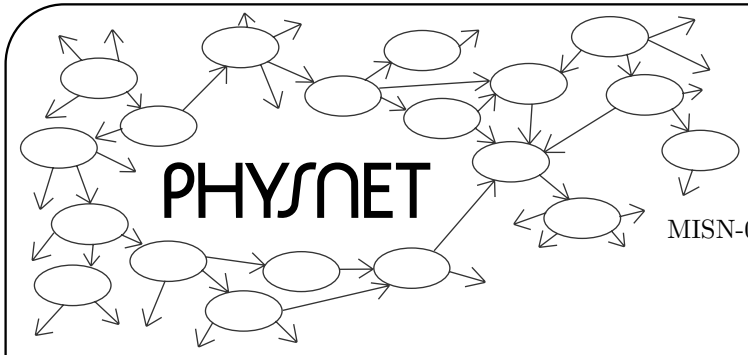
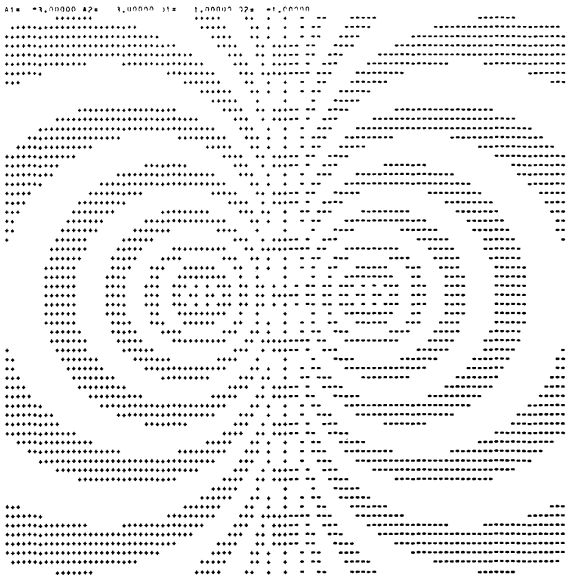


MISN-0-348

ADVANCED PRINTER GRAPHICS AND SCALING



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

ADVANCED PRINTER GRAPHICS AND SCALING

by

Robert Ehrlich
George Mason University

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Title: **Advanced Printer Graphics and Scaling**

Author: R. Ehrlich, Physics Dept., George Mason Univ., Fairfax, VA 22030; (703)323-2303.

Version: 3/22/2002 Evaluation: Stage 0

Length: 2 hr; 16 pages

Input Skills:

1. Vocabulary: interpolation.
2. Write (or modify) and run programs that utilize advanced programming features such as arrays and nested loops in Fortran (MISN-0-347), Basic, or Cpp.

Output Skills (Knowledge):

K1. Vocabulary: equipotential curves, intensity scaling, linear scaling function, logarithmic scaling function.

Output Skills (Project):

P1. Run a computer program to calculate the electrical potential over a rectangular grid of points and draw by hand equipotential curves and electric field lines on the computer output.

P2. Run a computer program to produce computer plots of the equipotential curves from two point charges.

P3. Modify the program in P2 to handle more than two point charges and run the program to treat the case of four charges located at the vertices of a square.

External Resources (Required):

1. A computer with FORTRAN, BASIC, or C++.

Post-Options:

1. "Numerical Solution of Differential Equations for RC Circuits-Euler's Method" (MISN-0-350).
2. "Waves in Two Dimensions" (MISN-0-353).

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ADVANCED PRINTER GRAPHICS AND SCALING

by

Robert Ehrlich
George Mason University

1. Drawing Equipotential Curves

1a. Introduction. This module applies computer graphics and intensity scaling to the solution of electrostatic problems. In the first part of the module the computer is used to calculate the electrostatic potential arising from two point charges, and equipotential curves are drawn by hand on the computer output. In the second part of the module we discuss techniques for drawing the equipotential curves by computer.

1b. Equipotential Curves. Given a number of point charges q_1, q_2, ..., q_n, the electrical potential at any point in the xy-plane can be obtained from

$$V(x,y) = \sum_{j=1}^{n} \frac{q_j}{r_j} \tag{1}$$

where r_j is the distance from the charge q_j to the point (x,y) and the Coulomb force constant $k = 1/4\pi\epsilon_0$ has been omitted. For the case of two charges we have

$$V(x,y) = \frac{q_1}{r_1} + \frac{q_2}{r_2} \tag{2}$$

If the potential $V(x,y)$ is found at all points in the xy-plane, a family of equipotential curves can be obtained by connecting all points having the same values of the potential. In practice, we can calculate $V(x,y)$ at some finite grid of points (see Fig. 1), and then approximately draw equipotential curves. A computer program for carrying out such a procedure is now described.

1c. Calculation of Potential Over a Grid. The program first reads numerical values for the parameters `A1`, `A2`, `Q1`, and `Q2`, where:

`A1` = x-coordinate of charge q_1
`A2` = x-coordinate of charge q_2
`Q1` = value of charge q_1
`Q2` = value of charge q_2.

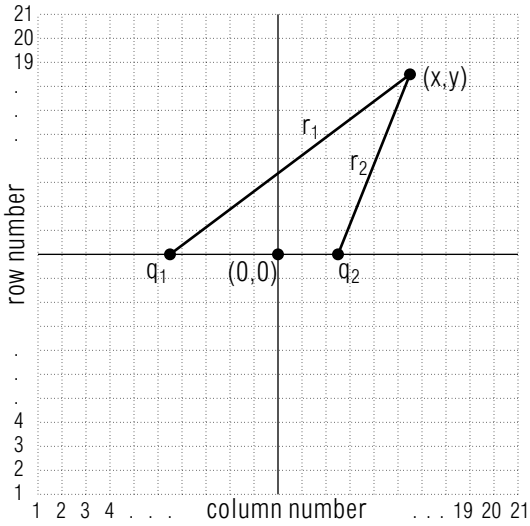

Figure 1. Grid of 21×21 points at which potential $V(x,y)$ is found.

It is assumed that the y-coordinates of both charges are zero. After the data has been read, the program proceeds to calculate the electrical potential V at all lattice points within the square defined by $-10 \le x \le +10$, $-10 \le y \le +10$. The points are identified by a pair of indices (j,k), where j = 1, 2, ..., 21 indicates the row number, and k = 1, 2, ..., 21 indicates the column number. At each lattice point, starting with (j,k) = (1,1), the program first determines the coordinates (x,y) in terms of the indices (j,k), using:

$$\mathtt{x} = \mathtt{k} - 11$$
$$\mathtt{y} = \mathtt{j} - 11$$

It then computes the distances r_1 and r_2 from the point (x,y) to each of the charges which are located at the points (`A1`,0) and (`A2`,0) (see Fig. 1). We add .000001 to r_1 and r_2, so that in computing the potential V according to Eq. (2), the computer never divides by zero. After finding the potential at one point, the program proceeds to the next point within the row. Once a row is complete, the array `V(K)`, `K` = 1, 2, ..., 21, which contains the values of the potential at each of the 21 points within the row, is printed out on a single line. The program then treats the next row of points in exactly the same manner, printing the array `V(K)`,`K` = 1, 2, ..., 21, for the next row of points on the line below the previous one. After it prints out the potential for all 21 rows, the process is complete, and the program reads the next set of parameters `A1`, `A2`, `Q1`, `Q2`, if it is

desired to repeat the entire process.

1d. Hand-Drawn Equipotential Curves. The output shown in Fig. 2 was obtained using the following numerical values for the parameters:

A1	A2	Q1	Q2
-3.0	3.0	2.0	-1.0

A number of equipotential curves have been drawn by hand directly on the computer output. To draw one such curve, for example, the one for $V = 0.10$ (drawn thickest), it is necessary to find, by interpolation, the approximate location of points for which $V = 0.10$, and then draw a smooth curve which passes through these points. To illustrate the interpolation procedure, suppose the sixteen values of the potential shown in Fig. 3 appear in some part of the output of Fig. 2. The sixteen points having these potentials may be assumed to lie at the intersections (nodal points) of an xy-coordinate grid drawn through the decimal points. Points for which $V = 0.10$ can be located approximately on the grid lines by a linear interpolation. These points can be found by

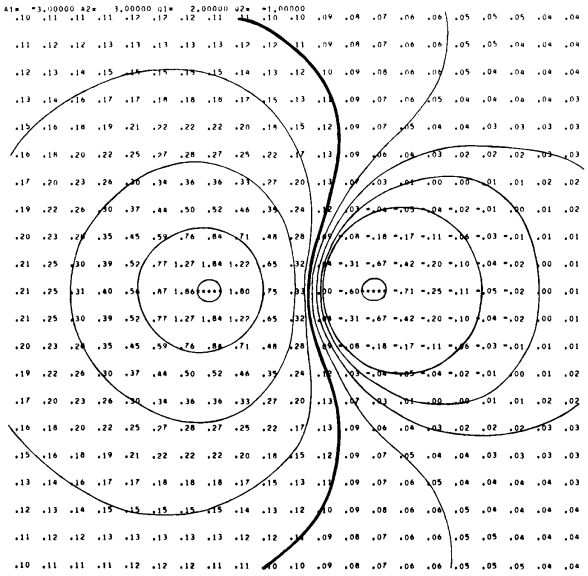

Figure 2. Equipotential curves drawn by hand on the output.

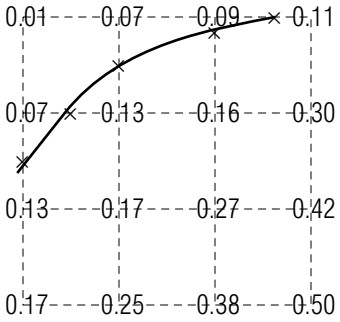

Figure 3. Interpolation procedure for drawing an equipotential curve.

eye and are shown as asterisks in Fig. 3. An equipotential curve can then be obtained by drawing a smooth curve through the asterisks. Following the same procedure for other values of the potential, we can obtain (with considerable effort) a set of equipotential curves.

2. Computer Graphics

2a. Introduction. Computers can be used to generate pictures, charts, and graphs for a wide variety of purposes. The level of detail in a picture drawn on a printer is limited by the character size; much more detailed pictures can be drawn on high resolution video terminals. The printer, however, will give quite adequate displays for our purposes.

2b. Intensity Scaling. The technique of intensity scaling permits the computer to draw equipotential curves. To illustrate the technique in its simplest form, suppose it is known that, in some region of the xy-plane, the potential V lies in the range $0.00 < V < 3.00$. Imagine that this region of the plane has been divided using a fine xy-grid, and that at every nodal point we place a single digit JV given by:

$$JV = 0 \text{ for } 0.00 < V < 1.00$$
$$JV = 1 \text{ for } 1.00 < V < 2.00$$
$$JV = 2 \text{ for } 2.00 < V < 3.00.$$

In the intensity scaling method we are able to print a much finer grid than in the hand-drawn method (see Sect. 1d), since now only a single digit is printed at each nodal point.

The method is illustrated in Fig. 4a, where you can see that single "intensity digits" JV have been printed at nodal points in a grid. You can easily draw two equipotential curves on this figure: the $V = 1.00$ equipotential

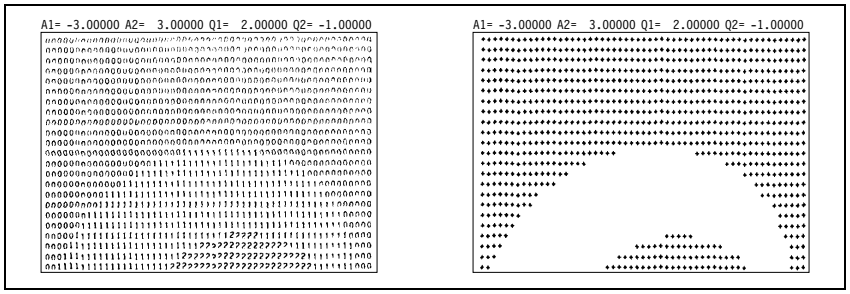

Figure 4. Intensity scaling using two methods.

is the boundary between the region of 0's and the region of 1's while the $V = 2.00$ equipotential is the boundary between the 1's and the 2's.

The method for $0 \leq V \leq V_{max}$ can be easily extended to deal with an arbitrary range of the potential. As an example, suppose that we wish to show nine equipotential curves for a series of equally spaced potentials between a V_{min} and a V_{max}. In this case, a single digit could be obtained for each nodal point by using the relation:

$$JV = 9.0 \left(\frac{V - V_{min}}{V_{max} - V_{min}} \right) \tag{3}$$

Note that the condition $V_{min} \leq V \leq V_{max}$ restricts JV to the range $0 \leq JV \leq 9$.

Although printing the scaling digit JV for all nodal points in a grid enables us to obtain a set of equipotential curves, we can modify the procedure to display the same equipotential curves in a more striking way. Instead of printing the digit JV itself, we can print a character C, where:

$C =$ ' ', a blank space, for JV odd,
$C =$ '+', a plus sign, for JV even, $V \geq 0$,
$C =$ '-', a minus sign, for JV even, $V \leq 0$.

The borders between regions of plus (or minus) signs and blanks now identify equipotential curves in a more graphic way (compare Figs. 4a and 4b).

2c. Linear and Non-Linear Scaling Functions. Equipotential curves can be found using any scaling function for the digit JV, but only linear functions provide a constant increment between adjacent curves. Equation 3 is an example of a linear scaling function since JV is a linear

function of the potential V. When a linear scaling function is used to compute the digit JV, the equipotential curves obtained correspond to a set of potentials having a constant increment ΔV: V_{min} + ΔV, V_{min} + 2ΔV, ..., V_{max} (see Sect. 2b). Nonlinear scaling functions can have advantages over linear ones. For example, consider the nonlinear logarithmic scaling function

$$JV = 2\, log_2|V|. \tag{4}$$

Fig. 5 was generated using this logarithmic scaling function, printing the characters '+', '-', and ' ' instead of the digit JV itself. The absolute value of V is used in the argument of the logarithm in Eq. 4 in order that the equation can be used for either sign of the potential V. The base-2 logarithm causes JV to increase additively by 2 if V is increased multiplicatively by 2. When base-2 logarithm scaling is used, each equipotential curve that is the inner boundary of a region of plus or minus signs represents a potential which is either twice or half as great as that of the equipotential curve that is the next inner boundary. Thus the equipotential curves defined by the inner boundaries of each region of plus or minus signs correspond to a series of potentials: V_{min}, $2V_{min}$, $4V_{min}$, $8V_{min}$, ... , etc. With logarithmic scaling the pattern of equipotential curves is not affected by the scale of the problem; that is, the shapes in the pattern would not change if, for example, the potential at every point were multiplied by any power of two. This "scale independence" means that for logarithmic scaling functions there is no need to state values for V_{min} and V_{max} as we must with linear scaling functions. The computer program used to generate the equipotential plots in Figures 5 and 6 is similar in its basic outline to that used for linear scaling, except now a much finer grid is used since only one character is printed per point. Now we use a $101 \times 61 (x, y)$ grid where previously it was 21×21. Now, however, the program is considerably slower than previously due to the added points and the evaluation of logarithms.

2d. Scaling of Plot Size. In the programs there are two scaling variables, `XSCLE` and `YSCLE`. They are set to default values in the programs but you can change them. If you generate a plot with `XSCLE` and `YSCLE` both set equal to 1.0, you will obtain the largest possible plot with the greatest amount of detail, but one that requires the greatest amount of time to produce. If the two scaling variables are set equal to 0.5 the plot will be half as large in each dimension and produced in one quarter the time. If you want to produce a square plot corresponding to the illustrations in this module you will probably need to use unequal values for `XSCLE` and `YSCLE`. A little trial and error may be necessary to find the

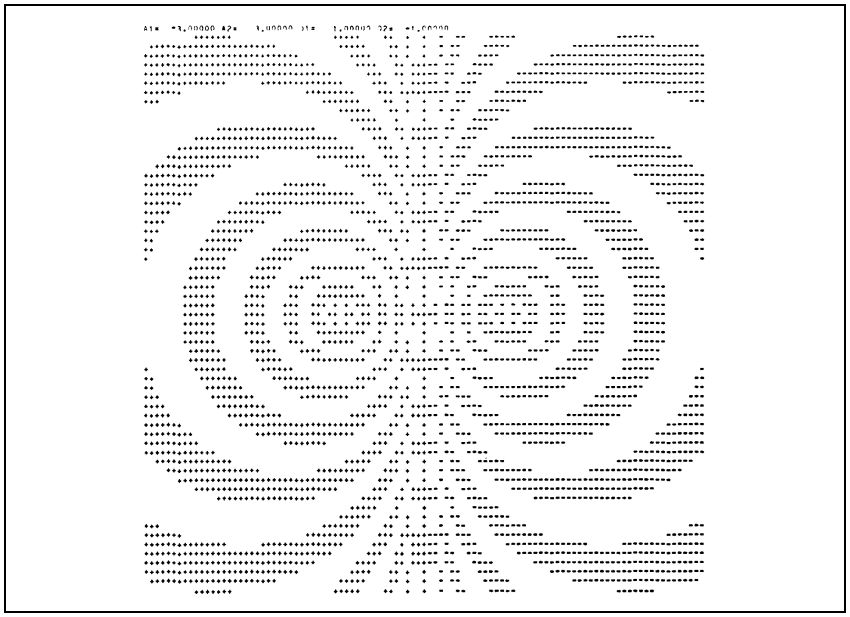

Figure 5. Equipotentials for equal but opposite point charges using logarithmic scaling.

best values. Remember that there is a tradeoff between plot size (and detail) and the amount of time needed to produce the plot.

3. Procedures

3a. Make a Check Run. First make a check run with each of the two programs using only one charge. Make sure the equipotential lines are circles at the right places.

3b. Equipotential Curves and Electric Field Lines. Enter and run the first program, the one that calculates and prints out $V(x, y)$. Try using the values:

$$\texttt{A1} = -3.0 \qquad \texttt{A2} = +3 \qquad \texttt{Q1} = 1.0 \qquad \texttt{Q2} = 1.0$$

Use the procedure discussed in Sect. 1d to construct (by hand) a number of equipotential curves directly on your output. (See the sample in Fig. 2.) After you have drawn the equipotential curves also draw electric field lines. Remember that electric field lines are everywhere perpendicular to

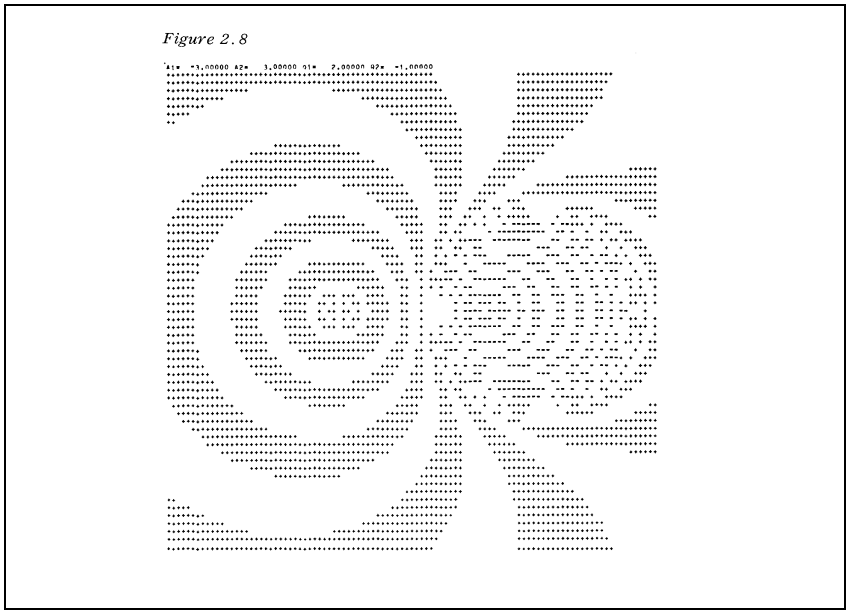

Figure 6. Equipotentials for two point charges having unequal magnitudes and opposite signs.

equipotential curves, and that they start on positive charges and end on negative charges or else go off to infinity. Now enter and run the second program, the one for which the computer generates equipotential plots. Use the same set of data as previously, and compare your hand drawn and computer produced plots.

3c. Unequal and Opposite Charges. The equipotential plots for two charges $Q1$, $Q2$ of unequal magnitude and opposite sign (Fig. 6) looks fairly complex. The situation is much less complex at large distances from the charges where the net charge $Q1 + Q2$ in effect produces a simple $1/r$ potential. To investigate the transition from small to large distances (compared to the charge separation), we can simply generate plots corresponding to three progressively smaller charge separations:

A1	A2	Q1	Q2
-3.0	+3.0	2.0	-1.0
-1.0	+1.0	2.0	-1.0
-0.1	+0.1	2.0	-1.0

About how far do you need to be from the charges in order that the equipotential curves "appear" circular in your output?

3d. More than Two Point Charges. Modify both of the programs so they can handle more than two point charges, located at up to four specified coordinate pairs (x_j,y_j). Run the plotting version of the program with four point charges located at the corners of the square 6.0 units on a side. Try one case where all four charges are identical and a second case where the charges at opposite ends of one diagonal have the value $+1.0$ while those at opposite ends of the other diagonal have the value -1.0.

Acknowledgments

Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

Glossary

- **Equipotential curve**: a line on all parts of which the potential has the same numerical value.
- **Intensity scaling**: the transforming of z-axis numbers plotted on an $x-y$ graph, leaving the $x-y$ positions of those z-axis numbers unchanged. This may involve severe truncation of the transformed z-axis numbers in order to plot them at a higher positional density than would otherwise be possible.
- **Linear scaling function**: a function that linearly transforms graphical output (for a non-truncated set of numbers, it is a constant multiplier on the numbers).
- **Logarithmic scaling function**: a function that logarithmically transforms graphical output (it is generally the logarithm of the numbers).

A. Fortran, Basic, C++ Programs

All programs are at

`http://www.physnet.org/home/modules/support_programs`

which can be navigated to from the home page at

`http://www.physnet.org`

by following the links: → `modules` → `support programs`, where the programs are:

`m348p1f.for`, program 1, Calculate V(X,Y), Fortran;

`m348p1b.bas`, program 1, Calculate V(X,Y), Basic;

`m348p1c.cpp`, program 1, Calculate V(X,Y), C++;

`lib351.h`, needed Library for both C++ programs;

`m348p2f.for`, program 2, Equipotential Plots, Fortran;

`m348p2b.bas`, program 2, Equipotential Plots, Basic;

`m348p2c.cpp`, program 2, Equipotential Plots, C++.

MODEL EXAM

1. See Output Skill K1 in this module's *ID Sheet*.

Examinee:
On your computer output sheet(s):

(i) Mark page numbers in the upper right corners of all sheets.

(ii) Label all output, including all axes on all graphs.

On your Exam Answer Sheet(s), for each of the following parts of items (below this box), show:

(i) a reference to your annotated output; and

(ii) a blank area for grader comments.

When finished, staple together your sheets as usual, but include the original of your annotated output sheets just behind the Exam Answer Sheet.

2. Submit your annotated computer output that shows the potential and electric field values, due to one point charge, on a rectangular grid. Make sure that you have drawn-in the equipotential curves and electric field lines, by hand, on your computer output. Check that the curves are circles and that at least one of them is at the right place.

3. Submit your annotated computer output that shows the potential and electric field values, due to two point charges, on a rectangular grid. Make sure that you have drawn-in the equipotential curves and electric field lines, by hand, on your computer output. Check that:

a. the curves are smooth;

b. the curves properly interpolate between the values at the grid points.

c. the curves are properly labeled and have directionality marked wherever appropriate.

4. Submit your annotated computer output that shows the equipotential curves, directly, due to two point charges. Check that the curves agree with the curves for the same case in No. 2 above.

5. Submit your annotated computer output that shows results for the modified program that calculates the equipotential curves due to more than two charges. Make sure your output shows the case of four charges located at the vertices of a square. Make sure you have put a short discussion, on your output, showing that the pattern you see in the output appears reasonable.

INSTRUCTIONS TO GRADER

If the student has submitted copies rather than originals of the computer output, state that on the exam answer sheet and **immediately stop grading the exam and give it a grade of zero.**

Note that the award of points is set up in such a way that a student will get 50% or less on the exam if the student does only computer work and no physics.

Brief Answers:

1. See this module's *text*.