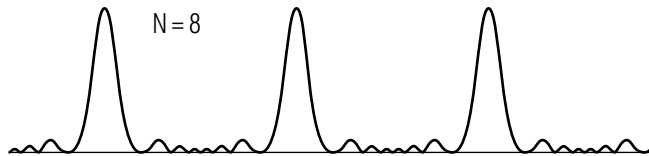
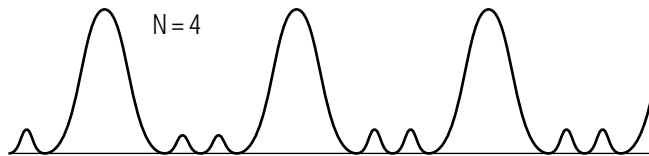
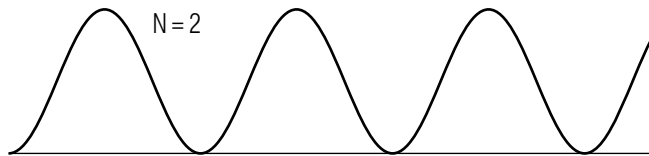


## INTERFERENCE OF MANY COHERENT WAVES; RADIO INTERFEROMETRY



## INTERFERENCE OF MANY COHERENT WAVES; RADIO INTERFEROMETRY

by  
Peter Signell and William C. Lane  
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**Input Skills:**

1. Vocabulary: electromagnetic spectrum, radio waves (MISN-0-212); coherent wave sources, constructive interference, destructive interference, phasor, interference pattern, maxima, minima, path difference, phase difference (MISN-0-230).
2. Name the major regions of the electromagnetic radiation spectrum (MISN-0-212).
3. Determine the wave disturbance at a given point resulting from two equal coherent wave sources (MISN-0-230).

**Output Skills (Knowledge):**

- K1. Vocabulary: angular sensitivity, node angle, radio interferometer, radio telescope, zenith.
- K2. Draw a diagram of a fixed linear radio interferometer consisting of  $N$  antennas in a row; indicate the crest line, the direction angle, and the differences in successive ray paths.
- K3. State the mathematical relationship between the successive receiver phase difference and the angular position (measured from the zenith) of the radio wave source.
- K4. State two of the criteria for a good radio interferometer and how they are achieved in practice.

**Output Skills (Problem Solving):**

- S1. Given the number of receivers, and the phase difference between the signals at successive receivers of a radio interferometer, sketch a phasor diagram and find the resultant phasor.
- S2. Given the number and the spacing of receivers in a radio interferometer, find the values of  $\delta$  and  $\theta$  corresponding to intensity minima and make a polar sketch of angular sensitivity.

**External Resources (Required):**

1. Ruler, protractor.

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# INTERFERENCE OF MANY COHERENT WAVES; RADIO INTERFEROMETRY

by

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## 1. Radio Astronomy

**1a. Jansky Discovers Radio Waves from Space.** In 1931 Carl Jansky, a scientist working at Bell Labs, was trying to discover the sources of radio interference which were disturbing transatlantic radio telephone communications. In addition to terrestrial causes of interference, such as static produced by thunderstorms, Jansky discovered a mysterious hiss from the sky. It was later ascertained that the signals were coming from the direction of the center of our galaxy. Thus began the field of “radio” astronomy.

**1b. Radio Waves Chart Our Galaxy.** Our galaxy contains clouds of dust which absorb light rays to such an extent that we cannot see even as far as the center of the galaxy. Radio waves are only slightly scattered and absorbed (because their wavelength is much larger than the size of the dust grains)<sup>1</sup> and permit radio astronomers to chart the structure of the galaxy by examining the intensity of radiation received from various parts. We have learned that our galaxy looks very similar to the spiral galaxy Andromeda, which is very close by (our galaxy and Andromeda are the largest members of what is called the Local Group of galaxies).

**1c. Radio Telescopes Have Advantages.** The size of the observable universe has been doubled by the use of “radio telescopes” since they can be used to detect radio waves from galaxies more than twice as far as the faintest galaxies observed with optical telescopes. In addition, new discoveries have been made since radio telescopes enable us to observe radiation in a new region of the electromagnetic spectrum. Finally, as far as radio astronomers are concerned, the daytime sky is as “black” as the nighttime sky. In fact, they can work on cloudy days as well as clear days.

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<sup>1</sup>For a discussion on the relationship between absorption efficiency and wavelength, see “Characteristics of Photons” (MISN-0-212).

**1d. Reasons for the Greater Range.** Radio telescopes detect radiation from greater distances for two principal reasons. First, although galaxies generally emit much less energy in the form of radio waves than of light waves, radio telescopes can be built much larger than optical telescopes. The largest radio telescope is sixty times the diameter of the largest optical telescope so it can collect 3600 times as much energy due to its larger collecting area. The second and most important reason is that there is a general illumination of the night sky which optically obscures visible light from the faintest galaxies. The background optical illumination is due to fluorescence of the upper atmosphere air molecules (night glow) and scattering of sunlight by interplanetary dust (zodiacal light). However, there is very little background radio illumination from the sky to obscure radio waves from distant galaxies.

**1e. Radio Astronomers Use Interferometry.** One problem with radio telescopes is that of poor angular resolution but this can be improved by using a number of receivers and analyzing the interference pattern they produce. In this module we will develop the appropriate interference ideas, then apply them to radio telescopes.

## 2. Interference: Several Sources

**2a. Phase Difference and Path Difference.** When, at some point in space, we combine wave disturbances from two coherent sources, the amount of interference between the two waves is determined by the difference in the phases of the two waves.<sup>2</sup> This phase difference, written  $\delta$  (lower case delta), is related to the path difference  $\Delta$  of the two waves by either

$$\delta = \left(\frac{\Delta}{\lambda}\right) 2\pi \quad \text{or} \quad \delta = \left(\frac{\Delta}{\lambda}\right) 360^\circ \quad (1)$$

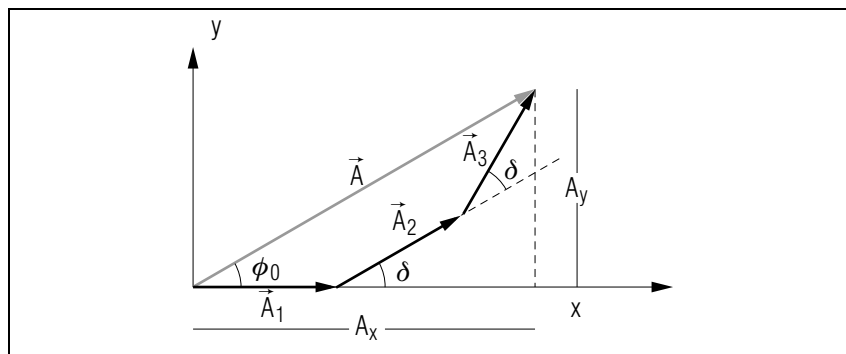
depending upon whether you express  $\delta$  in radians or degrees. The path difference of the two waves from their sources is:

$$\Delta = |\vec{r}_1| - |\vec{r}_2| \quad (2)$$

where  $|\vec{r}_1|$  and  $|\vec{r}_2|$  are the distances traveled from their coherent sources by waves #1 and #2 respectively. The same arguments can be easily extended to the case of interfering waves from three or more coherent sources.

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<sup>2</sup>See “The Interference of Two Coherent Sources” (MISN-0-230).



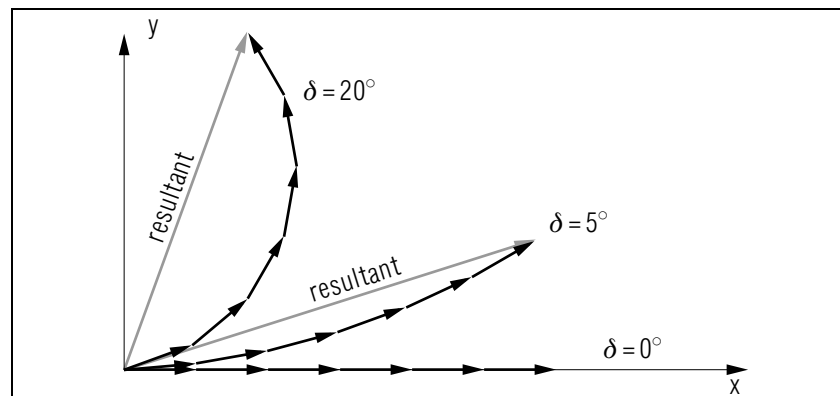
**Figure 1.** Three phasors plus their resultant phasor. All four phasors rotate counterclockwise rigidly at the frequency of the light wave. Only the  $x$ -components of the phasors have physical meaning.

**2b. Vector Sum of Phasors for the Net Intensity.** The intensity resulting from the interference of wave disturbances at some fixed point in space is determined by: (1) using vector addition to sum the individual-wave phasors to produce the “resultant” phasor; and (2) squaring the  $x$ -component of that resultant phasor. This procedure is illustrated in Fig. 1 for three wave disturbances of equal amplitude, assuming some phase difference  $\delta$  between the phasors from successive sources.

As time proceeds, the phasors all rotate in lock-step at the waves’ common frequency, so the resultant phasor rotates along with them at the same frequency. The  $x$ -component of a resultant phasor called  $\vec{A}$  varies from  $+A$  to  $-A$  as the phasors rotate, so the intensity reaches a maximum as  $\vec{A}$  instantaneously crosses the positive and negative  $x$ -axes and is instantaneously zero as the phasor crosses the positive and negative  $y$ -axes. The  $x$ -component actually varies as  $\cos(\omega t + \phi_0)$  so the net intensity varies as  $\cos^2(\omega t + \phi_0)$ . If one averages over time, the average intensity is just half the peak intensity.<sup>3</sup>

**2c. The Zero-Intensity Case.** Here is an interesting question: Is there any combination of phasors that yields zero intensity at all times? If the resultant phasor  $\vec{A}$  is zero, it has no  $x$ -component, hence no signal is received, and the intensity of the combined wave disturbance at that point in space is zero all the time. Such a point is an interference minimum. By determining the interference maxima and minima, the interference

<sup>3</sup>See Appendix: Time-Averaged Intensities (MISN-0-230).



**Figure 2.** Changes in the *resultant* phasor with increases in phase difference between successive individual phasors.

pattern of the wave sources may be predicted.

**2d. Maxima and Minima.** If we are given the number of coherent wave sources which contribute to an interference pattern, it is possible to use the simple geometrical constructs of phasor addition to determine the minima and maxima that occur in the intensity as the phase difference  $\delta$  is varied from  $0^\circ$  to  $360^\circ$ .

When  $\delta$  is zero, all the phasors are aligned parallel to each other. Then the magnitude of the phasor sum is merely the sum of the magnitudes of the individual phasors (see the  $\delta = 0^\circ$  case in Fig. 2). No other  $\delta$  could give a larger phasor-sum magnitude, so this value of  $\delta$  always corresponds to an intensity maximum. Since  $\delta$  is zero, all of the waves, initially in phase, have traveled equal distances so that the path differences and phase differences are zero. Therefore the waves are still in phase, yielding no destructive interference.

As one increases  $\delta$  away from zero, the phasor-addition diagram begins to “bow,” and the resultant phasor decreases in length as shown in Fig. 2. The first “zero” in intensity occurs when the phasors form a closed polygon so the resultant phasor is of zero length.

**2e.  $N$  Coherent Sources.** If there are  $N$  coherent sources, the first intensity zero will be for a value of  $\delta = 360^\circ/N$  or  $\delta = 2\pi/N$  radians. For example,  $\delta = 120^\circ$  for 3 sources,  $\delta = 90^\circ$  for 4 sources, etc. Further intensity minima (zeroes) occur at intervals of  $360^\circ/N$  until  $\delta = 360^\circ$ , which is mathematically equivalent to  $\delta = 0^\circ$ . At this point there is a

path difference of one wavelength between adjacent waves, and the pattern repeats itself. In these repeating patterns maxima must occur between the minima although they are not necessarily halfway between them.

▷ Show that, if there are an odd number of phasors combining when  $\delta = 180^\circ = \pi$  radians, successive pairs of phasors cancel and leave only the odd phasor as the resultant. Note that this maximum is much smaller than the maxima at  $\delta = 0^\circ$  and  $360^\circ$ . *Help: [S-1]*

▷ Show that if there are an even number of phasors combining when  $\delta = 180^\circ$ , all phasors cancel and leave a net result of zero (a minimum).

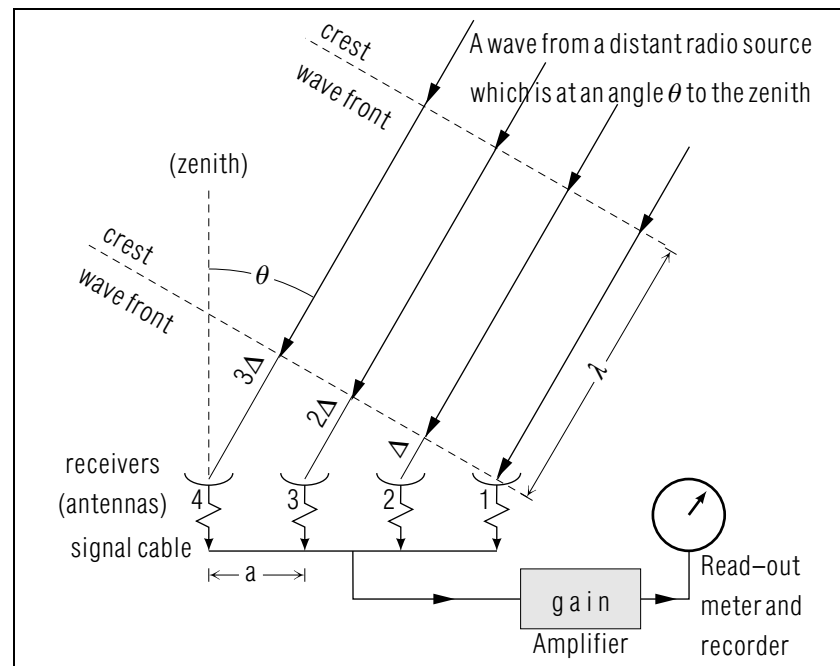
In general, for  $N$  identical phasors combining, there are  $N - 1$  minima (zeros), between  $\delta = 0^\circ$  and  $360^\circ$ , and there are  $N$  maxima, including the large maxima at  $\delta = 0^\circ$  and  $360^\circ$ .

### 3. The Radio Interferometer

**3a. A Linear Array of Antennas.** As an example of a linear array of coherent wave sources, consider a “radio interferometer” consisting of four equally spaced radio receiver-antennas, an amplifier, and a read-out device as shown in Fig. 3. The receivers absorb those parts of the single incident radio wave front which strike their individual surfaces, converting the energy of the incident electromagnetic wave of frequency  $\nu$  into an electrical signal of the same frequency  $\nu$  in each signal cable (wire). Each signal is in phase with its incident-wave segment and is proportional to its strength. The signal cable lengths are adjusted so as to provide equal paths from each of the receivers to the junction  $J$  and hence to the amplifier. Thus the receiver-antenna array acts as four coherent sources of wave disturbance, in a manner similar to Young’s use of a pair of slits to produce two coherent wave sources.<sup>4</sup>

**3b. Phase Difference Between Antenna Signals.** To find the net signal intensity from all four receivers we must first determine the phase difference between signals at successive receivers. At the particular instant of time shown in Fig. 3 a crest wave front of a plane radio wave has just reached Receiver #1, while the previous trough wave front has just reached Receiver #4. The path difference from the radio wave source to successive receivers is denoted as  $\Delta$ . Using the triangle formed by Receiver #1, Receiver #2, and the path-to-Receiver #2/crest wave front intersection, the path difference is:

<sup>4</sup>See “The Interference of Two Coherent Sources” (MISN-0-230).



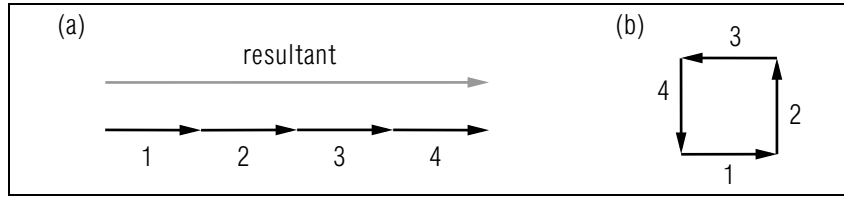
**Figure 3.** A radio interferometer consisting of 4 antennas in a line. A crest wave front has just reached Receiver #1.

$$\Delta = a \sin \theta. \quad (3)$$

The phase difference between the signals at successive receivers is therefore:

$$\delta = \left( \frac{\Delta}{\lambda} \right) (360^\circ) = \left( \frac{a \sin \theta}{\lambda} \right) (360^\circ) \quad (4)$$

**3c. Net Signal Intensity Varies With  $\theta$ .** The net signal intensity recorded by an antenna array varies with the angle of the signal source  $\theta$  (the angle that the incoming wave makes with the zenith). This variation in the net signal with the angle  $\theta$  is called the “angular sensitivity” of the radio interferometer. The reason for this angular variation in sensitivity is that as one changes  $\theta$  the phase difference  $\delta$  between successive antennas changes and of course the output of the array depends on  $\delta$  (it governs how much successive rays reinforce or cancel each other). Certain angular positions of the radio wave source, called “node angles” produce a phase



**Figure 4.** Phasor diagram for  $N = 4$ : (a)  $\delta = 0^\circ$ , (b)  $\delta = 90^\circ$ .

difference that results in complete cancelation, a sensitivity of zero.

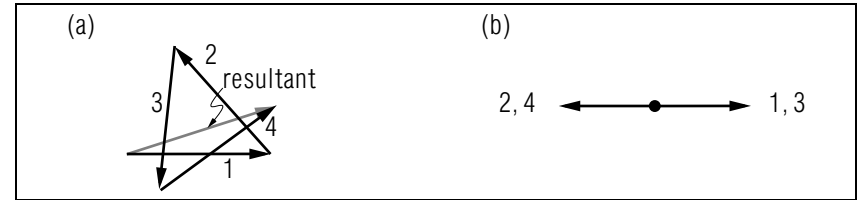
Suppose a radio source happens to be in the direction of a node so its waves produce no output in the interferometer: can the interferometer be turned so the object can be detected? Yes, by waiting for the rotation of the earth to do the turning! As the earth rotates, the zenith of the interferometer is swept through an angular displacement that “aims” the interferometer at different directions in the sky. Thus radio wave sources appear first at one horizon, move directly overhead, and then disappear over the other horizon as the earth turns.

**3d. Constructing the Angular Sensitivity Pattern.** We now turn to the problem of constructing the whole angular sensitivity pattern of the interferometer. At what angles will a radio interferometer give a maximum signal? A zero signal? We may follow the following general procedure for determining the angular sensitivity for a radio interferometer containing  $N$  receiver-antennas:

1. Specify  $N$ , use the method of phasor addition to find the values of  $\delta$  corresponding to maximum and minimum intensity, and construct a linear graph of intensity-vs- $\delta$ .
2. Specify the ratio of receiver-spacing to radio source wavelength being received and construct a polar graph of sensitivity-vs-source angle  $\theta$ . *Help: [S-8]*

**3e. Detailed Example for Four Receivers.** As a worked out example, consider a radio interferometer consisting of four receiver-antennas in a linear array.

1. Start with  $\delta = 0^\circ$  and  $\theta = 0^\circ$  (the radio wave source is directly overhead). Since  $\delta = 0^\circ$ , the individual phasors are all in line, resulting in a maximum value for the resultant phasor (see Fig. 4a).



**Figure 5.** Phasor diagram for  $N = 4$ : (a)  $\delta \simeq 132^\circ$ , (b)  $\delta = 180^\circ$ .

This means there is a large intensity peak for  $\delta = 0$ . Now as  $\delta$  is increased from zero, the resultant phasor gets smaller until at  $\delta = 90^\circ$  it is zero (see Fig. 4b).

As we increase  $\delta$  beyond  $90^\circ$ , the resultant phasor rises to a maximum value somewhere between  $90^\circ$  and  $180^\circ$  (Fig. 5a). From the length of the resultant in the figure it is apparent that this is a small intensity peak compared to the one for  $\delta = 0$ . As we increase  $\delta$  we will obtain a resultant phasor of zero length at  $\delta = 180^\circ$  (see Fig. 5b). Now use the values of  $\delta$  corresponding to peak and null intensity to plot a intensity-vs- $\delta$  graph, as shown in Fig. 6.

2. Now relate the phase difference  $\delta$  to the angular position of the radio wave source,  $\theta$ . For this example we will use a spacing between successive receivers of  $a = \lambda/2$ . Then:

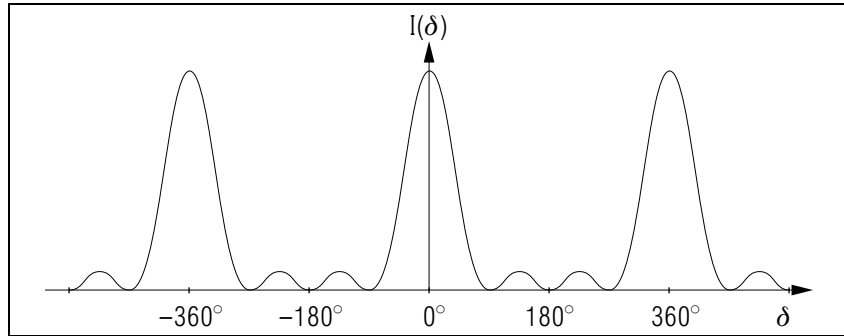
$$\delta = \frac{a \sin \theta}{\lambda} 360^\circ = (\sin \theta)(180^\circ) \quad (5)$$

For various radio wave source angular positions  $\theta$ , the corresponding phase differences  $\delta$  are shown in Table 1.

$\theta$	$0^\circ$	$30^\circ$	$90^\circ$	$180^\circ$	$270^\circ$	$360^\circ$
$\delta$	$0^\circ$	$90^\circ$	$180^\circ$	$0^\circ$	$-180^\circ$	$0^\circ$

Plot the values of  $\theta$  shown in Table 1 at the appropriate places on the intensity-vs- $\delta$  graph, as shown in the lower part of Fig. 7. By following the intensity as  $\theta$  is continuously increased from  $0^\circ$ , you can make a polar graph of sensitivity-vs- $\theta$  as in Fig. 8.

Note that the only physically meaningful values for  $\delta$  correspond to values of  $\theta$  from  $-90^\circ$  to  $+90^\circ$ , i.e. from one horizon to the



**Figure 6.** Net signal intensity vs. phase difference between signals from successive receivers, for  $N = 4$ .

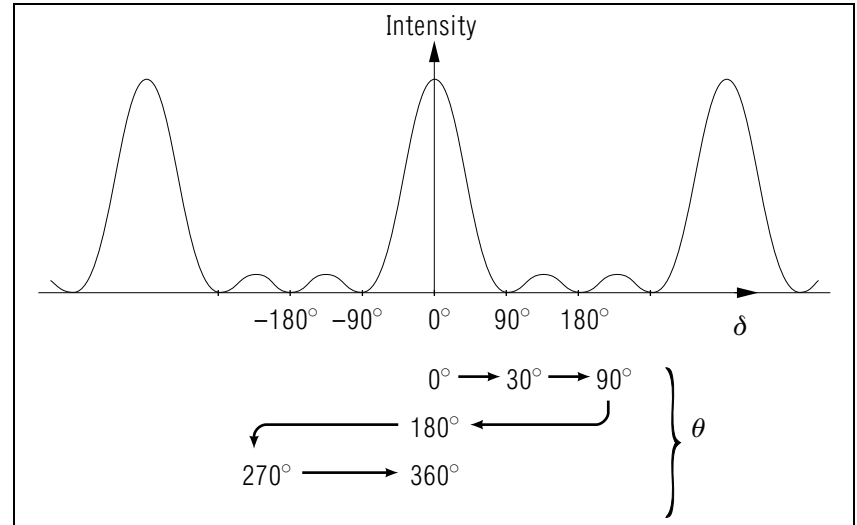
other. Thus for a receiver spacing of  $a = \lambda/2$  and 4 receivers, phase differences greater than  $180^\circ$  are not observed.

▷ Now you solve the 4 receiver problem for the sketch of the angular sensitivity, with  $a = \lambda/2$ , without looking at the solution just presented. When you have finished, check your sketch against the one presented.

## 4. Focus

**4a. Introduction.** The question we deal with here is how accurately an interferometer can pinpoint the direction, in the sky, of a radio source it is detecting. This means we want to examine the *angular sensitivity* of the interferometer and the best way to do that is by means of a polar plot (see Figs. 3 and 8). First, note in the figure that the angle  $\theta$  is defined as the angle from the zenith (directly overhead). Then the clockwise horizon is at  $90^\circ$  and the counterclockwise one at  $270^\circ$ . This is shown in Fig. 8, along with the angular sensitivity pattern implied by Fig. 7. The conversion from Fig. 7 to Fig. 8 was performed by noting the angles  $\theta$  of maxima and minima in Fig. 7 and then sketching them at the appropriate polar angles in Fig. 8 and by then just sketching in the rest of the curves free-hand.

The dashed sensitivity lines in Fig. 8 are for directions where the incoming waves would have to go through the earth to reach the telescope. The earth absorbs those waves before they reach the antenna, so the sensitivity of the telescope in those directions is zero.

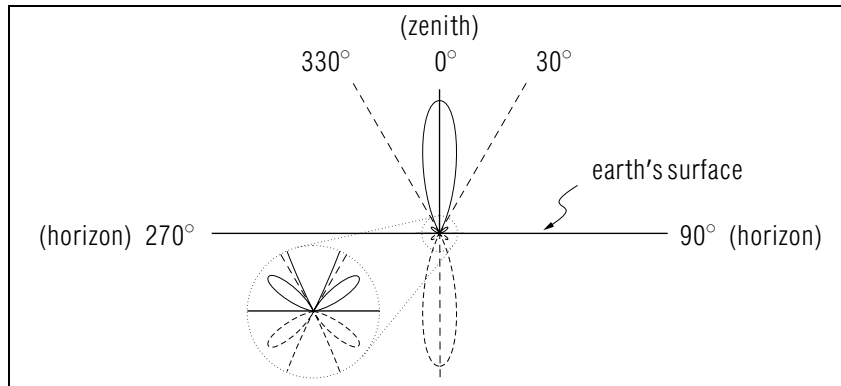


**Figure 7.** Intensity vs.  $\delta$  for  $N = 4$ , but with corresponding values of  $\theta$  also shown. The arrows indicate the path taken as  $\theta$  is increased smoothly from  $0^\circ$  to  $360^\circ$ .

**4b. Few Intensity Maxima.** The polar graph of angular sensitivity in Fig. 8 shows: (1) a main lobe of sensitivity (corresponding to the central maximum of the interference pattern) as the radio wave source crosses the zenith; (2) zero sensitivity at each horizon; and (3) zero sensitivity at  $\pm 30^\circ$  to the zenith. Additionally note the secondary interference maxima of intensity lower than that of the central maximum, located somewhere between  $\pm 30^\circ$  and each horizon.

Ideally it would be desirable to remove the secondary maxima so they would not be mistaken for the central maxima of other weaker radio wave sources angularly displaced from the one directly overhead. This may be accomplished by “tuning” the interferometer. The spacing  $a$  in Fig. 3 can be adjusted to make the intensity minima immediately flanking the central maximum actually occur at  $\theta = \pm 90^\circ$ .

**4c. Sharp, Narrow Central Maxima.** As more and more receivers are added to an antenna array, the orientations for which the signals from all receivers are in phase become more rare, usually occurring only when a radio wave source is directly overhead (see Fig. 8). This is because the central maximum has become very sharp and narrow, and much higher in intensity than any of the secondary maxima (see Fig. 9). *Help: [S-3]*



**Figure 8.** A polar graph of the angular sensitivity for  $N = 4$ ,  $a = \lambda/2$ . Polar angles are taken as increasing from  $0^\circ$  as one moves clockwise from the zenith.

This increases the ability of the telescope to distinguish, or “resolve,” signals from two radio wave sources separated by a small angular displacement. This principle is also used in diffraction gratings, where the number of “receivers” or coherent wave sources may be on the order of  $N = 100,000$ .<sup>5</sup>

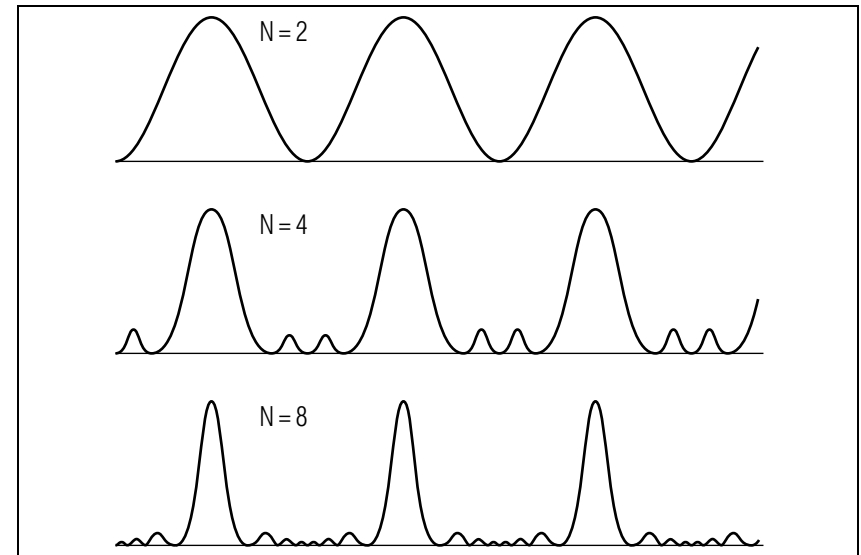
### Acknowledgments

Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

### Glossary

- **angular sensitivity:** the variation in net signal intensity of a radio interferometer for different angular positions of a radio wave source.
- **node angle:** an angular position of a radio wave source, measured with respect to the zenith, where the sensitivity of the interferometer drops to zero.
- **polar plot:** a graph in which the coordinates of a point are specified by: (1) the point’s distance from the origin; and (2) the angle of a line

<sup>5</sup>See “Diffraction Gratings and X-ray Scattering From Crystals” (MISN-0-237).



**Figure 9.** Increasing the number  $N$  of coherent sources decreases the widths of the large interference maxima.

going through the origin and the point, as measured counter-clockwise from the  $x$ -axis unless otherwise specified on the graph.

- **radio interferometer:** a radio telescope that uses an array of radio receiver/antennas to create an interference pattern of a single radio wave traveling to the earth from an extraterrestrial source.
- **radio telescope:** an instrument which collects radio waves instead of visible light from the stars.
- **zenith:** a reference axis for the measurement of angular positions, defined as the radial direction from the center of the earth through the point on the earth’s surface where the zenith is being defined. In other words, the zenith is the direction called “straight up.” If you are standing on your time zone’s *meridian* line, then at noon the sun is at its point of closest approach to the local zenith. If you are near the equator and on your meridian, the sun may be *at* the local zenith at noon, i.e. directly overhead.



### PROBLEM SUPPLEMENT

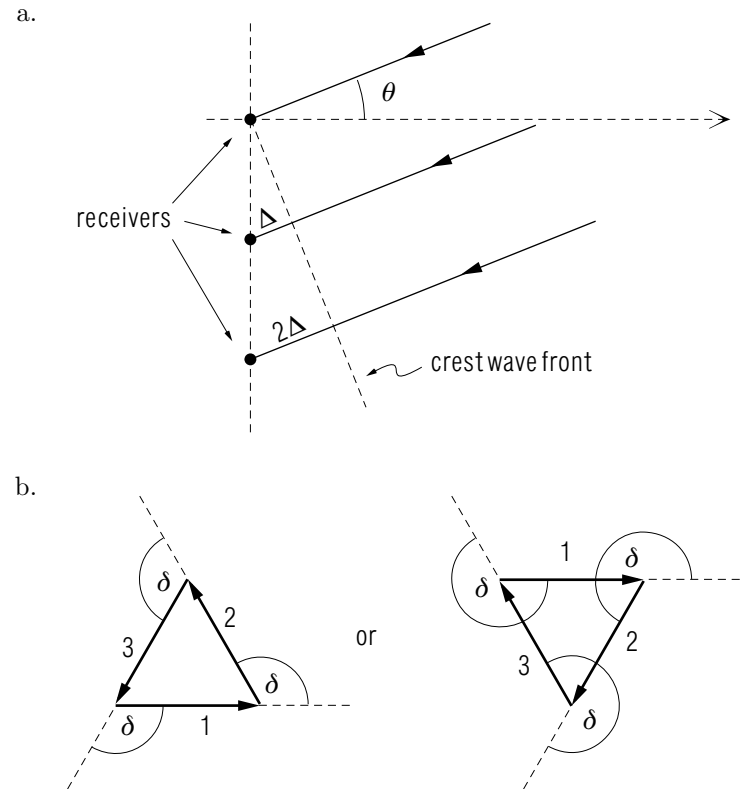
▷ **Warning:** first, do the three problems in this module's *text*. If you do not do those problems as prescribed, you will likely have trouble with the problems below.

1. Consider a fixed linear radio interferometer receiving radio waves of wavelength  $\lambda$ , with 3 receivers spaced  $\lambda$  apart.
  - a. Draw a diagram showing radio wave fronts arriving at the receivers from a distant radio wave source. Indicate a crest wave front on the diagram, shown just as the wave front reaches the first receiver. Label the angle between the direction to the radio wave source and the direction the interferometer is pointing with the symbol  $\theta$ . Mark the differences in wave paths to successive receivers as integer multiples of the symbol  $\Delta$ .
  - b. Sketch the phasor addition diagram for the above case for a value of  $\theta$  such that the angular sensitivity of the interferometer is 0. Be sure to mark  $\delta$  on the diagram. What do the individual phasors in the diagram correspond to, physically?
  - c. Determine the value of  $\delta$  for part (b), in degrees.
  - d. Sketch a graph of Intensity vs.  $\delta$  (see Fig. 7).
  - e. Sketch a polar graph of the angular sensitivity of the interferometer to objects which are in various angular positions with respect to the direction the interferometer is pointing (the zenith). *Help: [S-5]* Mark the zenith and horizon directions on your sketch. Is this a good receiver spacing, or would another be better?
2. Consider a fixed linear-array radio interferometer consisting of 4 receivers with unknown spacing, otherwise as described in Sect. 3 of this module's *text*.
  - a. If  $\delta = 60^\circ$ , calculate the amplitude  $A'$  of the resultant signal reaching the amplifier in terms of the signal amplitude  $A$  that would be received by any single receiver.
  - b. What is the phase of the *resultant* wave relative to the signal being received by antenna #4 (the last to receive a wave front)?
  - c. Find the ratio of antenna spacing to wavelength so that a radio source at  $30^\circ$  from the zenith will not be seen.

3. A radio interferometer has 7 receivers in its linear array of antennas. There is a phase difference of  $\delta = 20^\circ$  between successive receivers. Sketch a phasor addition diagram and find the resultant phasor's magnitude and direction relative to the first phasor in the sum.
4. For the radio interferometer described in Question 3, find the values of  $\delta$  and  $\theta$  corresponding to intensity minima and sketch a graph of the angular sensitivity of the radio interferometer. Assume the receiver spacing is  $a = \lambda/2$ . Indicate the zenith and each horizon on the sketch.

**Brief Answers:**

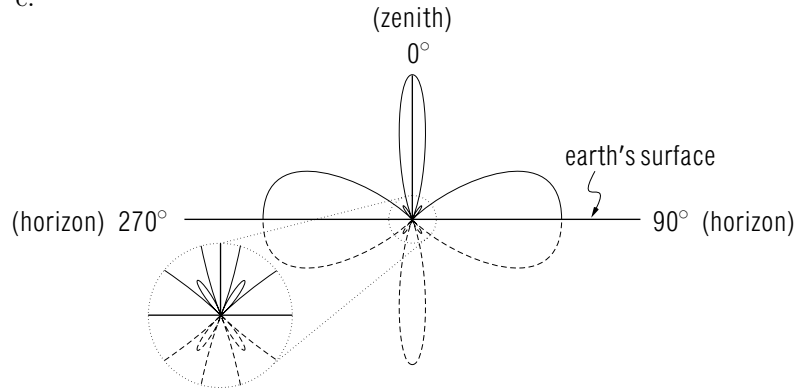
1. A linear radio telescope,  $N = 3$ , spaced one wavelength apart:



$x$ -components: individual contributions to wave disturbances at this instant, this angle.

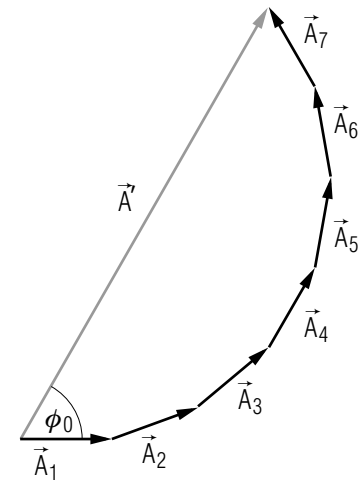
$y$ -components: fictitious, adjusted to make length of phasor be amplitude of that wave.

- c.  $\delta = 120^\circ$  or  $\delta = 240^\circ$
- d. (You can check your answer through part (e), below.)
- e.



This is not a good spacing: too many maxima to clearly define the position of a single radio source. *Help:* [S-4]

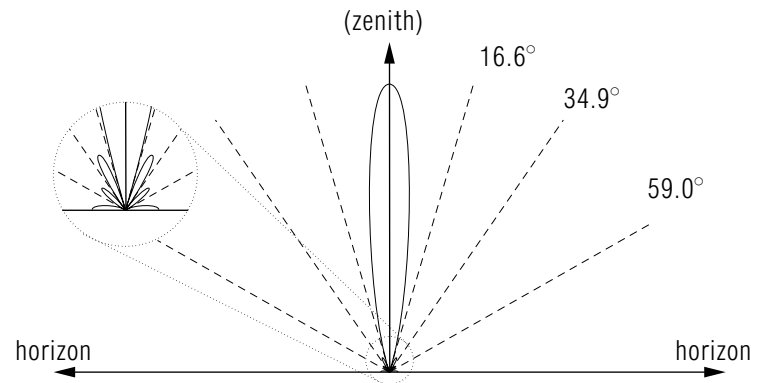
- 2. a.  $A' = 1.732 A$ . *Help:* [S-6]
- b. Resultant is  $90^\circ$  behind the signal from Receiver #4.
- c.  $a/\lambda = 1/2, 1, \text{ or } 3/2$ .
- 3.  $|\vec{A}_1| = |\vec{A}_2| = |\vec{A}_3| = |\vec{A}_4| = |\vec{A}_5| = |\vec{A}_6| = |\vec{A}_7| = A$   
 $|\vec{A}'| = 5.41 A, \phi_0 = 60^\circ$  *Help:* [S-7]



- 4. Nodes at:  $\delta = 51.4^\circ, 102.9^\circ, 154.3^\circ, 205.7^\circ, 257.1^\circ, 308.6^\circ$ ;

$\theta = 16.6^\circ, 34.9^\circ, 59.0^\circ$ ;

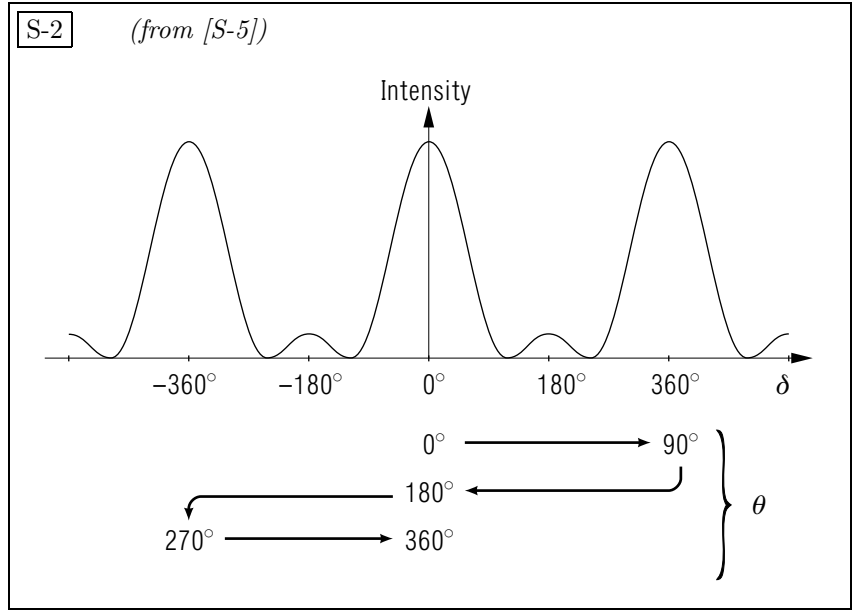
for  $|\theta| \leq 90^\circ, \delta \leq 180^\circ$ , so only the first three nodes are seen.



**SPECIAL ASSISTANCE SUPPLEMENT**

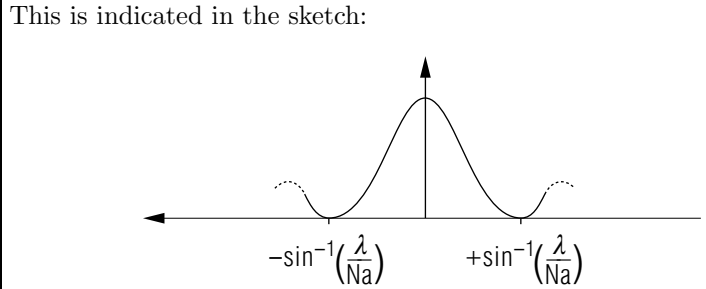
**S-1** (from TX-2c)

For  $\delta = 0^\circ$ , the phasors are all aligned, so the resultant's magnitude is merely  $N$  times the magnitude of any individual phasor. Since the intensity is proportional to the square of the phasor magnitude, the intensity of the wave when  $\delta = 0^\circ$  is  $N^2$  times greater than the intensity contributed by one wave. Thus the maxima at  $\delta = 0^\circ$  is  $N^2$  times stronger than the maximum occurring for  $\delta = 180^\circ$  with an odd number of phasors.



**S-3** (from TX-4c)

The width of the central maximum may be taken as the angular difference in the position of the first minima immediately flanking the central maximum. Since these occur at  $\delta = \pm 2\pi/N = \pm 360^\circ/N$ , the corresponding values of  $\theta$  can be found using Eq. (4) with  $360^\circ = 2\pi$  radians:

$$\delta = \left( \frac{a \sin \theta}{\lambda} \right) (2\pi) = \pm \frac{2\pi}{N} \Rightarrow \theta = \pm \sin^{-1} \left( \frac{\lambda}{Na} \right).$$


The width of the peak is therefore:  $\Delta\theta = 2 \sin^{-1}(\lambda/Na)$ . To get an idea of how  $\Delta\theta$  depends on the number of receivers  $N$ , use a simple configuration where  $\lambda = a$  and insert successively larger values of  $N$  in the equation:

$N$	$\Delta\theta$
3	38.9°
4	29.0°
10	11.5°
100	1.1°
1000	0.1°

**S-4** (from PS-Problem 1d)

Note that the signals received when the radio source is on either horizon are just as strong as the signal received when it is overhead. How do you distinguish between the three?

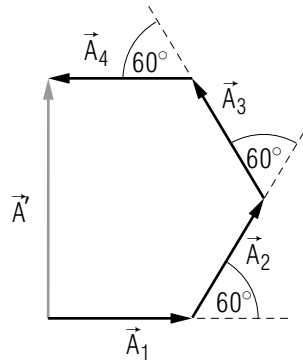
Answer: By changing the  $a/\lambda$  ratio, the field of view ( $|\theta| \leq 90^\circ$ ) can be adjusted to exclude the maxima at  $\delta = \pm 360^\circ$  ( $\theta = 90^\circ, 270^\circ$ ), and only one strong signal is received when the object is overhead.

**S-5** (from PS-Problem 1d)

First determine all values of  $\delta$  which yield zero resultant phasors. Find out how many of these phase differences are in the field of view,  $|\theta| \leq 90^\circ$ , by substituting  $\theta = 90^\circ$  into Eq. 4 ([S-2]). Convert those values of  $\delta$  into the corresponding  $\theta$ 's which are the node angles and draw dotted lines on the upper half of a Cartesian coordinate graph representing the node angles. Then beginning at  $\theta = 0^\circ$  plot a point on the  $y$ -axis far from the origin to indicate high sensitivity at  $\theta = 0^\circ$ . As  $\theta$  increases, the radial distance of plotted points should decrease to zero as a node is approached. After crossing a node, the radial distance increases slightly as the sensitivity increases slightly between nodes, yielding maxima smaller than the one at  $\theta = 0^\circ$ . Remember that  $\theta$  is measured with respect to the  $y$ -axis.

**S-6** (from PS-Problem 2a)

$A$  is the magnitude of the individual phasors:  $\vec{A}_1, \vec{A}_2, \vec{A}_3, \vec{A}_4$ .



$$\begin{aligned} A'_x &= A + A\cos 60^\circ + A\cos 120^\circ + A\cos 180^\circ \\ &= A(1 + \cos 60^\circ + \cos 120^\circ + \cos 180^\circ) = 0 \end{aligned}$$

$$\begin{aligned} A'_y &= A\sin 60^\circ + A\sin 120^\circ + A\sin 180^\circ \\ &= A(\sin 60^\circ + \sin 120^\circ + \sin 180^\circ) = 1.732 A \end{aligned}$$

$$\text{so: } A' = (A_x'^2 + A_y'^2)^{1/2} = 1.732 A,$$

$$\phi_0 = \tan^{-1}(A'_y/A'_x) = 90^\circ.$$

These values can also be estimated easily using a ruler and protractor.

**S-7** (from PS-Problem 3)

Virtually all parts of this problem are contained in the two problems in this module's *text*, and problems 1-2 in this *Problem Supplement*. Do those problems first. Do not merely copy their answers: make sure you understand their solutions thoroughly.

**S-7** (from TX-3d)

If you are not familiar with polar plots, see the relevant Glossary item and then examine the figures in the Text and Problem Supplement Answers until you understand the use of the term in this module.

**MODEL EXAM**

1. See Output Skills K1-K4 in this module's *ID Sheet*. One or more of these skills, or none, may be on the actual exam.
2. A radio interferometer has 7 receivers in its linear array of antennas. There is a phase difference of  $\delta = 20^\circ$  between successive receivers. Sketch a phasor addition diagram and find the resultant phasor's magnitude and direction relative to the first phasor in the sum.
3. For the radio interferometer described in Question 2, find the values of  $\delta$  and  $\theta$  corresponding to intensity minima and sketch a graph of the angular sensitivity of the radio interferometer. Assume the receiver spacing is  $a = \lambda/2$ . Indicate the zenith and each horizon on the sketch.

**Brief Answers:**

1. See module text.
2. See this module's *Problem Supplement*, Problem 3.
3. See this module's *Problem Supplement*, Problem 4.

