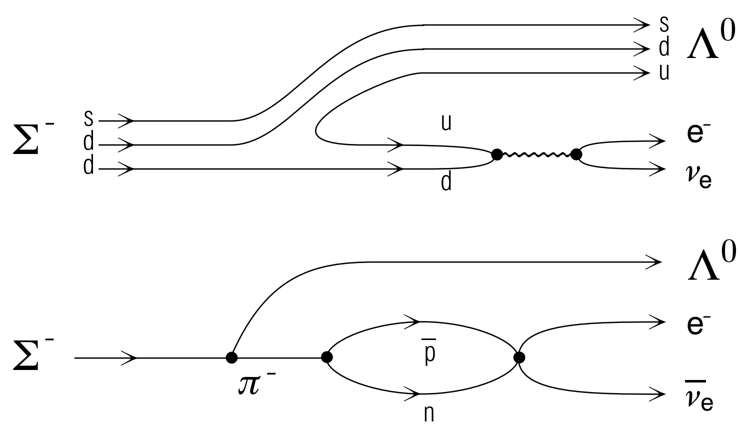
MISN-0-279

STRENGTHS OF INTERACTIONS AND PARTICLE DIAGRAMS



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

STRENGTHS OF INTERACTIONS AND PARTICLE DIAGRAMS

by

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Title: **Strengths of Interactions and Particle Diagrams**

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Input Skills:

1. State the energy-time uncertainty principle (MISN-0-241).
2. Given a decay or interaction use the conservation laws to determine if it can occur and, if so, which force is responsible (MISN-0-278).

Output Skills (Knowledge):

K1. State rough values for these coupling constants: g_s, g_{em}, g_w.

K2. Define: virtual particle, real particle.

K3. Relate "coupling constant" to "intrinsic rate."

Output Skills (Rule Application):

R1. Given any two of exchanged mass, time, and range for an interaction, estimate the third.

R2. Given a Feynman Diagram, estimate the couplng constant for the process.

R3. Given a Feynam Diagram, describe the process in words.

Post-Options:

1. "The Strong Interaction" (MISN-0-280).
2. "SU(3) and the Quark Model" (MISN-0-282).

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1. Diagrams

1a. Description of Particle Diagrams. An interaction is sometimes schematically represented on a diagram with position plotted vertically and time plotted horizontally. For example, β decay of the neutron might be represented by the diagram in Fig. 1. This shows a neutron which decays at point A into a proton, an electron, and an antineutrino. Such diagrams are called *time-ordered diagrams* but we will call them *particle diagrams* or simply *diagrams*.[1] Note that they are only schematic in that they show only one space dimension. Furthermore, the slopes of the lines are drawn for clarity of pictorial representation and are not meant to represent particle velocities as one might expect. The coordinate axes, x and t, are normally omitted from the diagram.

1b. Cataclysmic Nature of the Vertex. Particle diagrams emphasize the cataclysmic view the physicist has of the microscopic world. The diagrams always show the annihilation and creation of particles at a single point in space and time. As far as is known there is not a continuous change from one set of particles to another but rather a discontinuous change. Any point on the diagram at which this occurs is called a vertex.

[1] These diagrams are very simply related to the more modern Feynman diagrams: See "Feynman Diagrams: An Introduction" (MISN-0-364). Generally, an n-vertex Feynman diagram sums n! time-ordered diagrams. However, some of the nice developments in this module cannot be based upon Feynman diagrams.

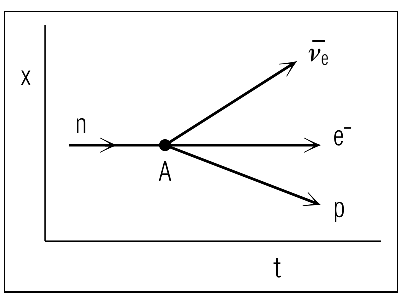

Figure 1.

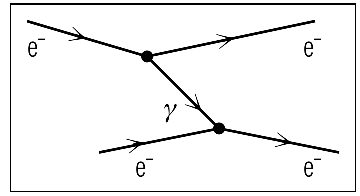

Figure 2.

The diagram above has one vertex, labeled A.

1c. Universality; "Continuous" Interactions. According to the current picture, all interactions in nature take place via cataclysmic vertices. Those interactions which appear to be continuous, in reality consist of many cataclysmic events separated by time intervals too small to be detected by the apparatus at hand. This is analogous to wind pressure, which gives the appearance of being continuous. On a small enough scale, however, the wind pressure you feel can be seen to be due to many small blows of air molecules as they strike you. Similarly, particle interactions may appear either continuous or cataclysmic, depending on the scale of the interaction. However, the cataclysmic interaction is the fundamental one since the other can be derived from it, just as in the case of wind pressure.

1d. Example: Electromagnetic Scattering. Scattering, in which the particle identities do not change, is believed to occur by the mechanism of the exchange of a particle between the two primary scattering particles. For example, charged particles do not respond to the electric field of other charged particles the way classical theory prescribes. Charge still creates electric and magnetic field intensities but these must be interpreted as a measure of the probability of finding a photon. It is the photon, transferred from one particle to another, that transfers energy and momentum and thus is the "cause" of the electromagnetic force. For example, the mutual deflection of two electrons is described by the diagram in Fig. 2. The photon is created at one vertex, annihilated at the other.

1e. Summing Diagrams, Diagram Interpretation. Diagrams are a pictorial representation of the mathematical theory used to predict outcomes of an interaction. In fact, the first step in such a calculation is to draw all possible diagrams which connect the desired initial and final states of the system being studied. Then rules tell you how to calculate the contribution of each diagram. They include an integration over all possible times for each vertex. Finally, the contributions of the diagrams

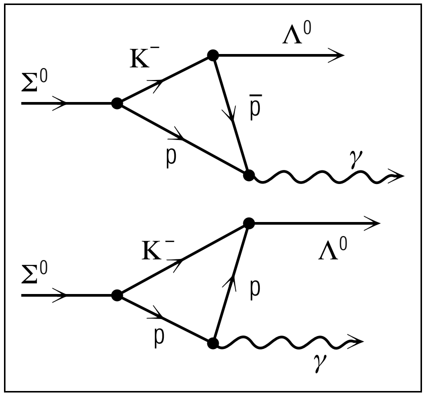

Figure 3.

are summed to obtain the total probability amplitude for the process. As a concrete example, in Fig. 3 we show two diagrams which contribute to the decay of the Σ^0 into a Λ and a γ. These diagrams' contributions must be summed, along with those of other diagrams not shown, in order to get the probability amplitude and hence the total probability transition rate.

1f. Creating Other Diagrams by Line Reversal. You can use a diagram to create other allowed diagrams by changing one or more of the particle lines to an antiparticle line. This is accomplished by merely changing a line's particle label, if necessary,[2] and reversing the line's directional arrow. It may also be convenient to alter the slope of the line on the diagram, as illustrated in the sequence in Fig. 4. There one starts off with bremmstrahlung (photon emission in electron-proton scattering) in the top diagram and winds up with photoelectric pair production on a photon in the bottom diagram. One major change is made in going from each diagram to the next. All of the diagrams shown are allowed ones.

2. Coupling Constants

2a. Decay Rate; Coupling Constant, Density of States. The transition rate for a decay or interaction is proportional to the product of the square of a *coupling constant* g and a *density of states* factor ρ:

$$R \propto g^2 \rho(E).$$

[2]No label change is necessary if the particle is a neutral boson (γ, π^0, ρ^0, etc.) hence is its own antiparticle.

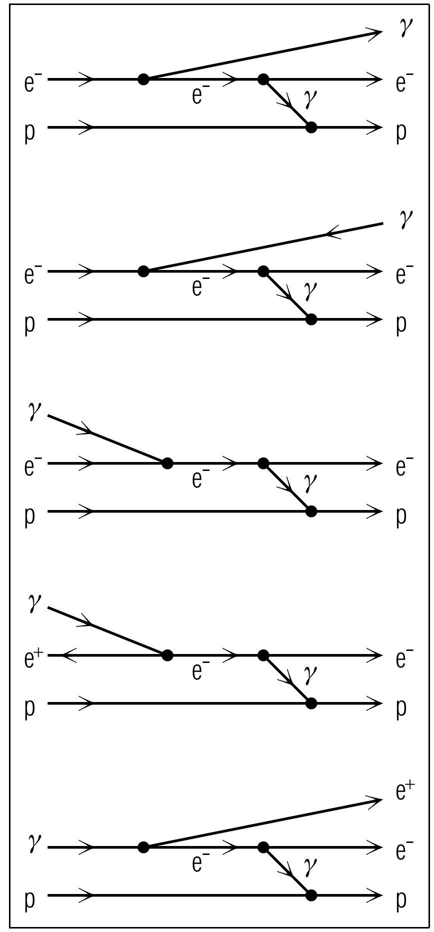

Figure 4.

The transition rate R is a measure of the probability per unit time that the interaction will occur. For decays, $R \propto 1/T$, where T is the lifetime of the particle.

The density of states ρ is a measure of the number of quantum mechanical states per unit energy range that are available for the final products. The more states that are available, the higher the transition rate to them.

The coupling constant g^2 can be interpreted as an *intrinsic rate*. For example, in β decay, g^2 measures the intrinsic rate that a neutron will

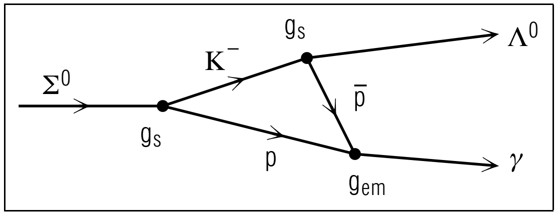

Figure 5.

decay into a proton, positron, and antineutrino. But the original energy can also be distributed in many different ways among the products, and $\rho(E)$ measures the number of different ways per unit energy interval.

2b. The Fundamental Coupling Constants. The coupling constant for a process, g, is the product of a number of *fundamental coupling constants*. The fundamental coupling constants are associated with the types of forces and are

$$\begin{aligned} &\textit{strong}: && g \approx 1, && (1)\\ &\textit{electromagnetic}: && g_{em} = e/\sqrt{\hbar c} \simeq 1/\sqrt{137} \approx 8.5 \times 10^{-2}, && (2)\\ &\textit{weak}: && g_w \approx 10^{-7}. && (3) \end{aligned}$$

2c. Coupling Constants and Vertices. One fundamental coupling constant is associated with each vertex of a diagram and g is the product of all the fundamental coupling constants which appear in the diagram.

2d. A Diagram Contributing to Neutral Sigma Decay. One contribution to the decay of a Σ^0 is shown in the three-vertex diagram of Fig. 5. Here Σ^0 decays strongly to a proton and a kaon; the kaon in turn decays strongly to a lambda and an antiproton. The latter annihilates the proton electromagnetically to produce a γ. The overall coupling constant is $g = g_s g_s g_{em} \simeq 1/\sqrt{137}$ where the last approximation holds since $g_s \simeq 1$.

A note: the decay $\Sigma^0 \to \Lambda^0 + \gamma$ cannot proceed according to the one-vertex diagram of Fig. 6. This is because uncharged particles can-

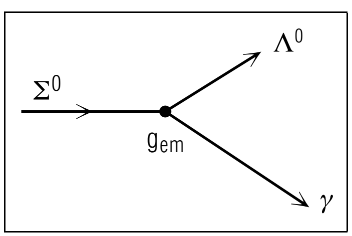

Figure 6.

not produce photons directly. There must be an intermediate decay to produce charged particles, one of which produces the γ. There are other diagrams for the decay of the Σ^0; these make use of other intermediate particles besides the kaon and the antiproton.

2e. Other Diagrams Also Contribute. Often many different diagrams can lead to the same final state.[3] If there are in fact many diagrams which lead to the same final state, all will contribute. The number of vertices is called the *order* of the diagram. As more and more weak or electromagnetic vertices are added, the contribution represented by the diagram decreases; more factors of g_w or g_{em} are contributing and these are small numbers. On the other hand, strong vertices can be added without appreciably changing the transition rate. The possibility of many strong vertices complicates particle interactions. For example, an interaction may occur on a time scale characteristic of a weak interaction but its diagram may in fact have several strong vertices (along with a weak one).

2f. Other Influences. Calculation of the transition rate or decay time may involve other quantities such as mass and momentum, spin, or isotopic spin. However, the product of the fundamental coupling constants gives a reasonable indication of the order of magnitude of the rate.

3. Intermediate States

3a. A Simple Intermediate State. The process $\pi^+ + \mathrm{p} \to \pi^+ \rho$ has contributions from many diagrams, but the simple-looking one in Fig. 7 is a major contributor. It shows a π^+ and a proton coming together and becoming a virtual Δ. The latter exists for awhile, then decays into a π^+ and a proton. An external observer-physicist would say that the process was π^+-p scattering. We will use this diagram for illustrative purposes.

[3]See "Feynman Diagrams; An Introduction" (MISN-0-364) for six diagrams contributing to Σ^0 decay.

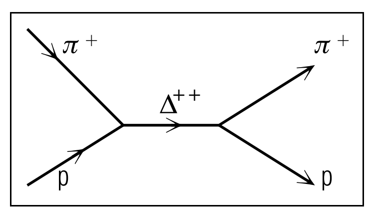

Figure 7.

3b. Conserved Quantities at a Vertex. At each vertex, all appropriate quantities are conserved except energy. By "appropriate" is meant all universally conserved quantities, except energy, plus those additional quantities conserved by the vertex's type of interaction. For example, strangeness must be conserved across a strong-interaction vertex but need not be conserved across a weak one. Parity is conserved across an electromagnetic vertex but not necessarily across a weak one. Momentum, angular momentum, baryon number, etc. are conserved across all vertices.

3c. Vertex ΔE, Virtual Particle Δt. According to the energy-time uncertainty relation,[4] conservation of energy can be violated in the amount ΔE for times less that $\Delta t \simeq \hbar/\Delta E$. Over the entire period of experimental observation of a process Δt is long and energy is conserved. However, at some vertices an intermediate particle is produced and shortly thereafter absorbed at another vertex (for example the Δ^{++} in the π^+-p scattering diagram above). If the intermediate product exists for only a short period of time, the energy can deviate considerably from the value it would have if it were conserved. In the above example, the mass of the Δ^{++} may be greater than the total energy of the original π^+-p system. At low kinetic energies the π^+-p cannot produce the Δ^{++} as a final particle but can produce one as an intermediate particle. If the energy is increased so that Δ^{++} can be produced without violating conservation of energy, the Δ^{++} may appear as a final product. Intermediate particles whose creation violates conservation of energy are called *virtual* particles. Note that the γ is real in the diagrams in Sects. 1e and 2d but virtual in the one in Sect. 1d.

3d. Transmission of ΔE Across a Diagram. In order for energy to be conserved across a whole diagram, the energy surplus or deficit at one vertex, ΔE, must be transmitted across the diagram to later vertices. The latter can then provide equal but opposite compensation, $-\Delta E$. The ΔE information is transmitted via the time phase of the intermediate particles' wave function. Thus in the above diagram the Δ^{++} wave function has an extra time phase proportional to the ΔE across the first vertex. This phase is used at the second vertex to produce a compensating shift and hence overall energy conservation.

[4]See "The Uncertainty Relations" (MISN-0-241).

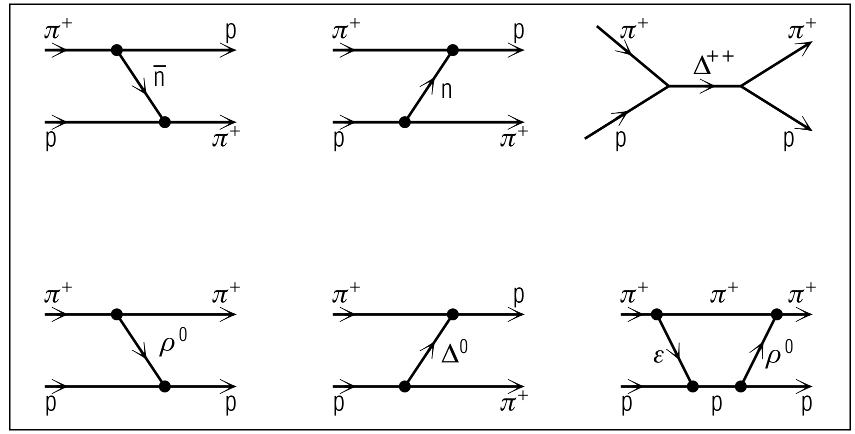

Figure 8.

4. Examples

4a. Pion-Proton Scattering. In the process $\pi^+ + \text{p} \rightarrow \pi^+ + \text{p}$, the initial and final particles will all experience the strong interaction since they are hadrons. This means that the process will be dominated by diagrams with strong interaction vertices and intermediate-state hadrons. Here are examples:

4b. Neutral Pion Decay to Two Gammas. The decay $\pi^0 \rightarrow \gamma + \gamma$ cannot occur directly since the π^0 is uncharged. Uncharged particles do not experience the electromagnetic interaction, and that is the only interaction experienced by the γ. Thus the π^0 must first undergo a strong decay to charged mesons or to a charged baryon-antibaryon pair (Fig. 9).

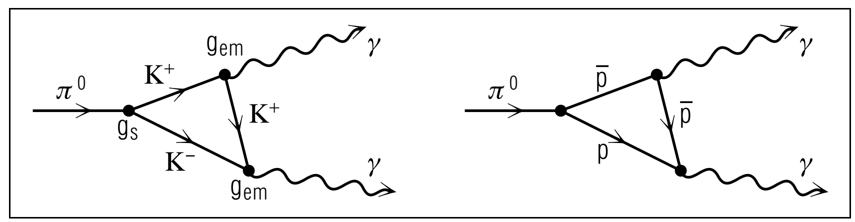

Figure 9.

Acknowledgments

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PROBLEM SUPPLEMENT

1. The following reactions are the same as those used in Problems 4-1, 4-2 of M. J. Longo, *Fundamentals of Elementary Particle Physics*, McGraw-Hill Book Co., N.Y. (l973).

   Devise simple diagrams for:

   a. K^--p elastic scattering
   b. e-p elastic scattering
   c. $\pi^- + \mathrm{p} \to \mathrm{K}^+ + \Sigma^-$
   d. $\pi^- + \mathrm{p} \to \mathrm{K}^0 + \Lambda^0$
   e. γ-p elastic scattering
   f. $\gamma + \mathrm{p} \to \pi^+ + \mathrm{n}$
   g. $\overline{\mathrm{p}} + \mathrm{p} \to \pi^+ + \pi^-$
   h. $\overline{\mathrm{p}} + \mathrm{p} \to \mathrm{e}^+ + \mathrm{e}^-$

2. Estimate the following rate ratios and compare to experimental fractional decay mode ratios from the latest "Review of Particle Properties," April *Physics Letters*.

   $(\pi^0 \to \gamma + \gamma)/(\pi^0 \to e^- + e^+ + \gamma)$

   $(\Sigma^+ \to \mathrm{p} + \pi^0)/(\Sigma^+ \to \mathrm{p} + \gamma)$

   $(\eta^0 \to 3\pi)/(\mathrm{K}^0 \to 3\pi)$

MODEL EXAM

1. Definitions and relations:

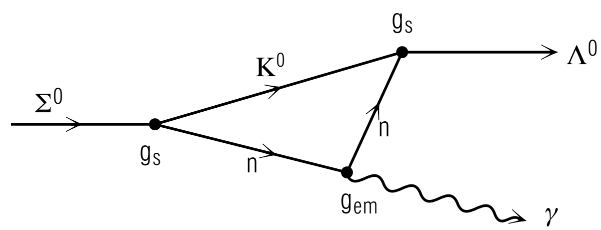
   a. Relate the "intrinsic rate" of an interaction to its coupling constant.

   b. Define "virtual particle."

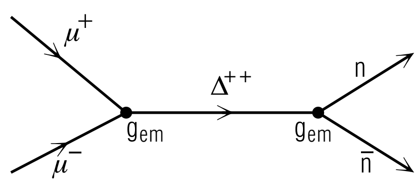
2. Give rough values for the fundamental rates $g_{\rm s}$, $g_{\rm em}$, $g_{\rm w}$.

3. Describe, in words, the processes shown in these diagrams:

   a.



   b.



4. For the process shown in part 3a), estimate the intrinsic rate.

5. A hypothetical interaction takes place by exchange of an X particle. Real X particles are produced when the center of mass energy is about 250 MeV. Estimate:

   a. the range of the interaction

   b. the typical time of the interaction

Brief Answers:

1. a. The intrinsic rate equals the coupling constant squared.

   b. A virtual particle is one in an intermediate state, produced by an interaction which violated conservation of energy.

2. $g_{\rm s} \simeq 1$; $g_{\rm em} \simeq 1/\sqrt{137}$; $g_{\rm w} \simeq 10^{-7}$.

3. a. A sigma to a neutron and a kaon via the strong interaction. The neutron (which has a magnetic moment!) emits a photon via the electromagnetic interaction, then strongly combines with the kaon to form a lambda via the strong interaction.

   b. A muon pair annihilates to form a photon which then turns into a neutron-antineutron pair.

4. $g^2 \simeq (g_{\rm s}^2\, g_{\rm em})^2 \simeq \dfrac{1}{137}$

5. a. $R \simeq c\Delta t \simeq c\dfrac{\hbar}{\Delta E} \simeq \dfrac{200\,\text{MeV fm}}{250\,\text{MeV}} \simeq 0.8\,\text{fm} = 8 \times 10^{-16}\,\text{m}.$

   b. Use $1\,\text{fm} = 10^{-15}\,\text{m}$ to get:
   $\Delta t \simeq \dfrac{\hbar}{\Delta E} \simeq \dfrac{(200\,\text{MeV})(10^{-15}\,\text{m})}{(3 \times 10^8\,\text{m/s})(250\,\text{MeV})} \simeq 0.3 \times 10^{-23}\,\text{s}$