PHYSNET

MISN-0-358

ANALYSIS OF BUBBLE CHAMBER PHOTOGRAPHS

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

ANALYSIS OF BUBBLE CHAMBER PHOTOGRAPHS
by
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Title: **Analysis of Bubble Chamber Photographs**

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Input Skills:

1. Vocabulary: chord, sagitta, rest mass, relativistic mass (MISN-0-23).
2. Write and run simple programs in FORTRAN (MISN-0-346) or BASIC.

Output Skills (Project):

P1. Recognize the production and decay of the Sigma-Minus particle in a bubble chamber photograph.

P2. Perform measurements on the particle tracks in bubble chamber photographs.

P3. Using measurements from a bubble chamber photograph of a Sigma-Minus production decay event, enter and run a canned program to compute the mass of the unseen neutral particle and identify it from a table of known masses.

P4. Write and run a simple program to calculate the lifetime of an elementary particle given its track length and momentum.

External Resources (Required):

1. A computer with FORTRAN or BASIC, measuring scale (ruler), protractor.

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1. Particle Reactions

1a. Introduction. A convenient way to study the properties of the fundamental subatomic particles is through observation of their bubble trails, or tracks, in a bubble chamber. Using measurements made directly on a bubble chamber photograph, we can often identify the particles from their tracks and calculate their masses and other properties. In a typical experiment, a beam of a particular type of particle is sent from an accelerator into a bubble chamber, which is a large liquid-filled vessel. To simplify the analysis of the data, the liquid used is often hydrogen, the simplest element. The use of liquid hydrogen, while it simplifies the analysis, complicates the experiment itself, since hydrogen, a gas at room temperature, liquifies only when cooled to -246° C. For charged particles to leave tracks in passing through the chamber, the liquid must be in a "super-heated" state, in which the slightest disturbance causes boiling to occur. In practice, this is accomplished by expanding the vapor above the liquid with a piston a few thousandths of a second before the particles enter the chamber.

1b. Particle Tracks in Bubble Chambers. A great deal of information can be obtained by carefully observing the tracks (or bubble trails) particles create in passing through the chamber. The chamber is usually placed in a very strong magnetic field B which causes charged particles to travel in curved paths. A particle of charge q and momentum p which moves perpendicular to a uniform magnetic field of magnitude B travels in a circular arc of radius R, given by

$$R = \frac{p}{qB}$$

For all known charged long-lived subatomic particles, the magnitude of their charge q is the same as that of the electron, $e = 1.6 \times 10^{-19}$ coulombs. Therefore, if the magnitude of the magnetic field B is known, we can determine the momentum of a particle by measuring the radius of curvature of its track. We can also infer the sign of its charge if we know the direction of the magnetic field and we observe in which sense the track curves

(clockwise or counterclockwise). In the bubble chamber photographs we shall examine, the magnetic field points out of the paper, so that the sense of curvature is clockwise for positive particles and counterclockwise for negative particles. Note that to determine the sign of a particle's charge, it is necessary to know in which direction the particle traveled along the track. This is not always obvious from looking at a bubble chamber photograph (see Fig. 1).

1c. Subatomic Particles. There are a large number of kinds of subatomic particles, many of which are produced only in very high energy collisions. In this module we shall be particularly interested in the following four subatomic particles: proton (p), kaon (K), pion (π), and sigma (Σ). Of these, only the proton is a stable constituent of ordinary matter. Each of the others comes in all three charges plus, minus and neutral.

1d. Subatomic Particle Reactions. In a typical particle physics experiment a beam of one type of particle is allowed to hit a target and various reactions are observed. Figure 1 shows a bubble chamber photograph in which a number of kaons are incident from the bottom of the photograph. The magnetic field B points out of the photograph. Thus the counterclockwise curvature of the tracks incident from the bottom clearly shows that the incident particles are negatively charged. If we follow the path of each K^- track, in Fig. 1, we see that some K^- particles passed through the chamber without incident; others interacted with a hydrogen nucleus (a proton) and initiated various reactions. The simplest type of interaction between two particles is elastic scattering, analogous to a billiard ball collision. Symbolically, the elastic scattering of a K^- particle and a proton may be written

$$K^- + p \rightarrow K^- + p \,.$$
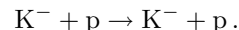
An incident K^- meson can also initiate reactions in which new particles are created. In many cases, the created particles live for only 10^{-10} seconds or less, and then disintegrate, or decay, into other particles which themselves may be unstable. We shall be particularly interested in the sigma production reaction

$$K^- + p \rightarrow \Sigma^- + \pi^+ \,,$$
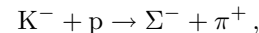
in which the K^- and proton disappear and a sigma minus particle (Σ^-) and a pi plus meson (π^+) are created. Of the four particles involved in the above reaction: K^-, p, Σ^-, and π^+, only the proton p is stable. Among the other three, the Σ^- is the most unstable, as it lives the shortest time
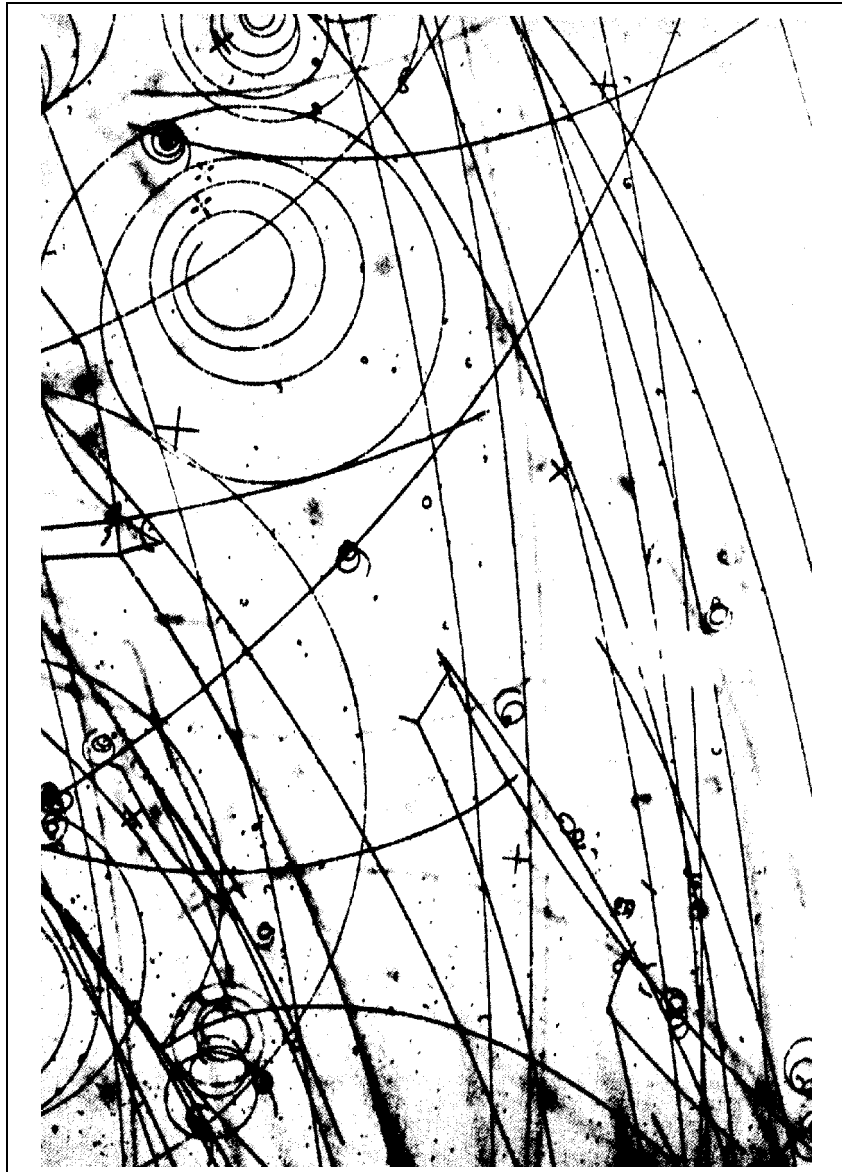
Figure 1. Bubble chamber tracks of a beam of kaons.

after it is created. After about 10^{-10} seconds a Σ^- particle decays into a π^- meson and a neutral particle (which leaves no track in the bubble
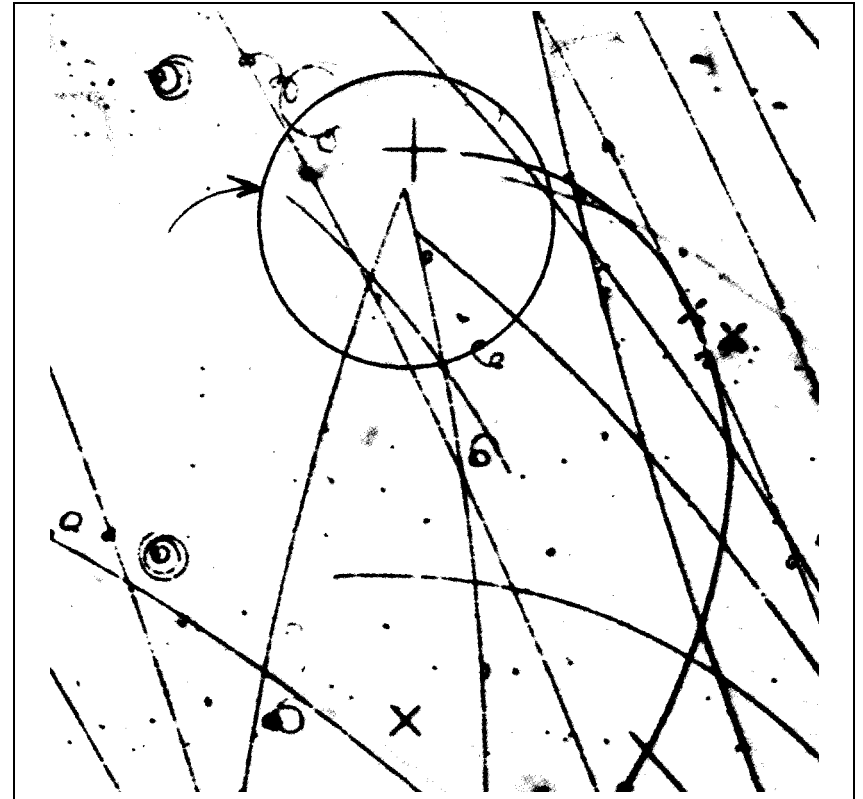
Figure 2. An example of a Σ^- production and subsequent decay.

chamber). We symbolically write the sigma decay reaction as

$$\Sigma^- - \rightarrow \pi^- + X^0 \, ,$$
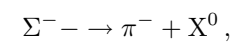
where X^0 designates the unseen neutral particle. We shall refer to a particular example of the two reactions, sigma production followed by sigma decay, as an event. Thus, in this experiment our attention will be particularly focussed on events of the type

$$K^- + p \rightarrow \Sigma^- + \pi^+$$
$$\hookrightarrow \pi^- + X^0$$
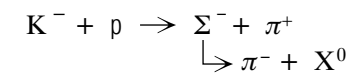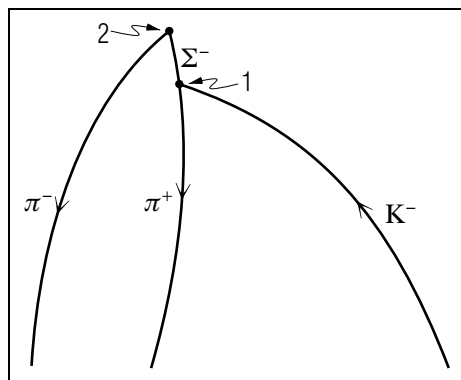
Figure 3. A schematic diagram of the event.

1e. Recognizing Bubble Chamber Events. Upon first examination, bubble chamber photographs may appear to be extremely confusing. Once you know what to look for, however, it is very easy to spot events corresponding to the Σ^- production and decay reactions in the photograph. In this experiment, the incident K^- mesons have sufficiently low energy that many of them slow down and come to rest within the chamber. In order for the sigma production reaction to take place, the K^- meson and the proton must be very close to one another (about 10^{-15} meters apart). Due to the attractive force between the negatively charged K^- and the positively charged proton, this is much more likely to happen when the K^- is moving slowly or at rest. Since the initial momentum of the two-particle (K^-/proton) system is zero, the law of conservation of momentum requires the Σ^- and π^+ particles to have equal and opposite momentum, which makes the Σ^- and π^+ tracks appear as one continuous track. The considerable energy of the Σ^- and π^+ particles in the energy-liberating sigma production reaction is the result of the conversion of mass into energy. (The combined rest mass of the Σ^- and π^+ particles is appreciably less than that of the K^- and proton). In Fig. 2, we have one such event which has been circled. For clarity, the circled event is shown schematically in Fig. 3 with the identity and direction of each track indicated. Note how the Σ^- and π^+ tracks appear to be one continuous track, implying that the sigma production reaction took place after the K^- came to rest (at point 1). Notice also the kink in the Σ^- track less than 1.0 cm from the point of its creation. It is at the kink (point 2) that the Σ^- decayed into a π^- meson and the unseen neutral particle X^0. In all events, the length of the Σ^- track will invariably be quite short due to its short lifetime. The details of an event, such as the length, direction, and curvature of each track, generally vary from event

to event. However, an event can usually be identified by its general configuration, or topology. Look through each of the photographs in Figs. 4 through 8 and see if you can identify events.

2. Measurement of Photographs

2a. Overview. A careful quantitative analysis of measurements made on tracks in bubble chamber photographs can reveal much more than can a simple visual inspection of the photographs. First, while reactions can often be unambiguously identified by their topology, such an identification can be confirmed if we make measurements of the length, direction, and curvature of each track, and then analyze these data by computer. Second, through such a procedure we can determine whether an unseen neutral particle was present. Third, we can determine properties of an unseen neutral particle. In each of the photographs in Figs. 4-8, there are one or more events. The circled event in each photograph is the one of particular interest because all of its tracks lie very nearly in the plane of the photograph and this considerably simplifies the analysis. Such "almost coplanar" events are a rare occurrence since all directions are possible for the particles involved. For events that are more non-coplanar we must analyze at least two stereoscopic photographs of each event in order to completely describe its three dimensional kinematics.[1] For each of the circled events we will first determine three quantities:

(a) the momentum of the π^- particle (p_π);

(b) the momentum of the Σ^- particle (p_Σ) at the point of its decay; and

(c) the angle θ between the π^- and Σ^- tracks at the point of decay.

Methods for finding these quantities are discussed in the remainder of this section (Sect. 2). For each of the circled events we will then find the lifetime of a visible paricle and the mass and hence identity of an invisible one. Methods for finding the mass and lifetime are discussed in Sect. 3, program input and output are shown in Sect. 4, and project procedures are discussed in Sect. 5.

2b. Determination of Momentum from Curvature. We can determine the momentum of the π^- particle if we measure the radius of curvature of the π^- track and use the relation:

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[1]In practice three photographs are often used in order to overdetermine the results and hence provide a check on the determinations.
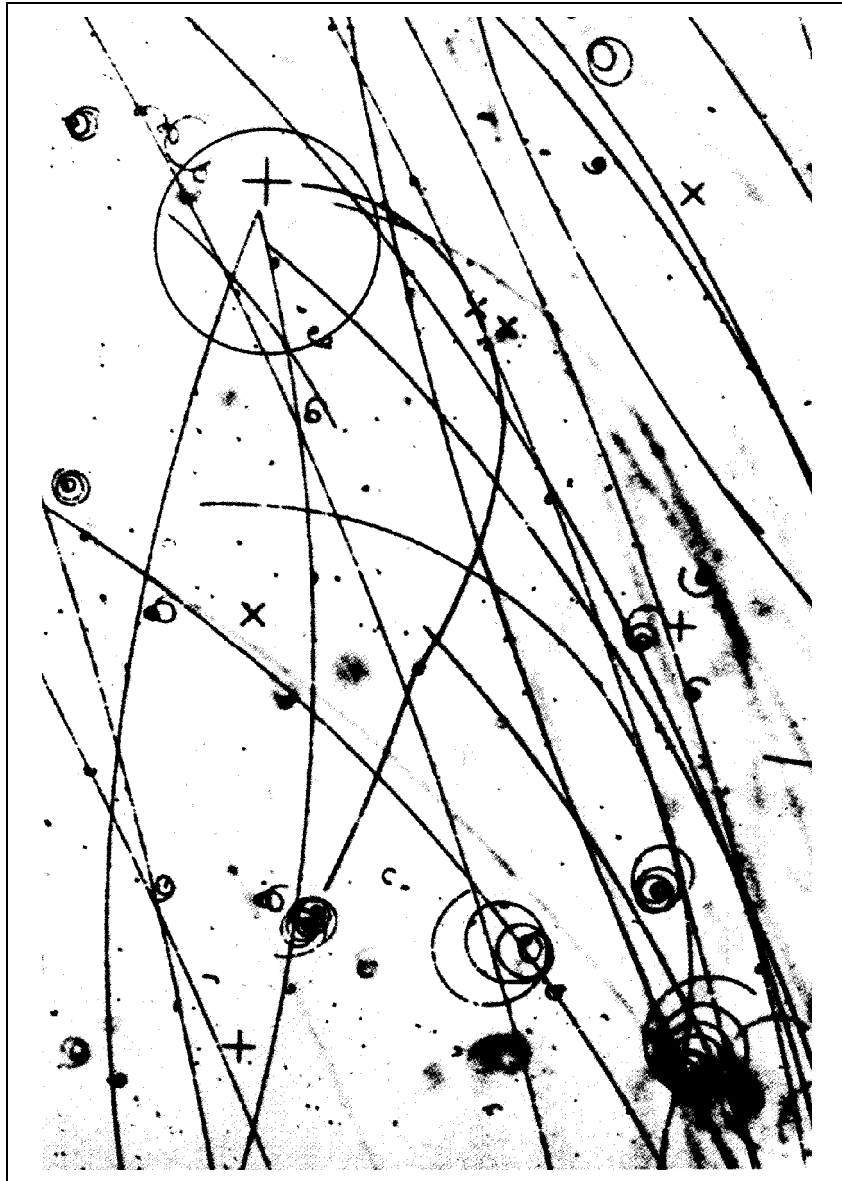
Figure 4. .
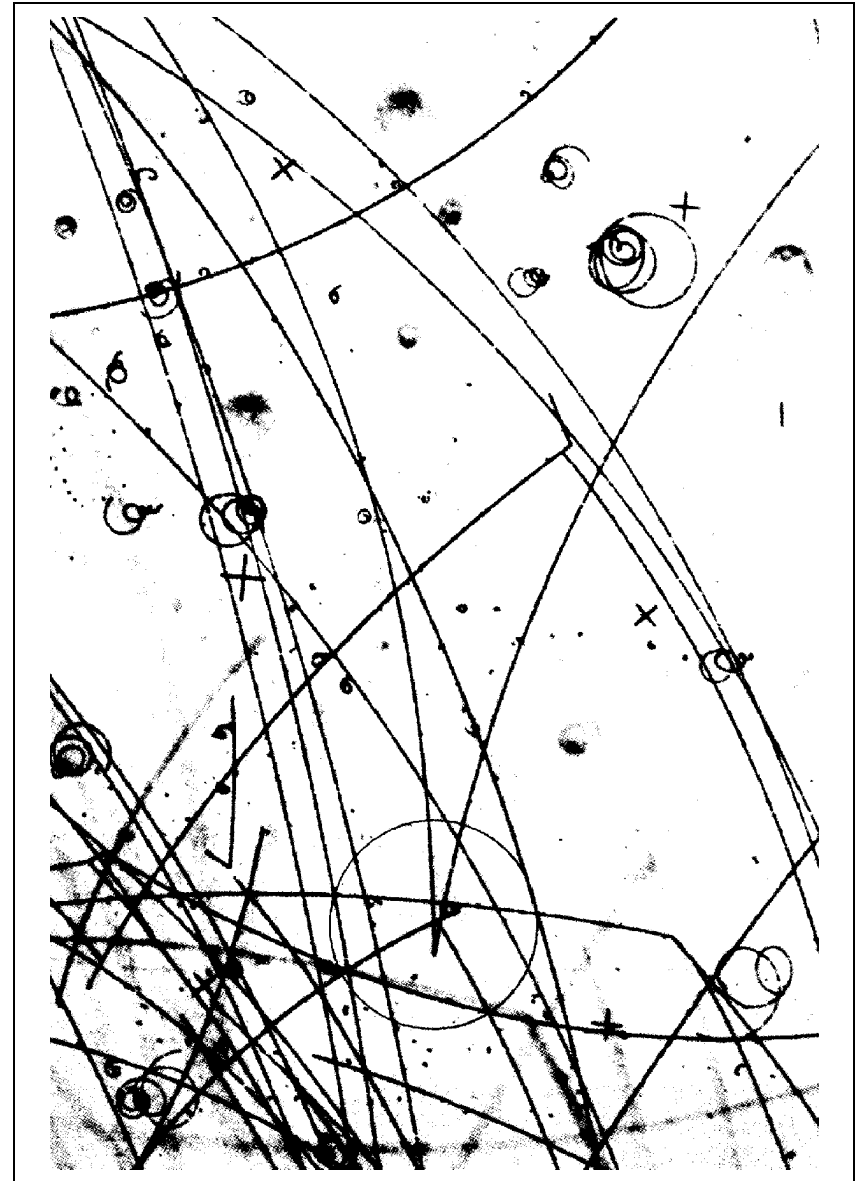
Figure 5. .

In our system of units, this can be written

$$p = q\,R\,B. \tag{1}$$

$$p = 6.86\,\mathrm{R}\,, \tag{2}$$
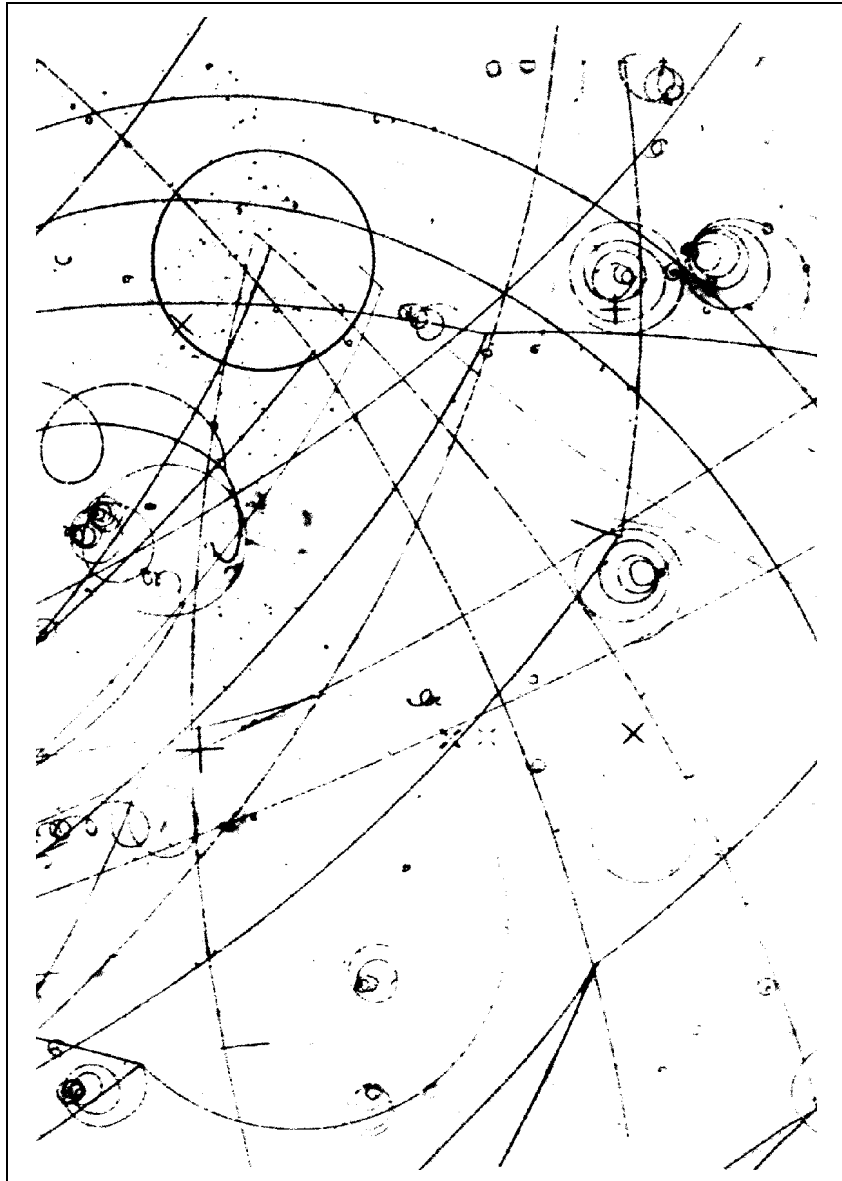
Figure 6. .

with R in units of cm. (The units for p will be discussed in the next section). For a curved track it is not easy to measure R directly, since
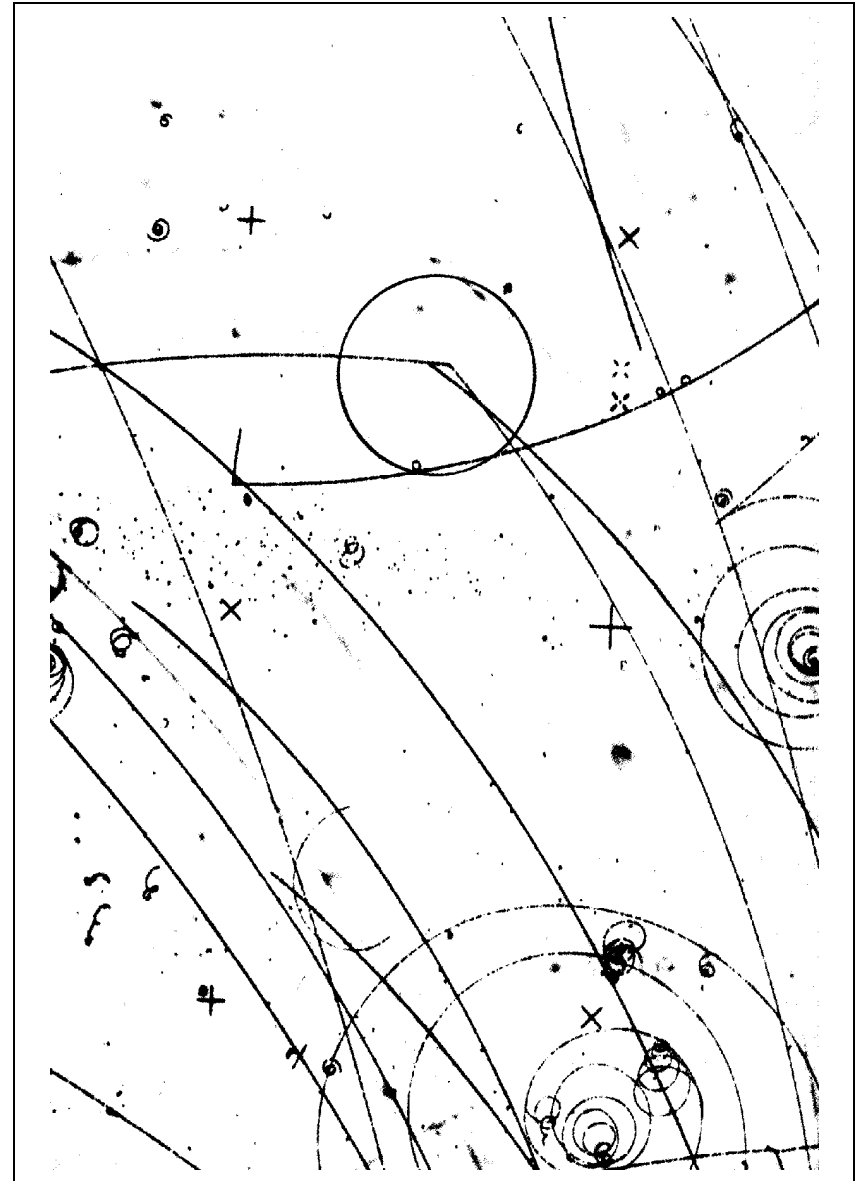
Figure 7. .

the center of the circular arc is not known. However, if we draw a chord
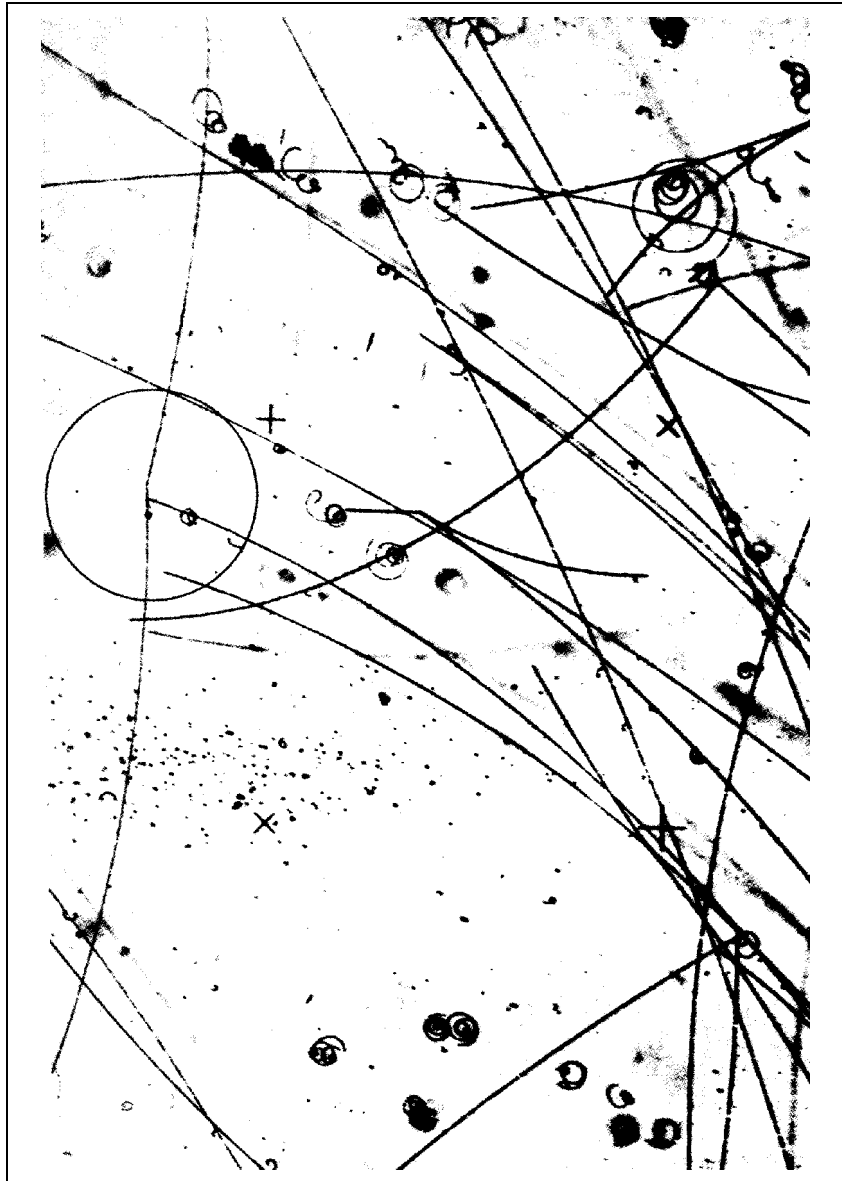
Figure 8. .

on the track and measure both the length of the chord, ℓ, and the length
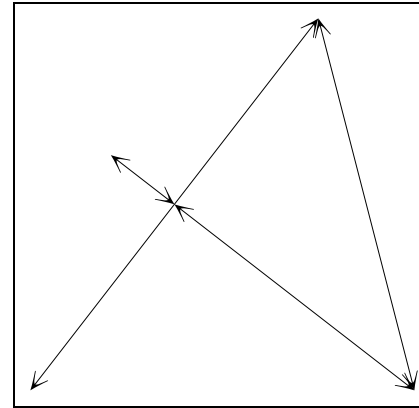
Figure 9. Sagitta method for determining the radius of curvature.

of the sagitta, s, (see Fig. 9), the radius R can then be found in terms of ℓ and s. As can be seen in Fig. 9, the Pythagorean theorem gives the following relation between the radius R, chord length ℓ, and sagitta s:

$$R^2 = (R - s)^2 + (\ell/2)^2$$

which can be solved for R to obtain

$$R = \frac{\ell^2}{8s} + \frac{1}{2}s. \tag{3}$$

Thus we can find the radius R by drawing the longest possible chord on the track, measuring the chord length ℓ and the sagitta s, and then applying Eq. (3). We want to draw the longest possible chord in order to obtain the greatest accuracy of measurement for the chord length ℓ and sagitta s.

**2c. Determination of Momentum from Range.** We cannot determine the momentum of the Σ^- track from its radius of curvature because the track is much too short. Instead, we make use of the known way that a particle loses momentum as a function of the distance it travels. In each event in which the K^- comes to rest before interacting, energy conservation applied to the process requires the Σ^- particle to have a specific momentum of $174\,\mathrm{MeV}/c$. The relatively massive Σ^- particle loses energy rapidly, so its momentum at the point of its decay is appreciably less than $174\,\mathrm{MeV}/c$ even though it travels only a short distance. It is known that a charged particle's range, d, which is the distance it traveled before coming to rest, is approximately proportional to the fourth power of its initial momentum, i.e., $d \propto p^4$. For a Σ^- particle traveling in liquid

hydrogen, the constant of proportionality is such that a particle of initial momentum 174 MeV/c has a range of 0.597 cm. The relationship:

$$d = 0.597 \left(\frac{p_\Sigma}{174} \right)^4 . \qquad (4)$$

clearly gives the correct range for $p_\Sigma = 174$ MeV/c and has the correct proportionality $d \propto p^4$. In most cases the Σ^- particle decays before it comes to rest, so the distance it travels, the Σ^- track length ℓ_Σ, is less than the maximum range $d_0 = 0.597$ cm.

Next we find the "residual range," which is the difference between the maximum range d_0 and the Σ^- track length ℓ_Σ. The relation between the residual range, p_Σ, and the momentum of the Σ^- particle at the point of decay is:

$$d_0 - \ell_\Sigma = 0.597 \left(\frac{p_\Sigma}{174} \right)^4 .$$

This can be solved for p_Σ to yield:

$$p_\Sigma = 174 \left(1 - \frac{\ell_\Sigma}{0.597} \right)^{1/4} . \qquad (5)$$

Note that when $\ell_\Sigma = 0.597$ you get $p_\Sigma = 0$ as you would expect.

2d. Determination of the Angle θ. The angle θ between the π^- and Σ^- momentum vectors can be determined by drawing tangents to the π^- and Σ^- tracks at the point of the Σ^- decay. We can then measure the angle between the tangents using a protractor. In Fig. 10 we show an alternative method which does not require a protractor. Let AC and BC be the tangents to the π^- and Σ^- tracks respectively. Drop a perpendicular (AB) and measure the distances AB and BC. The ratio AB/BC gives the tangent of the angle $180° - \theta$. It should be noted that only some of the time will the angle θ exceed 90°, as shown here.

3. Relativistic Dynamics

3a. Overview. In this section we discuss how the mass of the invisible "X^0" particle is calculated from measurements made on the photographs. The general approach is to first use the three measurements discussed in the previous section to determine the energy and momentum of both of the charged particles present in the sigma decay (the Σ^- and the π^-). We then use the laws of conservation of energy and momentum to determine the energy and momentum of the invisible X^0 particle. Finally, from its
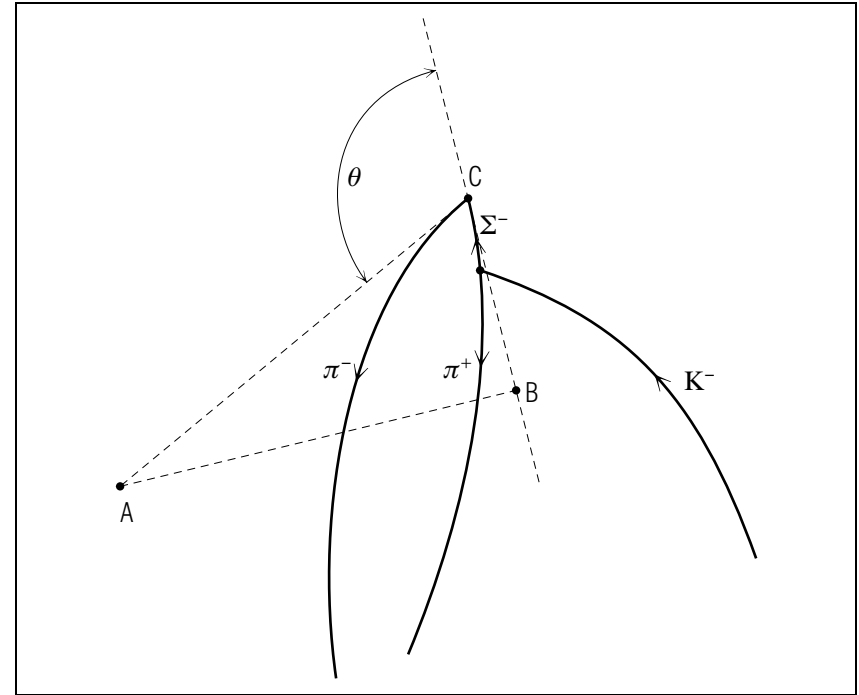
Figure 10. Determination of the angle θ.

energy and momentum, we calculate its mass and then identify it from a table that lists the masses of known particles.

3b. Relativistic Formulas and Units. Since the particle speeds here are all a significant fraction of the speed of light, c, we must use relativistic formulas. In conventional SI units we have the relations:

$$E = mc^2 \quad and \quad E^2 = p^2 c^2 + m_0^2 c^4,$$

where E is the total energy, m is the relativistic mass, m_0 is the rest mass and p is the momentum of a particle. Particle physicists prefer to use this system of units: MeV for energy, MeV/c for momentum, and MeV/c^2 for mass. In such units all factors of c conveniently disappear, giving:

$$E = m \quad and \quad E^2 = p^2 + m_0^2 . \qquad (6)$$

3c. Calculation of the Unknown Neutral Mass.

The unseen X^0 particle, whose mass we wish to find, is produced in the Σ^- decay:

$$\Sigma^- \to \pi^- + X^0 \,.$$

Conservation of momentum therefore requires that the X^0 has a momentum:

$$\vec{p}_X = \vec{p}_\Sigma - \vec{p}_\pi \,.$$

Conservation of energy requires that the X^0 has an energy:

$$E_X = E_\Sigma - E_\pi \,.$$

Equation (6) can be solved for m_0 to obtain:

$$m_0 = \left(E^2 - p^2\right)^1 /2 \,.$$

Thus we can calculate the rest mass of the X^0 particle using

$$m_X = \left((E_\Sigma - E_\pi)^2 - (\vec{p}_\Sigma - \vec{p}_\pi)^2\right)^{1/2}$$

which can be alternatively written:

$$m_X = \left((E_\Sigma - E_\pi)^2 - (p_\Sigma^2 - 2\,p_\Sigma\,p_\pi\,\cos\theta + p_\pi^2)\right)^{1/2} \,. \tag{7}$$

Using Eq. (7), we have a way of calculating the mass of the unseen neutral particle in terms of quantities that can be determined from measurements made on the bubble chamber photograph:

θ is found from the tangent method described in paragraph 2d,
p_π is found from the sagitta method described in paragraph 2b,
p_Σ is found from the track length measurement in paragraph 2c,
E_π and E_Σ are found from the momenta p_π and p_Σ using Eq. (6).

4. Program to Calculate the X Mass

4a. Input.

The input consists of values for the quantities N, LSIG, TH, S, and LPI where

N	=	an arbitrary identifying number for the event
LSIG	=	ℓ_Σ (chord length of the Σ^- track, in cm)
TH	=	θ (angle between π^- and Σ^- tracks, in degrees)
S	=	s (sagitta of π^- track, in cm)
LPI	=	ℓ_π (chord length of π^- track, in cm).

The program then proceeds to calculate the momenta of the pion and sigma particles using Eqs. (3) and (5), respectively, and finally the mass of the unseen X^0 particle using Eq. (7).

4b. Sample Output.

The following is an example of the calculated X^0 mass using input quantities shown below obtained from measurements of the event in Fig. 4.

```
Input:
EVENT NUMBER          =      1.000
SIGMA TRACK LENGTH    =       .500
THETA                 =    142.000
SAGITTA OF PI         =       .970
CHORD LENGTH OF PI    =     14.800

Output:
MASS OF X PARTICLE    =    915.000
```

5. Procedures

5a. Calculation of the X Particle's Mass.

Make measurements on each of the photographs in Figs. 4-8. In particular, for each of the circled events measure these four quantities:

ℓ_Σ - the length of the Σ track,
θ - the angle between the Σ^- and π^- track,
s - the sagitta of the π^- track,
ℓ_π - the chord length of the π^- track.

Your values for the event in Fig. 4 should be close to those given in the sample input. Run the program using each set of measurements, and tabulate the computed X^0 mass from each event. Compute an average of the calculated masses and find the average deviation, expressing your result as $M_x \pm \Delta M_x$. Compare your final result with some known neutral particles listed below and identify the X^0 particle based on this comparison.

particle	mass(in MeV/c^2)
π^0	135
K^0	498
n	940
Λ^0	1116
Σ^0	1192
Ξ^0	1315

5b. Calculation of the Sigma Lifetime.

The Σ^- lifetime can be approximately determined using the measured values of the Σ^- track

lengths. The average momentum of the Σ^- particle can be found from its initial and final values:

$$\bar{p}_\Sigma = \frac{1}{2}(174 + p_\Sigma)\,,$$

where p_Σ is found from Eq. (5), using the measured track length ℓ_Σ. The length of time that the Σ^- lives (the time between its creation and decay) is

$$t = \frac{\ell_\Sigma}{v}\,,$$

where ℓ_Σ is the length of the Σ^- track and v is the average velocity of the Σ^- particle. This time interval is measured in a coordinate system at rest in the laboratory. In a coordinate system moving with the particle, the "rest system," the time is given by

$$t = \frac{\ell_\Sigma}{v}\left(1 - \frac{v^2}{c^2}\right)\,,$$

which can be put in the form

$$t = \frac{m_0 \ell_\Sigma}{p_\Sigma c}\,,$$

where $m_0 = 1193$ is the rest mass of the Σ^- particle and $c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$ is the speed of light. Write a program that calculates the amount of time that each Σ^- lives, and determine an average lifetime. The accepted value is 1.49×10^{-10} seconds. Since all our photographs are less than life-size, the computed times must be multiplied by the scale factor 1.71. This method of finding the Σ^- lifetime can only be expected to give a very approximate result because (a) only four events are used and (b) we have ignored the exponential character of particle decay.

Acknowledgments

A. Fortran, Basic, C++ Programs

All programs are at

`http://www.physnet.org/home/modules/support_programs`

which can be navigated to from the home page at

`http://www.physnet.org`

by following the links: $\rightarrow$ `modules` $\rightarrow$ `support programs`, where the programs are:

`m358p1f.for`, Fortran;
`m358p1b.bas`, Basic;
`m358p1c.cpp`, C++;
`lib351.h`, needed Library for C++ program;

MODEL EXAM

Examinee:

On your computer output sheet(s):

(i) Mark page numbers in the upper right corners of all sheets.

(ii) Label all output, including all axes on all graphs.

On your Exam Answer Sheet(s), for each of the following parts of items (below this box), show:

(i) a reference to your annotated output; and

(ii) a blank area for grader comments.

When finished, staple together your sheets as usual, but include the original of your annotated output sheets just behind the Exam Answer Sheet.

1. Submit your hand-annotated output for determination of the invisible-particle mass from the bubble chamber photographs. Be sure that it shows:

   a. your measurements of four quantities $(\ell_\Sigma, \theta, s, \ell_\pi)$ for each of the five events, including a comparison to the values listed in the module for the first of them;

   b. your computed neutral missing-particle mass for each of the five events;

   c. your calculation of the mean and average deviation of the mass in b);

   d. your identification of the neutral particle.

2. Submit your hand-annotated output for determination of the visible-particle lifetime from the bubble chamber photographs. Be sure that it shows:

   a. the program you created for the determination;

   b. the lifetime of each of the five Σ^- particles, the lifetime averaged over the five events, and a comparison to the accepted value.

INSTRUCTIONS TO GRADER

If the student has submitted copies rather than originals of the computer output, state that on the exam answer sheet and **immediately stop grading the exam and give it a grade of zero.**