PHYSNET

MISN-0-390

TIME-INDEPENDENT PERTURBATIONS

Quantum Physics

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

TIME-INDEPENDENT PERTURBATIONS

by

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Title: **Time-Independent Perturbations**

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Input Skills:

1. Vocabulary: diagonal matrix elements, off-diagonal matrix elements.
2. Unknown: assume (MISN-0-389).

Output Skills (Knowledge):

K1. Given any time-independent perturbation $\mathcal{H}_1$ and any complete set of non-degenerate eigenstates of $\mathcal{H}_0$, derive: (a) the energy shifts to first and second order in $\mathcal{H}_1$, (b) the corrections to the wave-functions to first and second order in $\mathcal{H}_1$; and using the results of (a) and (b) above, write approximate expressions for the eigenvalues and eigenfunctions of $\mathcal{H}$ to first or second order in $\mathcal{H}_1$.

Output Skills (Problem Solving):

S1. Solve problems such as 9-5, 9-9, 9-10, 9-11 and 9-13.

External Resources (Required):

1. E. E. Anderson, *Modern Physics and Quantum Mechanics*, W. B. Saunders (1971).

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1. Introduction

Relatively few Hamiltonians are solvable exactly; most of the time we have to find ways to approximate the energy eigenstates and eigenvalues. Often we can write the Hamiltonian as $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1$, where $\mathcal{H}_0$ has *known* eigenfunctions and eigenvalues, while $\mathcal{H}_1$ (which contains the new physics) can be regarded as small compared to $\mathcal{H}_0$. The procedure for approximating the eigenstates and eigenvalues of $\mathcal{H}$ for the case when $\mathcal{H}_1$ is independent of time is called "Time Independent Perturbation Theory."

2. Procedures

1. Read section 9.1. Follow the derivations carefully and be sure you can reproduce them. The key results to remember are equations (9.8), (9.10), (9.12), (9.13) and (9.14).

   Two key assumptions enter this derivation:

   (a) The eigenstates of $\mathcal{H}_0$ are *non-degenerate*. If this were not true, some of the "energy denominators" in the above equations would vanish, leading to nonsensical results. The Perturbation theory of degenerate states is a more advanced topic which we shall not have time to take up.

   (b) The eigenstates of $\mathcal{H}_0$ and of $\mathcal{H}$ are in one to one correspondence. This is *not* always true, and when it isn't things are much more complicated. We shall always make this assumption.

   Having finished the derivations, equation (9.5) tells you how to write the approximate expressions for the energies and the eigenstates.

2. Solve problems 9-5, 9-9, 9-10, 9-11, and 9-13 to gain practice in applying the newly developed techniques. You may wish to read section 9.2 to see an example done, but this is not required.

Acknowledgments

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