PHYSNET

MISN-0-393

# SCATTERING IN THE BORN APPROXIMATION

# Quantum Physics

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI



SCATTERING IN THE BORN APPROXIMATION

by

R. Spital

Title: **Scattering in the Born Approximation**

Author: R. Spital, Dept. of Physics, Illinois State Univ

Version: 2/1/2000 Evaluation: Stage B0

Length: 2 hr; 9 pages

**Input Skills**:

1. Vocabulary: microscopic system, macroscopic system, atomic weight, solid angle.
2. Integrate using spherical coordinates or polar coordinates.
3. Unknown: assume (MISN-0-392).

**Output Skills (Knowledge)**:

K1. Define the differential and total elastic cross sections and apply the definitions to problems similar to the one in the procedures.

K2. Derive the classical differential and total elastic cross sections for hard sphere scattering,

K3. State the asymptotic form of the wave-function in elastic scattering, giving the significance of each term. Use this asymptotic form to define the scattering amplitude and give the connection between the scattering amplitude and the differential cross section.

K4. State the appropriate form of Fermi's Golden Rule for elastic scattering and indicate how it suggests the Born approximation. State the Born approximation clearly.

**Output Skills (Problem Solving)**:

S1. Calculate differential cross sections in the Born approximation for potentials of the type given in the problems assigned in the procedures.

**External Resources (Required)**:

1. E. E. Anderson, *Modern Physics and Quantum Mechanics*, W. B. Saunders (1971).



THIS IS A DEVELOPMENTAL-STAGE PUBLICATION
OF PROJECT PHYSNET

The goal of our project is to assist a network of educators and scientists in transferring physics from one person to another. We support manuscript processing and distribution, along with communication and information systems. We also work with employers to identify basic scientific skills as well as physics topics that are needed in science and technology. A number of our publications are aimed at assisting users in acquiring such skills.

Our publications are designed: (i) to be updated quickly in response to field tests and new scientific developments; (ii) to be used in both classroom and professional settings; (iii) to show the prerequisite dependencies existing among the various chunks of physics knowledge and skill, as a guide both to mental organization and to use of the materials; and (iv) to be adapted quickly to specific user needs ranging from single-skill instruction to complete custom textbooks.

New authors, reviewers and field testers are welcome.

# SCATTERING IN THE BORN APPROXIMATION

by

R. Spital

## 1. Introduction

The calculation of scattering cross sections is one of the most important uses of Fermi's Golden Rule. Since Fermi's rule involves only 1 matrix element of the interaction, it is a *first-order* approximation to the exact result. This approximation in turn suggests an approximation to the *scattering amplitude*, a complex quantity closely related to the cross section. It is this last approximation that is called the Born approximation. In this unit we shall use the Born approximation to calculate elastic scattering amplitudes and cross sections for simple potentials.

## 2. Procedures

1. The references in this unit are to Anderson. To review the meaning of the scattering cross sections, read section 3.2 through the beginning of the discussion of Rutherford scattering. For a given target the cross sections are defined as follows:

$$\frac{d\sigma}{d\Omega} = \frac{\text{number scattered per unit time into solid angle } d\Omega}{\text{number incident per unit time per unit area}}$$

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = \frac{\text{number scattered per unit time}}{\text{number incident per unit time per unit area}}$$

$\sigma$ is the total *elastic* cross section. We shall not deal with *inelastic* scattering in which particles are created or destroyed (or in the internal state of the target or projectile changes).

The cross section is a measure of the probability that an interaction occurs; the larger the cross section, the greater the probability that an interaction will take place when a particle is incident on the target. In general the cross section depends on the initial and final states of both the target and the projectile (including energy, spin, angle of scatter, etc.). To calculate the cross section a knowledge of the *dynamics* (nature of the interaction) is required.

The above definitions treat the target as a single scatterer. We are almost always interested in extracting the cross section for a single microscopic scatterer from measurements made on a macroscopic target





consisting of a great many such scatterers. In this case one must divide the cross section of the macroscopic target by the *number of scatterers in the path of the beam.* Note that this recipe assumes the target is very thin. (Why?)

As an exercise do the following problem: A beam of protons of 3 centimeter diameter contains $10^{10}$ protons/(sec cm$^2$). It is normally incident on a thin foil whose thickness $t$ is such that $\rho t = 0.0001\,\text{gm/cm}^2$, where $\rho$ is the density of the foil. The scatterers in this problem are the atomic nuclei of the foil which have an atomic weight of $A = 200$. A counter placed at an angle of 45° with respect to the beam direction is located 1 m from the target. It accepts all protons that cross an area of 10 cm$^2$ which is perpendicular to the line joining the counter to the target. There are 2.1 protons observed in the counter per second. What is the differential cross section $d\sigma/d\Omega$ for proton-nucleus elastic scattering at 45°?

(Answer: 0.01 barns/ster; 1 barn $\equiv 10^{-24}$ cm$^2$)

2. To gain practice calculating cross sections classically, solve problem 3.1.

3. Now for some quantum scattering theory. The definitions of the cross section are the same but the method of calculation is different.

Begin by reading section 11.1 through equation 11.2. Equation 11.1 is the *asymptotic* form of the wave function for elastic scattering. That is; it describes the wave *far* from the interaction region; in the interaction region the wave is much more complicated. Note that the wave-function in this case represents the superposition of the ingoing and outgoing particle fluxes, rather than the state of a single particle as we have been used to. (We can achieve the same results in a more physical and intuitive manner by using single particle wave-packets which propagate in space and time - but this is a more difficult approach.) $f(\theta)$ is called the *elastic scattering amplitude*; "elastic" is often omitted when it causes no confusion to do so, and we shall do so here. $f$ depends only on the scattering angle $\theta$ in problems with azimuthal symmetry - this shall always be the case for us. (Of course, $f$ also depends on energy, which particles are scattering, etc.. The above statement refers only to the angular dependence of $f$.) Equation 11.2 is the connection between $f$ and the differential scattering cross section. Note that $f$ is a complex function, but $d\sigma/d\Omega$ is always real and positive.

You need not follow the derivation of equation 11.2; but you should memorize equations 11.1 and 11.2 and be sure you understand what the 2 terms in equation 11.1 represent.

4. Now read section 11.2 through the end of the paragraph containing equation 11.22. Equation 11.18 is the form of the Golden rule needed. In what sense is the interaction time - dependent? Equation 11.21 is the Born approximation to the scattering amplitude. The "derivation" given in the text is "hand-waving"; it clearly does not give the phase of the scattering amplitude (which is certainly specified in equation 11.21). However, the cross section is surely proportional to $R$, so that the amplitude must vary as $\sqrt{R}$. And certainly, it makes sense that the states $i$ and $f$ are plane-wave eigenstates of the "free" Hamiltonian (Hamiltonian far from the interaction region). (Needless to say, there are much better derivations of this approximation.) The upshot of the Born approximation in any case is that, to lowest order in the interaction, the scattering amplitude is proportional to the matrix element of $V$ between plane-wave states.

5. Use the Born approximation to solve problems 11.6, 11.8, and 11.10.

## Acknowledgments

The author would like to thank Illinois State University for support in the construction of this lesson. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.