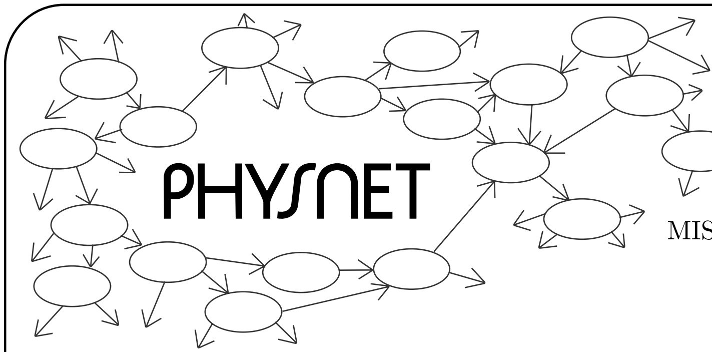

MISN-0-256

CONSERVATION LAWS FOR ELEMENTARY PARTICLE REACTIONS

$\pi^0 + p \rightarrow \Lambda^0 + K^+$

$\Lambda^0 \rightarrow p + \pi^-$

$\pi^- + p \rightarrow \Lambda^0 + K^0$

$n + p \rightarrow n + p + \gamma$

$K^+ \rightarrow \pi^+ + \pi^0$

$K^- + \Xi^0 \rightarrow \Omega^- + \pi^0$

$K^- + p \rightarrow \Lambda^0 + \pi^- + \pi^+$

$\Sigma^+ + p \rightarrow \Lambda^0 + p + \pi^0$

$n \rightarrow p + e^- + \nu_e$

$\pi^- + p \rightarrow n + \pi^0$

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

CONSERVATION LAWS FOR
ELEMENTARY PARTICLE REACTIONS

by

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Title: **Conservation Laws for Elementary Particle Reactions**

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Length: 1 hr; 24 pages

Input Skills:

1. Determine whether a given reaction goes via the strong, electromagnetic, or weak interaction from the nature of the particles involved in the reaction (MISN-0-255).
2. Given a two component system with angular momentum quantum numbers j_1 and j_2, determine the possible values of the resultant total angular momentum and of the projection of the angular momentum along any axis (MISN-0-251).

Output Skills (Problem Solving):

S1. Determine whether a proposed elementary particle reaction is allowed or forbidden by each of the absolute conservation laws (conservation of energy, momentum, charge, baryon number, electron number,and muon number).

S2. Determine whether a proposed elementary particle reaction is allowed or forbidden by conservation of isotopic spin or strangeness.

S3. Given a proposed elementary particle reaction, determine which interactions allow it, which forbid it.

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1. Introduction

Any particle reaction equation that you might write down won't necessarily represent a reaction that can occur in nature. Energy, momentum, and electrical charge, for example, must be conserved. These are conservation laws whose importance was discovered on the macroscopic scale, and whose absolute validity extends to the microscopic scale, the elementary particle interaction level. For example, consider the hypothetical reaction involving only baryons on both sides of the reaction equations

$$\mathrm{p} \Rightarrow \mathrm{n}\,,$$

or

$$\mathrm{n} \Rightarrow \mathrm{p}\,.$$

These reactions violate conservation of charge, and conservation of energy and momentum. These reactions are thus forbidden and won't go via the strong interaction, the weak interaction, the electromagnetic interaction, or any combination of these. There are other hypothetical reactions, which don't violate these familiar conservation laws which don't seem to occur, which appear to be absolutely forbidden. For example:

$$\mathrm{n} \Rightarrow \mathrm{e}^+ + \mathrm{e}^-$$

or

$$\mathrm{p} + \mathrm{e}^- \Rightarrow \gamma + \gamma$$

or

$$\mathrm{p} \Rightarrow \pi^+ + \pi^0\,.$$

These reactions conserve charge. Considering the last of these, if the π^+ and π^0 that resulted from the decay of the proton went off in opposite directions with equal speeds, then momentum would be conserved. Conserving energy as well would require each pion to have total energy equal to one half the proton's rest energy. Furthermore, all particles involved are hadrons so the reaction should go very quickly with a lifetime of

$\approx 10^{-23}$ sec. However, our existence testifies to the fact that this reaction does not occur at the expected rate; protons do not decay spontaneously! In fact, all attempts to seek any evidence of the instability of the proton result in the currently accepted conclusion that the proton is absolutely stable (its mean lifetime is infinite).[1] This suggests that there are other conservation laws operative on this fundamental interaction level which are not readily discernible on the macroscopic level. Such laws could account for the observed forbiddenness of certain reactions such as the decay of the proton. Such conservation laws are the topic of this module.

2. Universally Conserved Quantities

2a. Conservation of Momentum. If we consider the particles in a reaction and the forces between them as a closed, isolated system, then the system's total momentum must be conserved. Rigorous calculations to check conservation of momentum for most particle reactions require the use of relativistic dynamics and so are beyond the scope of this module. However we may make certain qualitative observations about the momenta of the products of a decay reaction. If the reacting particle decays into two products, then the two resulting ("product") particles must move in opposite directions with momenta of equal magnitude, when observed in the rest frame of the decaying particle. If the initial particle decays into three products, then the momenta of the three product particles must add to zero and hence must be coplanar, when observed in the rest frame of the decaying particle.

2b. Conservation of Energy. With the common assumption that the interacting system is closed and isolated, total energy is conserved in a particle reaction. Relativistic calculations are required to verify that the energy actually is conserved in any given reaction. However we may make these observations:

1. For two or more colliding particles, the reaction is energetically possible if sufficient kinetic energy is supplied to the reaction.

[1] However, current theories which propose to combine strong, weak and electromagnetic interactions into a unified fundamental theory, have as a consequence the possible very slow decay of the proton - with a characteristic lifetime of $\approx 10^{31}$ years (compare with the $\approx 4 \times 10^9$ years for the age of the universe!). Extensive research is now being conducted whose goal is detection of the decay of a proton.

2. For particle decays, conservation of energy requires that the mass of the decay products be less than or at most no greater than the mass of the decaying particle. This can be seen by considering the decaying particle in its rest frame. Before the decay its total energy is merely its rest energy, mc^2. After the decay, the product particles typically have some kinetic energy. To balance energy on both sides of the reaction equation, the total rest energy of the products must be less than the rest energy of the decaying particle. This may be stated mathematically as:

$$Mc^2 = \sum_{\text{products}} \left(mc^2 + E_k\right) . \tag{1}$$

Since $E_k \geq 0$, the total mass of the products must be less than the mass M of the initial particle.

2c. Conservation of Angular Momentum. In the absence of any external torques, the total angular momentum of an isolated system of interacting particles must be conserved. To check for conservation of angular momentum, you need to know the spins of the particles involved and the rules for adding quantized angular momenta.[2] As a quick check, use the following facts obtained from the rules for adding quantized angular momenta:

1. The total angular momentum of two particles of integer spin ($S = 0, 1, 2, \ldots$) is an integral multiple of $\hbar$.
2. The total angular momentum of two particles of half-odd-integer spin ($S = 1/2, 3/2, 5/2, \ldots$) is also an integral multiple of $\hbar$.
3. The total angular momentum of two particles, one of integer spin and the other of half-integer spin, is a half-odd-integer multiple of $\hbar$.

As an example, consider the reaction

$$\mathrm{p} + \pi^- \Rightarrow \mathrm{e}^+ + \mathrm{e}^- .$$

The spin of the proton is 1/2 and the spin of the π^- is zero, so the total angular momentum of the reacting particles is a half-odd-integer multiple of $\hbar$. However the electron and positron each have spin 1/2, so the resulting particles have a total angular momentum that is an integer multiple of $\hbar$. Angular momentum cannot be conserved, so this reaction will never take place.

[2] See "Quantized Angular Momenta" (MISN-0-251).

2d. Conservation of Charge. All elementary particle reactions must conserve charge. Unless you are considering the sub-hadronic world, all charged elementary particles have a charge that is a positive or negative integer multiple of the electron's charge.[3] Given the charge of all particles involved in a reaction, the net charge of the initial particles must equal the net charge of the final particles.

3. Family Particle Number

3a. Conservation of Baryon Number. A conservation law which accounts for the stability of the proton is the conservation of "baryon number," B. This also accounts for the fact that the neutron and all of the other heavier "elementary particles," the baryons, decay in such a way that the final product is the proton. This conservation law is similar to electrical charge conservation.

Just as all particles can be assigned electrical charge values of $0, \pm 1$, or ± 2, etc., (in units of the quantum of electric charge, the charge on the protons), every particle has a "baryon charge" of $B = 0$, $B = +1$, or $B = -1$. Furthermore, in any reaction the total baryon number of the products of the reaction must equal the sum of the baryon numbers of the initial particles. The proton is the lightest particle with baryon charge $B = +1$ so this accounts for the stability of the proton. All baryons have $B = 1$, their anti-particles have $B = -1$, and all mesons, leptons and the photon have $B = 0$. Thus

$$\begin{array}{rccccc} & \mathrm{p} & \Rightarrow & \pi^+ & + & \pi^0 \\ B: & +1 & & 0 & & 0 \end{array}$$

is forbidden by baryon conservation, while

$$\begin{array}{rccccc} & \Delta^{++} & \Rightarrow & \mathrm{p} & + & \pi^+ \\ B: & +1 & & +1 & & 0 \end{array}$$

is allowed by baryon number conservation as well as by all other conservation laws. This reaction involves only hadrons and it occurs with the characteristic time of 10^{-23} second. Similarly

[3] Current theories of elementary particles state that hadrons (baryons and mesons) are composed of particles called "quarks" which have charge 1/3 or 2/3 times the charge of an electron. See "SU(3) and the Quark Model" (MISN-0-282).

$$\begin{array}{lccccccc} & \mathrm{n} & \Rightarrow & \mathrm{p} & + & \mathrm{e}^- & + & \bar{\nu}_\mathrm{e} \\ B: & 1 & & 1 & & 0 & & 0 \end{array}$$

is also allowed by baryon number conservation. Since it involves leptons as well as hadrons, the reaction goes at a much slower rate (the neutron mean life being 1000 seconds).

3b. Conservation of Lepton Quantum Numbers. There appears to be a set of two quantum numbers associated with leptons that has similarities to baryon number. These two quantum numbers are similar to baryon number in the sense that the conservation laws associated with them are absolute; they must be satisfied in all processes. These quantum numbers are the electron lepton number and the muon lepton number. Their assigned values are:

Particle	**Electron No.**	**Muon No.**
e^-	+1	0
μ^-	0	+1
ν_e	+1	0
ν_μ	0	+1

The antiparticle to any of these particles has the opposite lepton number; for example, the e^+, the anti-electron, has an electron lepton number of -1. All other particles have zero value for both these lepton numbers. Consider the process: $\pi^- \Rightarrow \mu^- +$ "*neutrino.*" Which one of the 4 kinds of neutrino-type particles (neutrinos and anti-neutrinos) must this "neutrino" be if both electron lepton number and muon lepton number are to be conserved in this process? *Help: [S-7]*

4. Strangeness

4a. "Strange" Decay Modes. The failure of certain hadrons to decay via the strong interaction has led physicists to infer the existence of a hadron quantum number called "strangeness." Consider the hadron Λ^0 which is a baryon (the Λ^0 has a rest energy of 1116 MeV compared to 938 MeV for the proton). The Λ^0 is produced via Strong Interactions such as

$$\pi^- + \mathrm{p} \Rightarrow \Lambda^0 + \mathrm{K}^0$$

and

$$\pi^0 + \mathrm{p} \Rightarrow \Lambda^0 + \mathrm{K}^+ ,$$

where the K^0 and K^+ are mesons. The K's are $B = 0$ particles as are the pions, so baryon number is conserved in the reaction since the Λ^0 has $B = 1$. So Λ^0 must be a hadron (it's produced in a reaction which goes at the characteristic rate of strong interactions) and it is a baryon. The Λ^0 is unstable and decays via

$$\Lambda^0 \Rightarrow \mathrm{p} + \pi^- .$$

This reaction involves only hadrons and baryon number is conserved, so it should go at the SI (strong interaction) rate of $\propto 10^{-23}$ seconds. In fact the observed Λ^0 lifetime is much longer, about 10^{-10} seconds, which indicates that the reaction occurs via the WI (weak interaction) and not the SI. This is not an isolated case. For example, the K^0 and K^+ decay via:

$$\mathrm{K}^+ \Rightarrow \pi^+ + \pi^0 ,$$

Again these are hadrons, $B = 0$ on both sides, and you'd expect the reaction to be carried out by the strong interaction. Again the lifetime is about 10^{-8} seconds, which reveals that it is the WI that brings about the process. What forbids the occurrence, via the SI, of such reactions involving hadrons? Strangeness!

The answer is that these observations reveal the existence of another conservation law. This one is unlike the law of baryon number conservation or the law of charge conservation, which are absolute conservation laws (absolute in the sense that the laws are satisfied in all processes,[4] no matter what interaction meditates the process). This conservation law is absolute only for Strong Interaction and Electromagnetic processes; it may be violated in processes which go via the weak interaction.

4b. Conservation of Strangeness. Analogous to the assignment of baryon number, another quantum number is assigned to all hadrons, the Strangeness quantum number S. The strangeness assignments are to be made in a way which is consistent with what is observed. That is, those processes involving hadrons which are observed not to go via the SI must violate Strangeness conservation. As an example, consider the three processes

[4] That is to say, no processes violating these conservation laws have ever been observed.

$$\begin{aligned} \pi^0 + \mathrm{p} \Rightarrow \Lambda^0 + \mathrm{K}^+ &\qquad \text{(goes via SI)} \\ \Lambda^0 \Rightarrow \mathrm{p} + \pi^- &\qquad \text{(goes via WI)} \\ \mathrm{K}^+ \Rightarrow \pi^+ + \pi^0 &\qquad \text{(goes via WI)} \end{aligned}$$

Try an assignment scheme for yourself. Assign Strangeness quantum numbers to the particles involved in these three processes such that Strangeness is conserved in the first reaction but violated in the last two. As a simplifying ground rule, use only $S = 0$, 1, and -1 and assign the same S to each of the three members of the pion family *Help: [S-6]*. Of course, based on only these three experimental facts, there is no unique set of S values which will satisfy the observations. You could make assignments that would work just as well as the values you'll find in the physics literature. However, the assigned numbers must be restricted to those sequences that match the experimental results. Physicists always use the "commonly accepted" values found in the literature and we ask that you do likewise. The values you used in the above three reactions are indeed those values (see the complete table of them at the beginning of the Problem Supplement). There are many other hadrons and many other reactions, and the S quantum number assignments made to all the hadrons must be consistent with what is observed for all processes involving only hadrons: if a process is observed to go via the SI or the E-M (electromagnetic) interaction, Strangeness must be conserved; if it is to be allowed to go via the WI, then Strangeness must not be conserved.

5. Isospin

5a. Introduction to Isospin. There is yet another internal quantum number that is associated with elementary particles. This one, like strangeness, is associated with hadrons only. Note that the number of conservation laws that must be satisfied in a given interaction increases with increasing strength of the interaction, the SI having the most conservation laws, the WI the fewest (so hadrons have more quantum numbers than leptons). This internal quantum number is called the "isotopic spin," "I-spin," or "isospin," but it has nothing to do with angular momentum or spin.

5b. Assigning Isospin Quantum Numbers. The assignment of isospin quantum numbers may be made if we consider the situation which prevails with the spectrum of hadrons (baryons and mesons). If you

plot the masses of the baryons in a way similar to the plot of atomic energy levels and separately do the same for the meson masses you observe degeneracies, places where several particles have about the same mass. The spectrum is said to consist of degenerate multiplets. There are singlet levels (such as the Λ^0), doublets (such as the proton and neutron), triplets (such as the Σ^+, Σ^0, and Σ^-.), quartets (such as the Δ^{++}, Δ^+, Δ^0, Δ^-), etc.. Each grouping consists of particles of nearly equal mass and differing only in charge. This suggests that we can assign, to those particles, pairs of quantum numbers analogous to the (j, M_j) pair that are assigned to atomic levels. These quantum numbers have nothing to do with angular momentum, however. They are internal quantum numbers just as are B and S. These quantum numbers are usually labeled (I, I_3).[5] Consider the Σ^+, Σ^0, Σ^- triplet. The isospin I obviously is 1 and so I_3 has possible values of 1, 0, -1 *Help: [S-10]*. Note that I_3 is directly related to the electrical charge of the particle: values of I_3 are assigned in order of decreasing value as Q decreases. Thus I and I_3 for the Δ^{++} are 3/2 and +3/2, respectively; for the Δ^- they are 3/2 and $-3/2$, respectively *Help: [S-11]*.

5c. The Gell-Mann-Nishijima Formula. There is a relationship between electric charge Q and the quantum numbers I_3, S and B given by the Gell-Mann-Nishijima formula:

$$\frac{Q}{e} = I_3 + \frac{S+B}{2}\,.$$

Note that (Q/e) has integer absolute value. That is, (Q/e) may be 0, ± 1, ± 2, ... The sum $S + B \equiv Y$ is called "hypercharge."

5d. When and How Isospin is Conserved. The assignment of isospin quantum numbers would be an empty exercise except for the observation that I is conserved in all strong interactions and I_3 is conserved in all strong and electromagnetic interactions.

Conservation of I_3 is similar to the conservation of charge and of B and S (and not independent of these conservation laws because of the relationship $Q/e = I_3 + B/2 + S/2$): just add the I_3 values on the left side of a reaction equation and it must equal the sum of the I_3 values on the right side for the reaction to go via SI or the electromagnetic interaction.

Conservation of I is a little more complicated. To see how this works, consider first this reaction:

[5] In some references you'll see (T, T_z).

$$\pi^- + \mathrm{p} \Rightarrow \Lambda^0 + K^0 \qquad \text{(goes via SI)}$$

Given that: π^- belongs to a triplet, K^0 belongs to a doublet, both K and π are mesons, and Λ^0 has strangeness $S = -1$ and is a baryon, fill in this table *Help: [S-9]*:

	π^-	+	p	$\Rightarrow$	Λ^0	+	K^0
Q		&		$\Rightarrow$		&	
I_3		&		$\Rightarrow$		&	
S		&		$\Rightarrow$		&	
B		&		$\Rightarrow$		&	
I		&		$\Rightarrow$		&	

On the right side the total I is 1/2. Therefore on the left side the total I must be 1/2 for Isospin to be conserved. But the I-values on the left side are $I = 1$ and $I = 1/2$. How do you add these to get 1/2? Getting the resultant of these two is completely analogous to getting the resultant of two angular momenta.

We here summarize the rules for adding quantized angular momenta, which are treated in more detail elsewhere.[6] Because angular momentum is quantized, j is restricted to integer and half-integer values. This means that if you add two angular momenta, the resultant magnitude of this vector sum is also restricted to integer or half-integer values. However, the "third component" of angular momentum, with quantum number M_j, adds arithmetically. As an illustration, suppose we start with two angular momenta, each with $j = 1$. Associated with each $j = 1$ there are the three M_j values of 1, 0, and -1. What are the possible M_j values of the resultant? Each possible resultant M_j is the arithmetic sum of the possible individual M_j values. So with 3 possible M_j's in one group and 3 in the other there are nine possible sums. They are 2, 1, 0, 1, 0, -1, 0, -1, -2. Or rearranging them more suggestively:

$$\begin{array}{ccccc} 2 & 1 & 0 & -1 & -2 \\ & 1 & 0 & -1 & \\ & & 0 & & \end{array}$$

These are the possible M_j values of the resultant angular momentum. The upper line contains all the possible M_j values for $j = 2$, the middle

[6] See "Quantized Angular Momenta" MISN-0-251.

one represents $j = 1$, and the last $j = 0$. This then suggests that if you add a $j = 1$ to a $j = 1$ the possible values of the resultant are $j = 2$, 1, and 0, and this, indeed, is the case. Instead of going through the above analysis each time you add two angular momenta, you can remember the rule that if you add two angular momenta whose quantum numbers are j_1 and j_2, the resultant angular momentum quantum number will have possible values that occur in the interval from $j_1 + j_2$ to $|j_1 - j_2|$, with all possible values in between occurring in integer steps.

The addition of two isospins follows the addition of two angular momenta exactly. The example

$$\pi^- + \mathrm{p} \Rightarrow \Lambda^0 + \mathrm{K}^0\,,$$

does conserve I-spin. The right side has I-spin equal to 1/2 ($I = 0$ and $I = 1/2$ have only one possible sum). The left side is the resultant of $I = 1$ and $I = 1/2$. Therefore, the reaction can go via the SI because isotopic spin is conserved: both the left side and the right side can have $I = 1/2$. Note: It is very important that you know the distinction between I and I_3: adding two I's is done differently than adding two I_3's.

Thus for an odd number of particles in a multiplet, with each particle in a multiplet having its own I_3 value, the following list shows the I_3 values for members of quintuplets, trios, and singlets:

$$\begin{array}{ccccc} 2 & 1 & 0 & -1 & -2 \\ & 1 & 0 & -1 & \\ & & 0 & & \end{array}$$

while for an even number of particles the smallest three multiplets have particles with these I_3 values:

$$\begin{array}{cccccc} 5/2 & 3/2 & 1/2 & -1/2 & -3/2 & -5/2 \\ & 3/2 & 1/2 & -1/2 & -3/2 & \\ & & 1/2 & -1/2 & & \end{array}$$

Acknowledgments

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PROBLEM SUPPLEMENT

Lepton Numbers	
	Electron No.
e^-	$+1$
e^+	-1
ν_e	$+1$
$\bar{\nu}_e$	-1
	Muon No.
μ^-	$+1$
μ^+	-1
ν_μ	$+1$
$\bar{\nu}_\mu$	-1

Hadron Numbers			$\mathbf{I_3}$				
B	S	I	-1	$-1/2$	0	1/2	$+1$
1	0	1/2		n		p	
1	-1	0			Λ^0		
0	0	1	π^-		π^0		π^+
0	$+1$	1/2		K^0		K^+	
0	-1	1/2		K^-		$\overline{K}^0$	
1	-1	1	Σ^-		Σ^0		Σ^+
1	-2	1/2		Ξ^-		Ξ^0	
1	-3	0			Ω^-		
0	0	0			η		

$$\frac{Q}{e} = \frac{B}{2} + \frac{S}{2} + I_3$$

1. The Ω^- baryon is a singlet energy state whose electric charge is given by: $Q/e = -1$. The Ω^- is a hadron which eventually decays to the proton in a three stage decay process.

   a. What are (I, I_3) for the Ω^-? *Help: [S-3]*

   b. What is the strangeness of the Ω^-? *Help: [S-8]*

   c. What is the hypercharge of Ω^-? *Help: [S-4]*

   d. The Ω^- is observed to decay to $\Xi^- + \pi^0$. The Ξ^- has strangeness $S = -2$. Does this decay go via the SI or WI? *Help: [S-1]*

   e. What is I_3 for the Ξ^-? *Help: [S-5]*

   f. Ξ^- is one of the members of a doublet. What is the electric charge of the other particle in this doublet? *Help: [S-2]*

2. A multiplet of particles consists of three baryons with strangeness $S = +1$. What is the charge of each member of this three particle multiplet? *Answer: 1*

3. The μ^+ (positive muon) decays to an e^+ and two other particles. Using lepton conservation laws identify the other two particles. *Answer: 17*

4. In the process $A + B \Rightarrow C + D$ particles C and D belong to isospin zero multiplets. Particles A and B each belong to an isospin 1/2 multiplet. If I_3 for A is + 1/2 what is it for B? Can this reaction go via the strong interaction? *Answer: 2*

5. Suppose in the above reaction particle A belonged to an isospin zero multiplet and B was the $I_3 = 0$ component of an $I = 1$ multiplet. Would the reaction go via the SI? *Answer: 24*

6. Consider the following reactions. Determine whether or not each one is allowed via the strong interaction. If not allowed by the strong interaction determine whether or not it's allowed by the weak interaction or whether it's absolutely forbidden. If forbidden, state which conservation law forbids it.

   a. $\Xi^- \Rightarrow \Sigma^- + \pi^0$ *Answer: 3*

   b. $K^- + p \Rightarrow \Lambda^0 + \pi^- + \pi^+$ *Answer: 19*

   c. $n + p \Rightarrow n^- + p^-$ *Answer: 4*

   d. $K^- + \Xi^0 \Rightarrow \Omega^- + \pi^0$ *Answer: 7*

   e. $\Lambda^0 \Rightarrow n^0 + \pi^0$ *Answer: 22*

7. Determine the quantum numbers (I, I_3, S, B, Y, Q) for the antiparticle of the particle Δ^0 (one of the Δ^{++}, Δ^+, Δ^0, Δ^- multiplet). *Answer: 20*

8. What are the possible values for the isospin of the deuteron? *Answer: 5* (The deuteron is a nucleus consisting of a neutron and a proton).

9. A fictitious particle called Z is produced via the strong interaction in the process

$$K^+ + p \Rightarrow Z + \Lambda^0$$

   For this particle determine these quantum numbers:

   a. Charge *Answer: 21*

   b. Strangeness *Answer: 16*

   c. Baryon number *Answer: 9*

   d. I_3 *Answer: 11*

   e. With this value of I_3 what is the smallest value of I itself that the particle Z may have? *Answer: 8*

f. For this smallest value of I, how many companion particles are there for Z? *Answer: 15*

g. What are the charges of those companion particles? *Answer: 25*

10. Assuming that Z has enough mass for the following decays to be energetically possible, state which of the following might occur and which will be forbidden. If forbidden, state why. (Supply appropriate charges to balance the reaction).

a. $Z \Rightarrow \mathrm{p} + \pi$ *Answer: 12*

b. $Z \Rightarrow \mathrm{K} + \mathrm{K}$ *Answer: 18*

c. $Z \Rightarrow \mathrm{p} + \overline{\mathrm{p}} + \pi + \pi$ *Answer: 13*

d. $Z \Rightarrow \mu + \mu + \nu_\mu + \nu_\mu$ *Answer: 6*

11. Suppose (hypothetically) that strong interaction processes could go if the reactants were in the $I = 3/2$ state, but were forbidden in the $I = 1/2$ state. Which of the following processes would be allowed and which forbidden?

a. $\Sigma^+ + \mathrm{p} \Rightarrow \Lambda^0 + \mathrm{p} + \pi^+$ *Answer: 10*

b. $\pi^- + \mathrm{p} \Rightarrow \mathrm{n} + \pi^0$ *Answer: 23*

c. $\mathrm{p} + \Lambda^0 \Rightarrow \Sigma^+ + \pi^0$ *Answer: 26*

Brief Answers:

1. 2,1,0
2. $-1/2$; yes
3. Weak (Strangeness not conserved)
4. Forbidden (baryon no. and charge)
5. 1 or 0 (actually is zero)
6. Weak (if at all), if both μ's are μ^+ and neutrinos are ν_μ.
7. Strong.
8. +1

9. Zero
10. Allowed
11. +1
12. Forbidden (baryon conservation)
13. Weak if at all (strangeness)
14. $Q = I_3 + Y/2$
15. 2
16. +2
17. $\bar{\nu}_\mu$, ν_e
18. Allowed, strong
19. Strong
20. $I = 3/2$, $Q = 0$; $I_3 = 1/2$; $S = 0$, $B = -1$, $Y = -1$.
21. +2
22. Weak (strangeness)
23. Allowed
24. No
25. 1, 0
26. Forbidden

SPECIAL ASSISTANCE SUPPLEMENT

S-1 *(from PS, problem 1d)*

Weak

S-2 *(from PS, problem 1f)*

Zero

S-3 *(from PS, problem 1a)*

(0,0)

S-4 *(from PS, problem 1c)*

-2

S-5 *(from PS, problem 1e)*

$-1/2$

S-6 *(from TX-4b)*

Λ^0 ($S = -1$); π ($S = 0$); p, n ($S = 0$); K^+ ($S = 1$)

S-7 *(from TX-3b)*

$\bar{\nu}_\mu$

S-8 *(from PS, problem 1b)*

-3

S-9 *(from TX-5d)*

Left to right and top to bottom: -1, $+1$, 0, 0; -1, 1/2, 0, $-1/2$; 0, 0, -1, $+1$; 0, 1, 1, 0; 1, 1/2, 0, 1/2.

S-10 *(from TX-5b)*

The Σ occurs as a charge triplet with Σ^+, Σ^0, and Σ^-. It exists as three particles that differ only in charge. Then the isospin assignment is $I = 1$ with $I_3 = 1, 0, -1$ for the three charge states.

S-11 *(from TX-5b)*

The Δ occurs as a charge quartet with Δ^{++}, Δ^+, Δ^0, and Δ^-. Then the isospin assignment is $I = 3/2$ with $I_3 = 3/2, 1/2, -1/2$ and $-3/2$ for the four charge states.

MODEL EXAM

Lepton Numbers	
	Electron No.
e^-	$+1$
e^+	-1
ν_e	$+1$
$\bar{\nu}_e$	-1
	Muon No.
μ^-	$+1$
μ^+	-1
ν_μ	$+1$
$\bar{\nu}_\mu$	-1

Hadron Numbers			**I_3**				
B	S	I	-1	$-1/2$	0	$1/2$	$+1$
1	0	1/2		n		p	
1	-1	0			Λ^0		
0	0	1	π^-		π^0		π^+
0	$+1$	1/2		K^0		K^+	
0	-1	1/2		K^-		$\overline{K}^0$	
1	-1	1	Σ^-		Σ^0		Σ^+
1	-2	1/2		Ξ^-		Ξ^0	
1	-3	0			Ω^-		
0	0	0			η		

$$\frac{Q}{e} = \frac{B}{2} + \frac{S}{2} + I_3$$

1. For each of the following reactions and decays (assuming for the reactions that there is enough kinetic energy to start with) state whether it is allowed or forbidden. If it is allowed, explain by which kind of interaction it goes. If it is forbidden, explain why.

   a. $\mu^+ \Rightarrow e^+ + \nu_e$

   b. $\Omega^- \Rightarrow n + n + \pi^-$

   c. $K^+ \Rightarrow \pi^+ + \pi^0$

   d. $\Sigma^0 \Rightarrow \Lambda^0 + \gamma$

   e. $\pi^- + p \Rightarrow n + \pi^0$

2. Three fictitious particles, x_1, x_2, and x_3 all have approximately the same rest mass. They are observed to be created in strong interaction processes of which these are typical:

$$\begin{aligned} \pi^- + p &\Rightarrow x_1 + \overline{K}^0 \\ p + p &\Rightarrow x_2 + K^- + p \\ K^+ + n &\Rightarrow x_3 + \pi^0 + \pi^0 \end{aligned}$$

   a. What is the baryon number of x_1, of x_2, and of x_3?

   b. What is the charge of x_1, of x_2, and of x_3?

   c. What is the strangeness of x_1, of x_2, and of x_3?

   d. What is the third component of isospin of x_1, of x_2, and of x_3?

   e. These three x-particles are all observed to decay via the weak interaction. Write down a possible weak decay mode for particle x_2.

Brief Answers:

1. a. forbidden; muon and electron lepton numbers are not conserved.

   b. forbidden; baryon number not conserved.

   c. allowed; by weak interaction but not by strong because strangeness not conserved.

   d. allowed; by electromagnetic but not by weak.

   e. allowed; by strong.

2. a. $B = 1$ for each.

   b. $Q_1/e = 0$; $Q_2/e = 2$; $Q_3/e = 1$

   c. $S = +1$ for each.

   d. x_1 has $I_3 = -1$; x_2 has $I_3 = 1$; x_3 has $I_3 = 0$.

   e. In weak decay, S is not conserved but B and Q are. Therefore, a possible decay of x_2 is:

$$x_2 \Rightarrow p + \pi^+$$