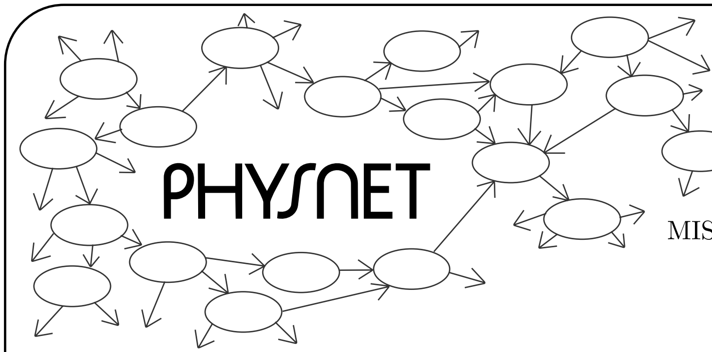
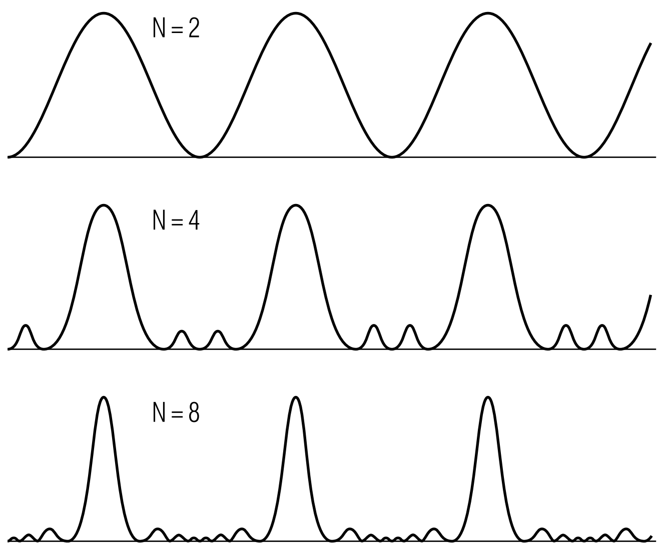


MISN-0-237

DIFFRACTION GRATING AND X-RAYS SCATTERING FROM CRYSTALS



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

DIFFRACTION GRATING AND

X-RAYS SCATTERING FROM CRYSTALS

by

J. S. Kovacs
Michigan State University

Title: **Diffraction Grating and X-Rays Scattering From Crystals**

Author: J. S. Kovacs, Michigan State University

Version: 2/1/2000 Evaluation: Stage B0

Length: 1 hr; 12 pages

Input Skills:

1. Explain how to find the maxima of an interference pattern produced by N synchronous sources (MISN-0-231).
2. State the condition for zero intensity in a diffraction pattern (MISN-0-235).

Output Skills (Knowledge):

K1. Describe the transition from a two-slit interference pattern to one with many slits having the same slit-to-slit separation. Explain why diffraction gratings' maxima are so sharp.

K2. For a diffraction grating, derive the expressions for the angles at which maxima are detected.

K3. Discuss scattering of X-rays by a crystal lattice using Bragg's equation, and describe the experimental arrangement for observing Bragg scattering.

Output Skills (Problem Solving):

S1. Given the wavelength of light incident upon a given size grating with N lines, calculate the angles of deviation of the principle maxima.

S2. For white light incident on a grating with N lines, calculate the angular separation for two given wavelengths of transmitted light.

S3. Given three of the following quantities for Bragg scattering, calculate the fourth: separation of crystal lattice planes, wavelength of light, angle of incidence, order of spectrum.

External Resources (Required):

1. M. Alonso and E. Finn, *Physics*, Addison-Wesley (1970). For access, see this module's *Local Guide*.

DIFFRACTION GRATING AND X-RAYS SCATTERING FROM CRYSTALS

by

J. S. Kovacs
Michigan State University

1. Introduction

The scattering of electromagnetic radiation from a grating of ruled lines, uniformly spaced, produces a diffraction pattern that has spacing dimensions which are characteristic of the grating spacing. These grating lines don't need to be artificially produced. The orderly spacing of the atoms in a crystal can also play the same role as the grating lines and the diffraction pattern thus observed can be interpreted to give information about the atomic spacing in these crystals. Because these are so closely spaced, radiation of very short wavelength, X-rays, is needed to observe the effect.

2. Diffraction Gratings

(see next page)

3. Scattering

(see next page)

4. X-Ray Scattering by Crystals

(see next page)

Suggestions for Study:

AF:[1]

(i) Read sections 29.6, 29.8, and 29.9

(ii) Answer study questions 10, 11, and 12

[1] AF: M. Alonso and E. Finn, *Physics*, Addison-Wesley (1970). For access, see this module's *Local Guide*.

(iii) Work through worked out problems 29.2 (p. 707) and 29.5 (p. 715). [Note: in Example 29.2, sin $\theta_{red} = 0.385$ hence $\theta_{red} = 22^\circ\, 39'$. Then the first order spectrum covers $11^\circ\, 24'$ and the second order covers $27^\circ\, 24'$.]

(iv) Work problems 29.13, 29.17, 29.25, and 29.27

Answers to Assigned Problems:

29.13 : $\sin\theta = 0.3, 0.6, 0.9$; $\theta = 17.4^\circ$, 36.9°, 64.2°

29.25 : Answer O.K.

29.27 : $\lambda = 1.24 \times 10^{-11}$ m is shortest wavelength of X-rays that you can get from stopping electrons of energy 10^5 eV. (Corresponds to X-ray carrying off all the kinetic energy the electron had.) Answer in text is O.K.

Acknowledgments

William Lane provided valuable feedback on an earlier version, Dean Eicher produced the cover on a computer-driven plotter. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

LOCAL GUIDE

The readings for this unit are on reserve for you in the Physics-Astronomy Library, Room 230 in the Physics-Astronomy Building. Ask for them as "The readings for CBI Unit 237." Do **not** ask for them by book title.

PROBLEM SUPPLEMENT

1. Suppose we consider a transmission grating of N slits in an opaque substance which, when a plane wave of monochromatic light falls on one side, acts as N synchronous sources of light transmitted through to the other side. Consider a point P on a screen a large distance from this system of N synchronous sources.

a. If the difference in path length for light to travel to P from two adjacent slits is exactly λ, do the amplitudes at P for these two wavelets add constructively, destructively or otherwise? (Draw a rotating vector diagram for these two sources to illustrate your answer). What is the phase difference δ in this case? [C]

b. What is the phase relation for the light arriving at P from three adjacent slits (at the same point P as in (a))? For all N slits? [E]

c. If the difference in path length for light to travel to another point P' from two adjacent slits is not quite λ, being λ - ϵ, when ϵ is small, explain the phase relation of the two waves from adjacent slits arriving at P'. Do these two wavelets add constructively, destructively, or otherwise (if otherwise, is it nearer to being constructive or destructive?) [B]

d. Suppose, in (c), $\epsilon = \lambda/100$. Considering the whole set of N sources (where N is very large), what is the overall effect on the amplitude and intensity of light arriving at P'? [F]

e. Using the result obtained above, explain in what ways the interference pattern differs from what you would expect if you had only 2 of the N slits contributing. Which interference pattern (2-slit or N-slit) is more useful for determining accurately the wavelength of light? [A]

2. If the N slits above are spaced such that there are M of them per meter of grating derive the expression for the angles (relative to normal incidence) at which you will be able to detect maxima for the transmitted light. [D]

Brief Answers:

A. See text

B. "almost" constructively, phase difference slightly less than 2π

C. constructive, 2π

D. See text

E. all in phase

F. Almost complete destructive interference, because a slit 50 slits away is $\lambda/2$ wavelength different from the one with which you start, hence out of phase by π. So slits which are 50 slits apart interfere destructively in pairs. The collective effects is that you get complete destructive interference.

MODEL EXAM

1. White light is incident on a grating that has 600,000 lines per meter. Find the angular separation in the second order for these two wavelengths of transmitted light: 8×10^{-7} meters and 5×10^{-7} meters. Which of them is diffracted more? [C]

2. a Discuss Bragg's equation for the diffraction of X-rays from a crystal lattice. Explain where the source of X-rays is placed relative to the crystal, where you detect the scattered X-rays and what the detected scattering pattern tells you. [D]

   b If for monochromatic X-rays of wavelength 2×10^{-10} meters, you observe the first large maximum of scattered X-rays from a crystal at 25°, what does that tell you about the crystal? [A]

   c. To add credence to the correctness of your conclusion to (b), at what angle would you expect to find the next maximum? [B]

3. See Exam Skills (Knowledge) K1-K2.

Brief Answers:

A. Separation of crystal lattice planes is 2.4×10^{-10} m.

B. 57.7°

C. 36.9° separation, $\lambda = 8 \times 10^{-7}$ m wavelength diffracted more.

D. See text by Alonso and Finn, Section 29.9.