat{x}$

     b. $\vec{v}\prime_{||} = 0\hat{y}$

105. a. $A_y = -3$ meter, $B_y = -1$ meter, $C_y = 1$ meter, $D_y = 3$ meter

     b. positive, negative

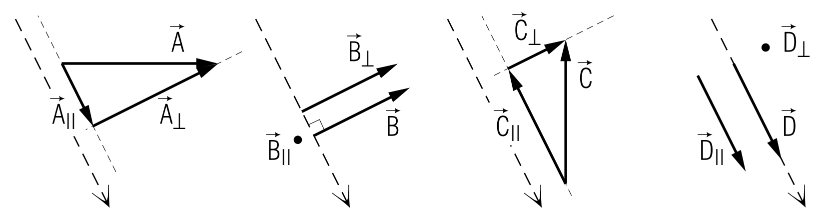
     c. The vector roughly along $\hat{y}$

106. a. $\vec{A} = (4\,\text{meter})\hat{x}$, $\vec{B} = (7\,\text{meter})\hat{x} + (4\,\text{meter})\hat{y}$

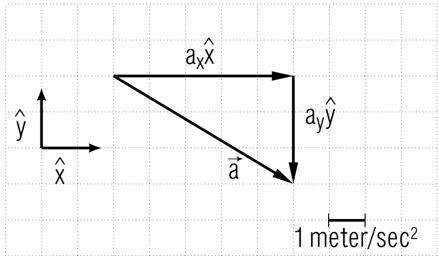
     b. $\vec{D} = (3\,\text{meter})\hat{x} + (4\,\text{meter})\hat{y}$

     c. $D = 5$ meter

107.



108. a.



b. $\vec{a} = (5\,\text{m/s}^2)\hat{x} + (-3\,\text{m/s}^2)\hat{y}$

c. $a_x = 5\,\text{m/s}^2$, $a_y = -3\,\text{m/s}^2$

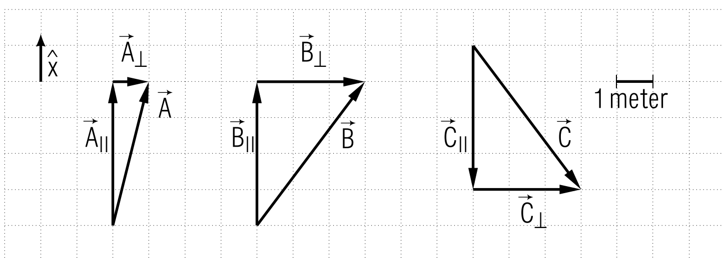
109. a. $P_x = Q_x = R_x = 2\,\text{Å}$, $P_y = 1.0\,\text{Å}$, $Q_y = -1.0\,\text{Å}$, $R_y = 1.0\,\text{Å}$

b. $L_x = 6.0\,\text{Å}$, $L_y = 1.0\,\text{Å}$, $|\vec{L}| = \sqrt{37}\,\text{Å} \approx 6.1\,\text{Å}$

c. $R'_x = 0$, $R'_y = -2.2\,\text{Å}$

d. $L'_x = 4.0\,\text{Å}$, $L'_y = -2.2\,\text{Å}$, $|\vec{L}\prime| = \sqrt{21}\,\text{Å} \approx 4.6\,\text{Å}$

110.



a. $\vec{A}_{||} = (4\,\text{meter})\hat{x}$, $A_x = 4\,\text{meter}$

b. $\vec{B}_{||} = (4\,\text{meter})\hat{x}$, $B_x = 4\,\text{meter}$, $\vec{C}_{||} = (-4\,\text{meter})\hat{x}$, $C_x = -4\,\text{meter}$

c. $\vec{A} \neq \vec{B}$, $A_x = B_x$

d. $\vec{B} \neq \vec{C}$, $B_x \neq C_x$

111. a. $\vec{C} = (-5.3\,\text{Å})\hat{x}$, $\vec{S} = (-2.7\,\text{Å})\hat{x} + (5.3\,\text{Å})\hat{y}$

b. $C_x = -5.3\,\text{Å}$, $C_y = 0$, $S_x = -2.7\,\text{Å}$, $S_y = 5.3\,\text{Å}$

112. a. Same

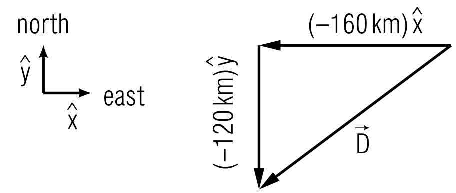
b. Same

c. No. Velocities have different horizontal components.

113. $\Delta y = 0$, $|\Delta x| = |2v_{Ax}v_{Ay}/g|$, 180 meter

114. (a.) $\vec{D} = (-160\,\text{km})\hat{x} + (-120\,\text{km})\hat{y}$

b.



c. $D = 200\,\text{km}$, southwest

115. a. $V_x = D_x/T$, $V_y = D_y/T$

b. $T = 0.5\,\text{hour}$

c. $D_y = 2\,\text{mile}$

116. 1.4 sec, 7.0 meter

117. Their motions are identical, and both reach the same height.

118. $|\Delta x| = v_0\sqrt{(2|\Delta y|/g)}$, $|\Delta x| = 400\,\text{meter}$

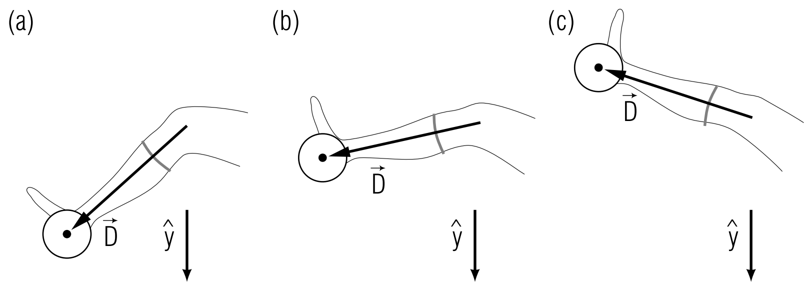
119. a. $v = 8.0 \times 10^3\,\text{m/s}$

b. $T = 5.0 \times 10^3\,\text{sec} = 83\,\text{minute}$

120. $|\Delta x'|/|\Delta x| = g/g'$, farther in Mexico City

MODEL EXAM

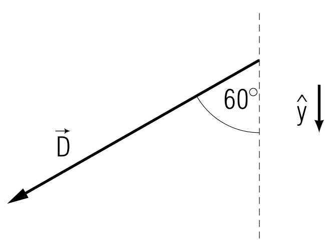
1. **Component description for an exercise device.** A "deLorme boot" with weights attached (sketched in the following drawings) is commonly used to increase the effort required to do exercises involving the leg and knee. If $\vec{D}$ is the displacement from the knee to the heel, the effort required to hold the foot (with the boot) at rest depends on the numerical component of $\vec{D}$ along the downward direction $\hat{y}$. Each of the following drawings shows the leg of one patient, and so the vector $\vec{D}$ has the same magnitude in each drawing.



   a. For each of the positions in the preceding drawings, is the numerical component of $\vec{D}$ along $\hat{y}$ positive, negative, or zero?

   b. Compare the values of this numerical component for positions (a) and (b). For which position is this component larger?

   Suppose the vector $\vec{D}$ has a magnitude of 0.6 meter and the angle between $\vec{D}$ and $\hat{y}$ is 60° as shown in this drawing:



   c. Construct arrows representing the component vectors of $\vec{D}$ parallel and perpendicular to $\hat{y}$.

   d. What is the value of the numerical component of $\vec{D}$ along $\hat{y}$?

2. **Motion of stones thrown from a bridge.** A boy and his sister throw stones at the same time from a bridge. The boy throws his stone vertically downward with an initial velocity of magnitude 3.0 m/s. The sister throws her stone at an angle to the vertical, so that her stone leaves her hand with the same initial *vertical* component velocity as her brother's, but with an initial *horizontal* component velocity of magnitude 4.0 m/s. Both stones have the same initial height above the water surface. The boy's stone strikes the water surface with a speed of 13 m/s at a time of 1.0 sec after he throws it. Assume the stones interact only with the earth while going through the air.

   a. At what time after she throws it does the sister's stone hit the water?

   b. What are the *magnitudes* of the horizontal and vertical components of the velocity of the sister's stone as it hits the water?

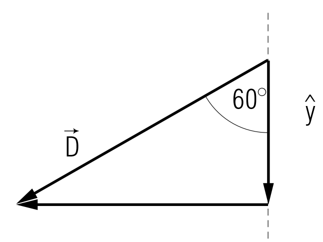
   c. At what *horizontal* distance from the point at which it left her hand does the sister's stone hit the water?

Brief Answers:

1. a. (a) positive (b) positive (c) negative

   b. (a)

   c.



   Both arrows, one parallel, one perpendicular to $\hat{y}$, with sum equal to $\vec{D}$.

   d. 0.3 meter

2. (a) 1.0 sec; (b) horizontal: 4.0 m/s; vertical: 13 m/s; (c) 4.0 meter