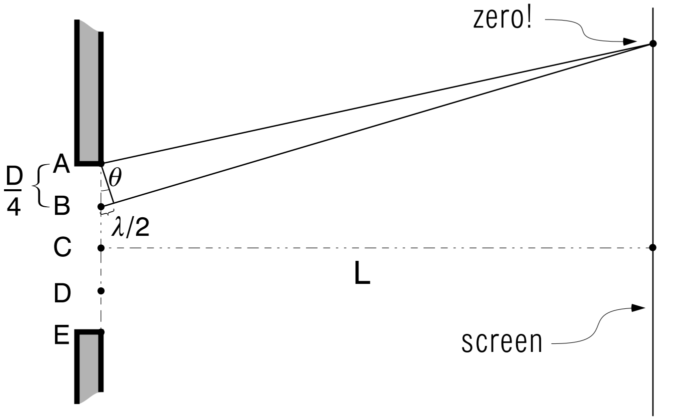
PHYSNET

MISN-0-235

# FRAUNHÖFER DIFFRACTION



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

FRAUNHÖFER DIFFRACTION
by
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Title: **Fraunhöfer Diffraction**

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**Input Skills**:

1. Explain how to find the resultant amplitude at an arbitrary point due to the interference of waves generated by $N$ equally spaced linear coherent wave sources (MISN-0-231).

**Output Skills (Knowledge)**:

K1. Derive the expression for the width of the central diffraction maximum for light of wavelength $\lambda$ falling on a screen at a distance $L$ away from a slit of width $D$.

K2. State Rayleigh's definition of resolving power for a slit and for a circular aperture.

K3. Describe the Fraunhöfer diffraction pattern due to a circular aperture.

**Output Skills (Problem Solving)**:

S1. Analyze the Fraunhöfer diffraction pattern due to a single rectangular slit, finding the regions of zero intensity on the detection screen.

S2. Use the expression for the Fraunhöfer diffraction by a circular lens aperture to determine the resolution of two distant point sources of light.

**External Resources (Required)**:

1. M. Alonso and E. Finn, *Physics*, Addison-Wesley (1970). For access, see this module's *Local Guide*.

**Post-Options**:

1. "Fresnel Diffraction" (MISN-0-236).
2. "Coherence" (MISN-0-287).
3. "Light Scattering" (MISN-0-293).

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# FRAUNHÖFER DIFFRACTION

by

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## 1. Introduction

A series of discrete coherent sources of monochromatic electromagnetic radiation result in the appearance of predictable interference patterns when that radiation is incident upon a detector (such as a screen or a series of receivers of radio waves). A generalization of this to a continuous distribution of source points along a wave front (which, by definition of the wave front, are coherent) leads to the prediction that interference effects (diffraction patterns) can be observed when a monochromatic wave front passes through spatially restricting apertures. This is accomplished by treating the continuous sequence of points on the wave front as sources of waves which then contribute to the wave disturbance at specified points in space and time. The construction of this wave disturbance is expressed in Huygens's principle. In this module we will consider Fraunhöfer diffraction, where the detector can be considered to be an infinite distance from the aperture through which the wave front passes.

## 2. Huygens' Principle

## 3. A Slit

## 4. A Circular Apperature

## 5. Two Equal Parallel Slits

## 6. Suggested Procedure

For Sections 2, 3, 4, 5, 6 (see above): In AF[1] read sections 29.1, 29.2, 29.3, 29.4 and 29.5.

In AF answer study questions (p. 715) 1, 2, 6, 7, and 8.

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[1] M. Alonso and E. J. Finn, *Physics*, Addison-Wesley (1970). For access, see this module's *Local Guide*.

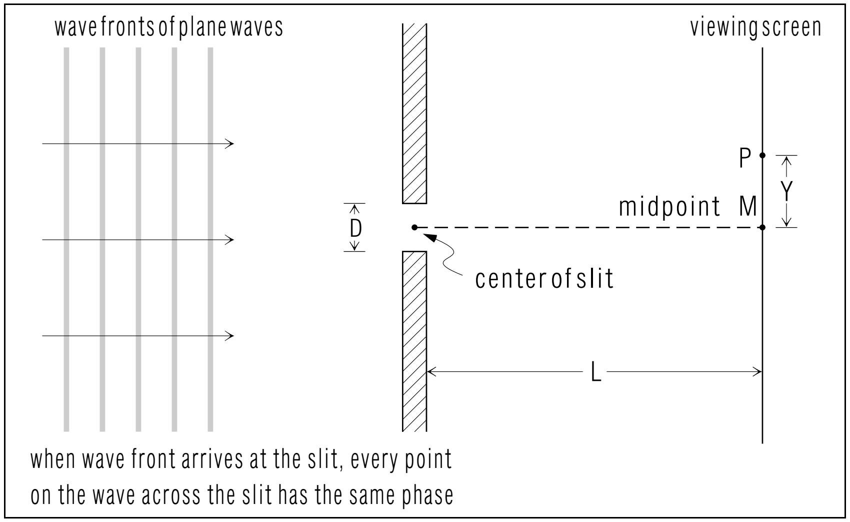

**Figure 1.** Geometrical description of Fraunhöfer diffraction apparatus.

## 7. Illustrative Example

Light of wavelength $\lambda$ from a distant source falls upon an opaque screen which has in it a long narrow slit of width $D$. On the other side of this opaque screen, a distance $L$ away is a screen upon which the transmitted light falls.

By dividing the slit successively into 2, 3, 4, 5, ... zones, analyze the relative phase of the waves arriving at points on the screen from corresponding points of neighboring zones and find where maxima and minima occur on the viewing screen. From this analysis, explain why the successive maxima get weaker in intensity. (Assume that $L \ll D$ so that you may use the approximation $\sin\theta \approx \tan\theta \approx \theta$).

**Discussion of Solution:**

At the midpoint $M$, rays arriving from all points across the wave front of the slit arrive in phase so that the resultant amplitude of the light at this point is as shown in Fig. 28.10 (b) of AF and the central spot is of maximum brightness. What about some point $P$ off the central point by a distance $Y$? There will be a phase difference between pairs of rays arriving from various points on the wave front across the slit and the resultant amplitude there will be the resultant of vectors such as in 28.10

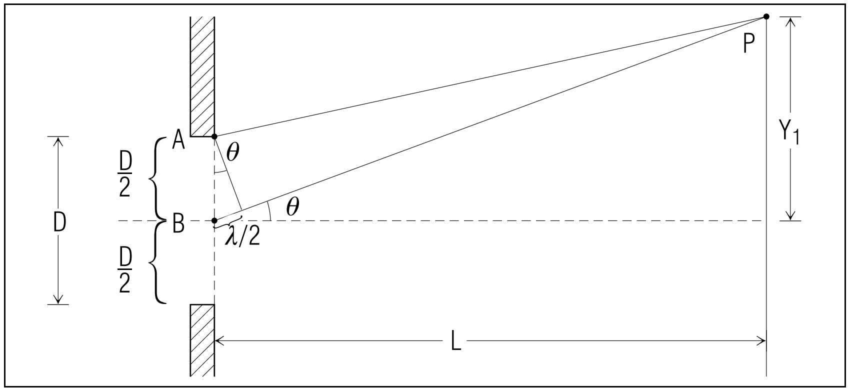

**Figure 2.** Wavelets from the edge and the middle of the slit completely cancel at point $P$.

(a) (except that there will be an infinite number of sources infinitesimally close to each other in phase for adjacent sources, $\delta \approx 0$, so that the broken curve of arrows in 28.10 (a) would now be a continuous arc). If that curve closes on itself (as in Fig. 28.11 for discrete source points) then the resultant amplitude will be zero. Where is the first such point $P$? This occurs when the wavelet arriving at $P$ from one edge of the slit is 180° out of phase with the wavelet arriving from the midpoint of the slit, as illustrated in Fig. 2.

If the wavelets starting from these two points, $A$ and $B$, are out of phase when they arrive at $P$, then a wavelet starting from a point just below A is out of phase with a wavelet starting from a point just below $B$ and so on all the way down the plane of the slit. Thus there is complete cancellation at the point $P$ which makes an angle $\theta$ with the line from the midpoint of the slit to the screen.

As can be seen in Fig. 2, $\theta$ is such that $(D/2)\sin\theta = \lambda/2$. Also, $\sin\theta \approx \tan\theta = Y_1/L$ so that you may relate to the wavelength and slit width the distance $Y_1$ from the central maximum to the first minimum. (There's also a symmetrically placed point $P$ below the center where the intensity vanishes, so that the full width of the central bright spot is $2Y_1$).

For a point slightly above $P$, to which the angle $\theta'$ is larger than $\theta$ above, the point $B'$ (from which the wavelet is 180° out of phase with the wavelet from $A$) is above the midpoint of the slit, as depicted in Fig. 3.

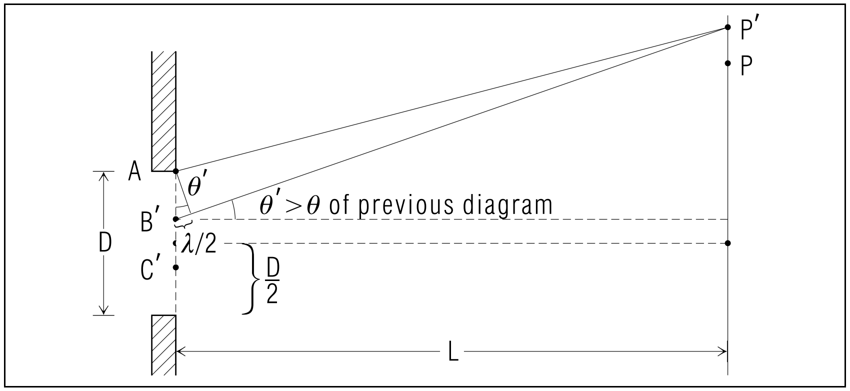

**Figure 3.** Wavelets from $A$ and $B'$ arrive out of phase at point $P'$

Hence wavelets arriving at $P'$ from all points between $A$ and $B'$ cancel (point by point) wavelets arriving from all points between $B'$ and $C'$. Only wavelets from the slit between $C'$ and the lower edge of the slit can contribute to the amplitude arriving at $P'$. So the intensity at $P'$, while not zero, is less than at the screen's center. Another zero in amplitude occurs when you can divide the slit into 4 zones such that

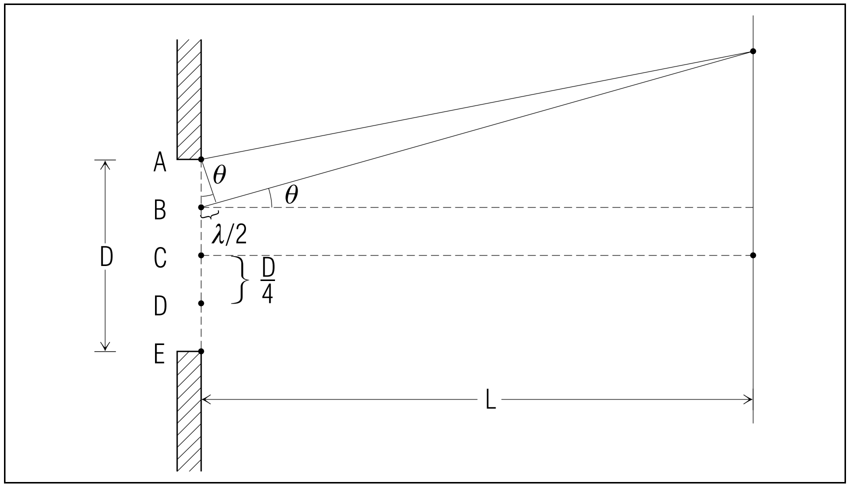

**Figure 4.** The division of the slit into four "zones" that yield intensity maximums upon the screen, less intense than the central maximum.

$(D/4)\sin\theta = \lambda/2$, as illustrated in Fig. 4.

Hence rays from points between $A$ and $B$ cancel rays from between points $B$ and $C$, while rays from points between $C$ and $D$ cancel those from points between $D$ and $E$.

You should be able to show thus that zeros of intensity occur at distances $Y_n = \pm n\lambda L/D$ (valid if $L \gg D$) from the center of the pattern where $n$ is any integer, and that the full width of the central bright region is $2\lambda L/D$. Notice that the narrower you make the slit, the broader the central bright line is! Figure 29.8 in AF shows the intensity distribution along the screen.

## Acknowledgments

John Johnston, Steve Cooper and Bill Smith gave valuable feedback on the first version of this module. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

# LOCAL GUIDE

The readings for this unit are on reserve for you in the Physics-Astronomy Library, Room 230 in the Physics-Astronomy Building. Ask for them as "The readings for CBI Unit 235." Do **not** ask for them by book title.

# PROBLEM SUPPLEMENT

Work these problems in AF:[2] 29.1, 29.2, 29.4, 29.6, 29.7, 29.9, 29.11

**Brief Answers**:

29.1: In the above example we assume $L$, the distance from the slit to the screen is very large so that all rays from the slit to point $P$ are essentially parallel. In this problem and others in the text, a lens is introduced to focus initially parallel rays to the point $P$ in question. The answer in the back of the book is okay.

29.2: 0.032 mm. *Help: [S-1]*

29.4: $\lambda_1 = 5 \times 10^{-7}$ m, $\lambda_2 = 4 \times 10^{-7}$ m (actually, they gave you more information than you needed).

29.6: Find the interference pattern assuming negligible width for the slits, then use method of Section 29.5 to find the diffraction pattern superposed on this pattern.

29.7: (b) 2.4 mm × 4.8 mm

29.9: (a): okay (b) $6.83 \times 10^{-6}$ m. Note that worked-out example 29.1 is done incorrectly. The denominator in the expression for $\theta$ should be the diameter, $2 \times 10^{-2}$ m, giving an answer $3.60 \times 10^{-5}$ radians. The radius of the central disk then should be $1.44 \times 10^{-3}$ cm.

29.11: okay

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[2] M. Alonso and E. J. Finn, *Physics*, Addison-Wesley (1970). For access, see this module's *Local Guide*.

# SPECIAL ASSISTANCE SUPPLEMENT

S-1 *(from PS-Problem 1)*

We get 2.2 cm, min-to-min.
Thus: $\theta_{min} L = 1.1$ cm, and of course $\theta_{min} = \lambda/b$, where $L$ is the distance to the screen and $b$ is the slit width.

# MODEL EXAM

1. Using a figure similar to Fig. 2 in the module, there is a point $P$ on the screen such that a wavelet reaching $P$ from point $A$ (at the top of the slit) is exactly 180° out of phase with light reaching $P$ from a point 0.5 millimeters below point $A$.

    a. For what values of the slit width will there be exactly zero intensity of light arriving at $P$? [K]

    b. For what value of the slit width will the maximum intensity arrive at $P$ (for a given incident intensity from the left of the slit)? Note: $P$ does not coincide with the midpoint $M$ in this case. [G]

    c. If the slit width is 0.6 millimeters, what fraction of the slit's width contributes to the intensity arriving at $P$? How does the intensity arriving at $P$ compare with the intensity arriving at $M$ in this case? [C]

    d. If the light is of wavelength $\lambda$, the slit width is $D$, and the screen is a distance $L$ ($L \gg D$) from the slit, derive the expression for the width of the central maximum of light falling on the screen. [H]

    e. If light were not "of a wave nature", but were classically behaving photons, what would be the width of the central maximum? Compare this with your answer to (d) in the case where $D \to 0$. [L]

2. A circular aperture of diameter $D = 5$ millimeters is illuminated by monochromatic light from a distant source. The wavelength of the light is $6 \times 10^{-7}$ meters.

    a. Assuming that light is produced by "rays of particles moving in straight lines" as in part (e) of Question 1, what is expected to be observed on a screen 2 meters from the aperture (describe the circle of light seen on the screen; what's inside and what's outside)? [E]

    b. With light actually of a wave nature what is observed on the screen? Is there any light falling outside of the central maximum? [M]

    c. Suppose there are two point sources of light (each of monochromatic, $6 \times 10^{-7}$ meters wavelength light) 2 meters apart sending light incident upon this aperture. If these sources are 2 kilometers from the aperture, what angle (in radians) do they subtend at the aperture? [I]

    d. What angle do the centers of their images subtend on the screen 2 meters from the aperture? [A]

    e. How far apart (in meters) are the centers of their images on the screen? [J]

    f. What is the Rayleigh criterion for the resolving power of this or any aperture? What is meant by "resolving power"? [B]

    g. By this criterion are these two spots on the screen resolved? [F]

    h. What is the distance of these two sources from the aperture for which the two images are just resolved by this criterion? [D]

**Brief Answers**:

A. $10^{-3}$ radians.

B. See text of this module.

C. Two-thirds of the slit contributes, but because there is some phase difference $\delta$ from various points here, this does not mean that the intensity at $P$ is 2/3 of the intensity at the central maximum; 2/3 is the upper limit.

D. 13.66 kilometers.

E. A circle of light $D$ in diameter, surrounded by darkness.

F. Yes.

G. 1.5 mm.

H. See text of this module.

I. $10^{-3}$ radians.

J. $2 \times 10^{-3}$ m.

K. 1 mm, 2 mm, etc.

L. Exactly $D$. The result from part (d) goes as $D^{-1}$, so as the slit width approaches zero, the central "maximum" broadens to infinite width.

M. A central maximum of radius $L\theta$, $L = 2$ meters, $\theta = 1.22\,(\lambda/D)$, so radius equals $2.93 \times 10^{-4}$ meters (as compared with $2.50 \times 10^{-3}$ meters, radius of aperture). Surrounding this is a dark ring, then light ring, etc. See AF, Figs. 29.12 and 29.13.