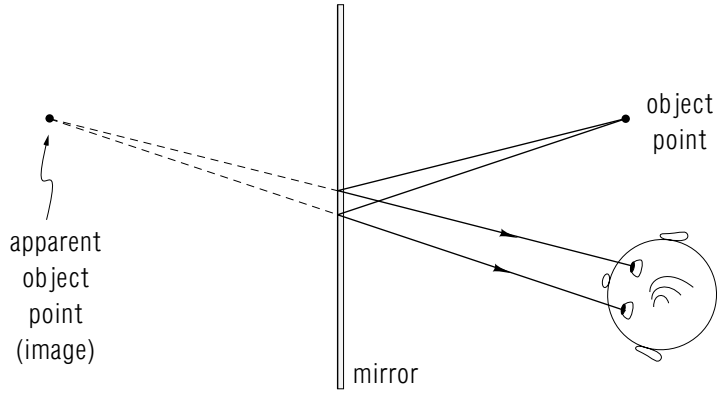
PHYSNET

MISN-0-260

REFLECTION AND PLANAR MIRRORS



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

REFLECTION AND PLANAR MIRRORS

by

Kirby Morgan

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Title: **Reflection and Planar Mirrors**

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Version: 2/1/2000 Evaluation: Stage 0

Length: 1 hr; 24 pages

Input Skills:

1. Vocabulary: wave front (MISN-0-203) or (MISN-0-430).
2. Prove given distances are equal by the method of congruent triangles.
3. State some general properties of photons and of electromagnetic waves (MISN-0-212).

Output Skills (Knowledge):

K1. Vocabulary: angle of incidence, angle of reflection, geometrical optics, image, incident ray, magnification, parallel ray, planar mirror, ray, ray tracing, real image, reflected ray, virtual image.

K2. State the law of reflection.

K3. Describe the difference between real and virtual images.

K4. Describe the image formed by a plane mirror.

K5. Prove that the image distance is equal to the object distance for a plane mirror.

Output Skills (Problem Solving):

S1. Determine the image of an object placed in front of a plane mirror, using the ray-tracing method.

Post-Options:

1. "Spherical Mirrors" (MISN-0-221).

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REFLECTION AND PLANAR MIRRORS

by

Kirby Morgan

1. Introduction

Have you ever stood in front of a flat mirror and wondered how your reflection was formed; why it was the same size as you, as far behind the mirror as your distance to the mirror, and why your right and left sides were reversed? The answers to these questions can be found through the study of Geometrical Optics, which is that area of optics where a light wave can be represented by a single line and its wave nature need not be specifically taken into account.

2. Law of Reflection

2a. Light Waves Can be Represented by Rays. The direction in which a light wave travels or propagates can be indicated by drawing lines called "rays." Rays are always normal (i.e. perpendicular) to the "wave fronts," which are surfaces that represent the progression of a wave as time passes. One can think of the wave fronts as surfaces of maximum and minimum disturbance; for a water wave, these would be crest ridges and trough positions. This is illustrated in Fig. 1 for plane and spherical wave surfaces.

For light passing through a medium which is homogeneous throughout, the rays are always straight lines since there is no reason for them to bend to one side or the other (a symmetry argument). Rays prove to be a very helpful tool for the tracing of light waves; as, for example, in the case of light that is reflected at the boundary between two media such as at the surface of a mirror.

2b. Definition of Angles of Incidence and Reflection. A ray reflecting from a surface forms an "angle of incidence" and an "angle of reflection" with the normal to the surface. These angles are shown in Fig. 2 for both a plane and a spherical surface. These angles are related by the Law of Reflection.

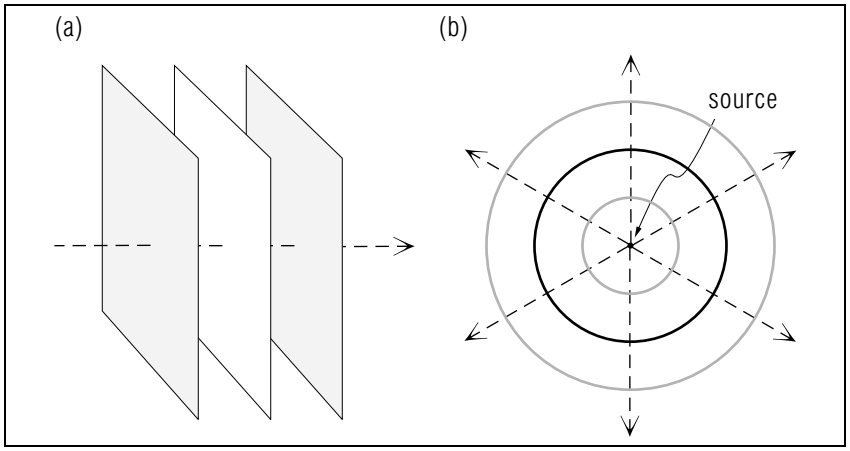


Figure 1. Wave crest and trough positions for: (a) plane and (b) spherical waves. The crest and trough surfaces are planes and spheres respectively. Rays are always normal to these surfaces.

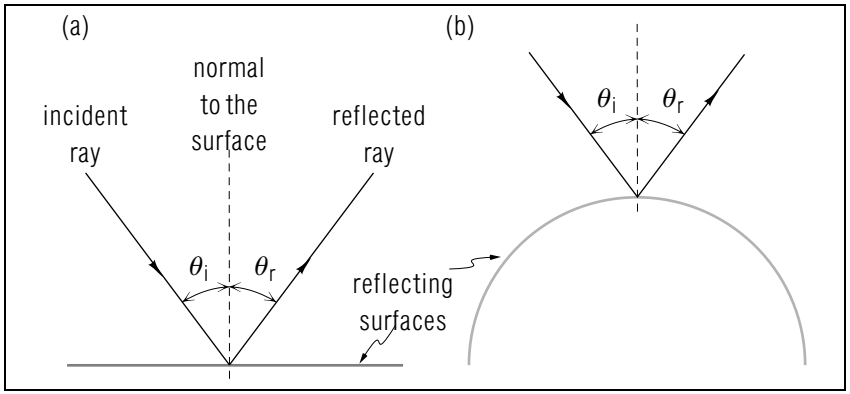


Figure 2. The angles of incidence (θ_i) and reflection (θ_r) are shown for (a) a plane and (b) a spherical reflecting surface.

2c. Angle of Incidence Equals Angle of Reflection. The Law of Reflection states:

1. The reflected ray lies in the plane formed by the incident ray and the normal to the surface.

2. The angle of incidence equals the angle of reflection.

These two rules of reflection are crucial for the tracing of rays when light strikes the surface of a mirror.

3. Objects and Images

3a. Definition of Object and Image. If an object, i.e. a source of light, is placed in front of a mirror, the Law of Reflection can be used to construct the reflected rays, which if extended backward intersect to form an "image" of the object. This is shown for the simplest of all cases, the plane mirror, in Fig. 3. If the rules of reflection are applied, triangles AOC and AIC can be shown to be congruent [S-1]. Thus the image is the same distance behind the mirror as the object is in front of it. If the object is an extended source such as a person's head or an arrow, every point of the object has a corresponding image point that lies an equal distance behind the plane of the mirror. Thus the image reproduces the object point for point.

3b. Example: Image of an Upright Arrow. If an upright arrow (i.e. an arrow whose head is situated above its tail) is placed in front of

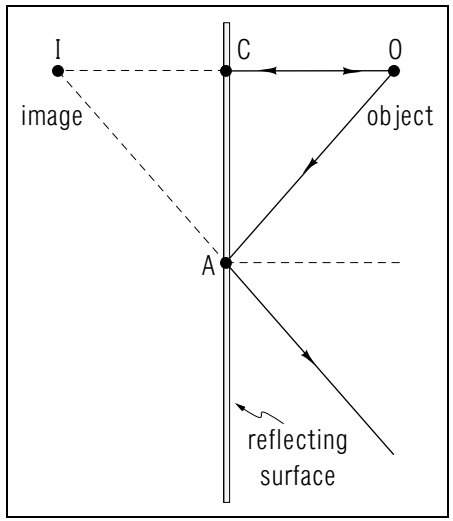


Figure 3. Distances OC and IC are equal since triangles AOC and AIC are congruent.

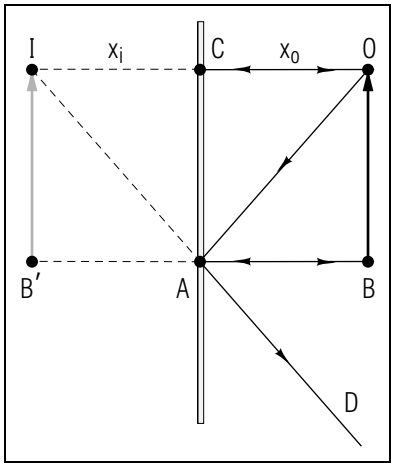


Figure 4. The image of an object arrow is found by tracing the rays for its head (O) and its tail (B). Note that one ray in the figure goes from O to A to D.

a plane mirror, its image may be easily found by geometrical construction, using the rules of reflection to trace the paths of the rays. This is illustrated in Fig. 4, where the arrow is situated a distance x_o in front of the mirror. Because the arrow is a straight line, it is sufficient to find the image points corresponding to its head and its tail and infer that points in between trace accordingly. Thus one finds that the image is upright and located a distance $x_i = -x_o$ from the mirror, where the minus sign is used to signify that the image is on the opposite side of the mirror from the object.

3c. How People Gauge Distances. A person uses the "known" distance between the person's eyes and the observed angle between the directions the eyes are pointing to estimate the distance to an object being looked at. This is illustrated in Fig. 5a. If a person is looking at an image produced by a mirror, the distance the person perceives is observed by extrapolating back along the reflected rays (see Fig. 5b). This produces the position and distance registered in the brain.

3d. Real and Virtual Images. Images may be either "real" or "virtual," depending on the paths followed by the rays originating from a point on the source. When the image is a "real" one, the light rays converge as they head toward the point on the image and then, after passing through the point, they diverge from it. When the image is a "virtual" one, the rays do diverge from the image point but in fact they never passed through that point. Thus a real image will appear on a sheet of paper placed at the image point because the rays are there to fall on the

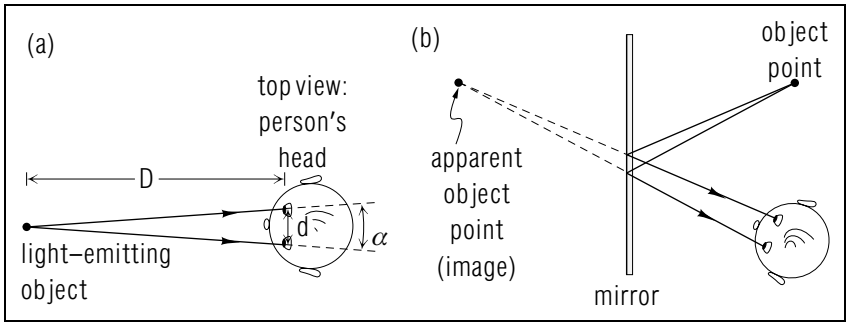


Figure 5. : (a) Person uses α and d to gauge D; (b) Person estimates reflected image position the same way.

paper. However, a piece of paper placed at the position of a virtual image will show nothing: no rays are there.

All images from planar mirrors are virtual because a sheet of paper placed behind the mirror will show no image at all (all the light rays coming from the mirror were reflected back away from the image point). However, the lenses in slide and movie projectors *do* produce real images: the image will show on a screen or a piece of paper placed at the image point.

Suppose rays are diverging from either a real or virtual image, but are not diverging too much. A strong enough lens can focus those rays (by altering their directions so instead of continuing to diverge they start to converge).

This is what happens when we look at the light rays coming from a mirror: the diverging light rays that appear to be coming from the virtual image are focused by the eye's lens to produce a real image on the back surface of the eye. This is the image we "see."

3e. Description of Plane Mirror Images. The image of an object in front of a plane mirror is virtual, upright, of the same size, and an equal distance from the mirror. The ratio of the size of the image to that of the object is called the "magnification." For a plane mirror the magnification is always equal to one, as can be proved by using the congruent triangles AOB and AIB′ of Fig. 4 [S-2]. An object which also has width, such as a printed page or a human being, will have a corresponding image whose left and right sides are reversed—a fact to which we can all attest from personal experience.

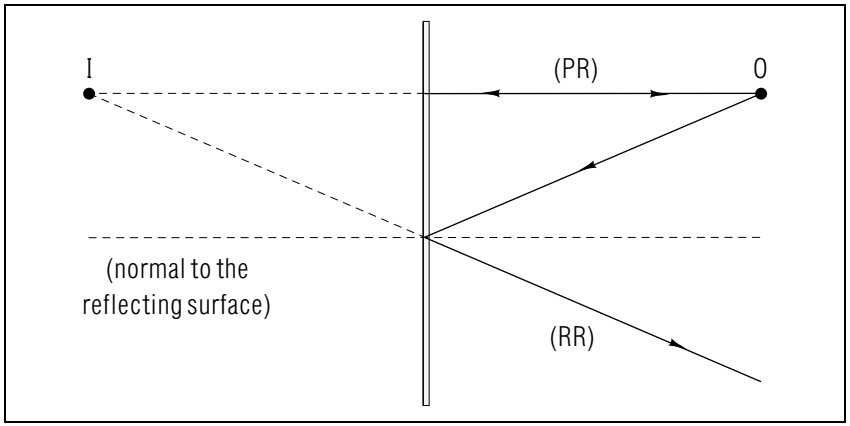


Figure 6. Construction of the Parallel Ray (PR) and a Reflected Ray (RR) gives the location of the image point.

4. Ray-Tracing Technique

4a. Graphical Means of Finding Image Points. A plane-mirror image point can be found graphically by constructing two rays and finding their intersection when the reflected parts are extended backwards. For simplicity use:

1. The Parallel Ray (PR) which, after it leaves the object point, travels along a path perpendicular to the reflecting surface and hence is reflected back along its incident path (see Fig. 6).

2. Any other Reflected Ray (RR) (see Fig. 6).

4b. Finding the Image of an Arrow. The image of an arrow can be found graphically by finding the intersection point of two rays for the tail and two rays for the head of the arrow. If the tail is always located on the axis, then its image will always be located an equal distance behind the mirror, on the axis. The image of the object arrow's head can be found by constructing two rays, and then the rest of the image arrow can be sketched in as shown in Fig. 7. The image is upright and the same size as the object.

4c. Ray-Tracing Can be Applied to Other Mirrors. Although the images formed by plane mirrors are readily predicted, the technique of ray tracing can also be used to find images formed by mirrors which

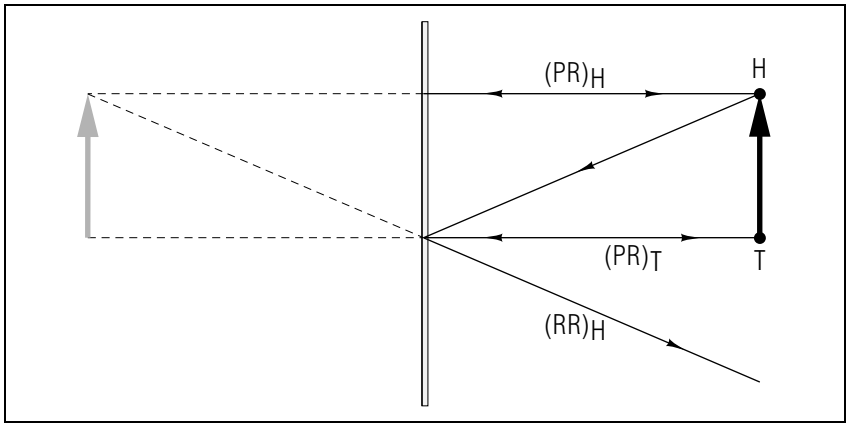


Figure 7. Ray tracing can be used to find the image of an arrow. A ray from the head of the arrow is marked H while one from the tail is marked T.

are spherical or, in fact, have any shape. The topic of spherical mirrors is covered elsewhere.[1]

Acknowledgments

We wish to thank Professor James Linneman for a helpful suggestion. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

Glossary

- **angle of incidence**: the angle formed by the incident ray and the normal to the mirror's surface.
- **angle of reflection**: the angle formed by the reflected ray and the normal to the mirror's surface.
- **geometrical optics**: the branch of optics in which light waves can be represented by rays.
- **converging (rays)**: two or more rays that are heading toward a common point of intersection. In contrast to diverging rays that head away from a common point of intersection. The rays need not actually arrive at the point of intersection; at some later point in their trajectories a mirror or a lens may intercept and divert them. In that case one must decide for the subsequent parts of their trajectories whether the rays are converging or diverging.

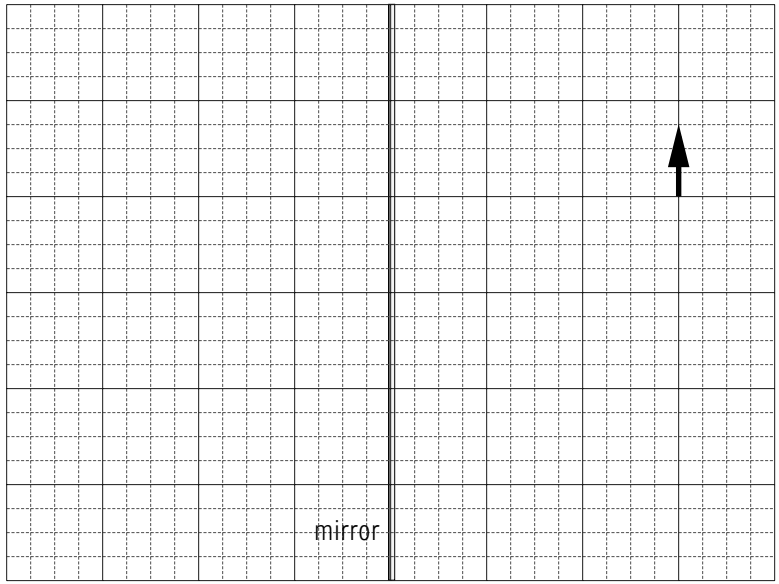
[1] See "Spherical Mirrors" (MISN-0-221) or (MISN-0-261).

- **diverging (rays)**: two or more rays that appear to be heading away from a common point of intersection. In contrast to converging rays that head toward a common point of intersection. The diverging rays need not have actually come from the point; they may have been put into their present trajectories by reflection or refraction.
- **image**: a likeness of an object, seen at a position other than the actual position of the object, which can be observed by looking into the object-emitted light rays coming from, or appearing to come from, the image position.
- **incident ray**: a ray that is approaching a surface.
- **magnification**: for an optical device, the ratio of the image size to the object size. This ratio is dimensionless and is negative if the optical device inverts the image.
- **parallel ray**: ray that travels parallel to a plane mirror's axis, hence perpendicular to its reflecting surface.
- **planar mirror**: a mirror that lies in a plane (a mirror that is "flat"). For such a mirror the image is virtual, upright, and the same size as the object (Magnification = 1).
- **ray**: a line that represents the trajectory of a point on a wave front.
- **ray tracing**: the act of drawing, to scale, the paths of the principal light rays from an object, to an optical device, to an image.
- **real image**: an image formed by light rays actually converging to a point. A real image can be seen by looking at a screen placed at the image position.
- **reflected ray**: ray that has reflected from a surface.
- **virtual image**: an image from which light rays appear to originate, but is actually an extrapolation of the rays diverging from an optical device. A virtual image cannot be seen by looking at a screen placed at the image position.

PROBLEM SUPPLEMENT

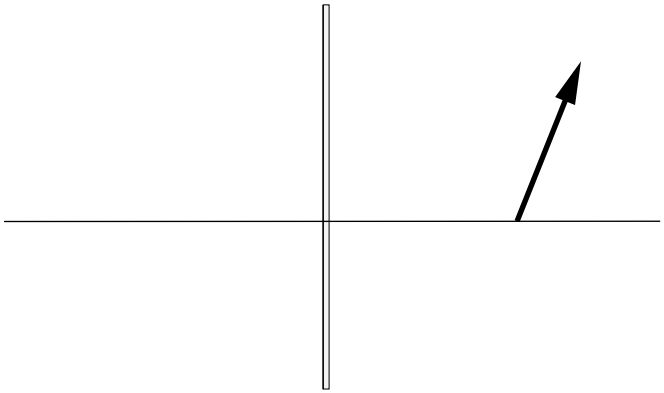
Note: Problems 7 and 8 also occur in this module's *Model Exam*.

1. Locate the image point for the tip of the arrow, graphically, using the law of reflection and ray tracing.

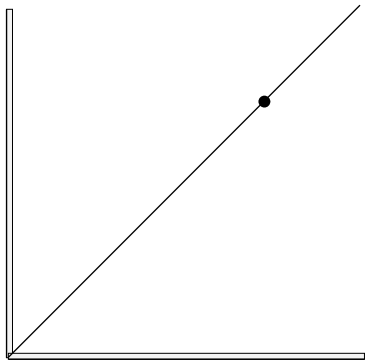


2. Draw a ray diagram to find the image of an upright object arrow which is situated 12.0 cm from a plane mirror. The object arrow is 3.0 cm high.

3. Use a ray diagram to find the image arrow corresponding to the object arrow shown below. Label the main rays for both the head and tail of the arrow.



4. Two plane mirrors are placed at right angles as shown below. Locate the image(s) of a point object which is located on the bisector of the angle, 10.0 cm from the mirrors' intersection.



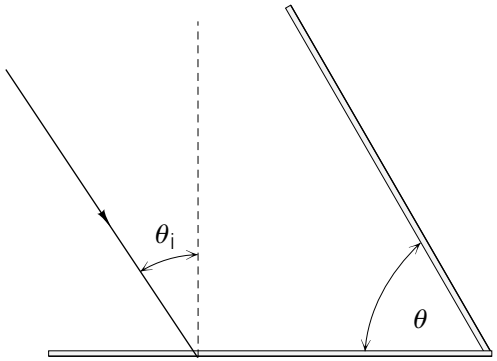
5. A point object is placed halfway between two parallel mirrors. Find the location of the four images closest to the object.

6. Demonstrate with a sketch that the right and left sides of an object are reversed when viewed in a planar mirror.

7. Construct a ray diagram to find the path of the reflected ray, and describe it in terms of the path of the incident ray, for the two plane mirrors shown in the sketch:

   a. if $\theta_i = 45°$ and $\theta = 45°$

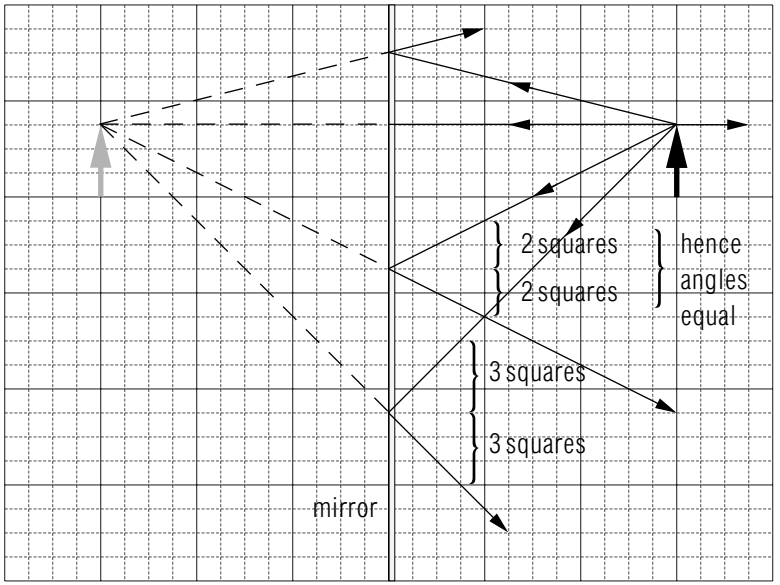
   b. if $\theta_i = 45°$ and $\theta = 90°$



8. Describe the image of your right hand if it is held 1 ft from a plane mirror: where is the image, is it real or virtual, is it left-right reversed from the direct (non-mirror) view?

Brief Answers:

1. You need only draw the horizontal line and one other, but we have drawn some more as a check. The "equal angles" in the figure are (of course) from the law of reflection.



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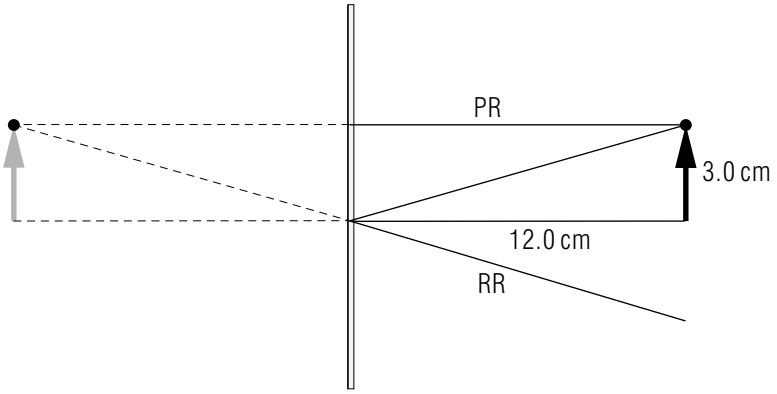

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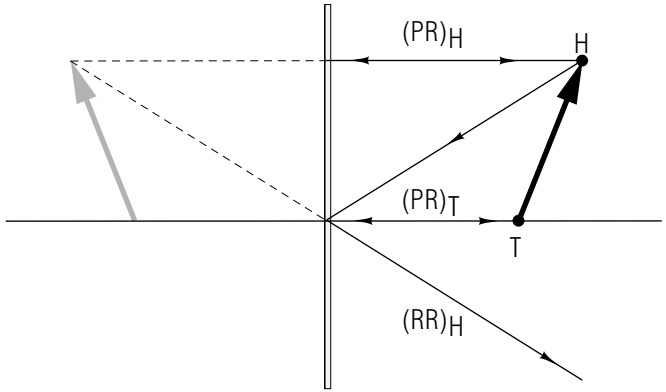

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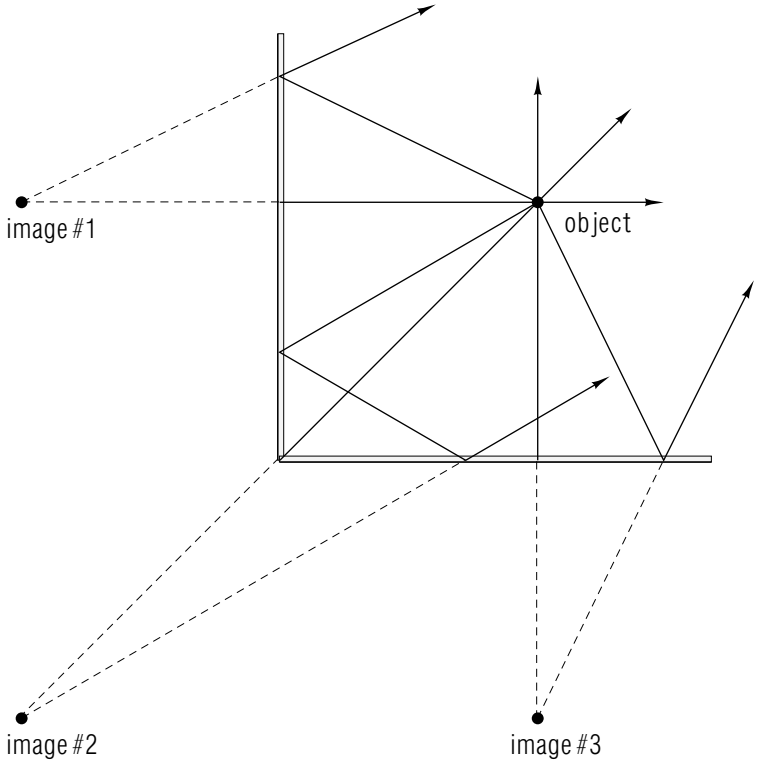

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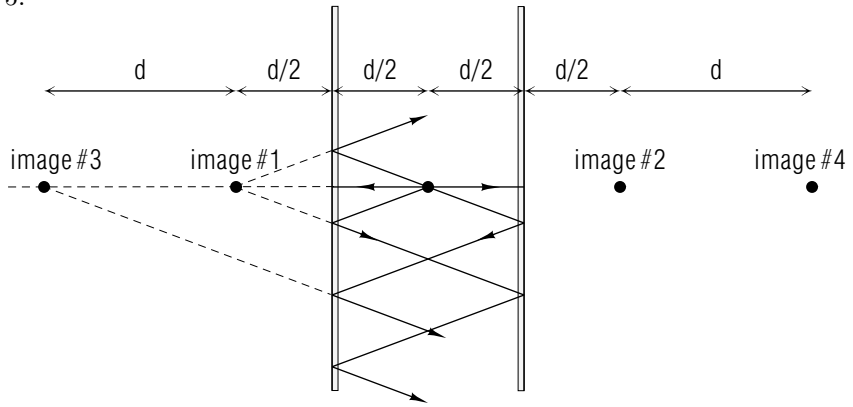
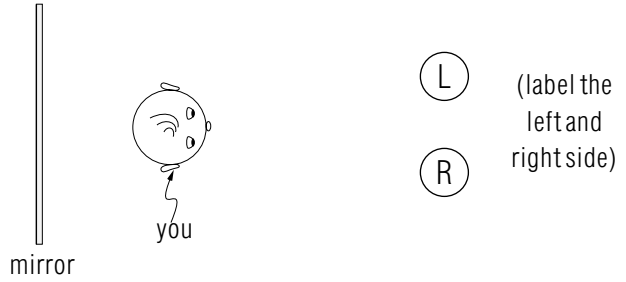
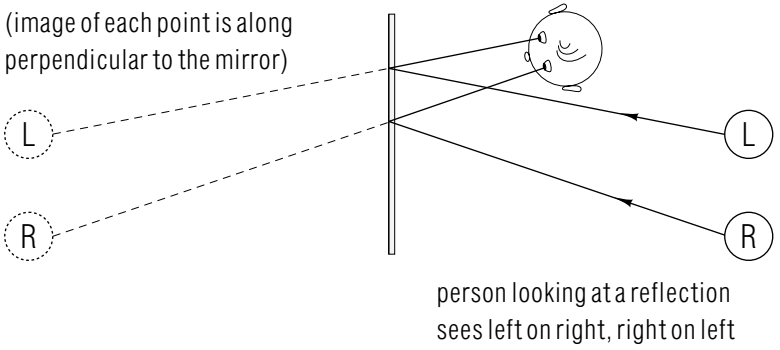


6. a. You, looking at the object directly:



b. You, looking at the object by reflection:



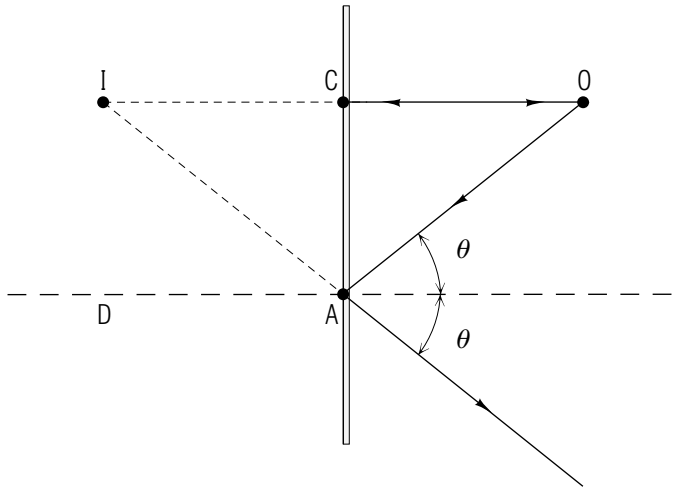
7. a. same path, opposite directions

   b. parallel paths, opposite directions

8. Virtual, upright, same size, 1 ft behind mirror, looks like a left hand seen directly.

SPECIAL ASSISTANCE SUPPLEMENT

S-1 *(from TX-3a)*



$\angle COA = \theta$: alternating interior angles

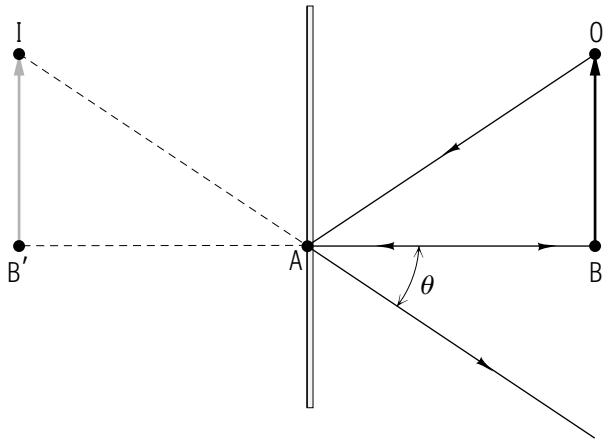
$\angle IAD = \theta$: vertical angles

$\angle CIA = \angle IAD$: alternate interior angles therefore $\angle COA = \angle CIA$

$CA = CA$

$\Delta AOC = \Delta AIC$: leg and angle of right triangle therefore $OC = IC$.

S-2 *(from TX-3e)*



$BA = AB'$

$\angle OAB = \theta$: law of reflection

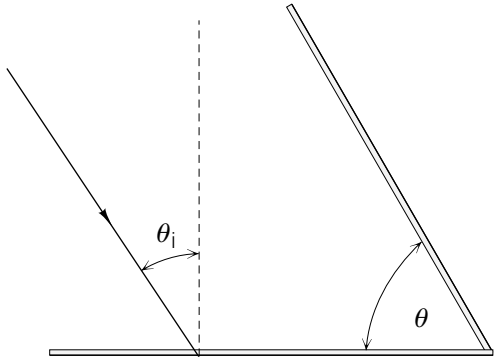
$\angle IAB' = \theta$: vertical angles therefore $\angle OAB = \angle IAB'$ and $\Delta OAB = \Delta IAB'$

$OB = IB'$

$m = \dfrac{IB'}{OB} = 1$

MODEL EXAM

1. See Output Skills K1-K4 in this module's *ID Sheet*.
2. Construct a ray diagram to find the path of the reflected ray, and describe it in terms of the path of the incident ray, for the two plane mirrors shown in the sketch:
   a. if $\theta_i = 45°$ and $\theta = 45°$
   b. if $\theta_i = 45°$ and $\theta = 90°$



3. Describe the image of your right hand if it is held 1 ft from a plane mirror: where is the image, is it real or virtual, is it left-right reversed from the direct (non-mirror) view?

Brief Answers:

1. See this module's *text*.
2. See Problem 7, this module's *Problem Supplement*.
3. See Problem 8, this module's *Problem Supplement*.