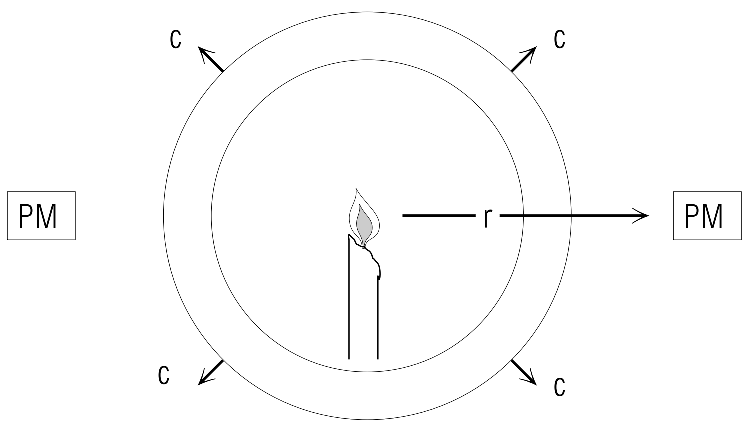
MISN-0-246

WAVE-PARTICLE DUALITY: LIGHT



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

WAVE-PARTICLE DUALITY: LIGHT
by
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Title: **Wave-Particle Duality: Light**

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Input Skills:

1. Vocabulary: particle trajectory, Newton's second law (MISN-0-15) or (MISN-0-409); photon energy, momentum, wavelength, frequency (MISN-0-212); two-source interference, diffraction (MISN-0-235) or (MISN-0-432); Snell's law of Refraction, polarization of light (MISN-0-222); electrostatic force (MISN-0-115) or (MISN-0-419); Maxwell's Equations (MISN-0-132) or (MISN-0-429).
2. Apply the photon energy-momentum-frequency relations (MISN-0-212).

Output Skills (Knowledge):

K1. Vocabulary: quantum field theory, wave-particle dualism.

K2. Contrast the quantum field theory description of light with that of the classical particle and wave models. Describe the predictive power of each of these.

Output Skills (Rule Application):

R1. Give reasons why any given light phenomenon can or cannot be satisfactorily described as each of these:

   a. classical particle
   b. classical EM wave
   c. quantum field

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New authors, reviewers and field testers are welcome.

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1. The Problem Posed by Light

1a. Overview. In classical physics, especially mechanics and electricity and magnetism, two distinct and very useful concepts have been invented and used repeatedly, that of particles and that of waves. However, to explain phenomena on the atomic level of size, the distinction between waves and particles becomes blurred, and an important modification, involving probability, must be made. Because the separate concepts of particle and of wave are so natural and satisfying for phenomena on the human scale it is somewhat difficult to explain why and how they must be modified and combined in order to describe the behavior of very small objects. Our most complete theory of light is a "quantum field" theory having both wavelike and particle-like properties—that is, incorporating "wave-particle dualism." We will clarify the relationships between classical particles, classical electromagnetic waves and quantum field theory by asking what each of them predicts for a single slit experiment.

1b. Classical Particles. A particle is characterized by having infinitesimal size and a definite location in space at each instant of time. It may have mass and an electrical charge, it certainly has momentum and carries kinetic energy. As time increases, the particle moves along a definite trajectory, which is determined entirely by local conditions at the point where the particle is. For example, in Fig. 1 we have a particle with charge q moving near a charged plate. At each instant of time, the particle experiences a force $\vec{F} = q\vec{E}$ determined by the electric field at the location of the particle. The subsequent motion of the particle in the

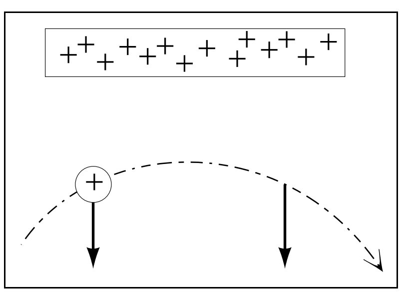

Figure 1. Local influences on the trajectory of a charged particle near a charged plate.

next infinitesimal interval of time is governed by: $\vec{a} = \vec{F}/m = (q/m)\vec{E}$. If there are no forces acting on it, the particle travels in a straight line at constant speed.

1c. Classical Waves. The "wave" concept is abstracted from many different phenomena that are only geometrically similar, such as waves on the surface of water, sound in gas, liquid or solid, electromagnetic waves, displacement of a taut string or a stretched membrane (drumhead, flapping sail) etc. In each case there is some "field" quantity varying smoothly in 1, 2 or 3 dimensions. (For water waves, vibrating strings and membranes it is the displacement of particles from their equilibrium position; for sound in air it can be density or pressure variations; for electromagnetic waves there are six varying fields, the three components of $\vec{E}$ and the three components of $\vec{B}$.)

We will not need to consider the detailed electromagnetic wave picture for light that can be derived from Maxwell's differential equations of the electromagnetic field, and which correctly predicts the light's velocity, its polarization (i.e., the fact that the $\vec{E}$ vector is perpendicular to the direction of propagation of the light wave) and the wave's behavior at surface boundaries with conducting and dielectric materials. Rather, we will consider the most characteristic property of any wave, that it undergoes diffraction at an obstacle.

The momentum and energy carried by a wave are not concentrated at a point, but are spread out smoothly over a finite volume. This is in contrast to the very small sizes of atomic particles, and this contrast is what makes it hard to imagine an entity which is simultaneously a "wave" and a "particle." In fact, we will see that light is composed neither of classical waves nor of classical particles, but is a new kind of entity that behaves in certain ways like a particle and in other ways like a wave. A probabilistic description ties these aspects together.

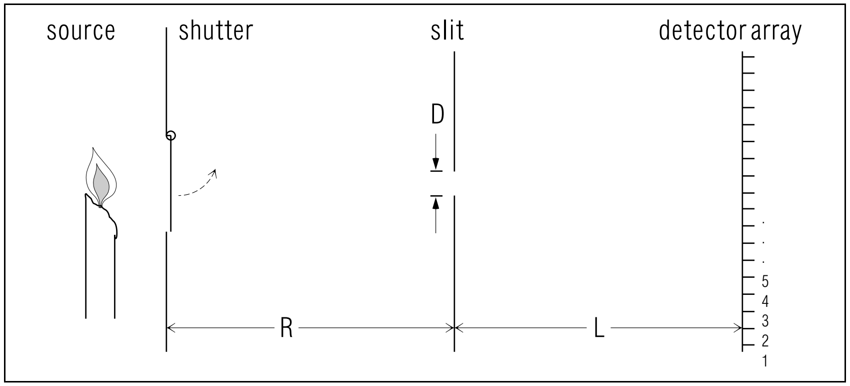

Figure 2. Single slit diffraction; experimental apparatus.

1d. The Nature of Light. Is light:

A wave?	Evidence: diffraction and interference experiments.
An electromagnetic wave?	Evidence: Maxwell's equations predict such behavior as its speed in vacuum, Snell's law of refraction, polarization, generation of long waves by simple electromagnetic circuits, etc.
A particle?	Evidence: Compton effect, photoelectric effect, photochemical reactions, etc.

The seemingly contradictory wave and particle natures of light are reconciled by quantum field theory. Its basic statement is: "The intensity (at a given point in space and time) of an electromagnetic wave of frequency ν gives the probability, and only the probability, that energy in the amount $h\nu$ may be transferred between the wave and an external object at that point in space." The wave, particle and quantum field theory ideas can be contrasted, compared and explained by using each to predict the results of a Fraunhöfer single slit diffraction experiment.

2. Both Aspects in One Experiment

2a. The Single Slit Apparatus. Light's seemingly contradictory wave and particle aspects can be made to appear simultaneously in the single-slit experiment of Fig. 2. The general idea of this experiment is to

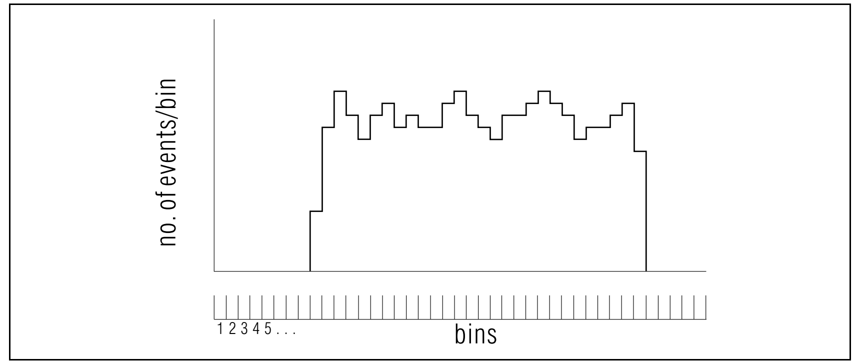

Figure 3. Particle Model, computer generated single-slit bar histogram; 41 detectors, 2305 particles.

open the shutter for a certain time interval, then examine the distribution of light energy among the various detectors.

We will assume that $R \gg L$ so the rays of light arrive at the slit as a parallel bundle. This allows the Fraunhöfer condition to be fulfilled so that the mathematical description is simpler.

The detection of light takes place in a two dimensional array of individual, finite-sized detectors, each with a detection threshold E_0; that is, a minimum amount of energy E_0 must reach the detector in some time Δt in order that the detector registers the presence of light. Examples of such arrays include the eye's retina, with rods and cones; a photographic plate, with individual silver halide crystals as detectors (size typically $1\,\mu$m); a stack of photomultiplier tubes; an array of thermocouples, sensitive to the heat energy from the light. Any of these could be used in the experiment.

Now let the shutter open for a time interval $T \gg \Delta t$, and look at the energy accumulated by each detector, as predicted by each of the models for light.

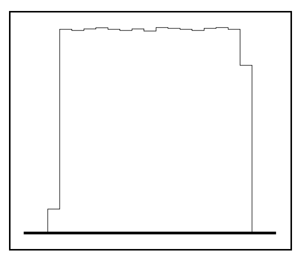

Figure 4. As in Fig. 3, but a much larger number of particles.

2b. Particle Model Predictions. We assume that all the light particles carry the same energy E_p, which is greater than the detector threshold E_0. To each detector we attach a counter, called a "bin," which is advanced one unit by the detector when it detects a particle. In Fig. 3 the results of a computer calculation simulating the results of the experiment are shown. (Randomly spaced straight line paths of the particles from the source to the detectors were assumed). Shown on the graph is the number of particles detected by each of 41 detectors in a row across the array. A total of 2305 particles arrived, but only 16 detectors show any particles (an average of 144 each); the rest are in shadow. (The end detectors of the 16 were partly in and partly out of the shadow and, especially on the left, show fewer counts). The random spacing between trajectories accounts for the fluctuations in the numbers of particles arriving at different detectors.

Simple statistical theory predicts that the average fluctuation, Δn, in number from one detector to another is given by $\Delta n \simeq \sqrt{< n >}$ where $< n >$ is the average number of particles reaching any one detector in the time interval, Δt, that the shutter is open. As the total number of particles reaching the detectors becomes very large, this fluctuation becomes comparatively unnoticeable, and so a strip of width D appears uniformly illuminated, with a sharp edged shadow, as shown in Fig. 4. This is the characteristic illumination pattern given by particle theory ("ray," or "geometrical optics" theory), and in fact it represents rather well everyday (every sunny day at least) observations made under certain circumstances. (What are they?)

2c. Wave Model Predictions. Continuing the above experiment (but with real light) let us replace the detector array with one whose individual detectors are much smaller and closer together. We will notice that the boundary between shadow and light is not completely sharp, as predicted by the particle model, but has a width $\Delta s \simeq \sqrt{L\lambda}$ where λ is the wavelength of the light. This feature is explained by a wave model of light; but instead of pursuing it further, we will move on to a mathematically simpler case by shrinking D until it is much smaller than the width of the light-shadow boundary. Then the wave model predicts a Fraunhöfer diffraction pattern in which the light of intensity I smoothly varies along the detector array according to the equation

$$I(y) = I_0(\sin^2\beta)/\beta^2, \qquad \beta = \pi D y/\lambda L,$$

where y measures distance along the detector array from the center of the pattern.

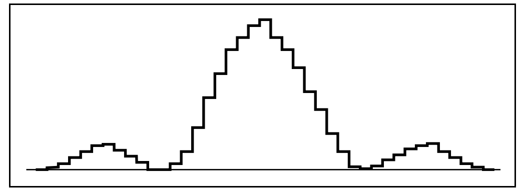

Figure 5. Fraunhöfer intensity pattern for 41 detectors, 1405 energy units.

Now we must look at the process of wave detection. For particles, our detectors "counted." But wave theory assumes that the energy is distributed smoothly in space and time and one could give a continuous readout of the energy collected versus time. However, real detectors must have finite size and finite time resolution, and so we assume, again, that the detector must absorb energy until it collects an amount E_0. It then advances its bin counter by one unit and is reset to begin the accumulation of another amount E_0 of energy.

For an array of 41 detectors which has registered the arrival of 1405 units of energy, wave theory predicts that we get the Fraunhöfer pattern of Fig. 5. But if the array contains many more detectors, and the threshold E_0 is very small, a smooth curve results (Fig. 6).

2d. Both Aspects Observed Simultaneously. Both the wave and the particle aspects of light will show up simultaneously in our single slit experiment if:

1. $D \ll \sqrt{L\lambda}$, so the diffraction pattern is present.
2. the intensity is low enough so we can have one photon at a time.

We also require that the detectors be sensitive enough that their threshold $E_0 \ll E_p = h\nu$. That is, they can detect single quanta of light. With all bins empty, we open the shutter. The detectors begin to detect photons at random intervals of time. As the counts build up, however, we see neither the sharp-edged pattern appropriate for classical particles (Fig. 3) nor the smooth diffraction pattern of a wave theory (Fig. 5) but

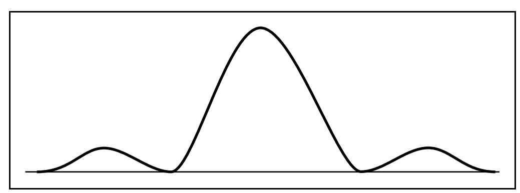

Figure 6. As in Fig. 5, but more detectors, small threshold.

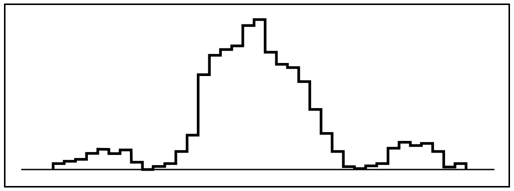

Figure 7. A non-particle, non-wave pattern, observable under appropriate circumstances.

rather a noisy Fraunhöfer pattern such as Fig. 7 shows after 1405 have been detected.

3. The Quantum Field Theory of Light

3a. Light Is Not Particles, Not Waves. Light cannot be categorized as either classical particles or classical waves. If it were classifiable as a classical particle it would not produce a diffraction pattern, but light diffracts. If it were a classical wave, its energy and momentum would have a continuous distribution in space and time, but light often acts as if it consists of localized photons having discrete energy and momenta. Thus light cannot be said to be intrinsically one or the other.

3b. Quantum Field Theory's Description. The principle features of the quantum field theory description of light may be stated thusly:

1. Electromagnetic field components, $\vec{E}$ and $\vec{B}$, still obey Maxwell's equations. When crossed and sinusoidally fluctuating, they still form an electromagnetic wave traveling at speed c. However, the $\vec{E}$ and $\vec{B}$ values no longer give exact forces on charged particles, hence no longer predict exact energy and momentum transfer rates to those particles.
2. An electromagnetic wave can only transfer energy and momentum to and from charged particles in increments fixed by the wave's frequency: $\Delta E = h\nu$; $\Delta p = h\nu/c$.
3. The actual times and places of these energy and momentum transfers cannot be predicted exactly, but the probabilities that they will occur in specified time and space intervals can be precisely calculated. These precise probabilities are linearly proportional to the wave's electromagnetic field intensities.

The first statement delineates wavelike properties of light; the second, particle-like properties. The third statement shows how we reconcile those

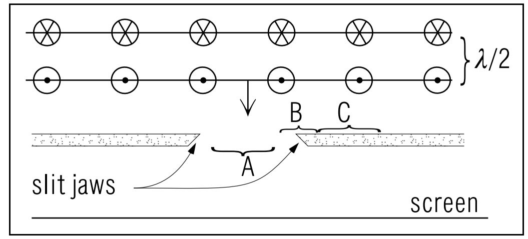

Figure 8. .

properties by including the observed random nature (unpredictableness) of the energy and momentum transfers. It allows us to (correctly) predict particle-like properties for small numbers of events and for short time intervals, and wavelike properties for large numbers of events where probabilistic distributions and actual distributions merge.

Here are some more details on the ways each classical model fails for light:

1. Light is not a classical wave. For example, consider the photoelectric effect using a "dust cathode." When light of low intensity is shown on fine metallic dust, photo-electrons start coming off immediately, whereas wave theory predicts that a long time must elapse before a dust particle can collect enough energy over its area from a wave to emit a photoelectron.[1]
2. Light is not a classical particle. That is, the photon cannot be thought of as having a straight-line trajectory of infinitesimal width. Experiments designed to establish which part of the slit opening a given photon crossed (for example by having a second, narrower slit) will cause a further diffraction or other disturbance of the photon and no definite path can be established. It is most correct to think of photons only at their instant of creation or destruction, and to consider light to be a probability wave in between these times, although in the geometrical limit (resolution of the path is less than $\sqrt{L\lambda}$) an approximate trajectory can be assigned to the light wave, as though it were a particle.

3c. Classical EM Wave Theory. Figure 8 shows a slit with shiny metal jaws. A light wave is incident on the slit, its $\vec{E}$ vectors perpendicular to the page.

[1]See "The Photoelectric Effect" (MISN-0-213).

In region A the wave is propagating as in free space.

In region B, where the wave is near the conducting metal, the time varying $\vec{E}$ field causes a current to flow in the metal. This current produces a $\vec{B}(t)$ field which produces (Faraday's law) another $E(t)$ field. All these $\vec{E}(t)$ and $\vec{B}(t)$ fields add to yield final $\vec{E}$ and $\vec{B}(t)$ fields that satisfy Maxwell's equations for a wave different from the incident one. This is the diffracted wave. In addition, the electrical resistance of the metal causes some of the light energy to be absorbed at the slit jaws.

In region C, the currents produce the $\vec{B}(t)$ [and thus also the $\vec{E}(t)$] fields of a reflected wave; some light is also absorbed.

The above description uses only the electromagnetic wave theory. All we need to add to the above is that energy is absorbed in terms of photons, each of energy $h\nu$ and transferring momentum hν/c to the slit. (The reflected photons transfer momentum $2h\nu/c$ to the slit). These photons are absorbed at random places and times on the metal in proportion to the intensity of the light ($\propto E^2$) and to a transition probability that depends on the electrical properties of the metal (such as its resistance).

Consider a little volume of light at region B in time interval Δt. There is a probability P_a that a photon will be absorbed by the slit jaws. Then there is a probability $P_b = 1 - P_a$ that no photon will be absorbed, in which case, the light in region B propagates down to the screen where new probabilities of the light being reflected, transmitted and absorbed can be calculated, depending on E^2 there and on the electrical properties of the screen.

4. Field Theory Applications

4a. Strange One- and Two-slit Predictions. A single photon must "go through" both slits of a double slit diffraction apparatus (see Fig. 9). With low intensity light (say an average of 1 photon/sec) a distinctive "double slit" diffraction pattern builds up as many photons are detected. If one slit is closed, the pattern becomes that of a single slit. It is difficult to explain this on a particle model; why would the use or non-use of one slit affect a particle through the other one? (The low intensity is specified to avoid possible cooperative effects between several photons and the slits, e.g. interactions between photons of one slit and those of the other.)

4b. Strange "Spherical Wave" Predictions. Although a classical electromagnetic wave may extend over a large volume of space, the pho-

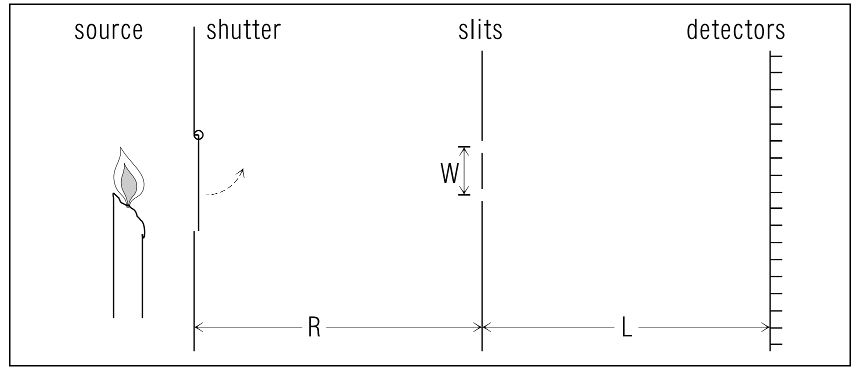

Figure 9. Apparatus for a 2-slit experiment.

ton is created or destroyed in a small, localized region. Consider a gas discharge tube in the center of the lab, and have photomultiplier detectors on the opposite walls (Fig. 10).

Assume that, on the average, one atom per second of the source emits a photon, considered a spherical electromagnetic wave pulse that expands in all directions with speed c. After a time $T = r/c$, either detector 1 or detector 2, or neither, will detect a photon containing all the energy of the emitted electromagnetic wave. In no case will detector 1 and detector 2 both detect parts of the same "spherical wave" pulse if the intensity is sufficiently low.

4c. Designing and Using an Astronomical Telescope. We can use the design and uses of an astronomical telescope to illustrate when it is appropriate to use the wave and when the particle approximations for light, and when it is necessary to use the more exact quantum field theory:

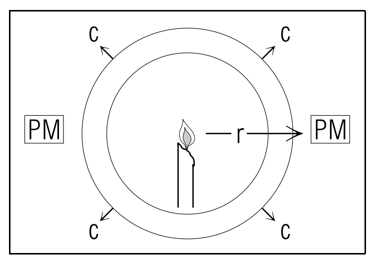

Figure 10. Photomultipliers (PM) detecting a flash of light.

1. Size of lenses and mirrors, their placement, image size, magnification, etc. Use "light rays," geometrical optics, laws of reflection and refraction (classical particle theory).
2. Resolving power of the telescope, diffraction pattern of a point source (star) processed through a circular aperture (lens). Use wave theory.
3. Analysis of the light for information about distance and speed of the source, media through which the light passed (interstellar space); Doppler shift, etc. Use electromagnetic wave theory.
4. For very faint sources, the telescope may detect only a few hundred photons appearing at random points in a diffraction pattern. Use quantum field theory to make a statistical analysis of the pattern.

Acknowledgments

We thank Professor James Linneman for helpful suggestions. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

Glossary

- **quantum field theory**: a theory that reconciles the wave and particle "duality" of light by stating that the intensity of an electromagnetic wave at a given point in space is related to the probability of an energy and momentum transfer by a photon.
- **wave-particle duality**: a model that encorporates both wave-like and particle-like properties.

PROBLEM SUPPLEMENT

Note: these problems also occur on this module's *Model Exam.*

1. For each application given below, give reasons it can or cannot be satisfactorily described as each of these: (a) a classical particle; (b) a classical EM wave; and (c) a quantum field.

   The Phenomena are:

   A. design of binoculars
   B. architectural analysis of light in and around a building
   C. study of bee navigation (sky polarization effects)
   D. analysis of light from very distant galaxies using a photomultiplier tube
   E. study of formation of vitamin D by "suntanning"
   F. using a light microscope to study bacteria 1μm long
   G. installation of a "magic eye" door opener
   H. building a "pin hole" camera

2. An atom emitting light in the visible typically takes about 10^{-8} seconds to do it. How long is the resulting wave packet in space?

Brief Answers:

1. A. a, geometrical optics
   B. a, geometrical optics
   C. b, polarized EM wave
   D. c, photon statistics
   E. c, quantum of light energy converted to bond energy
   F. b, size of diffraction effects
   G. c, photon, photo-electric effect
   H. a and b

2. About 3 meters.

MODEL EXAM

1. See Output Skills K1-K2 on this module's *ID Sheet*. One or more of these skills, or none, may be on the actual exam.

2. For each application given below, give reasons it can or cannot be satisfactorily described as each of these: (a) a classical particle; (b) a classical EM wave; and (c) a quantum field.

   The Phenomena are:

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3. An atom emitting light in the visible typically takes about 10^{-8} seconds to do it. How long is the resulting wave packet in space?

Brief Answers:

1. See this module's *text*.

2. See problem 1, this module's *Problem Supplement*.

3. See problem 2 this module's *Problem Supplement*.