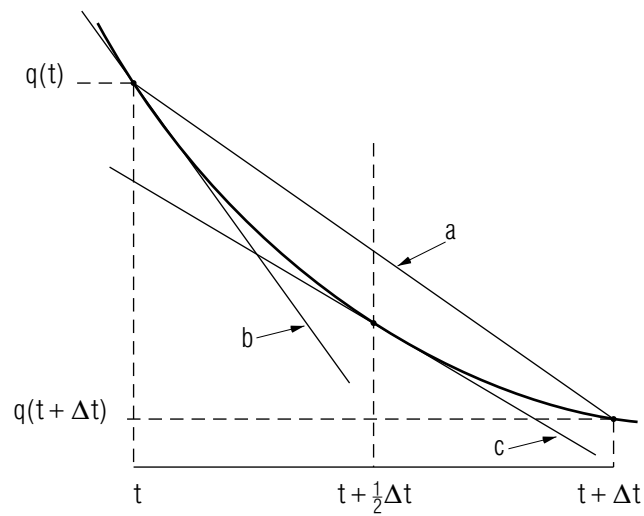


EULER'S METHODS FOR SOLVING DIFFERENTIAL EQUATIONS; RC CIRCUITS



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by
Robert Ehrlich

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Input Skills:

1. Vocabulary: linear scaling function (MISN-0-348).
2. State the expression for the potential difference across a capacitor in terms of the charge stored on the capacitor's plates (MISN-0-135).

Output Skills (Knowledge):

- K1. Describe the algorithm for solving first order differential equations using Euler's method.

Output Skills (Project):

- P1. Enter and run a program to compute the exact charge as a function of time on a discharging capacitor in an RC circuit and the approximate charge as a function of time resulting from a numerical solution of the differential equation for an RC circuit using Euler's method.
- P2. Modify the P1 program (above) so that it computes the charge on a capacitor in an RC circuit when a square wave input is applied.
- P3. Modify the P1 program (above) so that it computes the charge on a capacitor in an RC circuit using the improved Euler Method.

External Resources (Required):

1. A computer with BASIC or FORTRAN or C++.

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EULER'S METHODS FOR SOLVING DIFFERENTIAL EQUATIONS; RC CIRCUITS

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Robert Ehrlich

1. Euler's Method

1a. The Problem. In this module, an algorithm using Euler's method will be developed for the numerical solution of any first order differential equation which can be put into the form,

$$\frac{dq}{dt} = f(q, t), \quad (1)$$

where $f(q, t)$ is some known function of q and t . The method will be used to numerically solve the differential equation for an RC circuit. In this case, $q(t)$ gives the charge on the capacitor as a function of time.

1b. The Method of Solution. To apply Euler's method, the derivative dq/dt is written as a finite difference $\Delta q/\Delta t$, where $\Delta q = q(t + \Delta t) - q(t)$. In this case, Eq. (1) becomes:

$$\Delta q = f(q, t) \Delta t \quad (2)$$

so that:

$$q(t + \Delta t) = q(t) + f(q, t) \Delta t. \quad (3)$$

Repeated use of Eq. (3) allows us to numerically find the function $q(t)$ at a series of times: $t = \Delta t, 2\Delta t, 3\Delta t, \dots$, in terms of some initial value $q(0)$.

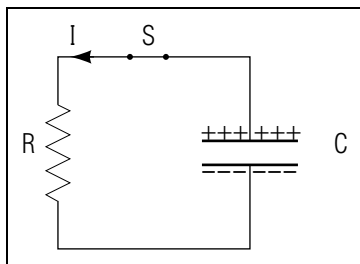


Figure 1. The RC Circuit (Capacitor Discharging)

1c. Application to the RC Circuit. If an initially charged capacitor is connected across a resistor, as shown in Fig. 1, it begins to discharge as soon as the switch S is closed ($t = 0$). In this problem we are interested in studying the time dependence of the charge on the capacitor. Once the switch is thrown, the voltage across the resistor V_R must at all times have the same magnitude as the voltage across the capacitor V_C . Since crossing a resistor in the direction of the current represents a potential drop, we may write

$$V_R + V_C = 0.$$

With the substitutions $V_R = IR = R dq/dt$ and $V_C = q/C$ we have:

$$R \frac{dq}{dt} + \frac{q}{C} = 0$$

so that:

$$\frac{dq}{dt} = -\frac{q}{T} \quad (4)$$

where $T = RC$ is the so-called "time constant" of the RC circuit and has units of time (e.g. seconds). This first order differential equation has been expressed in a form permitting its solution by Euler's method. This particular differential equation also has a simple exact solution: $q(t) = q_0 e^{-t/T}$, as can be verified by direct substitution in Eq. (4). The existence of this exact solution permits a direct check on the accuracy of Euler's method.

1d. Accuracy of Euler's Method. The solution of Eq. (4) using Euler's method here uses:

$$\Delta q = \frac{dq}{dt} \Delta t = -\frac{q}{T} \Delta t$$

so that the value of the charge q at time $t + \Delta t$ can be found in terms of the charge at time t using:

$$q(t + \Delta t) = q(t) - \frac{q}{T} \Delta t \quad (5)$$

Suppose, for example, we assume that $T = 10$, $\Delta t = 1$, and that at time $t = 0$ we have $q(0) = 1$. We can then use Eq. (5) to find $q(1)$ in terms of $q(0)$:

$$q(1) = 1 - \frac{1}{10}(1) = 0.900.$$

A second use of Eq. (5) to obtain $q(2)$ from $q(1)$ yields:

$$q(2) = 0.900 - \frac{0.900}{10}(1) = 0.810.$$

Repeating this process allows us to find $q(t)$ at a series of times $t = 0, 1, 2, 3, 4, 5, 6$ yielding these values for $q(t)$:

$$1.000, 0.900, 0.810, 0.729, 0.656, 0.590.$$

These values may be contrasted with the exact solution from $q(t) = q_0 e^{-t/T}$, at the same seven times, for $q(t)$:

$$1.00, 0.904, 0.818, 0.740, 0.670, 0.606.$$

1e. Limitations of Euler's Method. The above agreement between the numerical solution using Euler's method and the exact solution is only fair. It could be improved by making the size of the time step Δt a smaller fraction of T . However, as we shall see in the next section, the method itself can be changed to give better agreement without using smaller values of Δt . This can be of considerable practical importance, since the smaller we make Δt , the more time steps we need to reach a given time, and hence the longer it takes to carry out the computation. In addition, steps that are too small can result in round-off or truncation error.¹

2. Improved Euler's Method

2a. A Better Approximation. The basis for the Euler method of solving a differential equation is the replacement of the derivative dq/dt at time t by the finite ratio $\Delta q/\Delta t$,

$$\frac{dq}{dt}(t) \approx \frac{\Delta q}{\Delta t} = \frac{q(t + \Delta t) - q(t)}{\Delta t} \quad (6)$$

so that the solution using Euler's method can only be as good as this approximation. An improvement can be made based on the fact that the right side of Eq. (6) is actually a better approximation to the derivative at time $t + (1/2)\Delta t$ than at time t :

$$\frac{dq}{dt}\left(t + \frac{\Delta t}{2}\right) \approx \frac{q(t + \Delta t) - q(t)}{\Delta t}$$

This can easily be seen from Fig. 2; the slope of line "a," $\Delta q/\Delta t$, is much closer to that of line "c," $dq/dt(t + (1/2)\Delta t)$, than to that of line "b,"

¹For examples, see Appendix B of module MISN-0-245.

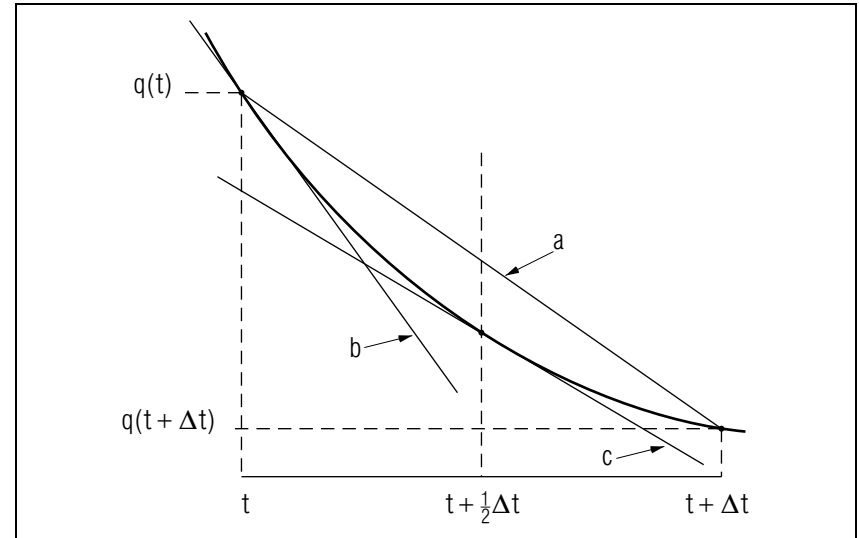


Figure 2. Graphical basis of the improved Euler's method. See text for details.

$dq(t)/dt$, independent of the detailed shape of the function $q(t)$. The above equation can be rearranged to yield

$$q(t + \Delta t) \approx q(t) + \frac{dq}{dt}\left(t + \frac{\Delta t}{2}\right) \Delta t. \quad (7)$$

In order to use Eq. (7) to find $q(t + \Delta t)$, the value of the charge at the "new" time $t + \Delta t$, we need to know the value of $q(t)$, the charge of the "old" time t , and also the derivative dq/dt at the intermediate time $t + (1/2)\Delta t$. An approximate value for the derivative at time $t + (1/2)\Delta t$ can be found by averaging the values at times t and $t + \Delta t$:

$$\frac{dq}{dt}\left(t + \frac{\Delta t}{2}\right) \approx \frac{1}{2} \left(\frac{dq}{dt}(t + \Delta t) + \frac{dq}{dt}(t) \right) \quad (8)$$

This approximation assumes, in effect that the derivative at the "average" time $t + (1/2)\Delta t$ is approximately equal to the average of the derivatives at times t and $t + (1/2)\Delta t$. This is only exactly true if $q(t)$ has at most a quadratic dependence on time. Substitution of Eq. (8) into (7) yields

$$q(t + \Delta t) = q(t) + \frac{\Delta t}{2} \left[\frac{dq}{dt}(t) + \frac{dq}{dt}(t + \Delta t) \right] \quad (9)$$

In order to clarify the meaning of Eq. (9), we use the subscripts “new” and “old” to signify quantities evaluated at times $t + \Delta t$ and t , respectively. Therefore Eq. (9) can be rewritten in the form

$$q_{\text{new}} = q_{\text{old}} + \frac{\Delta t}{2}(q'_{\text{new}} - q'_{\text{old}}) \quad (10)$$

where the primes indicate derivatives with respect to time. Note that to calculate a value for q_{new} , Eq. (10) requires a value for q'_{new} . However, we cannot obtain q'_{new} from Eq. (4) because that requires knowledge of the very q_{new} we are trying to find! Euler’s improved method gets around this circular set of requirements by specifying that we first calculate an approximate value for q_{new} using the original Euler’s method, then use this value in Eq. (4) to find q'_{new} , then use this q'_{new} in Eq. (10) to obtain our final value of q_{new} . One could alternately recompute q_{new} and q'_{new} for the same value of time until the values were unchanged, to the desired accuracy, from one iteration to the next. However, that is rightly rejected in the improved Euler method as a waste of time.

2b. Flow Diagram for Euler’s Methods.

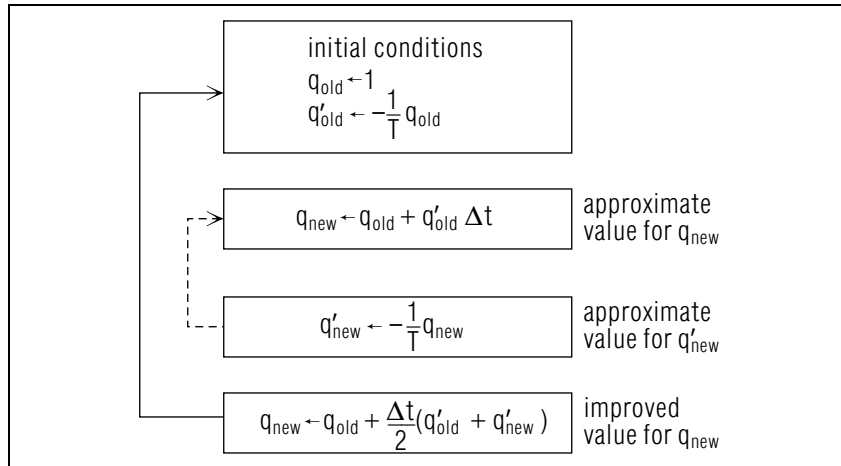


Figure 3. Flow diagram for Euler’s method. The dashed line shows the return path in the loop for the original method, the solid line for the improved method.

2c. Sample Calculation Using Improved Euler’s Method. We can illustrate the improved Euler’s method using the same values for the parameters used in the illustration of the original Euler’s method: $q_0 = 1$, $T = 1.0$, and $t = 0.1$. As in the original Euler’s method, we first find an approximate value for q_1 using

$$q_1 = q_0 + q'_0 \Delta t,$$

where, for brevity, we have written $q(0)$ as q_0 and $q(1)$ as q_1 . Substituting $T = 1.0$ in the differential equation, $q' = -(1/T)q$, permits us to write $q' = -q$ for any time. We then find this approximate value of q_1 :

$$q_1 = 1.0 + (-1.0)(0.1) = 0.9.$$

We find an improved value for q_1 using

$$q_1 = q_0 + \frac{\Delta t}{2}(q'_0 + q'_1)$$

where the derivative is $q'_1 = -q_1$. We therefore obtain

$$q_1 = 1.0 + \frac{1}{2}(0.1)(-1.0 - 0.9) = 0.905.$$

We can repeat the whole procedure in order to find q_2 , first finding an approximate value for it from

$$q_2 = q_1 + q'_1 \Delta t,$$

which yields

$$q_2 = 0.905 + (-0.905)(0.1) = 0.8145.$$

We then find an improved value using

$$q_2 = q_1 + \frac{\Delta t}{2}(q'_1 + q'_2)$$

which yields

$$q_2 = 0.905 + \frac{1}{2}(0.1)(-0.905 - 0.8145) = 0.819.$$

The values for q_1 and q_2 using the improved Euler’s method are much closer to the exact values ($q_1 = 0.904$, $q_2 = 0.818$) than are the values obtained using the original Euler’s method ($q_1 = 0.900$, $q_2 = 0.810$).

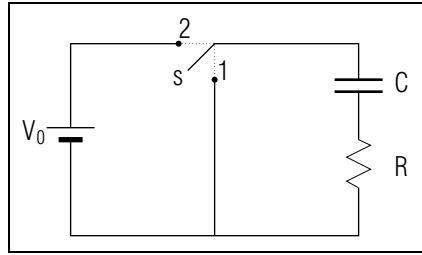


Figure 4. Circuit to charge capacitor

3. Other RC Circuits

3a. Charging Capacitor. In addition to considering the case of a capacitor being discharged (Fig. 1), we may also examine the time dependence of the charge $q(t)$ for a capacitor being charged by a constant voltage source V_0 . Consider the circuit shown in Fig. 4. If switch s is in position 1 for a long time, and then thrown to position 2 the capacitor C , initially uncharged, will over time become charged. After a very long time it will acquire a charge approaching $q_\infty = CV_0$.

When the switch s is in position 2 we have:

$$V_R + V_C = V_0$$

so that

$$R \frac{dq}{dt} + \frac{q}{C} = V_0$$

which can be rewritten as:

$$q' = \frac{dq}{dt} = \frac{(CV_0 - q)}{T} \quad (11)$$

where $T = RC$ is the time constant. Equation (11) can be used to solve for $q(t)$ using Euler's method just as was done previously for the case of the discharging capacitor.

3b. Square Wave Voltage Input. The voltage across the RC series combination in Fig. 4 is zero when switch s is in position 1 and V_0 when it is in position 2. Suppose the switch is repeatedly thrown back and forth at a regular frequency, and that any one throw takes a negligible time to complete compared to the time between throws. In that case, the voltage across the RC series combination has the form of a square wave such as the one depicted in Fig. 5. A continuous function that closely approximates a square wave is:

$$V(t) = \frac{1}{2}V_0 \left[1 + \frac{\sin \omega t}{|\sin \omega t| + .00001} \right] \quad (12)$$

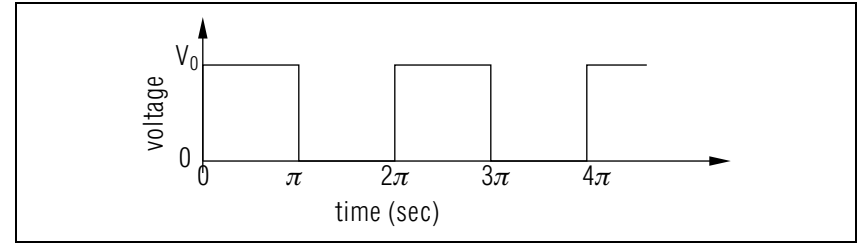


Figure 5. Square wave input to the circuit in Fig. 4 due to throwing the switch every π seconds.

The small term .00001 in the denominator is included to avoid the indeterminate form $0/0$.

3c. Euler Method Solution with Square Wave Input. As far as the Euler method rules are concerned, the only difference introduced by solving the RC circuit problem for a square wave input rather than a constant voltage input is that we must replace the constant voltage V_0 appearing in Eq. (11) with the square wave function $V(t)$ given by Eq. (12), thus obtaining:

$$q' = \frac{dq}{dt} = \frac{CV(t) - q}{T} \quad (13)$$

The Euler method algorithm shown in the flow diagram in Fig. 3 will again give charge q at a series of times, given some initial value q_0 . However, Eq. (13) must now be used to obtain q'_{old} and q'_{new} at the appropriate places in the flow diagram.

Although the Euler methods can be used to find approximate solutions for cases where discontinuous functions are involved, such as in the present square wave case, those methods must be used with great caution. The reason is that the methods' step, Δt , should be small compared to distances over which the driving function changes appreciably. There can be no such step for a discontinuous function.

4. Computer Implementation

4a. Input and Calculation. A program to carry out the calculation of $q(t)$ for a discharging capacitor using the original Euler method is listed in this module's Computer Program Supplement. The program first reads numerical values for three parameters TAU, DT, and N, where:

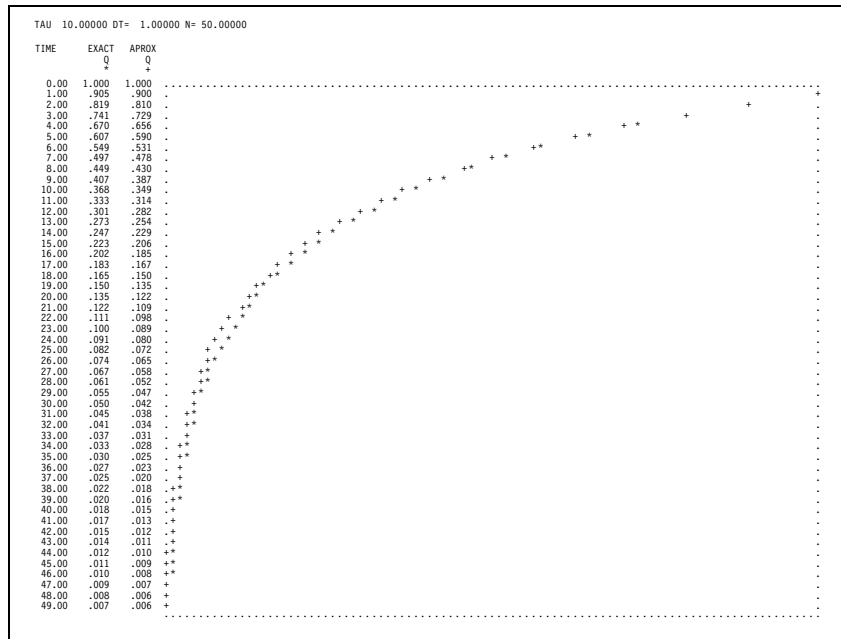


Figure 6. Sample output with TAU = 10, DT = 1, N = 50.

$$\begin{aligned}
 \text{TAU} &= T \text{ (time constant)} \\
 \text{DT} &= \Delta t \text{ (time step)} \\
 \text{N} &= \text{number of time steps.}
 \end{aligned}$$

After reading the input data, the program proceeds to advance the time t , step by step, in the sequence: $0, \Delta t, 2\Delta t, 3\Delta t, \dots, N\Delta t$. For each value of t the program calculates an exact value for the charge, labeled $Q1$, and an approximate value for the charge, labeled $Q2$, according to Euler's method (the original, not the improved method).

4b. Output. On the output the values of time are listed in the first column (see Fig.6). The exact values of the charge at each time are listed in the second column and are plotted graphically as asterisks. The approximate values of the charge at each time are listed in the third column and are plotted as plus signs. To read the graph as q against t , the sample printout in Fig. 6 should be rotated 90° counterclockwise.

4c. Scaling of Linear Plots. The graph is produced using a scaling technique that we shall use for all graphs made on the printer. The printer can only display characters at a discrete set of positions on a line, and is

therefore not a very accurate plotting device. Given a variable z which can take on a continuous range of values, we use a linear scaling function to determine the position on a printed line corresponding to a particular value of the variable z . We normally use a linear scaling function in graphing a function of one variable so as not to distort the plot:

$$M = az + b.$$

The values of the constants a and b are chosen so that the value of M stays within some desired range corresponding to values of z in some specified range. The value of M is truncated using integer arithmetic, so that M can be used as the subscript of an array X_4 . Suppose the array X has 101 elements X_1, X_2, \dots, X_{101} , which can be printed on one line as 101 single characters. Assume the character “ ” (blank) is first stored in every element of the array X . We then use the scaling function to store some other character (for example, “+” or “*”) in a particular element of the array X corresponding to the value of some variable. When the entire array X_1, X_2, \dots, X_{101} is printed on a line, the position of the (non-blank) character on the line corresponds to the value of the variable.

4d. Plotting the RC Discharge. In the RC circuit program, we have two variables ($Q1$ and $Q2$) to plot, and we shall use scaling functions of the same form for each:

$$\begin{aligned}
 M1 &= 100 Q1 + 1 \\
 M2 &= 100 Q2 + 1
 \end{aligned} \tag{14}$$

The form of these scaling functions insures that $M1$ and $M2$ are always within the desired range from 1 to 101. (The smallest values that $Q1$ and $Q2$ assume are $Q1 = Q2 = 0$ (at $t = \infty$) and the largest values are $Q1 = Q2 = 1$ (at $t = 0$.) Is is essential that the correct scaling function is used, otherwise $M1$ and $M2$ might take on values outside the range from 1 to 101, which can have disastrous consequences when the program is run on the computer. We can produce the entire graph, with each successive line of characters corresponding to a different value of the time t . Once values of $M1$ and $M2$ are computed for a specific value of the time, according to Eq. (14), the program prints out a line of characters which are all blanks, except for the $M1$ th, which is an asterisk and the $M2$ th, which is a plus sign.² Thus the positions of the asterisk and the plus sign on the line are linearly dependent on the values of $Q1$ and $Q2$

²The first and last characters on the line are not blanks. The character “.” is stored in elements X_1 and X_{101} of the array X to mark the boundaries of the graph.

respectively. After it prints a line of characters, the program repeats the procedure for a new value of the time; after it prints N lines the graph is complete. The program then reads another set of data, if any remain, with a new set of values for TAU , DT , and N . **IMPORTANT NOTE:** The scaling in the program assumes 122 characters can be printed out on a line. If your printer can only print 80 characters on a line, you will need to change scaling in Eq. (14) by replacing 100 by 50. Also change the program statement in which 101 elements of the array X are printed so that you instead print only 51.

5. An Investigative Project

5a. Dependence of Error on Step Size. Enter and run either the BASIC, FORTRAN, or C++ program, for the original Euler method, as listed in this module's Computer Program Supplement. Use at least three sets of values for TAU , DT and N , with DT varied and TAU held fixed (you decide what to do with N). Note that TAU , DT and N need to be appropriately chosen so that the program can handle the computation and plotting. In particular note that the scaling function used assumes that the charge varies from 0 to 1. In particular:

- For one of the runs, use the sample data given in Fig. 6 and check your output for that case against the output shown in the figure.
- For some one time (you choose it), plot the error against DT and show that, for DT small enough, the error varies approximately linearly with DT and extrapolates to zero error at zero DT .
- For the time in part (b), plot the approximate answer against DT , extrapolate DT to zero, determine the corresponding answer, and compare that answer to the exact answer.

5b. Using a Rapidly Changing Driving Term. Modify the listed program to deal with a square wave voltage input waveform. The statement in the program where Δq is found needs to be changed to be consistent with Eq. (13). In other words we want you to use:

$$\Delta q = \left(\frac{CV(t) - q}{T} \right) \Delta t$$

rather than:

$$\Delta q = -\frac{q}{t} \Delta t$$

which appears in the original version. Use the value $C = 1/2$, and, for $V(t)$, the square wave function given by Eq. (12) with $\omega = 0.8$ and $V_0 = 1.0$. Try running the program using $N = 100$, $\text{TAU}=3$, and $\text{DT}=.15$. Then find an answer good to two digits by reducing DT . Be sure to remove the $Q1$ solution, as it is inappropriate for this case (why?). For convenience in reading the plot you might want to simply set $Q1$ equal to $1/4$. Describe the problem(s) associated with solving the problem when there is such a fast-changing driving term.

5c. One Euler Method vs. Another. Restore the program to its original state (the discharging capacitor), and modify it to give a solution corresponding to the improved Euler method using the flow diagram of Fig. 3. Run the modified version of the program using various values for the parameters TAU , DT , and N , and discuss the relative agreement of the original and improved methods with the exact solution. Plot the error and the approximate solution against the square of DT and extrapolate to $\text{DT} = 0$ as in part 5a. Compare the actual computation time for the unimproved and improved Euler methods for a case where the two methods give the same error and have the same TAU and time. Is it more efficient to use the unimproved method that requires more cycles or is it more efficient to use the improved method that requires a greater number of calculations per cycle?

Acknowledgments

Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

A. Fortran, Basic, C++ Programs

All programs are at

http://www.physnet.org/home/modules/support_programs

which can be navigated to from the home page at

<http://www.physnet.org>

by following the links: → `modules` → `support programs`, where the programs are:

`m350p1f.for`, Fortran;
`m350p1b.bas`, Basic;
`m350p1c.cpp`, C++;
`lib351.h`, needed Library for C++ program;

MODEL EXAM

MODEL EXAM

1. Briefly describe the complete algorithm for using Euler's method to solve equations of the form: $dq/dt=f[q(t),t]$.

Examinee:

On your computer output sheet(s):

- (i) Mark page numbers in the upper right corners of all sheets.
- (ii) Label all output, including all axes on all graphs.

On your Exam Answer Sheet(s), for each of the following parts of items (below this box), show:

- (i) a reference to your annotated output; and
- (ii) a blank area for grader comments.

When finished, staple together your sheets as usual, but include the original of your annotated output sheets just behind the Exam Answer Sheet.

2. Submit your annotated output from your run that computed the exact charge as a function of time on a discharging capacitor in an RC circuit and the approximate charge as a function of time resulting from a numerical solution of the differential equation for an RC circuit using Euler's original method. Your runs will show:
 - a. one run using data of Sect. 4a, checked against Fig.6;
 - b. a graph showing that error decreases linearly with Δt for small Δt and that it extrapolates to the origin;
 - c. a graph showing that the answer changes linearly with Δt for small Δt and comparing the extrapolated answer to the exact answer (should be close!).
3. Submit your program changes and annotated output from your run(s) with a square wave input, showing: (a) how the output wave-form changes as the time constant changes; and (b) remarking on any problems that arise in dealing with a rapidly-changing driving term.

4. Submit your annotated output from your run with the program (above) modified so that it computes the charge on a capacitor in an RC circuit using the improved Euler Method, including:
 - a. run(s) with the same constants as in item 3 above, but now using the improved Euler method, showing that it is indeed an improvement over the simple Euler method;
 - b. a plot showing that error is linear in $(\Delta t)^2$ for small Δt and that the error extrapolates to the origin;
 - c. a plot showing that the solution is linear in $(\Delta t)^2$ for small Δt and that the value extrapolates to a value close to the exact one;
 - d. a comparison of the relative efficiencies of the simple and improved Euler methods for achieving the same accuracy for the same constants other than Δt .

INSTRUCTIONS TO GRADER

If the student has submitted copies rather than originals of the computer output, state that on the exam answer sheet and **immediately stop grading the exam and give it a grade of zero.**