PHYSNET

MISN-0-392

HARMONIC PERTURBATIONS

Quantum Physics

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

HARMONIC PERTURBATIONS

by

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Title: **Harmonic Perturbations**

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Input Skills:

1. Vocabulary: dipole moment, binding energy, ionization, photo-ionization.
2. Unknown: assume (MISN-0-391).

Output Skills (Knowledge):

K1. Derive an equation for the probability $|c_m(t)|^2$ that the system, starting in the eigenstate k of $\mathcal{H}_0$, is in state m at time t. The system is assumed to be under the influence of the interaction $\mathcal{H}'(x,t) = \mathcal{H}'(x)e^{\pm\imath\omega t}$.

K2. Discuss the result of K1 answering the following questions:

(a) For large times, what requirement must the energy of the state m meet?

(b) How can this requirement be interpreted in terms of absorption or emission of energy quanta by the system.

(c) The perturbing influence is usually the radiation field. How can absorption or emission of an energy quantum $\hbar\omega$ be interpreted in terms of the photon picture?

Output Skills (Problem Solving):

S1. For an electron of charge $-e$ bound in a 1-dimensional potential well, find an expression for the transition probability per unit time for photo-ionization under the influence of $\mathcal{H}' = -e\mathcal{E}e^{-\imath\omega t}$. Evaluate this expression for initial states of the type given in the procedures.

S2. Solve problems relating to harmonic perturbations of the type given in the procedures.

External Resources (Required):

1. D. S. Saxon, *Elementary Quantum Mechanics*, Holden-Day, Inc. (1968).

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1. Introduction

We now take up the topic of perturbations which vary harmonically in time - this will allow us to treat the processes of stimulated emission and absorption of photons by charged particles. We shall also study the process of *photo-ionization* in which an electron is removed from the system to which it was bound. This will allow us to apply Fermi's Golden Rule and gain a better understanding of the "density of states."

2. Procedures

All references refer to Saxon. These pages are available in the PA library. Ask for them as "the readings for CBI Unit 392."

1. Equation 72 of Chapter VII is the desired result. To get it, simply repeat the steps leading to equation 63 with the assumed form for $\mathcal{H}$'.

   Note that if $\mathcal{H}'(x)$ is Hermitian, $\mathcal{H}'$ of equation 71 is not a Hermitian operator. (Why is this physically unacceptable?) Both positive and negative frequency components must be present *together* in equation 71. As an exercise, derive $|c_m(t)|^2$ for $\mathcal{H}'(x,t) = \mathcal{H}'(x)(e^{\imath\omega t} + e^{-\imath\omega t})$ where $\mathcal{H}'(x)$ is Hermitian. You will get 3 terms. For large times (what does "large" mean here?), which terms are negligible for absorption? for emission? What happens to the curve $|c_m(t)|^2$ vs. E_m as $t \Rightarrow \infty$?

   It can be shown from a careful consideration of the measurement process that the *minimum* time needed to determine whether a system is in one of two energy eigenstates (energies E_1 and E_2) is given by $t \geq \hbar/|E_1 - E_2|$. This is reminiscent of, but not the same as, the time-energy uncertainty relationship we discussed in early units. How does this fact affect energy conservation for the transitions we are considering?

2. Read the discussion after equation 72 through the beginning of the last paragraph on page 214. The answers to the questions can be found there, provided you remember what you learned about photons in unit 381.

3. Continue reading through equation 79, which is the desired result. Make sure you can reproduce all the steps including the derivation of the density of states, equation 77. {"Counting" states in the continuum is particularity important in scattering theory; and this example, while only 1-dimensional, contains all the essential ideas. Don't be surprised if you find it tricky; everyone does until they've been through it many times.}

   Evaluate equation 79 for an electron in the ground state of a harmonic oscillator. Neglecting possible recapture of electrons, how many oscillators will be ionized after time t, if the original number of oscillators was N_0?

4. Solve problem 15-b on page 223. Also solve the problem given at the end of MISN-0-391 with this new harmonic perturbation replacing the old "on and off" perturbation.

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