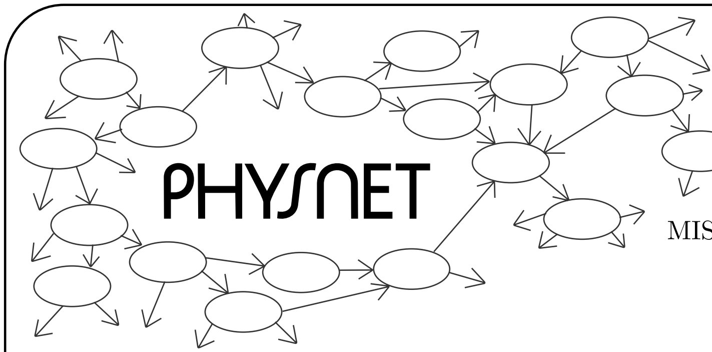
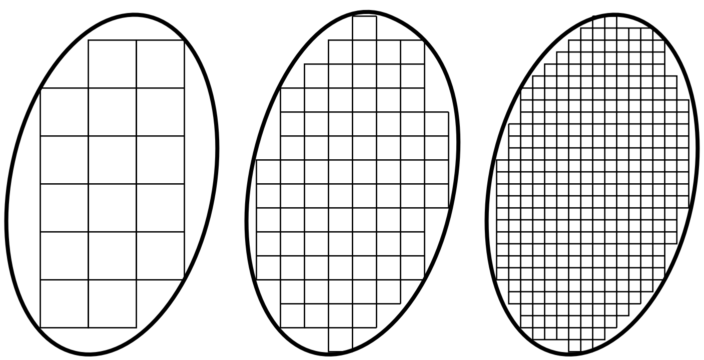


MISN-0-403

PHYSICAL DESCRIPTION AND MEASUREMENT



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

PHYSICAL DESCRIPTION AND MEASUREMENT

by

F. Reif, G. Brackett and J. Larkin

CONTENTS

Title: **Physical Description and Measurement**

Author: F. Reif, G. Brackett and J. Larkin, Department of Physics, University of California, Berkeley.

Version: 4/30/2002 Evaluation: Stage 0

Length: 1 hr; 40 pages

Input Skills:

1. Given two numbers, express them in scientific notation and calculate their sum, difference, product, and/or quotient (MISN-0-401).

Output Skills (Knowledge):

K1. Vocabulary: length, time, clock, period of a clock, standard, unit, basic standards, basic units, variable, function, independent variable, dependent variable.

K2. State the condition of unit consistency.

K3. Describe the SI system of units. State how many basic units it has and give two examples of how these units are defined.

K4. Define random errors and systematic errors and give an example of each.

K5. State the difference between the value and the magnitude of a number.

Output Skills (Problem Solving):

S1. Express the value of a given quantity in terms of any specified set of units.

S2. Given an equation, determine whether its units can be made consistent.

S3. Given the units of all quantities but one in an equation, find the units of that one.

S4. Given an arithmetic expression, calculate its correct value with the appropriate number of significant digits and the units in simplest form.

S5. Given two signed numbers, state which has the larger value and which has the larger magnitude.

THIS IS A DEVELOPMENTAL-STAGE PUBLICATION
OF PROJECT PHYSNET

The goal of our project is to assist a network of educators and scientists in transferring physics from one person to another. We support manuscript processing and distribution, along with communication and information systems. We also work with employers to identify basic scientific skills as well as physics topics that are needed in science and technology. A number of our publications are aimed at assisting users in acquiring such skills.

Our publications are designed: (i) to be updated quickly in response to field tests and new scientific developments; (ii) to be used in both classroom and professional settings; (iii) to show the prerequisite dependencies existing among the various chunks of physics knowledge and skill, as a guide both to mental organization and to use of the materials; and (iv) to be adapted quickly to specific user needs ranging from single-skill instruction to complete custom textbooks.

New authors, reviewers and field testers are welcome.

MISN-0-403

PHYSICAL DESCRIPTION AND MEASUREMENT

Abstract:
Since science deals with observations, the minimum requirement for any scientific progress is a symbolic language capable of describing observations conveniently and unambiguously. Indeed, some of the most far-reaching scientific advances (such as the relativity and quantum theories) originated from the realization that the description of some seemingly simple observations had been inadequate. In the present unit we shall discuss some general methods of description useful throughout all the sciences. This discussion is intended to review, and to define more precisely, some basic concepts which are probably already familiar from previous experience.

SECT.

A QUANTITATIVE DESCRIPTION

The aim of description is to make apparent similarities and differences between observations by assigning the same symbol to observations which have common features and different symbols to observations which have different features. To avoid ambiguities, any procedure used to associate symbols with observations must be operationally well defined, i.e., it must specify precisely what one must actually *do* in order to assign particular symbols to particular observations.

The symbols used for description may be words, called "properties," which can be assigned to something observable according to some specified procedure. (For example, the word "long" might be assigned to a car if it extends beyond both ends of a particular curbstone next to which the car may be parked.) Such a "qualitative" description in terms of properties is useful, but may often be inadequate because it is not sufficiently detailed. (For example, both a passenger car and a trailer truck might be designated by the same word "long" according to our definition of this property.)

A description is "precise" or "detailed" if any distinction which one wishes to make between observations can be represented by a corresponding distinction in the description of these observations. Such a precise description can be achieved by using as symbols numbers as well as words. We call a "quantity" a property to which it is possible to assign a number (or a set of numbers) according to some specified procedure. The procedure whereby this number is assigned is called a "measurement." (Familiar examples of such quantities are the number of bacteria in a drop of water or the length of an object.)

MEASUREMENT PROCEDURES

When some observable thing has distinct features, the quantity called the "number of these features" can be obtained by merely counting the features. Thus it is quite straightforward to count the number of fingers on a hand or the number of bacteria in a drop of water. The number obtained as a result of such a counting process is always some integer.

When one deals with a property which is not characterized by discrete features (e.g., with the length of some object), a number can be assigned to this property by specifying some comparison procedure which involves indirectly some counting process. The following two sections will illustrate such comparison procedures by discussing the two quantities called "length" and "time" which are of the most fundamental importance in all the sciences.

SECT.

B LENGTH

Consider a line (curved or straight) joining two points P and P'. We wish to describe this line by a quantity called its "length" (or the "distance" along the line). To refine our intuitive notion of length, we shall now specify an operationally clearly defined procedure for assigning a meaning and a number to this word "length." (This procedure is suggested by the common way of using a meter stick in everyday life.)

To carry out the procedure, we shall compare the line with a measuring instrument which we choose to be a rigid rod. The comparison can then be carried out in the following way by making repeated observations of the coincidence between two points in space: We place the rod so that one of its ends coincides with the beginning point P of the line and so that its other end also lies on the line. Then we continue in a similar way by placing the rod repeatedly end-to-end next to the line, as shown in Fig. B-1a. Finally, we count the complete number N of times that the rod can be placed in this way end-to-end next to the line without extending beyond the end point P' of the line.

[Suppose that we used the preceding procedure with a rod which is "shorter," i.e., which can be placed end-to-end next to the line a larger number of times (as illustrated in Fig. B-1b). Then the end point of such a rod, when placed next to the line the last time, is likely to coincide more closely with the end point P' of the line.]

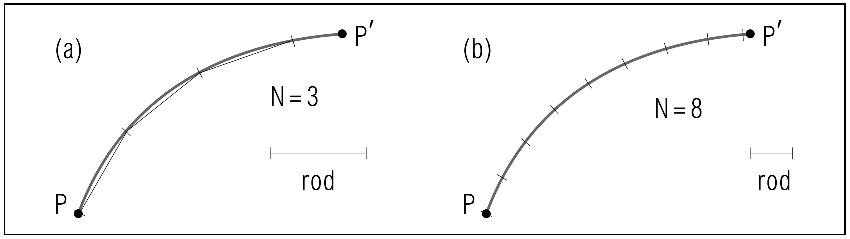


Fig. B-1: Measurement of a line with different rods. (a) Measurement with a rod which can be laid end-to-end $N = 3$ times. (b) Measurement with a shorter rod for which $N = 8$. (Only the successive positions of the ends of the rod are indicated.)

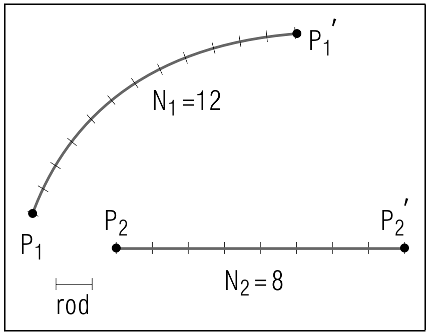


Fig. B-2: Comparison of the lengths of two lines with a measuring rod. Only the successive positions of the ends of the rod are indicated in the diagram. If the rod is short enough for the desired precision of measurement, $L_1/L_2 = 12/8 = 1.5$.

Now that we have described how to compare a line with any measuring rod, we can specify the following procedure for comparing the "lengths" L_1 and L_2 of any two lines 1 and 2 (such as the line 1 between P_1 and P_1' and the line 2 between P_2 and P_2' in Fig. B-2): Take any measuring rod. Count the number N_1 of times that this rod can be placed end-to-end next to line 1 and the number N_2 of times that it can be placed end-to-end next to line 2. Compare these measurements by calculating the ratio N_1/N_2. Repeat the preceding procedure with successively shorter rods (so that the numbers N_1 and N_2 become successively larger) until the value of the ratio N_1/N_2 remains unchanged (within the desired precision) if we use any shorter rod. Then we *define* the lengths L_1 and L_2 of the lines to be such that the ratio L_1/L_2 is equal to this final value of the ratio N_1/N_2. This definition is illustrated in Fig. B-2.)

To summarize, the quantity called "length" (and denoted by the symbol L) is defined to be such that the ratio of the lengths of any two lines is obtained by the preceding measurement procedure according to this specification:

$$\frac{L_1}{L_2} = \frac{N_1}{N_2} \quad \text{(when } N_1 \text{ and } N_2 \text{ are large enough)} \qquad \text{(B-1)}$$

This definition of length is an operational definition since it describes precisely what one must *do* to determine the ratio of any two lengths.

STANDARD OF LENGTH

Since the definition of length involves a comparison procedure, it specifies merely the *ratio* of the lengths of any two lines, but does not specify a unique value for the length of any one line. This ambiguity can be eliminated if one agrees always to measure the lengths of all lines by

comparison with the *same* particular object S (called the "standard" of length) whose length can be simply denoted by the algebraic symbol L_S.

To facilitate communication between all people, it is desirable that everyone in the world agrees about the choice of this standard. Such international agreements are nowadays reached by scientific meetings which select with great care a standard which has as many desirable properties as possible. (For example, a standard should be indestructible and should allow one to make precise measurements.)

Until a few years ago, international convention specified as the standard of length a carefully preserved metal bar kept in a vault near Paris. This standard was called the "standard meter bar" and its length L_S was denoted by the algebraic symbol "meter" by writing: L_S = meter.*

> * Note that the whole word "meter" is regarded as a single algebraic symbol, just as is L_S.

(The length of this standard meter bar is roughly the same as that of a stick a yard in length.) More recently, the length denoted by "meter" has been redefined as the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.[1] However, the redefinition of the meter is such that the old standard meter bar is still extremely close to 1 meter in length when compared to the new standard.[2]

The preceding discussion of the operational definition of length should enable you to answer these questions:

B-1 *Illustration: measuring area by comparison:* Using the definition of length as a model, we can construct a definition of area by describing how to compare the areas A_1 and A_2 of two regions of surfaces. To compare these areas, choose a very small square and lay it side by side as many times as possible within each of the regions 1 and 2, counting the number N_1 of times the small square can be placed within the region 1 and the number N_2 of times the small square can be placed within the region

[1] See `http://physics.nist.gov/cuu/Units/current.html`.

[2] from PEN, August 1998: IT'S ABOUT A YARD. The first official definition of the meter was decreed by the new Republican Government of France in August 1793 as, one-ten-millionth of the distance between the North Pole and the Equator. This rather pedestrian (terrestrial?) definition of the meter has been displaced by the mighty photon, so that now we conventionally define the meter as the length of the path traveled by light in a vacuum in 1/299,792,458 of a second. The intellectual distance between 1793 and 1998 is the subject of a Web site hosted by the National Institutes of Standards and Technology. This Web site tells the tale of the tape, so to speak, in narrative and chronological form, tracing the history of the standard meter. To learn more go to http://www.mel.nist.gov/div821/museum/length.htm.

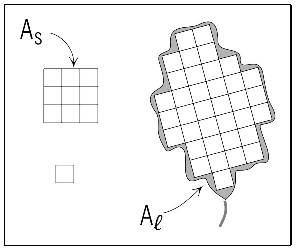


Fig. B-3: A peanut leaf with area A_l, a standard square with area A_s, and a small square used for comparing these areas.

2. Then the areas A_1 and A_2 are defined so that $A_1/A_2 = N_1/N_2$, where the numbers N_1 and N_2 are large enough that replacing our small square by a still smaller square will not change the value of the ratio N_1/N_2 within the precision we desire. Using this definition, compare the area A_l of the peanut leaf shown in Fig. B-3 with the area A_s of the standard square shown by finding the ratio A_l/A_s. For the precision desired here, the small square shown is small enough. *(Answer: 101)*

Operational Specification of Measurement

B-2 A dictionary defines the temperature of an object as "the degree of warmth or coolness of the object." Is this an operational specification of measurement? Briefly explain why or why not. *(Answer: 105)*

SECT. C TIME

Consider any two events occurring at the same place. We wish to describe the relationship between these events by a quantity called the "time" between these events. To refine our intuitive notion of time, we need again to specify a clearly defined procedure for assigning a meaning and a number to the word "time."

To carry out the procedure we shall compare the two events with a measuring instrument called a "clock" and defined as follows:

Def. **Clock**: Any device having a configuration (i.e., a measurable characteristic) which recurs repetitively. (C-1)

For instance, a wrist watch is a clock because its rotating big hand points recurrently to the same position (e.g., to the number 12 on the dial). The earth rotating about its axis is also a clock since one can observe the recurrent daily passages of the sun across the highest point on the sky.

To measure the time between two events E and E' occurring at the same place, we use a clock located at this place. We then carry out the following comparison procedure involving the observation of the simultaneity (i.e., temporal coincidence) of events: When the first event E occurs, we note the configuration of the clock. We then simply count the number N of subsequent repetitions of this configuration until the occurrence of the second event E' as indicated for $N = 2$ in Fig. C-1.

[Suppose that we used the preceding procedure with a clock which is "faster," i.e., which repeats its same configuration a larger number of times (see the $n = 6$ case, Fig. C-1). Then the initial configuration of the clock is likely to occur more nearly simultaneously with the second event E'.]

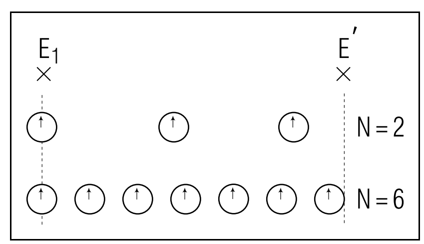


Fig. C-1: Measurement of the time between two events with different clocks. First, measurement with a clock which repeats its configuration $N = 2$ times. Second, measurement with a faster clock for which $N = 6$.

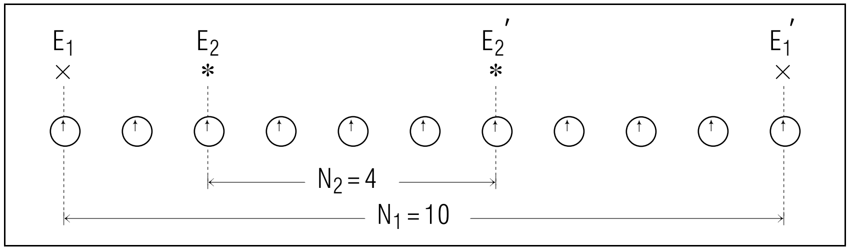


Fig. C-2: Comparison of two times with a clock. If the clock is fast enough for the desired precision of measurement, the time T_1 between the events E_1 and E_1' and the time T_2 between the events E_2 and E_2' are such that $T_1/T_2 = 10/4 = 2.5$.

Now that we have described how to compare the time between two events with any clock, we can specify the following procedure for comparing the "time" T_1 between two events E_1 and E_1' with the "time" T_2 between two events E_2 and E_2':
(i) Take any clock.
(ii) Count the number N of successive repetitions of this clock between the events E_1 and E_1' and the number N_2 of successive repetitions of this clock between the events E_2 and E_2'. (See Fig. C-2.)
(iii) Compare these measurements by calculating the ratio N_1/N_2.
(iv) Repeat the preceding procedure with a series of faster clocks (so that the numbers N_1 and N_2 become correspondingly larger) until the value of the ratio N_1/N_2 remains unchanged (within the desired precision) if we use any faster clock.
(v) Then we *define* the times T_1 and T_2 between these pairs of events to be such that the ratio T_1/T_2 is equal to this final value of the ratio N_1/N_2.

To summarize, the quantity called the "time" between two events (and denoted by the symbol T) is defined to be such that the ratio of the times between any two pairs of events is obtained by the preceding measurement procedure according to this definition:

$$\frac{T_1}{T_2} = \frac{N_1}{N_2} \text{ (when } N_1 \text{ and } N_2 \text{ are large enough).} \qquad \text{(C-2)}$$

This definition of time is an operational definition since it describes precisely what one must *do* to determine the ratio of any two times.

STANDARD OF TIME

Since the definition of time involves a comparison procedure, it specifies merely the *ratio* of any two times, but does not specify a unique value for any one time. This ambiguity can be eliminated if one agrees always to measure all times by comparison with the same particular clock S (called the "standard" of time). The "period" of this clock may then be simply denoted by the algebraic symbol T_S. Here the word "period" has this meaning:

Def. | **Period of a clock**: The time between successive repetitions of the same configuration of the clock. | (C-3)

Although the choice of a standard is basically arbitrary, it is desirable to choose a standard which has, as far as possible, the following properties: (a) Measurements with this standard should lead to a simple description of observed phenomena. (b) The standard should permit one to make precise measurements. (c) The standard should be permanent and should be readily available.

CHOICE OF DESIRABLE STANDARDS

The first of the preceding properties is the most important and can be illustrated by the actual choice of the standard of time. Until a few years ago, the clock chosen as this standard was the earth rotating about its axis. But then one began to compare this clock with many other clocks, including various very precise "atomic clocks" which rely on repetitive phenomena occurring within atoms or molecules. It was then found that all these other clocks speed up when compared to the earth (i.e., that all their periods become gradually smaller when compared to that of the earth), although they remain synchronized when compared to each other (i.e., the ratio of their periods remains unchanged). It then became apparent that the world can be described more simply by choosing a particular atomic clock as the standard of time. For then all the other clocks which we have considered remain always synchronized with this atomic clock. Only the earth seems then anomalous since it gradually slows down compared to the atomic clocks (by nearly one second per year).

When the earth rotating about its axis was adopted as the standard of time, the period T_S of the earth (i.e., the time between successive passages of the sun across the highest point of the sky) was assigned various algebraic symbols such as "day" or "second" defined so that

$$T_S = 1\,\text{day} = 86,400\,\text{second}.* \qquad \text{(C-4)}$$

* These definitions conform with the traditional use of familiar names used as algebraic symbols and defined so that day = 24 hour, hour = 60 minute, minute = 60 second.

Since the earth is not the most desirable standard of time, an atomic clock using cesium atoms has recently been internationally adopted as the standard of time.[3] Although the "second" is now defined in terms of this new standard, this redefinition has been made so that the period of rotation of the earth is still 86,400 second to an excellent approximation.

REMARK ON THE INADEQUACIES OF OUR DEFINITIONS

Although we have seemingly been quite careful in defining the basic concepts of length and time, we have failed to consider some important situations. In particular, we have *not* defined explicitly how to measure the time between two events occurring at *different* places, nor even what we mean by saying that two such events occur "at the same time." At present we shall merely *assume* that these notions can be adequately defined (e.g;, by using radio signals to compare clocks located at different places). But this assumption is questionable and will ultimately cause difficulties in making precise predictions or in describing motion with very high speeds. We shall then be forced to provide an adequate definition for the time between events occurring at different places. Indeed, this definition is the basic foundation of Einstein's theory of relativity and has very far-reaching implications.

[3]See http://physics.nist.gov/cuu/Units/current.html.

SECT. D

STANDARDS AND UNITS

All quantities (such as length or time) which are measured as a result of a comparison procedure have these properties: (1) They must involve a specification of the standard used in making the comparison, and (2) they must have some error associated with them (since no actual comparison can ever be carried out with unlimited precision). We shall discuss the specification of standards in this section and shall make some comments about errors in the next section.

As we have seen in the case of lengths or times, a comparison procedure merely specifies the *ratio* of the values of two quantities of the same kind. One can, however, agree to choose one particular object as a "standard" and then to specify the value of every other quantity by comparison with this object. Thus we introduce this definition:

Def. **Standard**: A standard for some quantity is a particular object with which all such quantities are to be compared. (D-1)

The quantity specified by the standard can be assigned a value denoted by an algebraic symbol called a "unit" defined as:

Def. **Unit**: An algebraic *symbol* denoting a quantitative value defined by a standard. (D-2)

For example, once the meter was chosen as the standard of length, its length L_S may be expressed in terms of various conveniently chosen algebraic symbols (such as "meter, " "centimeter," "inch") all of which are called "units of length." For example, the length L_S is assigned the values L_S = meter or L_S = 100 centimeter. Hence these units are related so that*

$$\text{meter} = 100\,\text{centimeter} \qquad \text{(D-3)}$$

* A unit, such as "meter," has no plural form in this book; we write "6.1 meter," not "6.1 meters." When one multiplies "6.1" by "x," one never writes the result as "6.1 xs" but rather as "6.1 x." Thus when one multiplies "6.1" by "1 meter," one should write the result as "6.1 meter." Nevertheless, almost all scientists and engineers say "6.1 meters."

The measuring procedure used to compare a quantity with a standard always determines a ratio. For example, one might find

$$\frac{L}{L_S} = 5.2$$

where L is the length of some line and L_S is the length of the standard meter. Then one can also write

$$L = 5.2 L_S \text{ or } L = 520\,\text{centimeter}$$

if one uses the definition $L_S = 100\,\text{centimeter}$ to express L_S in terms of the unit centimeter. Thus we see that any quantity measured by comparison with a standard can always be expressed as a "pure number" (i.e., a number without any associated units) multiplied by some unit corresponding to this standard. Note that the specification of the unit is essential to avoid ambiguity. A statement such as $L = 520$ is meaningless since it does not specify what kind of comparison was used to arrive at the number 520.

Since the quantity specified by a standard can be expressed in terms of various units (e.g., $L_S = meter = 100\,\text{centimeter}$), any quantity can be expressed in terms of various units. Since units are merely algebraic symbols, one can use the rules of algebra to convert between these units.

Example D-1: Conversion between units

The length of a certain line is $L = 2.7\,\text{centimeter}$. What is this length expressed in terms of the unit "meter"?

Since the units meter and centimeter are related so that meter $= 100\,$centimeter, one can solve this equation for centimeter to obtain

$$\text{centimeter} = 0.01\,\text{meter}.$$

Then one finds by direct substitution that

$$L = 2.7\,\text{centimeter} = 2.7(0.01\,\text{meter}) = 0.027\,\text{meter}.$$

BASIC STANDARDS AND UNITS

Any scientific discussion usually involves various different kinds of quantities which can be measured by comparison with several different

standards. One can then choose a set of "basic standards" according to this definition:

Def. **Basic standards**: A particular set of standards which can be chosen independently of each other and which are sufficient to permit the measurement of all quantities of interest. (D-4)

With each such standard one can then associate some "basic unit."

Def. **Basic unit**: The basic unit corresponding to a basic standard is a particularly chosen unit defined in terms of this standard. (D-5)

All other standards and units are then said to be "derived" from these basic standards and units. For example, centimeter, meter^2, and meter/second are all units derived from the basic units "meter" and "second."

The basic standards and units adopted by the most recent international agreement is called the "SI system" of standards and units. (SI stands for Systeme International.) In order to meet all scientific needs, this system has adopted seven convenient basic standards and corresponding basic units which are described in detail in the appendix. For example, the SI basic standard of length is the krypton atom and the SI basic unit of length is the "meter" defined in terms of a particular wavelength of light emitted by such a krypton atom. The basic SI standard of time is an atomic clock using the cesium atom and the basic SI unit of time is the "second" defined in terms of the period of this particular clock.

All other standards and units (including the old British units still used in the United States) can be related to the basic SI standards and units. (For example, when expressed in terms of SI units, a speed of 45 mile/hour $\approx$ 20 meter/second.) We shall use primarily SI units throughout this book.

QUANTITIES INDEPENDENT OF STANDARDS AND UNITS

Some quantities can be measured and assigned unambiguous numerical values *without* the need to use any standards. For example, this is the case of any quantity whose value can be obtained by direct computing. (When we say that a hand has 5 fingers, the word "fingers" is not a unit since no standard is required to count the number of fingers.) Similarly, the *ratio* of any two quantities of the same kind can be determined by direct comparison with each other without the need to use a standard or any units associated with such a standard. For example, the *ratio* L_1/L_2 of two lengths can be measured by direct comparison without a standard and is thus a pure number independent of any units. Even if we did use a standard to measure these lengths (thus finding values such as $L_1 = 6\,\text{meter}$ and $L_2 = 3\,\text{meter}$), the ratio $L_1/L_2 = (6\,\text{meter})/(3\,\text{meter}) = 2$ is properly a pure number since the unit meter has disappeared from this ratio.

DEPENDENCE OF EQUATIONS ON UNITS

Any equation, such as $A = B$, asserts that the quantities A and B are always equal. Such a generally valid relationship must remain true independently of what particular standards (or associated units) happen to be chosen for the measurement of A and B.*

> * This conclusion is also apparent by noting that $A = B$ implies that $A/B = 1$, which is a pure number independent of any units.

Suppose that both A and B are expressed in terms of the same set of basic units (e.g., SI units). Then A and B can each be written as some pure number multiplied by some combination of basic units. The equality $A = B$ then implies that the pure numbers on both sides of the equation must be equal and that the basic units appearing on both sides of the equation must also be equal. This conclusion about units can be summarized by this statement:

> *Condition of unit consistency*: Both sides of an equation must be expressible in terms of the same combination of basic units. (D-6)

This condition provides a very useful way of checking equations or of finding correct equations. For example, any lack of consistency between basic units on both sides of an equation immediately indicates that the equation must be wrong. Similarly, if a quantity (such as A) consists of a sum or difference of several terms, each of these terms must have the same combination of basic units since the condition of unit consistency could otherwise not be satisfied. The discussion of units and standards in this section should enable you to acquire these capabilities:

Knowing About Standards, Units, and SI Units

D-1 *Standards and units:* Which of the following phrases describes a standard and which a unit? (a) a thing used for comparison in measuring all quantities of a particular kind; (b) a symbol for a quantity; (c) a thing which should be permanent, indestructible, and readily accessible; (d) a symbol which is manipulated using the rules of algebra. *(Answer: 109)*

D-2 *SI units:* (a) Which of these values is expressed in terms of SI units: $50\,\text{mile/hour}$, $10\,\text{second}^{-1}$, $10\,\text{meter/second}$? (b) Which of these lengths is closest to your height: $20\,\text{meter}$, $2\,\text{meter}$, $0.2\,\text{meter}$? *(Answer: 113)*

Finding Values for Quantities (Cap. 1)

D-3 The volume flow rate F of blood through parallel capillaries is given by the equation $F = nas$, where n is the number of capillaries, a is the cross-sectional area of each capillary, and s is the average speed of blood in the capillaries. For the messentary (tissue supporting the intestine) of a dog, these quantities have the approximate values: $n = 1.0 \times 10^9$, $a = 5.0 \times 10^{-5}\,\text{millimeter}^2$, $s = 9.0 \times 10^{-3}\,\text{millimeter/minute}$. (a) Find the value of the flow rate F, using the units millimeter and minute. (b) Use the relations $\text{millimeter} = 1.0 \times 10^{-1}\,\text{centimeter}$ and $\text{minute} = 60\,\text{second}$ to express F using the more common units centimeter and second. *(Answer: 116) (Suggestion: [s-6])*

D-4 If an object travels with constant speed around a circle of radius R, the speed s of this object is given by the relation $s = 2\pi R/T$, where $\pi = 3.14$ and T is the time required for the object to travel once around its path. A test tube in a medical centrifuge travels once around a circle of radius $18\,\text{centimeter}$ in a time of $3.0 \times 10^{-4}\,\text{minute}$. Find the value of the speed of the test tube, expressing your answer in terms of SI units. *(Answer: 102) (Suggestion: [s-1])*

More practice for this Capability: [p-1]

Unit Consistency In Equations (Cap. 2)

The following problems illustrate the use of unit consistency to check an equation or an expression of which you are unsure.

D-5 Suppose you wish to find the flow rate R of pollutant particles through a smokestack. You remember that R is related to the cross-sectional area A of the stack, the speed S of the rising air, and the number N of particles per unit volume of air, but you cannot remember which of the following equations is correct: $R = AS/N$ or $R = ASN$. The units of the relevant quantities are: $R(\text{sec}^{-1}$ or $1/\text{sec})$, A(meter^2), S(meter/sec), N(meter^{-3} or $1/$ meter^3). Which of the two equations is correct? *(Answer: 106) (Suggestion: [s-4])*

D-6 The cost of a cylindrical blood collection bottle depends on its surface area S, while its volume V determines the amount of blood it can hold. If the collection bottle has height H and radius R, one of the following expressions equals the surface area S, one equals the volume V and one expression is meaningless. (Remember that $\pi = 3.14$). (a) $2\pi RH + 2\pi R^2$, (b) $2\pi R^2 H + 2\pi R$, (c) $\pi R^2 H$. Identify the correct expressions for S and V, and briefly describe why the remaining expression is meaningless. *(Answer: 110) (Suggestion: [s-3])*

More practice for this Capability: [p-2]

SECT.

E ERRORS

No measurement by a comparison procedure can ever be carried out with unlimited precision. (For example, in a measurement of length the observation of the coincidence between points can only be made with some residual uncertainly.) Repeated measurements of the same quantity under seemingly identical conditions lead thus to numbers which are not exactly the same, but which differ from each other slightly in some unpredictable way. The extent of the difference between such repeated measurements is called the "random error" involved in the measurement. This random error can be reduced, but never completely eliminated, by refining the method of observation (e.g., by using a magnifying glass to observe the coincidence between points in a measurement of length). However, the counting of separate, distinct objects results in an integer and there should be no uncertainty whatsoever about such a value.

Our inability to measure with infinite precision certainly creates an uncertainty about the "true" values of measured quantities, but there exist more subtle sources of error. For example, suppose that comparison of the length L of some object with a meter stick leads to the result $L = 1.473$ meter. Suppose now that a later check of this meter stick against a standard meter showed that the length of this meter stick is really 1.002 meter rather than 1.000 meter. Then the value $L = 1.473$ meter would be "inaccurate" (no matter how precisely it has been measured) because this value is not equal to the "true" value which one claims to be measuring. The "systematic error" of a measured quantity is the difference between the measured value of this quantity and the value one claims to be measuring. The magnitude of the systematic error can be estimated by checking the extent of consistency obtained when the same quantity is measured by significantly different methods.

The total error associated with any measured quantity consists ordinarily both of random and systematic errors, so both of them must be kept in mind. (For example, it would be foolish to use elaborate instrumentation to reduce the random error of a length measurement to 10^{-6} meter when one suspects systematic errors as large as 10^{-3} meter.)

An adequate specification of any measured quantity X requires thus not only a specification of its numerical value (and associated units), but also a specification of the error associated with this quantity. It is customary to specify this information by writing

$$X = E \pm e \qquad \text{(E-1)}$$

where E is the best estimate of the quantity X and where e is the "probable error" associated with this quantity. Statement (E-1) usually implies that there is about a two-thirds probability that the true value of X lies between $E-e$ and $E+e$. The quantity e/E is called the "relative error" or "fractional error" in the quantity X; it is useful since it compares the magnitude of the error relative to the magnitude of the quantity itself. (For example, an error of 1 centimeter in a length of 10,000 centimeters may be quite negligible, while an error of 1 centimeter in a length of 2 centimeters may be of substantial importance.)

Example E-1: Specification of error

The statement that length $L = (4.783 \pm 0.002)$ meter carries the implication that the true value of L has a two-thirds probability of lying between 4.781 meter and 4.785 meter. The probable error is then 0.002 meter and the relative error is $0.002/4.783 = 4 \times 10^{-4}$ (or 0.04 percent).

If one is satisfied with an approximate specification of the error, one may omit the explicit statement of the error and specify the error implicitly by the number of digits retained in stating the numerical value of the quantity. The digits thus retained, other than zeros to the left of the first non-zero digit, are called "significant figures." It is then implied that the last digit retained is correct and any uncertainty is in the next (unstated) digit. For example, the statement $X = 0.064$ implies that X is known to two significant figures and that its true value is between 0.0635 and 0.0645.[4]

If zeros are merely used to locate the decimal point, they may give a misleading impression of the magnitude of the error. For example: if $X = 400 \pm 50$ but one omits the "± 50" then the statement $X = 400$ would seem to imply the $X = 400 \pm 0.5$. To avoid such difficulty of interpretation, one should write numbers using scientific notation. For example, we can write $X = 4.0 \times 10^2$ to indicate two significant digits or $X = 4.00 \times 10^2$ to indicate three significant digits.

[4] John R. Taylor, *An Introduction to Error Analysis,* University Science Books, Mill Valley, CA (1982).

REMARK ON CALCULATIONS

When performing calculations with numbers which have errors associated with them, one must be careful that the final quoted result does not give a misleading impression of precision, i.e., one must "round off" the final result by discarding digits which imply an excessive precision. These guidelines are useful: When adding or subtracting quantities, the final result should not have more digits after the decimal point (i.e., "decimal places") than the quantity which has the smallest number of such digits (since this quantity has the largest error). When multiplying or dividing quantities, the final result should not have more significant figures than the quantity which has the smallest number of significant figures. In performing all calculations, it is usually safest to retain for every quantity more than (or all of) the digits which are ultimately needed and then to discard the excess digits in the final answer. (This avoids the possibility of introducing additional errors as a result of the calculation itself.)

Using these guidelines you should develop the habit of always expressing values in correct form, i.e., as a number with an appropriate number of significant figures multiplied by a unit in simplest form. Thus you should acquire these capabilities:

Significant Figures and Decimal Places

E-1 What is the number of significant figures and the number of decimal places in each of these values: (a) 1.2, (b) 1.20, (c) 0.012, (d) 0.0120? *(Answer: 114) (Suggestion: [s-5])*

Stating Values In Correct Form (Cap. 3)

E-2 Find and state in correct form the values of each of these expressions: (a) $(1.2 \times 10^9\,\text{meter}) + (2.2 \times 10^8\,\text{meter})$; (b) $(9\,\text{meter/sec}) - (1.7\,\text{meter/sec})$; (c) $(2.0\,\text{meter/sec})/(3.0 \times 10^1\,\text{sec})$; (d) $(2.50 \times 10^3\,\text{meter/sec}) \times (1.20 \times 10^{-6}\,\text{sec})$; (e) $(0.21\,\text{meter}) \times (4.50\,\text{meter})$. *(Answer: 103) ([s-7], [p-3])*

E-3 Suppose that the distance D' is the shortest distance between points on the surfaces of the earth and moon. (See Fig. E-1.) This distance can be measured using the relation $D' = ct/2$, where t is the travel time of a radar pulse which leaves the earth, is reflected by the moon, and then returns to the earth, c is the speed of the radar pulse,

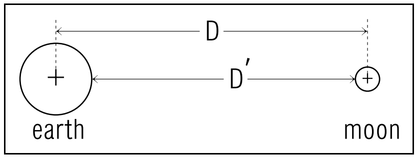


Fig. E-1: The distance D' between the nearest points on the earth and moon, and the distance D between the centers of the earth and moon. (This drawing is *not* to scale.)

and 2 is the counted number of times the radar pulse travels through the distance D'. (a) Use the typical values $c = 3.00 \times 10^8$ meter/sec and $t = 2.562$ sec to find the value of D'. (b) The distance D between the *centers* of the earth and moon is related to distance D' by $D = D' + R_e + R_m$, where $R_e = 6 \times 10^6$ meter is the radius of the earth, and $R_m = 1.7 \times 10^6$ meter is the radius of the moon. What is the value of D? (c) Suppose we were to use the distance D' as an approximation for D, thus neglecting the radii of the earth and moon. What is the error $D - D'$ in this procedure? (d) What is the relative error $(D - D')/D$ in this procedure? *(Answer: 107) (Suggestion: [p-4])*

SECT.

F RELATIONSHIPS BETWEEN QUANTITIES

The measurement of various quantities allows one to compare them in detail and thus to discover relationships existing between them. Let us examine briefly the description of such relationships.

The particular number (including units) assigned to any quantity is called the "value of this quantity". This value may also include a specification of sign, i.e., the value may be positive or negative. (For example, if x denotes the elevation of some place above sea level, the value of x is considered positive if this place is above sea level and negative if this place is below sea level. Thus the elevation of Mount Everest is $x = 8850$ meter while that of the Dead Sea is $x = -395$ meter.)

The "magnitude of a quantity" x is denoted by $|x|$ and is defined as the *positive* number obtained by disregarding any negative sign attached to this number. (For example, the magnitude of -8 meter is 8 meter.) Thus it is important to distinguish carefully between the *value* and the *magnitude* of a quantity.

When comparing the values of two quantities, it is important to pay proper attention to their signs as well as to their magnitudes. (The geometrical representation of numbers as points along a line, indicated in Fig. F-1, may be helpful in visualizing such comparisons.) For example, if $x_1 = -6$ meter and $x_2 = +3$ meter, the *value* of x_1 is smaller than the value of x_2, although the *magnitude* of x_1 is larger than the magnitude of x_2 (since 6 meter is larger than 3 meter).

A quantitative comparison between two numbers is most conveniently made by calculating their ratio. In particular, the ratio $|x_1|/|x_2|$ is called the "relative magnitude" of x_1 compared to x_2. (For example, a statement that $|x_1|/|x_2| = 3$ implies that the magnitude of x_1 is 3 times as large as that of x_2.)

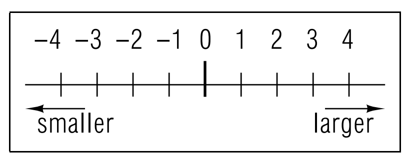


Fig. F-1: Geometrical representation of numbers as points along a line.

FUNCTIONAL RELATIONSHIPS BETWEEN QUANTITIES

A quantity which can assume various possible values is called a "variable." Most of science is concerned with explaining or predicting what relationships exist between such variables (e.g., in predicting how the concentration of a chemical substance changes with time). We shall say that two variables are "functionally related" (or "mutually dependent") if there exists for every value of one of the variables a corresponding value (or set of values) of the other variable. One may choose to focus special attention on one of these variables, which is then called the "independent variable" (or simply the "variable"). The other variable is then called the "dependent variable" (or simply the "function") which depends on the first variable.

It is important to note that the information available about observable quantities is necessarily always of limited precision and accuracy. Furthermore, it is also necessarily always incomplete (e.g., one can never measure the temperature of some object at *all* instants of time, but only at some selected times). Hence any functional relationship between observable quantities is always the *best estimate* describing this relation with some specified precision for all values of interest.

Various familiar methods are commonly used to describe the relationship between some variable t and some function x which depends on this variable. For example, corresponding values of the variable t and the function x can be displayed in a table (as illustrated in Table F-1). Alternatively, they can be represented by points in a graph (such as the points in Fig. F-2). The curve in the graph represents then the *best estimate* of the functional relationship for *all* values of the variables in a certain range (e.g., for values of t between 0.3 second and 1.1 second in Fig. F-2). The graphical representation makes the relationship between x and t immediately apparent in a readily visualized way, but does not allow as precise a description as that possible by means of many-digited numbers in a table.

If the relationship between x and t is sufficiently simple, it may also be summarized compactly by a formula which specifies explicitly how to calculate the value of x corresponding to every value of t. (For example, the best estimate of the functional relationship represented by Table F-1 or the graph in Fig. F-2 can be summarized by the formula $x = at - bt^2$ where $a = 8$ meter/second and $b = 5$ meter/second^2.)

t second (±0.02)	x meter (±0.05)
0.3	1.91
0.4	2.43
0.5	2.76
0.6	2.97
0.7	3.19
0.8	3.23
0.9	3.10
1.0	3.04

Table F-1: Height x above the ground of a ball observed at various times t.

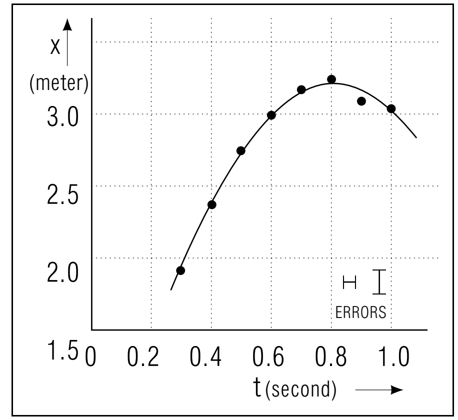


Fig. F-2: Graph showing the height x of a ball at various times t.

The discussion of magnitudes and values in this section should enable you to demonstrate these capabilities:

Finding and Comparing Magnitudes (Cap. 4)

F-1 Determine the magnitudes of each of the quantities in quotes in this sentence: On a day when the temperature is "-30 degree Fahrenheit," a man deposits a check bringing his bank balance from "-760 dollar" to "760 dollar." *(Answer: 111)*

F-2 For each of the following pairs of values, state which *value* is larger, and then state which value has the larger *magnitude*. (a) A car driving out of Death Valley travels from an elevation of -250 *foot* to an elevation of *50 foot*. (b) A physician conducting yearly physical exams notes changes in his patients' weight (as measured by his office scale). The change in a man's weight is *10 pound* while the change in his wife's weight is -7 *pound*. *(Answer: 104)*

F-3 *Illustration of relative magnitudes:* A physician may become concerned about a weight change when the magnitude of this change relative to the weight of the person is large. If the man described in the preceding problem weighs 200 pound and his wife weighs 100 pound, what is the magnitude of each person's weight change, relative to his or

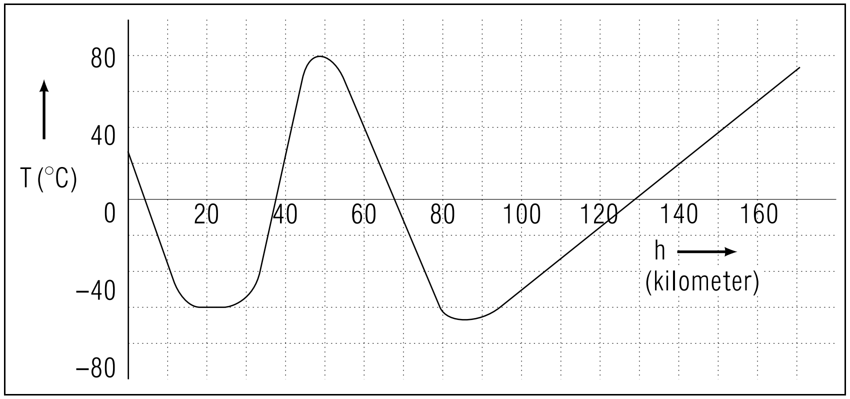


Fig. F-3: A graph showing the temperature T of the atmosphere at various heights h above the earth's surface.

her weight. *(Answer: 108) (Suggestion: [s-2])*

F-4 *Review: Using graphs to describe relationships:* The following questions concern the functional relationship described by the graph in Fig. F-2. (a) What is the algebraic symbol representing the function (the dependent variable) and that representing the independent variable? (b) What are the units of each of these variables? *(Answer: 112)*

F-5 Use the graph in Fig. F-3 to answer these questions: (a) What is the value of the function T corresponding to the value $h =$ 80 kilometer? (b) For each of the following regions of h, describe T as h increases by writing that T is *increasing, decreasing, constant*, or *none of these*: values of h between 0 and 10 kilometer, between 20 and 25 kilometer, between 70 and 100 kilometer, between 90 and 100 kilometer. *(Answer: 115)*

SECT.

G SUMMARY

DEFINITIONS

length (Sec. B)

time (Sec. C)

clock; Def. (C-1)

period of a clock; Def. (C-3)

standard; Def. (D-1)

unit; Def. (D-2)

basic standards; Def. (D-4)

basic units; Def. (D-5)

IMPORTANT RESULTS

Condition of unit consistency: rule (D-6)

Both sides of an equation must be expressible in terms of the same combination of basic units.

USEFUL KNOWLEDGE

operational definitions (Sects. B and C)

SI units (Sec. D)

random and systematic errors (Sec. E)

distinction between value and magnitude (Sec. F)

functional relationships and their description (Sec. F)

NEW CAPABILITIES

You should have acquired this module's capabilities:

(1) Manipulate units algebraically in order to express the value of any quantity in terms of any specified set of units. (Sec. D, [p-1])

(2) Determine whether the units of an equation are consistent and to use a correct equation to find the units of any quantity appearing in the equation. (Sec. D, [p-2])

(3) Habitually state the value of any quantity with the appropriate number of significant figures and with units in simplest form. (Sec. E, [p-3], [p-4])

(4) Compare two signed numbers by stating which has the larger value and which has the larger magnitude. (Sec. F)

PRACTICE PROBLEMS

p-1 *THE SI UNIT OF SPEED (CAP. 1):* When using SI units, we express speeds using the unit meter/sec. To develop some familiarity with this unit, let us express some common speeds in terms of meter/sec and in terms of mile/hour. Use the relations mile = 1600 meter and hour = 3600 second. (a) A person can stride a little less than a meter, so a typical walking speed is about 2.0 meter/sec. Express this speed in terms of mile/hour. (b) Express a typical automobile speed of 54 mile/hour in terms of meter/sec. *(Answer: 1) (Suggestion: Review text problems D-3 and D-4.)*

p-2 *THE SPEED OF FLUID EJECTED FROM A SYRINGE (CAP. 2):* The speed s with which a fluid emerges from the needle of a completely filled hypodermic syringe depends on the cross-sectional area A of the needle, the diameter D and length L of the syringe barrel, and the time T required to push the plunger. The SI units of these quantities are: $A(\text{meter}^2)$, $D(\text{meter})$, $L(\text{meter})$, $T(\text{second})$. One of the following equations correctly relates these quantities: $s = \dfrac{\pi D^2 T}{4LA}$, $s = \dfrac{\pi L D^2}{4TA}$, $s = \dfrac{\pi T L}{4D^2 A}$. Which of these equations is correct? *(Answer: 3) (Suggestion: Review text problems D-5 and D-6.)*

p-3 *STATING VALUES IN CORRECT FORM (CAP. 3):* Determine the values of each of these expressions:

(a) $(5.00 \times 10^5\,\text{centimeter}^3) - (2.5 \times 10^4\,\text{centimeter}^3)$

(b) $(25\,\text{meter}) - (0.9\,\text{meter})$

(c) $(2,000\,\text{sec})/(60\,\text{sec})$

(d) $(350\,\text{mile/hour}) \times (3\,\text{hour})$

(e) $(0.15)(2.50)$

(Answer: 2) (Suggestion: Review text problem E-2.)

p-4 *DISTANCE TRAVELED BY A SLOWING CAR (CAP. 3):* A careful driver traveling with a speed $s_0 = 16\,\text{meter/sec}$ (about 36 mile/hour) sees a stoplight and gently applies his brakes. Before coming to a stop, his car travels through a distance $x = s_0 t - (1/2)at^2$, where t is the time of 10 second required to stop the car, the integers "1" and "2" in the fraction "(1/2)" are completely accurate values obtained by counting procedures (see Text Section E), and a has the value $1.6\,\text{meter/sec}^2$. (a) What is the distance x traveled by the car? (b) Express x in terms of the unit mile, where mile $= 1.6 \times 10^3$ meter. *(Answer: 4) (Suggestion: Review text problems E-2 and E-3.)*

SUGGESTIONS

s-1 *from Text Problem D-4:* You wish to express the value

$$s = 2\pi R/T = 2\pi(18\,\text{centimeter})/(3.0 \times 10^{-4}\,\text{minute})$$

in terms of the SI units meter and second. Because meter = 100 centimeter and minute = 60 second, you can replace centimeter in the previous expression by (meter/100) and minute by (60 second). *If you need further help, review text problem D-3, and read the suggestions in [s-6].*

s-2 *from Text Problem F-3:* The magnitude of a quantity A relative to a quantity B is simply the magnitude of A divided by the magnitude of B. Thus if $A = 100$ foot and $B = -25$ foot, the magnitude of A relative to B is: $|100\,\text{foot}|/|-25\,\text{foot}| = +4$.

s-3 *from Text Problem D-6:* The quantities H and R are lengths, and have the SI unit meter, while $\pi = 3.14$ has no unit. Area has the SI unit meter^2 and volume the unit meter^3. To identify the correct expressions for S and V, just find the unit associated with each of these expressions. Remember that it is meaningless to add (or subtract) two quantities with different units. Thus if an expression is the sum (or difference) of two quantities, each of these two quantities must have the same unit. *(If you need further help, review text problem D-5, using the suggestions in [s-4].*

s-4 *from Text Problem D-5 and Suggestion [s-3]:* We can decide which equation is correct by checking which has consistent units. For example, in the equation $R = AS/N$, the left side has the SI unit 1/sec, while the right side has the unit:

$$\frac{(\text{unit of } A)(\text{unit of } S)}{(\text{unit of } N)} = \frac{(\,\text{meter}^2)(\,\text{meter/sec})}{1/\,\text{meter}^3} = \text{meter}^6/\text{sec}\,.$$

Because the units of the equation $R = AS/N$ are not consistent, this equation cannot be correct.

s-5 *from Text Problem E-1 and Suggestion [s-7]:* **Decimal places**: If a number is not multiplied by a power of 10 (i.e., not in scientific notation), then the number of decimal places in the value is just the number of digits (including zeros) which appear to the right of the decimal point. Thus 0.0120 has four decimal places. **Significant figures**: One easy way to find the number of significant figures in any value is to count the digits,

from left to right, beginning with the first *non-zero* digit. Thus 0.0120 has three significant figures. Alternatively, you may find it easier to express 0.0120 in scientific notation, 1.20×10^{-2}. Then the number of significant figures of the value is just the total number of digits in the decimal number preceding the power of ten.

s-6 *from Text Problem D-3 and Suggestion [s-1]:* **Part (a)**: The following sample calculation illustrates a procedure for multiplying or dividing values which include units and powers of 10. The calculation is greatly simplified by grouping separately numbers, powers of 10, and unit symbols. Remember that unit symbols are manipulated as algebraic symbols. Here "mm" means "millimeter."

$$\begin{aligned} F = nas &= (1.0 \times 10^9)(5.0 \times 10^{-5}\,\text{mm}^2)(9.0 \times 10^{-3}\,\text{mm/min}) \\ &= (1.0 \times 5.0 \times 9.0)(10^9 \times 10^{-5} \times 10^{-3})\,(\,\text{mm}^2 \times\ \text{mm/min}) \\ &= (45)(10^1)\,(\,\text{mm}^3/\text{min}) \end{aligned}$$

This result should ordinarily then be written in scientific notation.

Part (b): To express the preceding value for F in terms of centimeter and second, you can use the relations provided to substitute (1.0×10^{-1} centimeter) for millimeter and (60 second) for minute. Thus:

$$\begin{aligned} F &= 4.5 \times 10^2\,\frac{\text{millimeter}^3\ \text{minute}}{} \\ &= 4.5 \times 10^2\,\frac{(1.0 \times 10^{-1}\,\text{centimeter})^3}{60\,\text{second}} \\ &= \frac{4.5 \times 10^2 \times 1.0 \times 10^{-3}}{60}\ \frac{\text{centimeter}^3}{\text{second}} \end{aligned}$$

This result should be simplified and written in scientific notation.

s-7 *from Text Problem E-2:* **Part (a):**: This expression can be written as

$$(1.2 \times 10^9\,\text{meter}) + (0.22 \times 10^9\,\text{meter}) = (1.2 + 0.22) \times 10^9\,\text{meter}\,.$$

Find the value of this expression by adding the numbers within the parenthesis, remembering to round to the appropriate number of decimal places.

► ______________________

There is one problem which commonly arises in calculations like this. Suppose instead of expressing each term as a multiple of 10 (the largest power among these terms) we had expressed each as a multiple of 10^7:

$$(120 \times 10^7 \text{ meter}) + (22 \times 10^7 \text{ meter}) = (120 + 22) \times 10^7 \text{ meter}.$$

It now looks *as if* all the digits in 120 and 22 are significant and that the correct sum is 142×10^7 meter. In fact, the 0 in 120 was not present in the original number and is *not* significant. Thus the correct sum is 140×10^7 meter (where the 0 is not significant), the same result as before. Thus in adding and subtracting, be wary of zeros introduced as you change powers of ten, and remember that they are not significant.

What is this difference: $(2.34 \times 10^4) - (5.6 \times 10^2)$?

► ______________________

(Answer: 5) (Note: If you need further help, review your work in text problem E-1 and read the suggestions in [s-5].) Now: Return to text problem E-2.

ANSWERS TO PROBLEMS

1. a. 4.5 mile/hour
   b. 24 meter/sec
2. a. 4.75×10^5 centimeter^3
   b. 24 meter
   c. 33
   d. 1×10^3 mile
   e. 0.37 or 0.38
3. $s = (\pi L D^2)/(4TA)$
4. a. 80 meter or 8.0×10^1 meter
   b. 5.0×10^{-2} mile
5. 1.4×10^9 meter, 2.28×10^4

101. $A_l/A_s = 36/9 = 4$
102. $s = (63 or 62.8)$ meter/sec
103. a. 1.4×10^9 meter
   b. 7 meter/sec
   c. 6.7×10^{-2} meter/sec^2
   d. 3.00×10^{-3} meter
   e. 0.94 meter^2 or 0.95 meter^2
   (When the final digit is 5, you can round up or down.)
104. larger values: a. 50 foot; b. 10 pound

   larger magnitudes: a. $|-250 \text{ foot}| = 250$ foot; b. 10 pound
105. No, because does not specify how to assign a description to the "degree of warmth or coolness of the object."
106. $R = ASN$
107. a. 3.84×10^8 meter
   b. 3.92×10^8 meter
   c. 8×10^6 meter
   d. 0.02 or 2×10^{-2}

108. man: 5.0×10^{-2}; wife: 7×10^{-2}. (Check that significant figures are correct.)

109. a. standard
   b. unit
   c. standard
   d. unit

110. S is (a), V is (c), units of (b) are inconsistent.

111. 30 degree Fahrenheit, 760 dollar, 760 dollar. Note that the unit of a quantity is always part of its magnitude.

112. a. function: x, variable: t
   b. function: meter, variable: second

113. a. 10 second^{-1}, 10 meter/sec (These values include only the units meter and second.)
   b. 2 meter (1 meter $\approx$ 3 foot)

114. Significant figures: a. 2, b. 3, c. 2, d. 3

   Decimal places: a. 1, b. 2, c. 3, d. 4

115. a. -60 degree Centigrade
   b. between 0 and 10 kilometer, decreasing;
   between 20 and 25 kilometer, constant;
   between 70 and 100 kilometer, none of these (because in this region graph first decreases, then increases);
   between 90 and 100 kilometer, increases.

116. a. 4.5×10^{2} millimeter^3/minute
   b. 7.5×10^{-3} centimeter^3/sec.

MODEL EXAM

1. **Fluid speed for "needle-free" injection.** A "needle-free" injection device ejects a fluid stream which is sufficiently small in diameter and large in speed that it can penetrate human skin. The speed S of such a fluid stream is related by the following equation to the cross-sectional area A of the fluid stream, the total volume V of the fluid ejected, and the time T for this fluid to be ejected from the device:

$$S = \frac{V}{AT}$$

   Typical values for these quantities are: $A = 1.5\times10^{-4}\,\text{cm}^2$, $T = 0.2\,\text{sec}$, and $V = 0.10\,\text{cm}^3$.

   a. Use the values provided to find the speed S of the fluid ejected by a needle-free injection device.

   b. The fluid speed S for such a device can be as large as the speed of sound $S_0 = 3.3 \times 10^2$ meter/sec. Use the relations mile $= 1.6 \times 10^3$ meter and hour $= 3.6 \times 10^3$ sec to express the speed S_0 in terms of the familiar units mile and hour.

2. **Comparing altitudes.** Altitude is a number with a sign which indicates distance above or below sea level. The altitude of the lowest point in Death Valley is -280 foot (i.e., it is 280 foot below sea level). The nearby town of Sand Dune Junction has an altitude 150 foot.

   a. Which of these altitudes has the larger value?

   b. Which of these altitudes has the larger magnitude?

Brief Answers:

1. a. $S = 3 \times 10^3$ cm/sec
   b. $S_0 = 7.4 \times 10^2$ mile/hour

2. a. 150 foot
   b. -280 foot