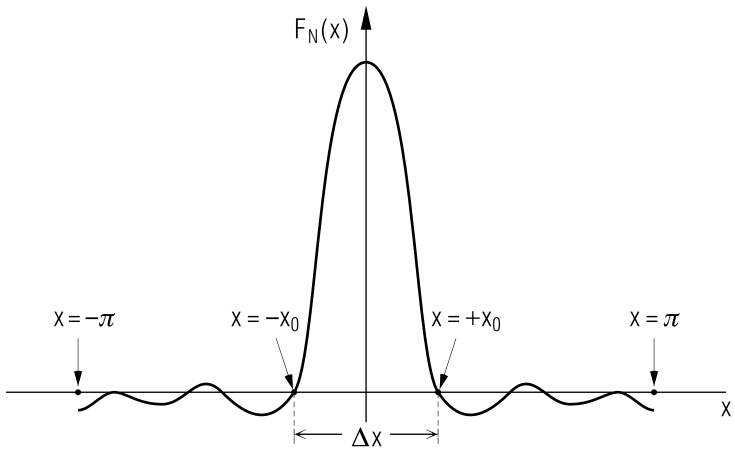
PHYSNET

MISN-0-352

FOURIER SYNTHESIS; SUPERPOSITION OF WAVES



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

FOURIER SYNTHESIS; SUPERPOSITION OF WAVES

by

Robert Ehrlich

Title: **Fourier Synthesis; Superposition of Waves**

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Input Skills:

1. Vocabulary: amplitude, period, wave (MISN-0-201).
2. Write (or modify) and run a program that utilizes advanced programming features such as arrays and nested loops in FORTRAN (MISN-0-347) or in BASIC.

Output Skills (Knowledge):

K1. Vocabulary: superposition principle, Fourier analysis, Fourier synthesized function.

K2. State Fourier's theorem and the expressions for the Fourier coefficients.

K3. State the expression for the average deviation of the Fourier synthesized fraction from the exact function.

Output Skills (Project):

P1. Compute and plot the synthesized wave form $F_N(x)$ for triangular, Fourier square and spiked wave forms and also compute the average deviation for $N = 100$. Based on your own output, discuss how the value of N and type of wave form effects the goodness of the approximation for the Fourier synthesized wave form $F_N(x)$ compared to the exact wave form $F(x)$.

P2. Fourier synthesize saw tooth and a square pulse wave forms. Based on your output, discuss how the behavior of the Fourier synthesized saw tooth and square pulse depends on N. In particular discuss the relation between the width of the pulse and N.

External Resources (Required):

1. A computer with FORTRAN or BASIC.

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1. Introduction

1a. Fourier Synthesized Functions. The present module explores the implication of Fourier's theorem which says that any function $F(x)$ defined in the interval $0 < x < 2\pi$ can be expressed as:

$$F(x) = \sum_{k=0}^{\infty} a_k \cos(kx) + \sum_{k=1}^{\infty} b_k \sin(kx)\,, \tag{1}$$

as long as $F(x)$ is sufficiently well behaved. In effect, $F(x)$ can be expressed as an infinite sum of sine and cosine functions with multiplying coefficients $a_k (k = 0, 1, 2, \ldots)$ and $b_k (k = 1, 2, 3, \ldots)$. The coefficients a_k and b_k depend on $F(x)$. They are given by:

$$\begin{aligned} a_k &= \frac{1}{\pi}\int_{-\pi}^{+\pi} F(x)\cos(kx)\,dx, k = 1, 2, 3, \ldots \\ b_k &= \frac{1}{\pi}\int_{-\pi}^{+\pi} F(x)\sin(kx)\,dx, k = 1, 2, 3, \ldots \\ a_0 &= \frac{1}{\pi}\int_{-\pi}^{+\pi} F(x)\,dx\,. \end{aligned} \tag{2}$$

Since the sum of sine and cosine terms in Eq. (1) is periodic with period 2π, the function $F(x)$ will only be given by Eq. (1) inside the interval $0 \le x \le 2\pi$ unless it too is periodic outside this interval. In general, there are infinite number of non-zero coefficients a_k and b_k. Practical calculations cannot sum an infinite number of terms in general so the sum in Eq. (1) is cut off at some integer N. This leads to a "Fourier synthesized function" $F_N(x)$ where:

$$F_N(x) = \sum_{k=0}^{N} a_k \cos(kx) + \sum_{k=1}^{N} b_k \sin(kx)\,. \tag{3}$$

Obviously,

$$F(x) = \lim_{N\to\infty} F_N(x).$$

The purpose of this module is to determine how well $F(x)$ is approximated by the Fourier synthesized function $F_N(x)$. The accuracy of the approximation will depend on $F(x)$ as well as N.

1b. Accuracy of the Synthesis vs. Number of Terms. The simplest way to judge the goodness of the approximation to a particular wave form, obtained using the Fourier expansion, Eq. (3), is to plot the function $F(x)$ and the Fourier synthesized function $F_N(x)$ on the same graph. An inspection of the two curves may be all that is necessary to determine if enough terms have been used to achieve the desired degree of closeness. To obtain a quantitative measure of the difference between the two functions $F(x)$ and $F_N(x)$, we calculate the average deviation, defined as

$$\Delta s_N = \frac{1}{M}\sum_{j=1}^{M} |F(x_j) - F_N(x_j)| \tag{4}$$

where M is some number of points at which the functions $F(x)$ and $F_N(x)$ are sampled. By calculating Δs_N for a range of N values for a given function $F(x)$, we can see how the goodness of the approximation depends on N.

1c. Physical Interpretation of Fourier Synthesis. A geometric or physical interpretation of the Fourier theorem and the Fourier synthesized function $F_N(x)$ can be obtained by appealing to the "superposition principle." The superposition principles states that whenever two or more waves arrive at some point in space at the same time, the amplitude of the resultant wave is obtained by simply summing the amplitudes for each of the individual waves. Thus, we can look at the Eq. (1) for $F(x)$ and Eq. (3) for the Fourier synthesized function $F_N(x)$ as simply adding up a series of sine and cosine waves of different wavelengths and with appropriately chosen amplitudes a_k and b_k in order to get the resultant wave $F(x)$ or $F_N(x)$. The terms "Fourier analysis" and "Fourier synthesis" are closely related. Fourier analysis involves the determination of the component wave amplitudes corresponding to a particular wave form F(x) using Eqs. (2). Fourier synthesis involves the combination of some number N of the component waves in order to approximate the wave form $F(x)$.

2. Fourier Synthesis of Simple Wave Functions

2a. Three Sample Wave Forms. The three wave forms displayed in Fig. 1, which we shall refer to as a triangle wave, a square wave, and a spike wave, can be defined in the interval $-\pi \le x \le +\pi$, as follows:

$$\text{triangular wave: } F(x) = 1 - \frac{2}{\pi}|x| \tag{5}$$

$$\begin{aligned} \text{square wave: } F(x) &= +1 \text{ for } 0 \le x < \pi \\ &= -1 \text{ for } -\pi \le x < 0 \end{aligned} \tag{6}$$

$$\text{spike wave: } F(x) = \lim_{\epsilon \to 0} F_\epsilon(x)$$

where:

$$\begin{aligned} F_\epsilon(x) &\equiv \frac{1}{\epsilon} \text{ for } |x| < \frac{\epsilon}{2} \\ &\equiv 0 \text{ for } |x| > \frac{\epsilon}{2}. \end{aligned} \tag{7}$$

Each of the three wave functions can be defined for all values of x by noting that they are periodic with period 2π radians.

2b. Their Fourier Coefficients. As shown in the Appendices the coefficients a_k and b_k may be found using Eqs. (2) for each of the three wave forms:

	$\mathbf{a_k}$	$\mathbf{b_k}$
triangular wave	$\left(\frac{2}{\pi k}\right)^2 [1-(-1)^k]$	0
square wave	0	$\left(\frac{2}{\pi k}\right) [1-(-1)^k]$
spike wave	$\frac{1}{\pi}$	0

The above formulas hold for all positive integral values of k. The leading coefficient a_0 is taken to be zero in all three cases. This is equivalent to assuming that the wave form has zero net area between $-\pi$ and $+\pi$. Clearly this is true for the square and triangle waves. It is only true for the spike wave if the wave form is moved down by an amount $1/(2\pi)$. The a_k coefficients are zero for the square wave and the b_k coefficients are zero for the triangle and spike waves, for all values of k, from symmetry considerations. A symmetric function (i.e., a function such that $F(-x) = F(x)$), such as a triangle or spike wave, is expressible in terms of a sum involving only symmetric functions, such as cosines. Similarly, an antisymmetric function (i.e., a function such that $F(-x) = -F(x)$),

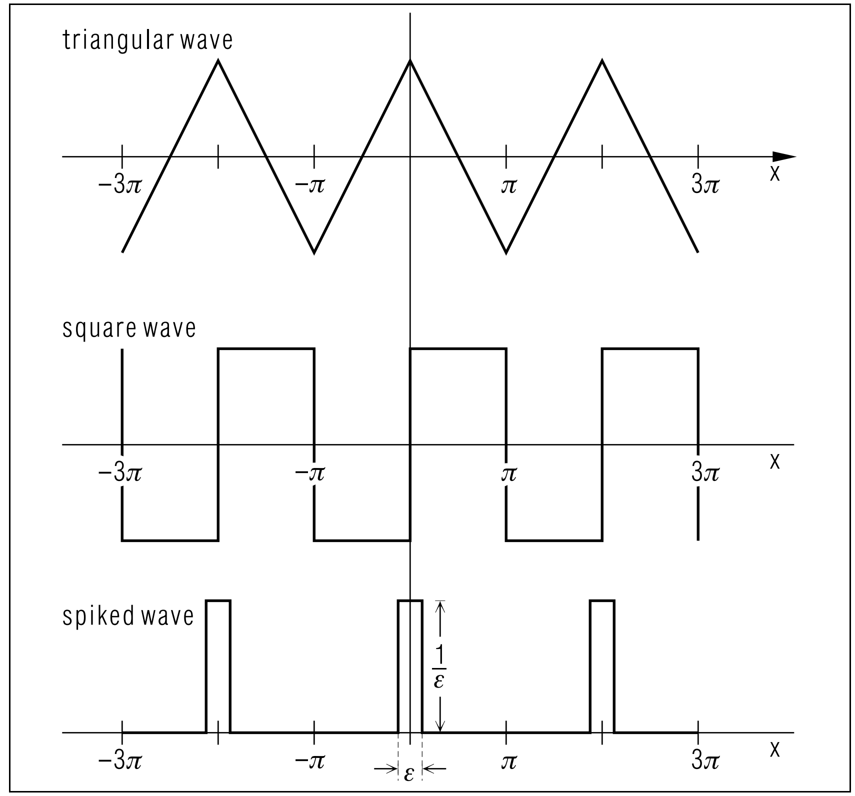


Figure 1. Three sample wave forms.

such as the square wave, is expressible in terms of a sum involving only antisymmetric functions, such as sines.

In the general case of a function $F(x)$ which is neither symmetric nor antisymmetric, both the sine and cosine sums are needed. That would be the case if any of the three wave forms are shifted to the right or left by a non-integral multiple of π. In comparing the three wave forms it is interesting to notice that the a_k or b_k coefficients have a dependence on k that can be expressed as: k^{-2} (square wave), k^{-1} (triangle wave), and k^0 (spike wave). This has an important consequence for how good an approximation the Fourier synthesized function $F_N(x)$ is to the original $F(x)$ in each case.

2c. Rate of Convergence. The dependence of the goodness of the approximation on the number of waves can be judged indirectly by seeing how a_k and b_k depend on k, since the faster a_k and b_k decrease with k, the faster Δs_N decreases with N. For the three functions depicted in Fig. 1, we would expect the approximation to improve fastest for the triangle wave, for which $a_k \propto k^{-2}$, next for the square wave, for which $b_k \propto k^{-1}$, and slowest for the spike wave, for which $a_k \propto k^0$. This result should not be surprising, in view of the fact that the three functions are increasingly "badly behaved" in this order. The triangle wave is everywhere continuous, but it has a discontinuous first derivative at the points $x = 0, \pm\pi, \pm 2\pi, \ldots$. The square wave has a discontinuity in the function itself, as well as the first derivative, at these points. Finally, the spike wave has two discontinuities, at $x = \pm\epsilon$.

2d. Examination of the Spike Wave Form. For any wave form, the more waves we add in the Fourier synthesized wave form $F_N(x)$, the more closely we approximate the wave form $F(x)$. In the case of the spike wave, this means that the width of the peak in the function $F_N(x)$ must become narrower as N increases, approaching zero as N approaches infinity. It is instructive to see how this comes about. Let us define the full width Δx of the central peak in the function $F_N(x)$, as the distance between the first zeros at $x = \pm x_0$, on either side of the peak, i.e., $\Delta x = 2x_0$, where $F_N(\pm x_0) = 0$ (see Fig. 2). For the spike wave, we have Fourier coefficients $a_1 = a_2 = a_3 = \ldots = a_N = 1/\pi$ and $b_1 = b_2 = b_3 = \ldots = b_N = 0$, so that

$$\begin{aligned} F_N(x) &= \sum_{k=1}^{N} a_k \cos(kx) \\ &= \frac{1}{\pi}(\cos(x) + \cos(2x) + \cos(3x) + \ldots + \cos(Nx)). \end{aligned} \tag{8}$$

There is a useful trick involving vectors for evaluating the sum in Eq. (8). Consider N vectors each of length $1/\pi$ arranged as shown in Fig. 3. Each vector makes an angle x with the preceding one. The sum in Eq. (8) is equal to the sum of the horizontal components of the N vectors. We can find the smallest angle between the vectors which yields a zero resultant horizontal component by requiring the resultant vector to be vertical (see Fig. 3). In this case, the angle x between one vector and the next is given by

$$x = x_0 = \frac{\pi}{N+1}. \tag{9}$$

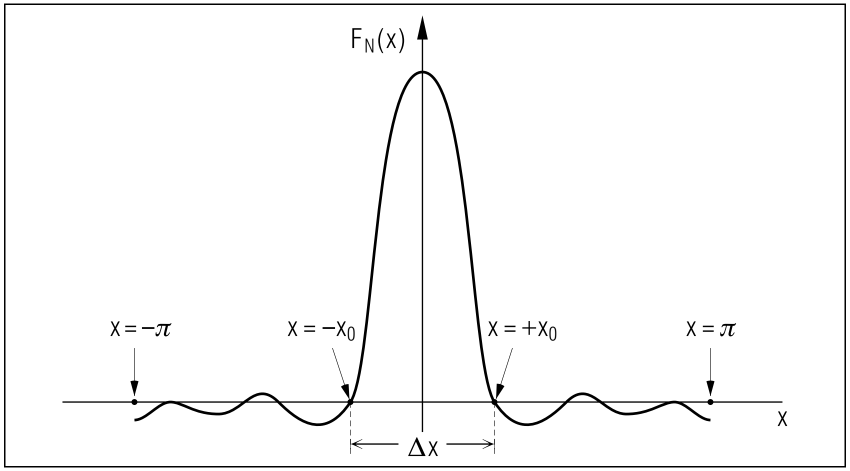


Figure 2. $F_N(x)$ for a spike wave, $N = 5$.

Thus, $x = x_0$ is the first zero in the function $F_N(x)$. The formula we have sought for the full-width of the peak is therefore

$$\Delta x = 2x_0 = \frac{2\pi}{N+1}. \tag{10}$$

Equation (10) is clearly consistent with the narrowing of the peak as N increases: the width approaches zero as N approaches infinity.

2e. Applications of the Spike Wave Form. The result indicated in Eq. (10) has important implications for a wide range of physical phe-

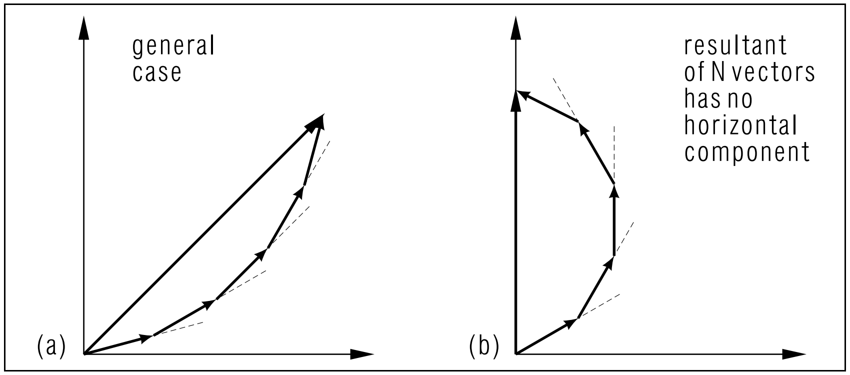


Figure 3. Sum of N vectors of length $1/\pi$ for $N = 5$.

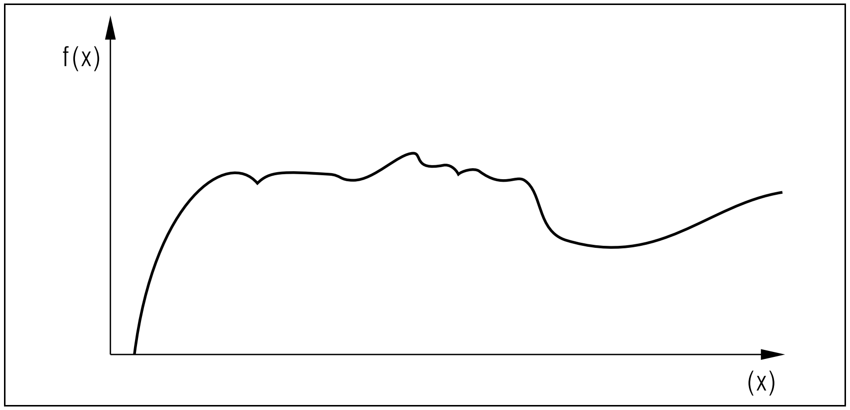


Figure 4. A wave form in the shape of a human profile.

nomena. In acoustics, for example, Eq. (10) implies that if we wish to Fourier synthesize a sound pulse of very short duration, we need to use a large number of frequencies (large band width in engineering terminology). Another interesting application of the spike wave form is to the problem of a wave packet, which may be represented by a nonperiodic function which has a single spike and is zero everywhere else. This may be considered a limiting case of the spike wave form, in which the wavelength or distance between spikes becomes infinite. Wave packets are of great utility in quantum mechanics, where this type of "localized" wave is used to represent a traveling particle. The width of the packet, Δx, is a measure of the uncertainty in the particle's position. The particle also has an uncertainty in its momentum Δp, which is proportional to the number of waves N, used in the Fourier synthesis of the wave packet. (This is because in quantum mechanics, a particle having a unique momentum p is represented by a sine (or cosine) wave of one specific frequency, so that a range of momenta Δp corresponds to a number of different frequencies.) Thus a consequence of Eq. (10) for this case is that the more localized the particle (smaller Δx) the greater the range in momentum Δp needed in the synthesis of the wave packet. This inverse relationship between Δx and Δp is the basis of the Heisenberg uncertainty principle.

3. Synthesis of Arbitrary Wave Forms

Fourier's theorem states that any periodic function is expressible in terms of a sum of sine and cosine functions. However, a formal expression

for the coefficients a_k and b_k can only be found for certain functions (such as the square, triangle, and spike wave forms). In many cases the function $F(x)$ is such that a formal evaluation of the integrals in Eqs. (2) is impossible. In other cases, even the function $F(x)$ itself may not be expressible in formally. Should $F(x)$ represent some natural phenomenon, then the function can only be given in a graph or its equivalent, a table of numbers, giving the value of the function at a series of x-values (where x usually represents either time or position). For example, to Fourier analyze human speech we must specify the sound intensity $F(x)$ at a series of x time values. In such cases, each of the coefficients, a_k, b_k, $k = 1, 2, 3, \ldots N$, must be calculated using a numerical integration technique, such as Simpson's rule.[1] As an example of wave form that needs to be treated numerically consider the function shown in Fig. 4. This function (a human profile turned sideways) has no known formal mathematical form, but it can be specified by a table of numbers (for various plotted x-values) obtained from the graph.

4. Synthesis by Computer

4a. Input. A computer program has been written to compute and plot Fourier synthesized wave forms specified by the two input parameters:

`N`	=	number of waves to use in the Fourier synthesis
`TYPE`	=	0, 1, 2, 3 to select which of the four wave forms is desired (square, triangle, spike, or human profile, respectively)

Any positive number may be used for `N`; however, for the case `TYPE` = 3, if `N` exceeds 48, only the first 48 waves will actually be included, since at present only the first 48 coefficients have been specified in the program. Note that if the value of `N` is 100 (for `TYPE` ≠ 3), then in addition to giving the usual output (the Fourier synthesized wave form), the program also generates a plot of the average deviations for `N` $= 2, 4, 6, \ldots,$ `N` waves. This provides a quantitative measure of how the goodness of the approximation depends on the number of waves.

4b. Output. The output shown in Figs. 5-10 was obtained using five sets of input parameters:

[1]See "Numerical Integration," (MISN-0-349).

	N	TYPE
Fig. 5	5	0
Fig. 6	5	1
Fig. 7	5	2
Fig. 8	100	2
Fig. 10	48	3

Recall that the values of `TYPE` on the first three lines specify the square, triangle, and spike wave forms, respectively. The output for the fourth data line (Fig. 8) is also for the spike wave form; the large value of `N` causes the spike to be extremely narrow (it is so narrow that its width cannot be determined from this figure). Since the value of `N` is 100 for this case, the computer produces an additional graph (Fig. 9) showing the average deviations for Fourier synthesized wave forms having `k` $= 2, 4, 6, \ldots, 100$ waves. The output shown in Fig. 10 is produced using the fifth set of input parameters for which `N` $= 48$ and `TYPE` $= 3$.

5. Procedures

5a. Dependence on N, the Number of Waves. Discuss the results you obtain when running the program using a series of `N` values between 1 and 100 for each of the four wave forms (`N` $= 48$ is the maximum value for the human profile wave form — `TYPE` $= 3$). An interesting phenomenon occurs for the case `N` $= 50$. Try using `N` $= 40$, 50, and 60 for the case of the square wave (`TYPE` $= 0$), and see if you can explain the differences. Discuss both the goodness of the approximation as a function of `N` for each wave form separately and how the goodness of the approximation depends on the nature of the wave form, for particular values of `N`. As was discussed in paragraph (2c) you should find that the triangle wave synthesis improves fastest with increasing `N`, and the spike wave increases slowest with increasing `N`.

5b. Plots of Average Deviation. A quantitative way to study the improvement in the Fourier synthesized wave form with increasing `N` is to examine the average deviation plots for each wave form. These plots are created when `N` is selected to be 100. Compare the three average deviation plots you obtain for the triangle, square, and spike wave forms, and relate your observations to the formulas for the Fourier coefficients (and the discussion in paragraph (2c)). An example of the results for the spike wave form is shown in Fig. 9 which is a plot of the average deviation versus the number of waves. As expected the average deviation decreases

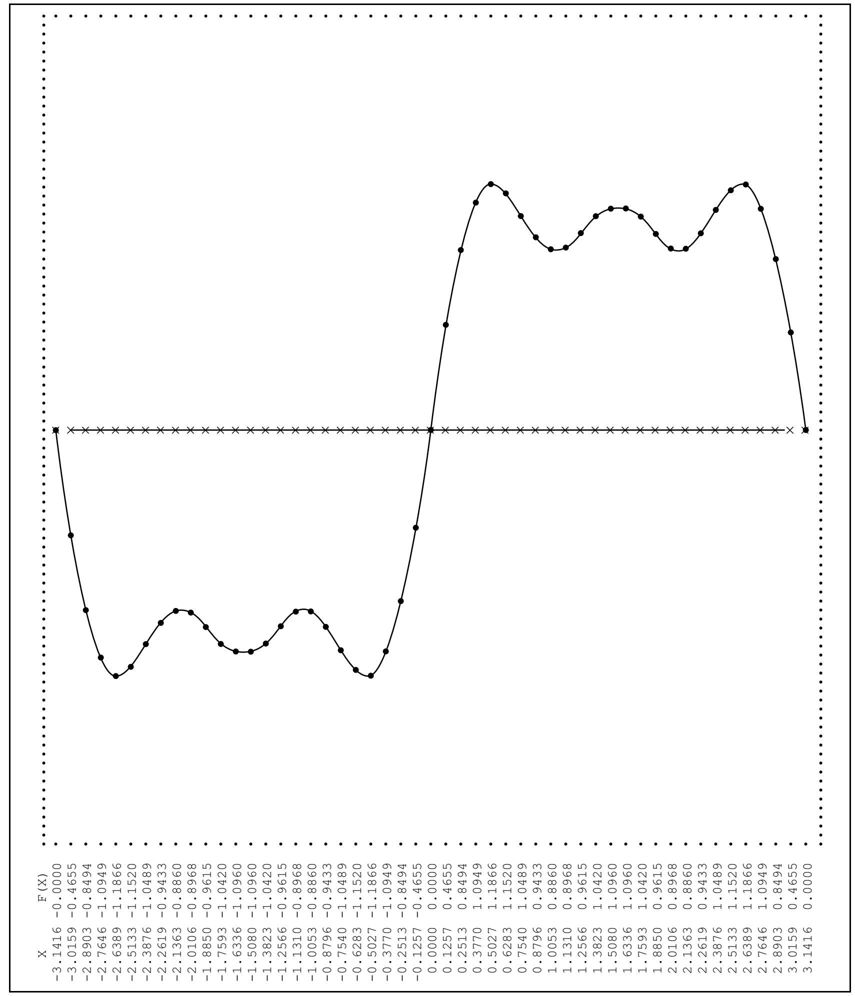


Figure 5. Computer output for Fourier synthesized square wave, `N` $= 5$.

with increasing `N`. One surprising feature of Fig. 9 is the sharp dips that occur at certain values of `N`. See if you can explain the cause of these irregularities. Hint: In computing the average deviation, the theoretical and Fourier synthesized wave forms are sampled at a finite number of points. The values of the wave forms are not sampled between those evenly-spaced x-points.

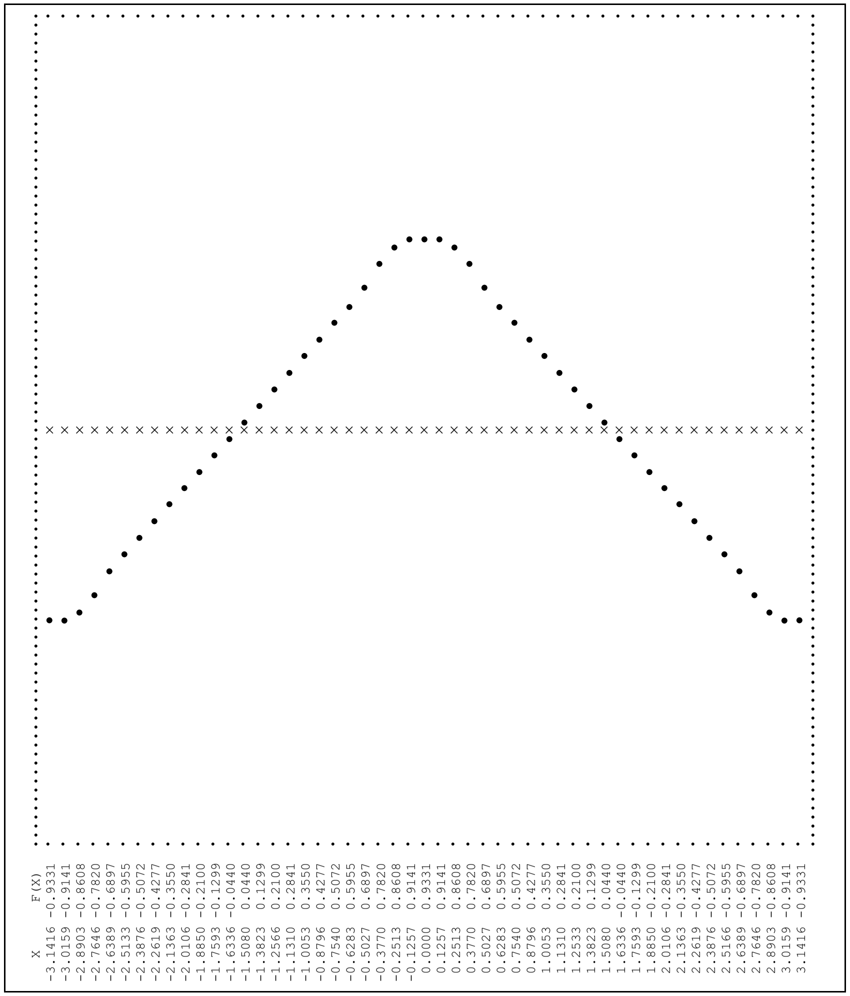


Figure 6. Computer output for Fourier synthesized triangle wave, N $= 5$.

5c. Two Other Wave Forms. Modify the program, so that it can Fourier synthesize the "saw tooth" wave form depicted in Fig. 11. The Fourier coefficients are given by

$$a_k = 0;\ b_k = \frac{2}{\pi k}(-1)^{k+1}\,.$$

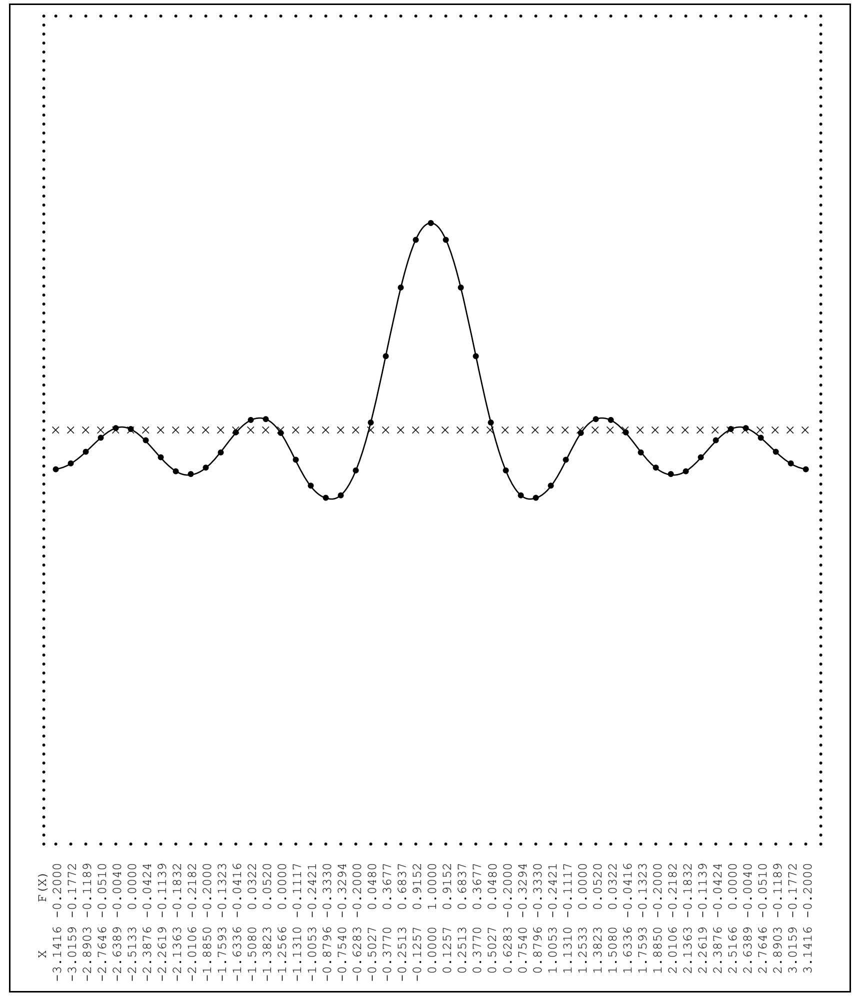


Figure 7. Computer output for Fourier synthesized spike wave, N $= 5$.

Also, modify the program so that it can Fourier synthesize the "square pulse" wave form depicted in Fig. 12. The Fourier coefficients are given by:

$$a_k = \frac{2}{\pi k}\sin(kx_0);\ b_k = 0\,.$$

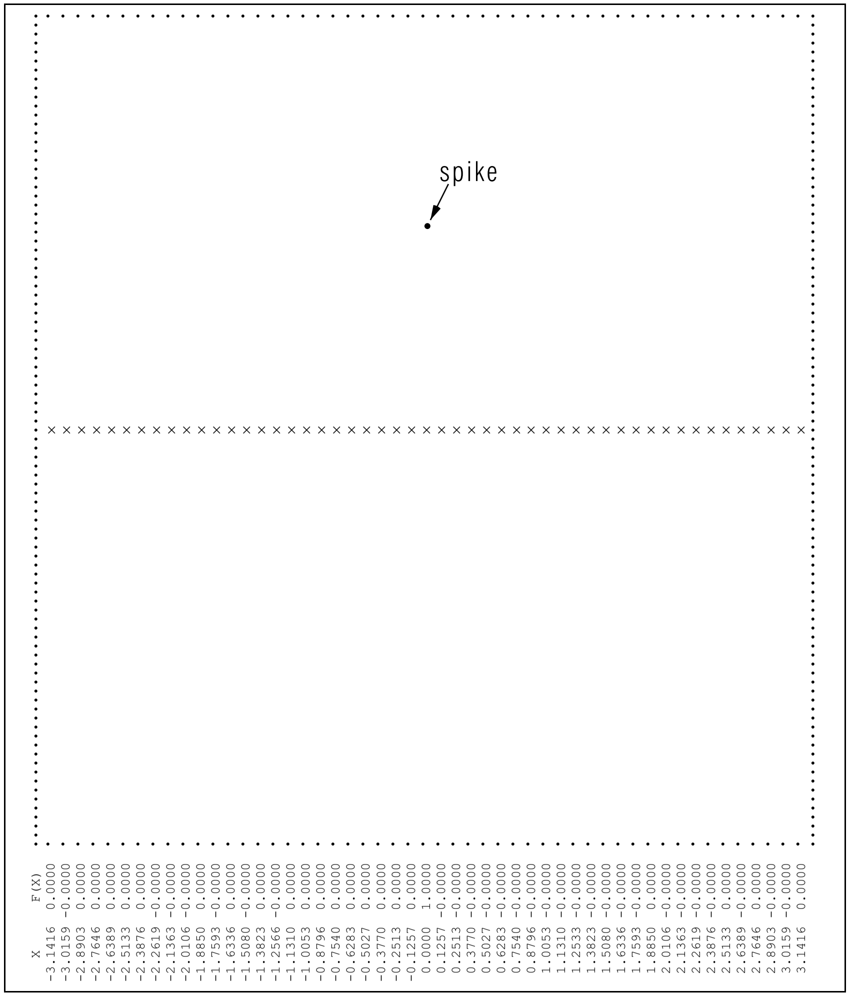
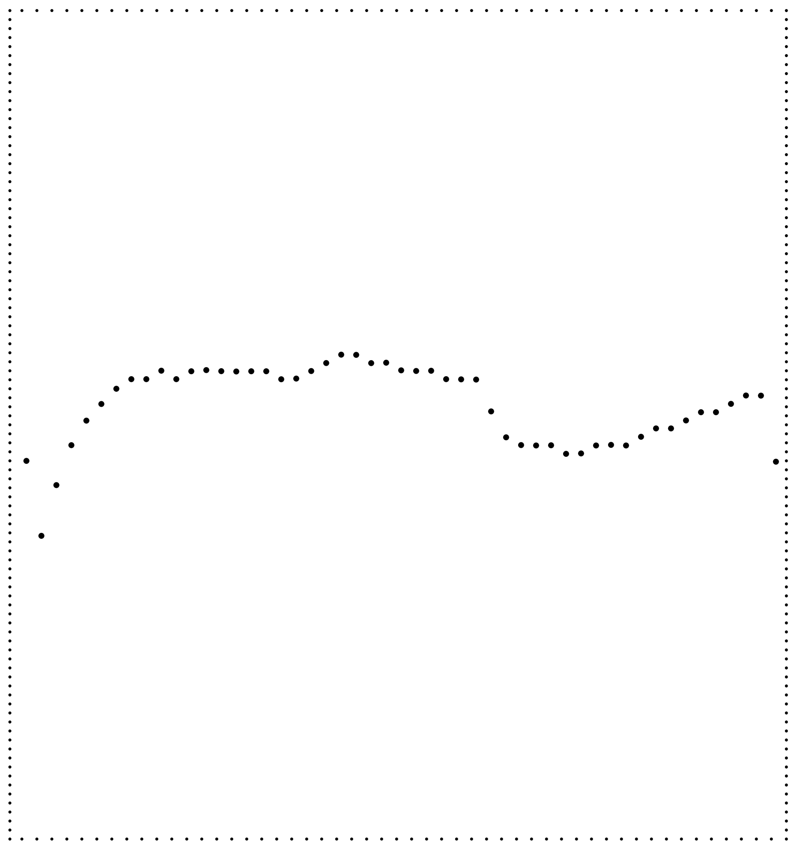


Figure 8. Computer output for Fourier synthesized spike wave, N $= 100$.

(Let TYPE $= 4.0$ for the saw tooth and $TYPE = 5.0$ for the square pulse.) Run the program using various values for the parameter x_0, which is the half-width of the pulse (see Fig. 12). For each value of x_0, use a number of values for N, the number of waves. Discuss how the number of waves required to get a reasonably good approximation to the wave form depends on x_0. A graph of the average deviations for k $= 2, 4, 6, \ldots, 100$

Figure 9. Computer output for average deviation of Fourier synthesized spike wave for N $= 2, 4, \ldots, 100$.

waves, which is printed when N $= 100$, would be useful for this discussion.



X	F(X)
−3.1416	−0.2800
−3.0159	−0.6204
−2.8903	−0.3812
−2.7646	−0.1872
−2.6389	−0.0713
−2.5133	0.0131
−2.3876	0.0692
−2.2619	0.1095
−2.1363	0.1392
−2.0106	0.1478
−1.8850	0.1384
−1.7593	0.1592
−1.6336	0.1661
−1.5080	0.1663
−1.3823	0.1658
−1.2566	0.1644
−1.1310	0.1585
−1.0053	0.1370
−0.8796	0.1358
−0.7540	0.1544
−0.6283	0.1897
−0.5027	0.2474
−0.3770	0.2325
−0.2513	0.1865
−0.1257	0.1806
0.0000	0.1562
0.1257	0.1681
0.2513	0.1550
0.3770	0.1333
0.5027	0.1302
0.6283	0.1138
0.7540	−0.0295
0.8796	−0.1532
1.0053	−0.1934
1.1310	−0.2116
1.2533	−0.2185
1.3823	−0.2225
1.5080	−0.2214
1.6336	−0.2120
1.7593	−0.1994
1.8850	−0.1814
2.0106	−0.1581
2.1363	−0.1393
2.2619	−0.1119
2.3876	−0.0842
2.5133	−0.0567
2.6389	−0.0286
2.7646	−0.0040
2.8903	0.0217
3.0159	0.0259
3.1416	−0.2800

Figure 10. Computer output of Fourier synthesized human profile, $\mathtt{N} = 48$.

Acknowledgments

Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

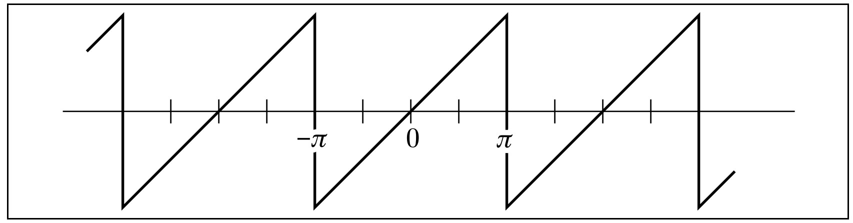


Figure 11. The saw tooth wave form.

A. Fourier Coefficients: Square Wave

For the square wave, we have

$$a_k = \frac{1}{\pi}\int_{-\pi}^{+\pi} F(x)\cos(kx)\,dx = 0;\ \text{for } k = 1, 2, 3, \ldots$$

The integral vanishes because the integrand is an antisymmetric function, being the product of an antisymmetric function $[F(x)]$ and a symmetric function ($\cos kx$). The b_k coefficients for the square wave can be found using

$$b_k = \frac{1}{\pi}\int_{-\pi}^{+\pi} F(x)\sin(kx)\,dx\,.$$

With the definition of the square wave,

$$F(x) = +1 \text{ for } 0 \le x \le \pi$$
$$= -1 \text{ for } -\pi \le x \le 0\,,$$

this becomes

$$b_k = -\frac{1}{\pi}\int_{-\pi}^{0} \sin(kx)\,dx + \frac{1}{\pi}\int_{0}^{+\pi} \sin(kx)\,dx\,.$$

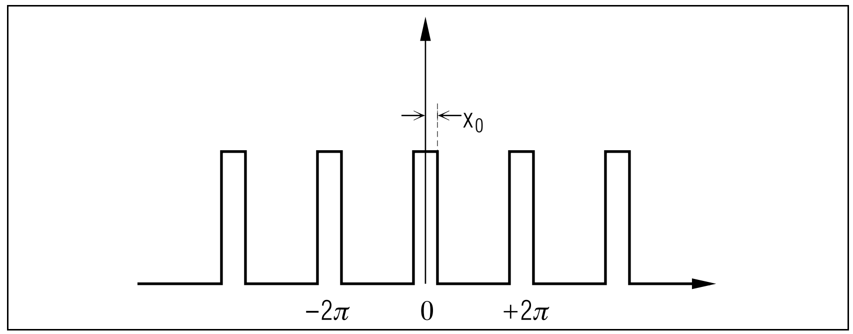


Figure 12. The square pulse wave form.

These integrals can be easily evaluated, giving

$$\text{square wave: } b_k = \frac{2}{\pi k}\left[1-(-1)^k\right] \text{ for } k = 1,2,3,\ldots$$

B. Fourier Coefficients: Triangle Wave

In the case of the triangle wave, we find

$$b_k = \frac{1}{\pi}\int_{-\pi}^{+\pi} F(x)\sin(kx)\,dx = 0$$

since the integrand is antisymmetric. The a_k coefficients for the triangle wave can be found using

$$a_k = \frac{1}{\pi}\int_{-\pi}^{+\pi} F(x)\cos(kx)\,dx\,.$$

With the definition of the triangle wave: $[F(x) = 1-(2/\pi)\,|x|]$, this becomes

$$a_k = \frac{1}{\pi}\int_{-\pi}^{0}\left(1+\frac{2x}{\pi}\right)\cos(kx)\,dx + \frac{1}{\pi}\int_{0}^{+\pi}\left(1-\frac{2x}{\pi}\right)\cos(kx)\,dx\,.$$

These two integrals can be easily evaluated, giving

$$\text{triangle wave: } a_k = \left(\frac{2}{\pi k}\right)^2\left[1-(-1)^k\right];\ \text{for } k = 1,2,3,\ldots$$

C. Fourier Coefficients: Spike Wave

Finally, in the case of the spike wave, we have

$$b_k = \frac{1}{\pi}\int_{-\pi}^{+\pi} F(x)\sin(kx)\,dx = 0$$

because the integrand is antisymmetric. The coefficient a_k for the spike wave can be found using

$$a_k = \frac{1}{\pi}\int_{-\pi}^{+\pi} F(x)\cos(kx)\,dx\,.$$

We define the spike wave as the limit, as ϵ goes to zero, of:

$$F(x) = \frac{1}{\epsilon} \text{ for } |x| < \frac{\epsilon}{2};\ F(x) = 0 \text{ for } |x| > \frac{\epsilon}{2}\,.$$

In the limit as ϵ goes to zero, this is called a "delta function." The area under the delta function, its area, is unity. The function's Fourier coefficients can be found by starting with:

$$a_k = \frac{1}{\pi}\int_{-\epsilon/2}^{\epsilon/2}\frac{1}{\epsilon}\cos(kx)\,dx\,.$$

As ϵ approaches zero, we have $cos\,(kx) \approx 1$ for all x in the interval $-(\epsilon/2) < x < +(\epsilon/2)$, so:

$$\text{spike wave: } a_k = \frac{1}{\pi} \text{ for } k = 0,1,2,3,\ldots$$

Note that it is only for the spike wave that the a_k coefficients are independent of k.

One problem with the synthesized delta function ("spike") wave is that the height of the central peak is infinite for the exact wave but finite for the synthesized wave. This makes the deviation for the central peak infinite and the same for the average deviation. In addition, the height of the central peak of the synthesized function depends strongly on the number of cosine waves that form it. This is in contrast to the other wave forms, where the "size" of the synthesized wave is roughly independent of the number of contributing waves. For plotting, the synthesized spike wave is the only one which needs a scale factor that depends on the number of contributing waves.

We could scale the spike wave plot by requiring that it have unit area, just as does the exact function. However, that gives the rather useless result that the average deviation, omitting the central peak value, is roughly independent of the number of contributing waves. A skinny central peak, resulting from the use of many cosine waves, has the same average deviation as does a fat central peak, resulting from use of only a few waves. Visually, however, the skinny peak looks more like what we are after.

In order that the average deviation shall roughly correspond to a synthesized function's visual quality, we multiply the synthesized function at all x-values by a single constant that makes the function have unit height at its peak. Then the plot with the desirable skinny peak has a smaller average deviation than does the plot with a fat peak. (The constant multiplier equals the number of participating cosines. Why?)

D. Fortran, Basic, C++ Programs

All programs are at

`http://www.physnet.org/home/modules/support_programs`

which can be navigated to from the home page at

`http://www.physnet.org`

by following the links: → `modules` → `support programs`, where the programs are:

`m352p1f.for`, Fortran;
`m352p1b.bas`, Basic;
`m352p1c.cpp`, C++;
`lib351.h`, needed Library for C++ program;

MODEL EXAM

1-3. See Output Skills K1-K3

Examinee:
On your computer output sheet(s):

(i) Mark page numbers in the upper right corners of all sheets.

(ii) Label all output, including all axes on all graphs.

On your Exam Answer Sheet(s), for each of the following parts of items (below this box), show:

(i) a reference to your annotated output; and

(ii) a blank area for grader comments.

When finished, staple together your sheets as usual, but include the original of your annotated output sheets just behind the Exam Answer Sheet.

4. Submit your hand-annotated output for the Fourier synthesis of each of the four wave forms, showing:

   a. a series of `N` values between 1 and 100 for each of the four wave forms, such that one can see the highlights of the progression with `N`;
   b. the interesting phenomenon for the case `N` = 50;
   c. how the goodness of the approximation depends on `N` for fixed wave form, for each wave form in turn;
   d. how the goodness of the approximation depends on the wave form for fixed `N`, for several `N` values.

5. Submit your hand-annotated output for the average magnitude of the deviation of the synthesized waves from the input waves, showing:

   a. how the details of the plots for the three wave forms relate to the discussion in 4c and 4d above.
   b. why there are sharp dips in the plots and an especially sharp dip at `N` = 50. Note: you must not just repeat the "hint" given in the text; you must give a complete argument.

6. Submit your hand-annotated output for the saw tooth case, showing that you achieved a good synthesis of the target form.

7. Submit your hand-annotated output for the square pulse case, showing how the number of waves required to get a reasonably good approximation depends on the pulse width.

INSTRUCTIONS TO GRADER

If the student has submitted copies rather than originals of the computer output, state that on the exam answer sheet and **immediately stop grading the exam and give it a grade of zero.**