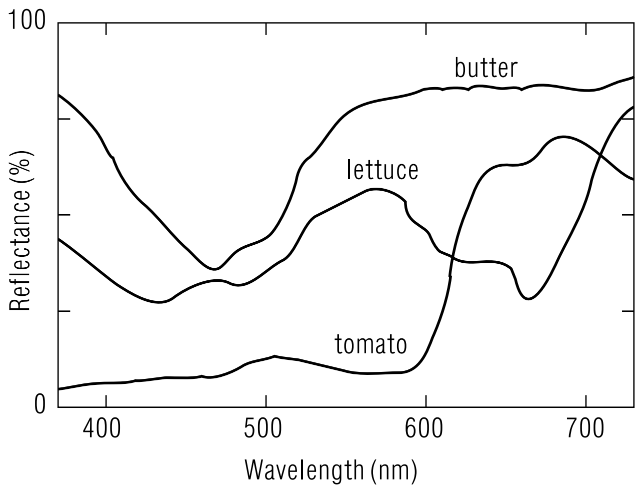
MISN-0-270

# PREDICTING AND SPECIFYING THE PERCEIVED COLORS OF REFLECTIVE OBJECTS




Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

PREDICTING AND SPECIFYING THE PERCEIVED COLORS OF REFLECTIVE OBJECTS

by

Peter Signell

Title: **Predicting and Specifying the Perceived Colors of Reflective Objects**

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Version: 2/1/2000 Evaluation: Stage 0

Length: 1 hr; 20 pages

**Input Skills**:

1. Vocabulary: standard observer, CIE, chromaticity coordinates, Chromaticity Diagram, tristimulus, spectral energy distribution (MISN-0-229).
2. Given the wavelength of any monochromatic light beam, plus the tristimulus curves, determine the relative responses of each of the three types of cone receptors (MISN-0-227).
3. Given the wavelength of any monochromatic light beam, state its approximate perceived color (MISN-0-212).

**Output Skills (Knowledge)**:

K1. Vocabulary: illuminance, spectral reflectance, spectral illuminance.

**Output Skills (Project)**:

P1. Use the wavelength distribution of an illuminating light, an object's reflectance distribution and the 1931 CIE tristimulus curves to determine the CIE Chromaticity Diagram location of the object's perceived color (see this module's *Local Guide*).

**External Resources (Optional)**:

1. "Light and Color," Lamp and Lighting Bulletin #TP-119, The General Electric Co., Cleveland (1974), especially the color plates. For access, see this module's *Local Guide*.



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## 1. Introduction

**1a. The Energy Path.** The color we perceive when looking at an opaque object depends on: (i) the particular mixture of wavelengths ("pure colors") present in the illuminating light; (ii) the change in that mixture as the object partially absorbs and partially reflects the various wavelengths; and (iii) the response of our eye-brain system to the wavelength mixture which reaches it. Thus to predict the object's color, as perceived by the brain, we need three wavelength distributions: that of the illuminating light, that of the object's ability to reflect light, and that of the eye-brain response mechanisms. Such distributions for common illuminants, for common-object reflectances, and for standard eye-brain responses, can often be obtained from published tables and graphs. Alternatively, such distributions can be measured in the laboratory or in the field.

**1b. Overview of the Method.** To use published illuminance and reflectance data to obtain the perceived color of an object, follow these five steps: (i) multiply the illuminating light's spectral illuminance data by the reflecting object's spectral reflectance data at corresponding wavelengths; (ii) plot the resulting reflected energy values vs wavelength, making sure there are enough points to draw the curves with the desired accuracy; (iii) multiply that reflected energy distribution by the eye/brain system's three "tristimulus" distributions, one distribution at a time and wavelength-by-wavelength within each distribution, giving you the three stimulation distributions; (iv) plot the three stimulation distribution curves and find the area under each curve; and (v) divide each area by the sum of all three areas. The three quotients add to unity: use two of them as coordinates to locate the color's point on a Chromaticity Diagram, printed in color.[1] All observable colors are on this diagram, gradually blending from one into another. The color at the point is the predicted color of the object.

[1] If you wish to see an example, look in "Light and Color," Lamp and Lighting Bulletin #TP-119, The General Electric Co., Cleveland (1974), pp. 18. For access, see this module's *Local Guide*.





**1c. Limitations of the Method.** Tristimulus predictions for perceived colors may be inaccurate due to the presence of one or more of these effects: (i) the observer's field of view may contain color intensity information which conflicts with that observer's subconscious expectations;[2] (ii) the light entering the observer's eye may be flickering at a rate which differentiates between the eye's three different stimulus response times[3] and (iii) the observer's eye-brain response curves may differ from those of the "standard observer" used in the calculation. The first effect, a psychological one, is produced by the brain in a predictable fashion for typical persons. It appears to be an effort to automatically correct for odd illumination such as that of the orange color of outdoor light near sunrise or sunset. This effect's discovery by Edwin Land, along with its applications in survival, photography, and art, are discussed elsewhere.[2] The second effect appears to be an artifact of the eye's electro-chemical response mechanisms and has no known use.[3] The third effect may be due to normal and/or abnormal variations among people, to errors in the tristimulus curves, and to a natural dependence of perceived color on intensity.

## 2. Spectral Data

**2a. Illuminance.** The perceived color of an object certainly depends on the color of the light illuminating it. How do we take this into account in predicting perceived color? Any particular illuminating light has a specific wavelength distribution with greater intensity at some wavelengths than at others. Given a source, its distribution of illuminating energy vs. wavelength can be generated using a prism or a grating and a detector, wavelength by wavelength. More usually, however, the distribution is both measured and plotted automatically by a device called a "recording spectrometer."[4] The resulting distribution is called that source's "spectral energy" distribution. Curves for three typical sources are shown in Fig. 1. Tables for common sources are given under *Data Sources* in Appendix B.

**2b. Reflectance.** A given object reflects different fractions of various pure-wavelength light rays: the set of such reflected fractions is called

[2] See "Land's Observations on Color Perception" (MISN-0-228).

[3] For a discussion of this effect, along with methods of simple observation (e.g. Benhan's Disk), see *The Intelligent Eye*, R.L. Gregory, McGraw-Hill, NY (1970), paperback, esp. p. 77.

[4] For more details see "Transitions and Spectral Analysis" (MISN-0-216)

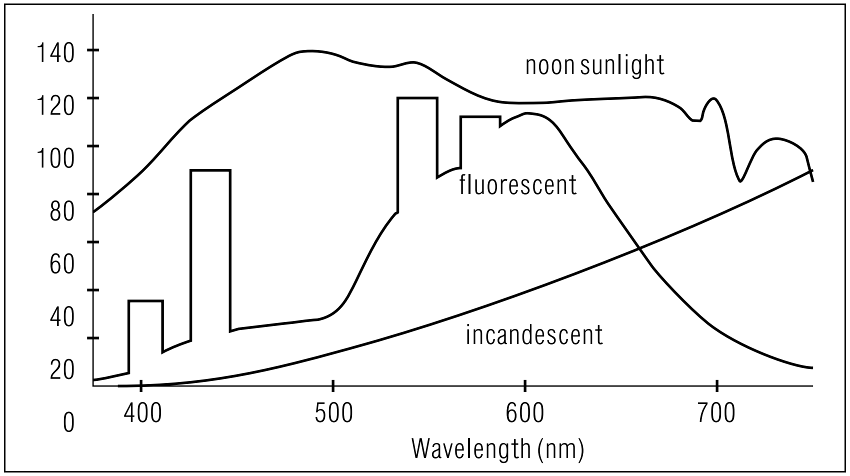


**Figure 1.** Spectral Energy Distribution curves for three typical light sources (see Appendix B). The numbering on the vertical axis is arbitrary: an absolute scale is not needed to determine perceived colors.

the object's "spectral reflectance." It is given as percent reflectance vs. wavelength, either as a table as in Appendix B or as a curve as in Fig. 2. These percent reflectances are obtained by dividing the amounts of reflected light at various wavelengths by the amounts of incident light at the same wavelengths. In practice the curve is generally measured one wavelength at a time, using a white light and a prism or a grating to produce the desired pure wavelengths. The single points so obtained are connected to form a smooth curve and are checked to make sure that there are enough of them to properly define the shape of the curve.

**2c. "Standard" Response.** The International Commission on Illumination (CIE) has set up standard tristimulus response curves based on two requirements: (i) the curves should be similar to Young and Helmholtz's supposed response curves for the average human eye-brain system; and (ii) the color "white" should be produced by equal responses in all three stimulus channels.[5] Fig. 3 shows the curves set up by the CIE in 1931. Without such standard curves, objects' color descriptors would vary according to whose eye-brain responses or whose measurement of

[5]The perception "white" is assumed to be produced by a light intensity which is constant over visible wavelengths. This is approximately true of noon sunlight (see Fig. 1).





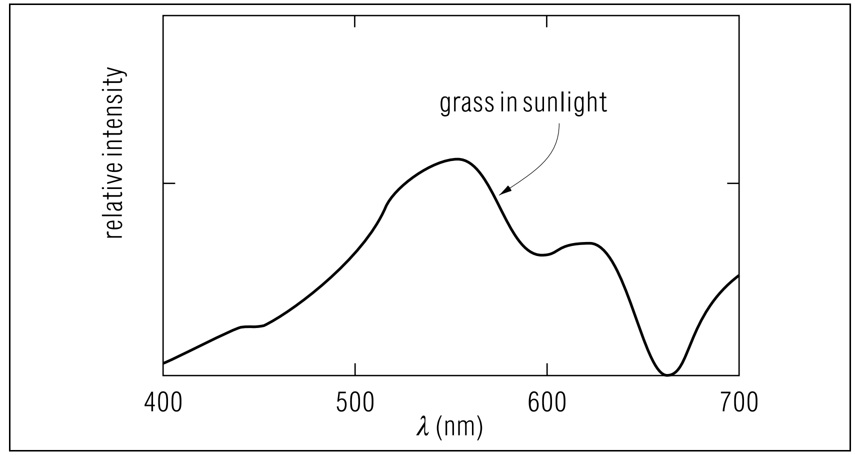


**Figure 2.** Spectral Reflectance for grass (see Appendix B). Note that the regions of high reflectance check with the object's common color.

them happened to have been used. The 1931 CIE curves, shown in Fig. 3 and tabulated in Appendix B, have become a common standard for color specification, although occasionally later CIE curves or one of a number of other systems are used.

## 3. Calculation

**3a. Reflected Energy.** To obtain the relative reflected energy distribution for a particular object in a particular light, one must either measure it or simply multiply the tabulated illuminance distribution by the reflectance distribution, wavelength by wavelength. Fig. 4 shows such a reflected energy curve for summer grass illuminated by sunlight. Note that the high point of the curve occurs in the green, as one might expect. Thus by examining the relative reflected energy curve one can obtain an estimate of color. To obtain a more precise perceived color, we must also fold in the response functions for the eye-brain system.

**3b. Tristimulus Values.** To obtain a particular light's standard color and brightness descriptors, calculate the three integrated response intensities[4] for the eye-brain system of the 1931 CIE "standard observer." Specifically: (i) multiply the energy distribution incident upon the eye by each of the three CIE tristimulus response distributions, wavelength by

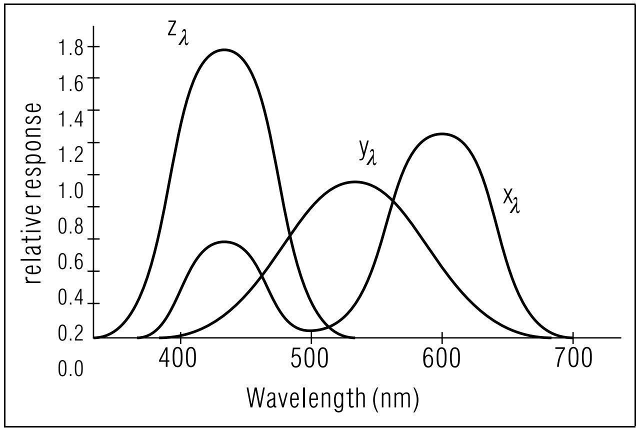


**Figure 3.** The 1931 CIE eye-brain tristimulus values for the "standard observer."

wavelength; (ii) plot the three resulting curves, as in Fig. 5; then (iii) determine the area under each of the curves.[6] The three resulting areas are labeled $X$, $Y$ and $Z$, as in Fig. 5, and are the integrated responses which would be produced by the retinal cones of an CIE "standard observer."

**3c. Chromaticity Coordinates.** In order to specify a light's color in the standard way, we convert its three integrated stimulus values into two numbers specifying color and one specifying intensity. This separation assumes that a light's perceived color is independent of the light's intensity. To perform the separation, divide the three stimulus values by

[6] See Appendix C.

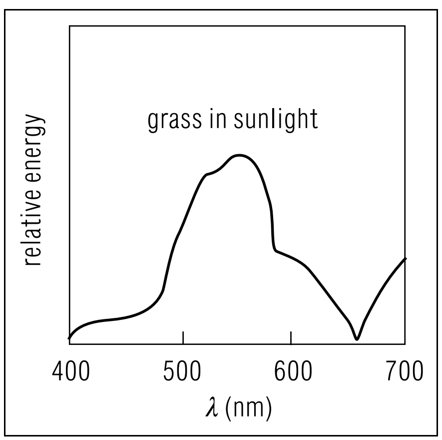


**Figure 4.** Relative Reflected Energy curve for summer grass in sunlight. This curve was obtained from Fig.'s 1 and 2 through point by point multiplication.





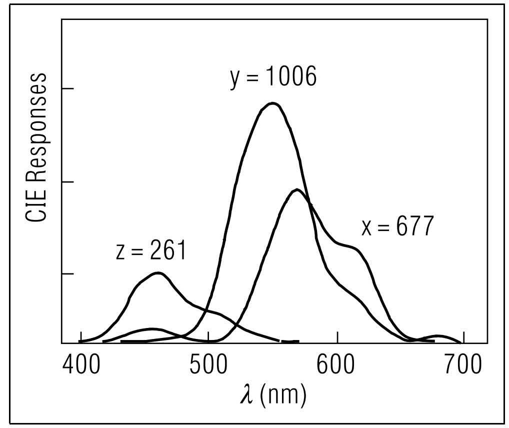


**Figure 5.** CIE Response distributions and integrated responses for summer grass in sunlight.

the total stimulus intensity:

$$x \equiv \frac{X}{X+Y+Z}; \qquad y \equiv \frac{Y}{X+Y+Z}; \qquad z \equiv \frac{Z}{X+Y+Z}$$

The resulting fractional stimuli, $x$, $y$, and $z$, are called the light's "chromaticity coordinates." Examples are given in Table I. Since the sum of $x$, $y$ and $z$ is always unity, only two of them are needed and it is customary to specify $x$ and $y$. Note that $X$, $Y$ and $Z$ can be reconstructed from knowledge of $x$ plus $y$ plus intensity.

Table I. Example chromaticity coordinates computed from data in Appendix B.

| **object** | **illumination** | $x$ | $y$ | $z$ |
|---|---|---|---|---|
| human skin | "60 W bulb" | 0.51 | 0.40 | 0.09 |
| summer grass | "sunlight" | 0.35 | 0.52 | 0.13 |

## Acknowledgments

Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

## Glossary

- **standard observer**: a person whose eye-brain system responds to light according to the 1931 CIE tristimulus curves. All persons differ from this standard in various ways: there is no real person who is precisely a "standard observer."
- **CIE**: the International Commission on Illumination which met in 1931 and proposed standards. The initials CIE are for the French name of the Commission.
- **illuminance**: radiant power per unit area, for light in the visible or photographic spectrum, normally expressed in lumens per square meter.
- **spectral illuminance**: illuminance per unit wavelength, expressed in lumens per square meter per micrometer; a "spectral density."
- **spectral reflectance**: the ratio of reflected light intensity to incident light intensity, as a function of wavelength, usually expressed in percent.
- $X(\lambda)$, $Y(\lambda)$, $Z(\lambda)$: in color science, the spectral response intensities computed using the CIE tristimulus spectral response functions, spectral illuminance, and spectral reflectance or transmittance.
- $x_\lambda$, $y_\lambda$, $z_\lambda$: in color science, the CIE tristimulus spectral response functions.
- $X$, $Y$, $Z$: in color science, the integrated spectral response intensities. $X \equiv$ area under curve of $X(\lambda)$ vs $\lambda$, etc.
- $x$, $y$, $z$: in color science, the relative integrated spectral response intensities. $x \equiv X/(X+Y+Z)$, etc. Note: $x+y+z=1$.





## A. The Chromaticity Diagram

The boundary of the Chromaticity Diagram (Fig. 6) is straightforwardly obtained by calculating the chromaticity coordinates of visible light at various pure wavelengths. All one need do is convert the CIE tristimulus values for light at individual wavelengths into chromaticity coordinates for those wavelengths and then plot them on an $x-y$ graph. Only these two coordinates need be used since $z$ can always be determined from $x$ and $y$: $x+y+z=1$. You can easily calculate a few points using mental arithmetic, then check them on Fig. 6. The diagram in three dimensions is shown in Fig. 6. All colors lie in a single plane defined by: $x+y+z=1$. The usual chromaticity diagram is this diagram's projection onto the $x$-$y$ plane.[7]

[7]See Fig. 2, MISN-0-229.

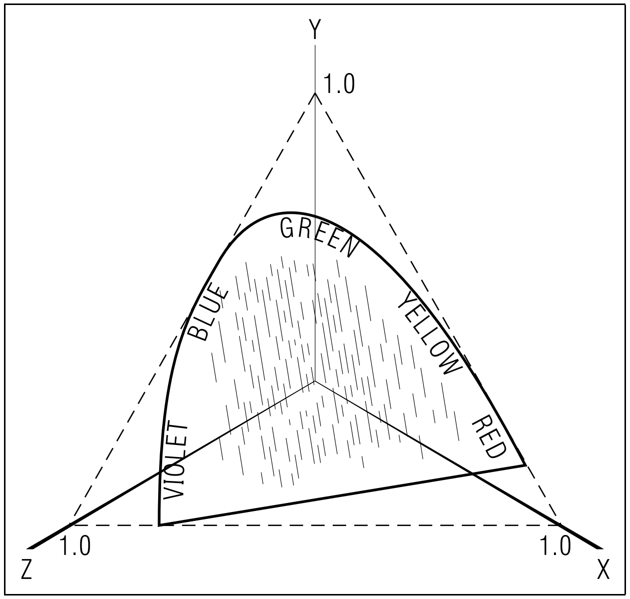


**Figure 6.** The locus of pure wavelengths as a function of chromaticity coordinates.

## B. Spectral Distribution Tables

| | Spectral Reflectances | | | | | Tristimulus Values | | | Rel. Spectral Illuminances | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\lambda$ | sg | ds | sn | wat | hs | $x_\lambda$ | $y_\lambda$ | $z_\lambda$ | su | fb | ib |
| 400 | 10 | 179 | 720 | 70 | 167 | 14 | 0 | 68 | 470 | 177 | 137 |
| 410 | 13 | 184 | 720 | 76 | 183 | 44 | 1 | 207 | 672 | 231 | 166 |
| 420 | 14 | 189 | 725 | 81 | 217 | 134 | 4 | 646 | 733 | 274 | 198 |
| 430 | 14 | 195 | 730 | 87 | 233 | 284 | 12 | 1386 | 787 | 334 | 234 |
| 440 | 18 | 202 | 730 | 92 | 250 | 348 | 23 | 1747 | 911 | 387 | 274 |
| 450 | 20 | 208 | 730 | 99 | 267 | 336 | 38 | 1772 | 1006 | 443 | 317 |
| 460 | 22 | 214 | 735 | 105 | 267 | 291 | 60 | 1669 | 1080 | 467 | 364 |
| 470 | 24 | 219 | 740 | 112 | 283 | 195 | 91 | 1288 | 1138 | 502 | 415 |
| 480 | 26 | 224 | 740 | 119 | 300 | 96 | 139 | 813 | 1183 | 497 | 469 |
| 490 | 37 | 231 | 740 | 126 | 300 | 32 | 208 | 465 | 1210 | 500 | 526 |
| 500 | 53 | 238 | 740 | 131 | 300 | 5 | 323 | 272 | 1215 | 484 | 586 |
| 510 | 78 | 247 | 740 | 138 | 317 | 9 | 503 | 158 | 1206 | 457 | 650 |
| 520 | 97 | 256 | 745 | 144 | 317 | 63 | 710 | 78 | 1199 | 474 | 715 |
| 530 | 107 | 265 | 750 | 152 | 300 | 166 | 862 | 42 | 1188 | 532 | 784 |
| 540 | 114 | 273 | 750 | 160 | 300 | 290 | 954 | 20 | 1198 | 647 | 854 |
| 550 | 115 | 282 | 750 | 169 | 317 | 433 | 995 | 9 | 1190 | 816 | 926 |
| 560 | 110 | 289 | 750 | 175 | 367 | 595 | 995 | 4 | 1182 | 1000 | 1000 |
| 570 | 96 | 295 | 750 | 181 | 350 | 762 | 952 | 2 | 1178 | 1150 | 1075 |
| 580 | 73 | 301 | 750 | 184 | 367 | 916 | 870 | 2 | 1168 | 1208 | 1151 |
| 590 | 53 | 307 | 750 | 183 | 400 | 1026 | 757 | 1 | 1161 | 1166 | 1228 |
| 600 | 45 | 314 | 755 | 177 | 467 | 1062 | 631 | 1 | 1167 | 1031 | 1305 |
| 610 | 47 | 320 | 760 | 167 | 500 | 1003 | 503 | 0 | 1168 | 848 | 1383 |
| 620 | 48 | 327 | 760 | 157 | 567 | 854 | 381 | 0 | 1165 | 698 | 1460 |
| 630 | 40 | 334 | 760 | 145 | 600 | 642 | 265 | 0 | 1176 | 527 | 1538 |
| 640 | 25 | 341 | 760 | 134 | 633 | 448 | 175 | 0 | 1175 | 405 | 1614 |
| 650 | 7 | 349 | 760 | 125 | 667 | 284 | 107 | 0 | 1173 | 305 | 1690 |
| 660 | 0 | 350 | 760 | 122 | 667 | 165 | 61 | 0 | 1166 | 220 | 1765 |
| 670 | 13 | 350 | 760 | 125 | 667 | 87 | 32 | 0 | 1160 | 155 | 1839 |
| 680 | 28 | 350 | 760 | 133 | 667 | 47 | 17 | 0 | 1149 | 95 | 1912 |
| 690 | 52 | 350 | 760 | 144 | 667 | 23 | 8 | 0 | 978 | 50 | 1983 |
| 700 | 88 | 350 | 760 | 157 | 667 | 11 | 4 | 0 | 1108 | 20 | 2052 |





### Table Key

$\lambda$: wavelength in nm.

**sg**: summer grass[a]; refl. $\times$ 1000

**ds**: dry sand[g]; refl. $\times$ 1000

**sn**: snow covered with layer of ice[h]; refl. $\times$ 1000

**wat**: water, very muddy[i]; refl. $\times$ 1000

**hs**: human skin[b] (color is approximately independent of race but reflectivity varies widely); refl. $\times$ 1000

**$x_\lambda$, $y_\lambda$, $z_\lambda$**: 1931 CIE Standard Observer Tristimulus Values[c] $\times$ 1000

**su**: sunlight, direct, normal incidence[d]; relative values (see below).

**fb**: fluorescent bulb, "cool white"; add the Hg line contributions[e]; relative values (see below).

**ib**: incandescent bulb, 60 watt frosted inside (2800 K)[f]; relative values (see below).

**relative values**: this notation means that all values in the column can be multiplied by the same intensity constant.

### Data Sources


a. E. L. Krinov, *Spectral Reflectance Properties of Natural Formations*, translation by G. Belkov issued as Technical Translation 439, National Research Council of Canada, Ottawa (1953), Appendix I, No. 164.

b. E. A. Edwards and S. Q. Duntley, "Science," *90*, 235 (1939), graph digitized by present author.

c. G. Wyszecki and W. S. Stiles, *Color Science*, John Wiley and Sons, Inc., New York (1967), Table 3.2.

d. P. Moon, "Proposed Standard Solar Radiation Curves for Engineering Use," J. Franklin Institute, 230 583 (1940). Units are $\mathrm{W\,m^{-2}\,\mu m^{-1}}$. Normal incidence, mean solar distance, sea level, all as quoted in Ref. c, Table 2.1.

e. Data from Ref. c, Table 1.12. Add 10 nm wide bands at these wavelengths with these intensities: (404.7 nm, 44.3), (435.8 nm, 102.6), (546.1 nm, 58.3), (577.8 nm, 12.4). Data are "typical U. S. lamp production for 1959."

f. Ref. c, Table 1.6.

g. Ref. a, No. 247.

h. Ref. a, No. 354.

i. Ref. a, No. 335.

## C. "Counting Squares" for Numerical Integration

Suppose you need the integral of a function that is only defined by a series of points. If you do not know a functional form for the curve, you must resort to some form of numerical integration or area measurement. An appropriate method for the present case is that of "counting squares."

As an example of the "counting squares" method, consider the curve of Fig. 7, for which the desired area is between the limits $x = 1$, and $x = 7$. The regions $A$ and $C$ must be evaluated separately from $B$, where the values of the curve are negative. To evaluate the integral in, say, region $A$, pick as basic squares either the small squares or the large darker-line ones which enclose 25 smaller squares. Then count the number of your basic squares that are enclosed by the boundaries of region $A$. If the curve cuts through a square, estimate what fraction of the square is under the curve. Normally, such estimates have surprisingly good accuracy. Using a large square as the basic unit of measure, you should find that region $A$ contains $1.82 \pm 0.02$ of those large squares. That's an accuracy of 1%!

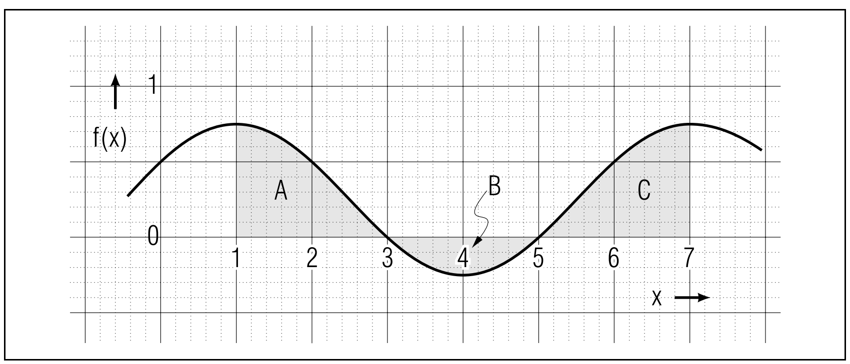


**Figure 7.** The integrals in each of the three regions shown are: $A = 0.91$, $B = -0.32$, $C = 0.91$.





Now obtain the scale factor between your squares and the $f(x)$ scale by either finding the amount of graph area for a given number of squares or by finding the number of squares between definite $x$- and $y$-coordinate intervals. For example, since there are two large squares between the intervals ($0 < x < 1$; $0 < y < 1$), there are two such squares per unit function area. This means that the true area of the region $A$ is 0.91. Similarly, you should be able to find the areas of $B$ and $C$ given in the caption to Fig. 7. The total integral is then:

$$\int_1^7 f(x)dx = 1.50$$

# LOCAL GUIDE

## Credit for this Module

Choose an object with a spectral reflectance and an illumination source with a spectral illuminance, as given in the table in Appendix B. Do not choose a combination that is worked out as an example in the module. You must construct two graphs:

1. relative reflected energy vs. wavelength
2. spectral response intensities X($\lambda$), Y($\lambda$), Z($\lambda$).

Then you must use these graphs to calculate the chromaticity coordinates of the object. Your calculations should be neat and clearly arranged.

Bring all your original material, not a copy, with you to the exam room and check in as with any other exam. Answer any exam questions pertaining to the Knowledge Output Skill of the module.

The last "question" of the exam will direct you to hand in your original materials relating to the project. Your "answer" to this question should be a reference to your material which you should attach to your exam sheets, immediately following the answer sheets.

The exam procedure is the same for both unit and block exams. In particular, in either case a lack of project originals will cause the grader to assign a grade of zero to the entire exam.

Now read all of this module's *Model Exam*.

***Equipment Needed:*** During the exam you will need a simple calculator and graph paper. Graph paper can be obtained from the Exam Manager.

The readings for this unit are on reserve for you in the Physics-Astronomy Library, Room 230 in the Physics-Astronomy Building. Ask for them as "The readings for CBI Unit 270." Do **not** ask for them by book title.





# MODEL EXAM

1. Define illuminance, spectral reflectance, spectral illuminance.

Examinee: As a total response to item #2, below, mark page numbers on originals of the calculation sheet(s) and two graphs. Staple those sheets just behind the Exam Answer Sheet, first making sure your annotations explain how your sheets answer item #2.

2. Hand in your project that shows, in detail, use of the spectral energy distribution of an illuminating light, an object's spectral reflectance, and the 1931 CIE tristimulus curves to determine the chromaticity coordinates of the object's perceived color. No credit will be given if your choice of illumination and reflective object matches one of the combinations worked out as examples in the module (summer grass in sunlight, human skin under a 60 W bulb)or if you hand in copies rather than originals.

INSTRUCTIONS TO THE GRADER

**Grader**! The student must have attached:

- the ORIGINAL of his/her calculation sheet(s)
- the ORIGINAL of her/his two graphs.

If the student did not hand in originals of both of the above, then **immediately give the student a grade of zero** on this entire exam. Write the reason on the Exam Answer Sheet and grade no further.