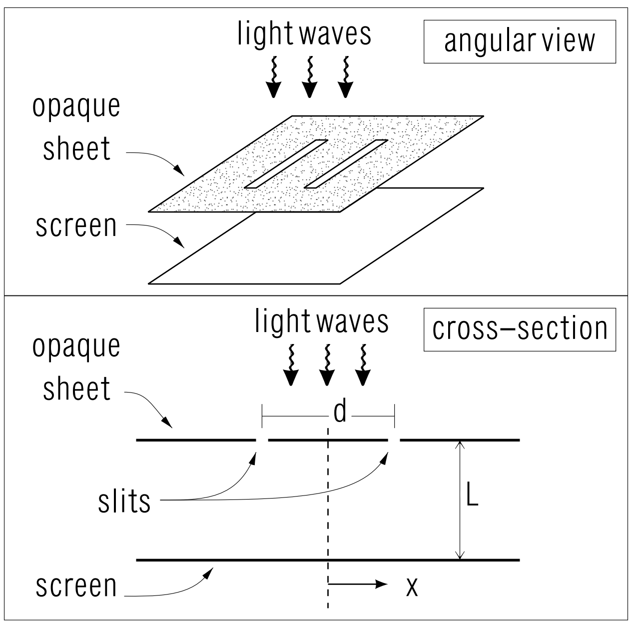
MISN-0-230

INTERFERENCE FROM TWO SYNCHRONIZED WAVE SOURCES



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

INTERFERENCE FROM TWO SYNCHRONIZED WAVE SOURCES
by

Peter Signell and William C. Lane, Michigan State University

Title: **Interference from Two Synchronized Wave Sources**

Author: Peter Signell and William C. Lane, Dept. of Physics/Astronomy, Mich. State Univ.

Version: 3/22/2000 Evaluation: Stage 0

Length: 1 hr; 28 pages

Input Skills:

1. Vocabulary: wave disturbance, amplitude, frequency, wavelength, wave number (MISN-0-201) or (MISN-0-430); intensity (MISN-0-203) or (MISN-0-430); light wave (MISN-0-212) or (MISN-0-430).
2. State the mathematical expression describing a one-dimensional single-frequency wave disturbance as a function of time and distance from the wave source (MISN-0-203).

Output Skills (Knowledge):

K1. Vocabulary: central maximum, coherent wave sources, constructive interference, destructive interference, interference pattern, path difference, phase difference, angular frequency.

K2. State the conditions for destructive and constructive interference, both in terms of phase difference and in terms of path difference.

K3. State the expression for the intensity of the net wave disturbance produced by two equal-amplitude waves from coherent sources, as a function of path difference.

Output Skills (Problem Solving):

S1. Determine by mathematical and graphical methods the amount of net wave disturbance at a given point in space produced by two equal-amplitude coherent-source waves.

S2. Determine points in space where two equal-amplitude coherent-source waves produce maximum or minimum resultant waves.

S2. Sketch the wave disturbance, produced at a given point in space by two equal-amplitude coherent-source waves, as functions of time, along with each of the contributing waves.

External Resources (Required):

1. Ruler, protractor.

THIS IS A DEVELOPMENTAL-STAGE PUBLICATION
OF PROJECT PHYSNET

The goal of our project is to assist a network of educators and scientists in transferring physics from one person to another. We support manuscript processing and distribution, along with communication and information systems. We also work with employers to identify basic scientific skills as well as physics topics that are needed in science and technology. A number of our publications are aimed at assisting users in acquiring such skills.

Our publications are designed: (i) to be updated quickly in response to field tests and new scientific developments; (ii) to be used in both classroom and professional settings; (iii) to show the prerequisite dependencies existing among the various chunks of physics knowledge and skill, as a guide both to mental organization and to use of the materials; and (iv) to be adapted quickly to specific user needs ranging from single-skill instruction to complete custom textbooks.

New authors, reviewers and field testers are welcome.

Views expressed in a module are those of the module author(s) and are not necessarily those of other project participants.

INTERFERENCE FROM TWO SYNCHRONIZED WAVE SOURCES

by

Peter Signell and William C. Lane, Michigan State University

1. Introduction

1a. Overview. The interference of waves is one of the most important and useful phenomena in physics. Historically, interference effects helped confirm the wave nature of light. The bending of sound and light waves around obstacles and corners, the fundamental limitation of the resolving power of optical instruments, the existence of "dead spots" in an auditorium, the appearance of colored rings in oil slicks, and the non-reflective properties of thin-metal coatings on lenses are all examples of interference and its related phenomenon, diffraction.

1b. Synchronized Wave Sources. Interference of two waves is generally only useful if the waves have the same frequency and hence a *constant* phase difference. This will happen if the two different waves come from sources that are "synchronized" so they emit crests at the same time, troughs at the same time, etc. As an illustration, think of an infinitely large swimming pool. Out in the middle are two flat horizontal discs just under the surface of the water (they are parallel to the surface). A vertical rod is connected to each disc. The two rods go straight up to an overhead crankshaft which causes the discs to move up and down together so they are synchronized wave sources. They move up and down with the same amplitude so they are equal sources of waves. The *combination* of waves from both wave sources is visually apparent over the surface of the water as an "interference pattern" with some areas that move up and down, some points in the centers of those areas that become first maximum-height peaks and then minimum-height troughs, and some points between the "up and down" areas that never move at all ("nodes").

1c. Source-to-Point Distances are the Key. All that is required to predict the interference pattern at some space point, due to two synchronous wave sources, is the differing distances of the two sources from the point where the interference is being examined. This is illustrated in Fig. 1 where the distance difference is called Δ.

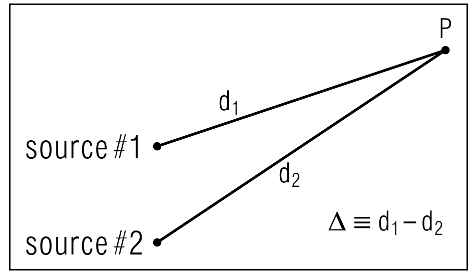


Figure 1. Two wave sources and a point P where we wish to examine the interference between waves from the two sources.

1d. Constructive interference. If there are two sources and their distances to the space-point in question are equal, the crest of the wave from one source will arrive at the same time as a crest of the wave from the other source. At that time, at that point, the two crests will add to make a single crest which has the height of the sum of the two individual crests. A quarter cycle later, at the same point, a node of one wave will arrive at the same time as a node of the other wave and these two nodes will produce a resultant node (two zeros add to zero). A quarter period later the troughs of the two waves will arrive and produce a trough that is as deep as the sum of the two. Because the various parts of the two waves arrive together, the two waves are said to "in phase." As time goes on at that space point, the two waves produce what looks like a single wave passing by which is twice as larger as either of the contributing waves alone (see Fig. 2). This is called "constructive interference." This completely constructive interference will occur at any point P where the distance difference Δ is an integer number of wavelengths since then the crest of the wave from one source will arrive at the same time as a crest from the other source.[1]

1e. Destructive Interference. If the distances from two synchronous sources to a space point differ by a half integral number (1/2, 3/2, 5/2, ...) of wavelengths, then a crest of one wave will arrive at the same time as a trough of the other wave and, if the two waves have equal amplitude, the two will cancel to produce zero wave disturbance at that point at that time. Half a cycle later the crest and trough will be reversed but the waves will still cancel. The addition of such waves will produce a node (zero wave height) at all times hence there will never be any observed wave

[1]For example if, at the point P, the distances d_1 and d_2 are such that $\Delta = 3\lambda$, where λ is the waves' common wavelength, then the 13th wave crest from one source will arrive at the point P at the same time as the 16th wave crest from the other source. Because crest arrives with crest, they will add to produce a crest that is twice as high as the crest of either one alone.

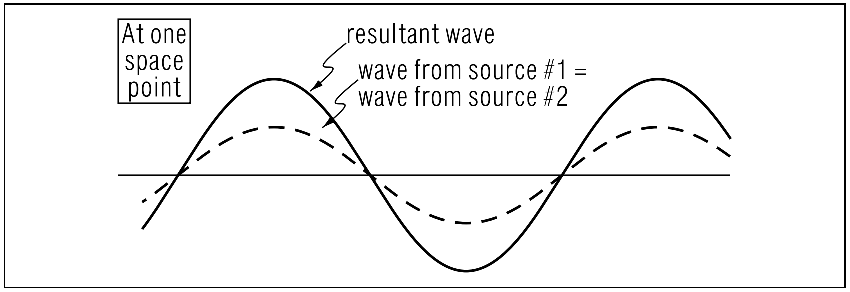


Figure 2. Two equal-amplitude waves from synchronous sources arriving in phase at the point P of Fig. 1.

at such space points (see Fig. 3). We say that the waves are "completely out of phase." This called "destructive interference."

1f. Goals of This Module. In this module we concentrate on finding the wave at some given space point resulting from two synchronous sources located some distance away, with the additional assumption that the amplitudes of the two arriving waves are equal.[2] These simplifications, synchronicity and equal arriving amplitudes, make problem-solving simpler while retaining the essential aspects of interference. Specifically, we consider these two types of problems: (1) the two sources and the space point are given and the goal is to find the observed resultant wave at the space point as a function of time; or (2) the sources are given and the goal is to find space points where the resultant wave *amplitude* is a maximum or a minimum (zero). In both these cases one must compute the sources-to-point distances difference and compare it to the waves'

[2]More-general cases are treated elsewhere (see, for example, MISN-0-231).

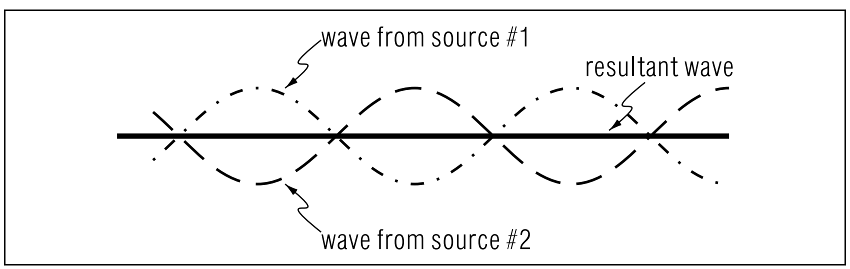


Figure 3. Two equal-amplitude waves from synchronous sources arriving completely out of phase at the point P of Fig. 1.

wavelength to see the extent to which the waves add constructively or destructively.

2. Mathematical Description

2a. The Phase Difference: Description. We combine two travelling waves that have the same frequency, wavelength, and amplitude, but differing phase. The reason the phases of the two waves are different is because their synchronized sources are *different distances* away from the point where we are examining the waves. If at some time we see that one wave arrives one-quarter cycle (one-fourth of a period) ahead of the other wave, that one-quarter wave difference between the two will be constant with time because the sources and the examination point are assumed to be stationary. One can describe the one-quarter-wave difference as a 90° phase difference between the two waves or, entirely equivalently, as a $\pi/2$ radian phase difference.

2b. The Phase Difference: Calculation. Equal-amplitude waves from two synchronous sources, examined at some specific space point, will have a constant phase difference which can be computed from the waves' wavelength and the difference between the distances to the two sources:

$$\delta = \frac{\Delta}{\lambda} 2\pi \text{ radians; or } \delta = \frac{\Delta}{\lambda} 360^\circ .$$

Here Δ is the sources-to-point distances difference and λ if the waves' common wavelength. We can simplify the phase equation by introducing the waves' common *wave number*, $k \equiv 2\pi/\lambda$:

$$\delta = k\Delta . \tag{1}$$

2c. Derivation of the Phase Difference. We can write the individual heights, ξ_1 and ξ_2, of the two waves arriving at the examination point P as:

$$\begin{aligned} \xi_1(P,t) &= \xi_0 \sin(kd_1 + \omega t)\,, \\ \xi_2(P,t) &= \xi_0 \sin(kd_2 + \omega t)\,, \end{aligned} \tag{2}$$

where ω is the waves' common "angular frequency" ($\omega \equiv 2\pi\nu \equiv 2\pi/\tau$), ν is their frequency, τ is their period (the time for one complete wave to be emitted by the source or to pass the point P), the two ξ_0's are the waves' amplitudes, the $\xi(d_1,t)$ and $\xi(d_2,t)$ are the waves' disturbances at their distances d_1 and d_2 from the source at time t. Note that we have taken time zero on our stop-watch ($t = 0$) as being when the waves are rising through a node as they come out of their sources (zero phases at that time). We can eliminate $k\,d_2$ betwen Eqs. (1) and (2) to get:

$$\begin{aligned} \xi_1(P,t) &= \xi_0 \sin(kd_1 + \omega t)\,, \\ \xi_2(P,t) &= \xi_0 \sin(kd_1 + \omega t + \delta)\,. \end{aligned} \tag{3}$$

This form makes it clear that the two waves are identical at any space point except for a phase difference δ which depends only on the distance difference and not on time.

2d. The Resultant Wave. Two equal-amplitude synchonous-source waves, ξ_1 and ξ_2, simply add at any point to give a resulting wave ξ_R which is a function of time at that point:

$$\xi_R(P,t) = \xi_0 \sin(kd_1 + \omega t + \phi) + \xi_0 \sin(kd_1 + \omega t + \phi + \delta)\,, \tag{4}$$

where we have used Eqs. (3). We now use trigonometric identities on the right-hand side of Eq. (4) to get the equivalent single-wave form:

$$\xi_R(P,t) = \xi_{R0}(P) \sin\left(kd_1 + \omega t + \frac{\delta}{2}\right)\,, \tag{5}$$

where

$$\xi_{R0}(P) = \xi_0 2\cos(\delta/2)\,. \tag{6}$$

Thus the resultant disturbance is itself just a wave like either of the two incident waves, with their wavelength and their frequency, but with a different amplitude and phase. Note that, for this case of equal-amplitude incident waves, the resultant wave's phase constant is just half way between the two incident waves' phases (compare Eq. (5) to Eqs.(3)).

▷ Evaluate Eq. (5), using Eq. (6) to evaluate $\xi_{R0}(P)$, for the cases: (i) $\delta = 0$; (ii) $\delta = \pi$; (iii) $\delta = 2\pi$. Also determine the difference between the two source-to-point distances for each case. *Help: [S-1]*

3. Solving Problems

3a. Finding Maximum and Minimum Points. To find places where the resultant wave is always zero, we need only solve Eq. (5) for those δs which make the wave's amplitude always zero, ($\delta = \pi, 3\pi, ...$), and then solve Eq. (1) for the corresponding distance differences, ($\Delta = \lambda/2, 3\lambda/2, ...$). To find the places where the resulting wave disturbances

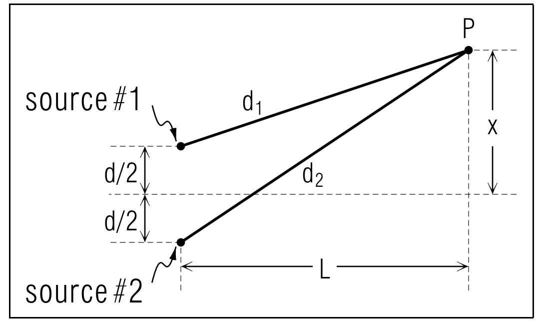


Figure 4. Specification of dimensions for Fig. 1.

are maximum, we go through the same process using those δs which make the resulting wave's amplitude a maximum, ($\delta = 0, 2\pi, 4\pi, ...$) with the corresponding distance differences, ($\Delta = 0, \lambda, 2\lambda, ...$).

3b. Finding the Resulting Wave at a Point. To find the resulting wave at a point we need only measure the distances from the point to the sources, then calculate δ from Eq. (1), then calculate $\xi_{R0}(P)$ from Eq. (6), then calculate the resultant wave itself from Eq. (5).

3c. Intensity. If wave intensity $I = |\xi_R(P,t)|^2$ is needed or given rather than wave amplitude, note that a place of maximum, minimum, or zero amplitude will be a place of maximum, minimum, or zero intensity as well.

3d. An Example. Suppose there are two synchronous sources located a distance d apart as in Fig. 4: we are given the source waves' wavelengths and are asked to find the first spot P a distance x up the line in the figure where the interference pattern between the equal-amplitude waves shows a constant zero disturbance.[3] One way to solve the problem would be to get out a scale (a "ruler") and measure the distances from the sources to some point P at an x and use the distance difference to see if $\Delta = \lambda/2$; if not, keep moving to new points P and recalculating Δ until the point with a wave minimum is found. Alternatively, one can solve the equation

$$d_{1,min} - d_{2,min} = \lambda/2\,, \tag{7}$$

where $d_{1,min}$ and $d_{2,min}$ are the distances to the P that shows the minimum. Equation (7) can be written in terms of the variables in Fig. 4 as:

$$\sqrt{L^2 + (x_{min} + d/2)^2} - \sqrt{L^2 + (x_{min} - d/2)^2} = \lambda/2\,. \tag{8}$$

[3]The line containing P is perpendicular to the perpendicular bisector of the line connecting the two sources; x is measured from the perpendicular bisector as shown.

Here we solve for x_{min}, the position of the first minimum resultant disturbance found as x increases from zero; the first position where the distances differ by a half integral number of wavelengths. Solving Eq. (8) directly is somewhat involved.[4]

Most applications of Eq. (8) to light make the very good assumption that $(x \pm d/2) \ll L$ whereupon Eq. (8) reduces to the very manageable:

$$x_{min} \approx \frac{\lambda L}{2d}\,. \tag{9}$$

4. Young's Experiment

4a. Historical Overview. In 1801 Thomas Young demonstrated the interference of light, thus confirming the wave nature of light. Until this time, the particle theory of light proposed by Sir Isaac Newton was the viewpoint accepted by the majority of the scientific community, despite the claims of theorists such as Huygens that light was a wave phenomenon. The results of Young's experiment, though disbelieved by many at the time, gave indisputable proof that light has wave-like properties such as wavelength and frequency. The discoveries of Quantum Mechanics in the present century have shed further light on the nature of light and on its wave-like and particle-like properties.[5]

4b. Coherent Sources Were the Key. An absolutely essential requirement for interference to be observed is the availability of coherent sources of light waves; that is, sources that have constant phase differences over periods of time long enough for observation. Until Young's experiment, no one had succeeded in producing a pair of coherent sources. By illuminating two narrow parallel slits in an opaque shield, separated by distance d (measured from the center of each slit), with a single wave front of light, these slits act as coherent sources, causing an interference pattern to appear on a screen a distance L away from and parallel to the plane of the two slits (see Fig. 5). The slits act as cylindrical wave sources, producing "cylindrical waves" which only drop off in intensity as r^{-1} instead of r^{-2} as for a spherical wave. Thus the assumption of equal-amplitude waves from equal sources is usually a better approximation for the cylindrical waves produced by line sources than for the spherical waves produced by point sources.

[4]To solve Eq. (8) formally, square both sides and then isolate the one remaining square root on one side of the equation. Square both sides again and the square root will be gone. The resulting equation is a quartic (fourth-order) polynomial equation. A similar case is solved in Problem 6 of this module's *Problem Supplement*, using shortcuts available for this case. To solve any quartic equation brainlessly, follow the directions in *Handbook of Mathematical Functions*, Ed. by Abromowitz and Segun, U.S. Department of Commerce, National Bureau of Standards Applied Mathematics Series No. 55, U.S. Government Printing Office, Washington D.C. (1970), p. 17. The method involves combining the quartic equation's coefficients in three ways and using those combinations as the coefficients in a cubic equation. The cubic equation is then solved using the technique given many places, including in Abromowitz. The real solution of the cubic equation is then combined with the quartic equation's coefficients in four ways to produce the coefficients of two different quadratic equations. Each of those quadratic equations has two solutions and the four solutions together are the four solutions to the original quartic equation. On the other hand, the equation can be solved numerically through use of polynomial-root-finding computer programs.

[5]See *Characteristics of Photons*, MISN-0-212, and *Wave Particle Duality for Light*, MISN-0-246.

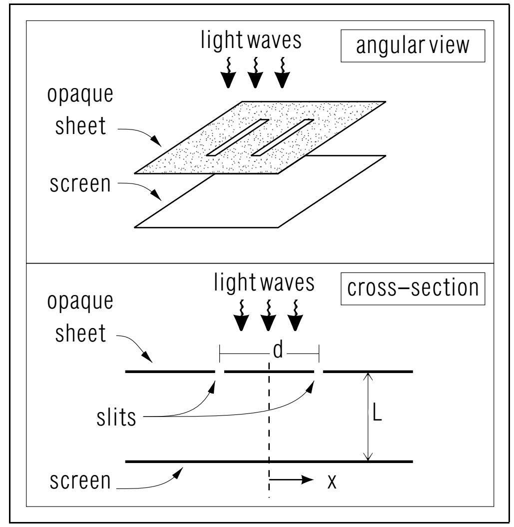


Figure 5. A schematic diagram of Young's two-slit experiment.

4c. Angular Measure for the Path Difference. For Young's experiment, and for any geometry like Fig. 5 where x and d are much smaller than L, it is useful to combine x and L and merely specify the angle θ shown in Figs. 6 and 7. The path difference is then:

$$\Delta = d_2 - d_1 \simeq \theta\, d \text{ for } \theta \ll 1\,. \tag{10}$$

where we have used the fact that $\sin\theta \approx \tan\theta \approx \theta$ for $\theta \ll 1$. Note that the two rays are nearly parallel. Combining this geometrical expression for the path difference of two waves with the criteria for interference maxima

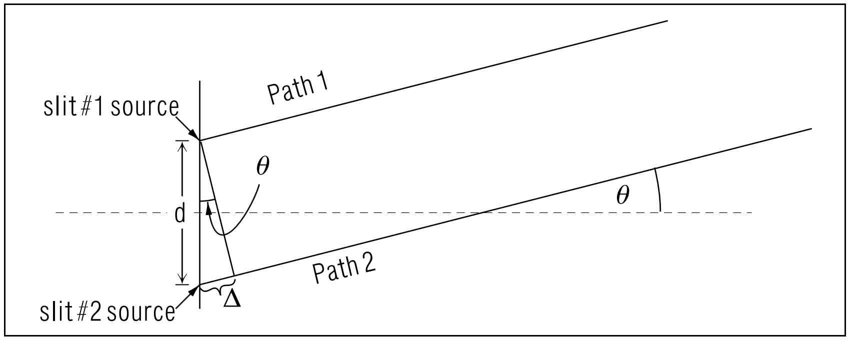


Figure 6. The calculation of the path difference for parallel path distances.

and minima, we obtain the result:

$$\theta_n\, d = \left\{ \begin{array}{ll} n\,\lambda, & \text{for maxima} \\ \left(n + \frac{1}{2}\right)\lambda, & \text{for minima} \end{array} \right\} n = 0, 1, 2, \ldots \text{ and } d \ll L\,. \tag{11}$$

Here θ_n is the angular position of the n^{th} maximum or minimum.[6]

4d. Positions of Maxima and Minima. For fixed slit separation d and wavelength λ, the angular positions of the maxima and minima depends only on the value of n. The value of n determines the "interference order" of maxima and minima on the screen. For $n = 0$, there is only one angle at which there is a maximum; at $\theta = 0$. This maximum is called the "central maximum" and lies on the perpendicular bisector of the slits (see Fig. 7). There is a minimum on either side of the central maximum at the angular positions $\theta = \pm\lambda/2d$. Further maxima and minima appear on the screen at larger angles. These are referred to as the "n^{th} order" maxima and minima, corresponding to the number of complete wavelengths in the path difference. Figure 8 shows a graph of intensity as a function of the angle to the screen position x. We assume $\theta \ll 1$ and plot (θd) so the positions of the minima and maxima will have simple values.

[6] If a lens is used to focus the rays on the screen, one can use exactly parallel rays in Fig. 6 and then the left side of Eq. (11) becomes $d \sin\theta_n$.

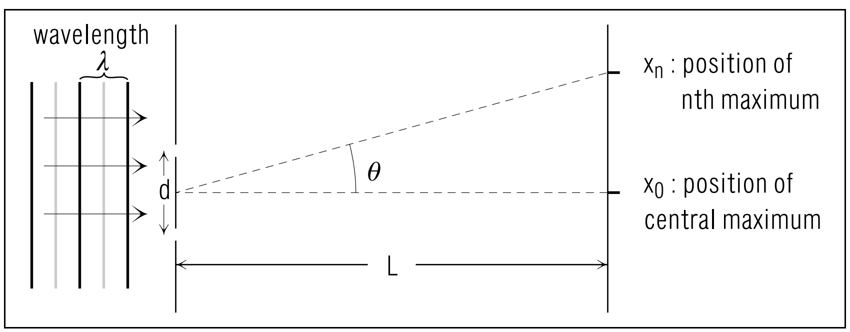


Figure 7. The position of maxima for Young's two-slit experiment.

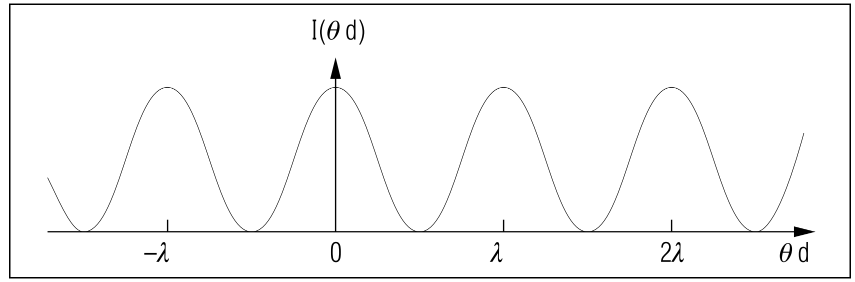


Figure 8. The intensity as a function of screen position (scaled by a convenient factor) for the two-slit experiment.

Acknowledgments

We would like to thank Dr. J.S. Kovacs for his contributions to an earlier version of this module. Dr. James Linnemann gave us a suggestion that resulted in an improvement to the module. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

Glossary

- **central maximum**: in the interference pattern on a screen whose surface is parallel to the line joining a set of evenly spaced synchroized sources: the intensity peak directly in front of the center of the row of sources.
- **coherent wave sources**: a group of wave sources whose waves have constant phase differences (among them) over a period sufficient for

interference patterns to be observed.

- **constructive interference**: a combination of two or more wave displacements at a particular point in space where the net wave intensity is greater than the intensity of any of the individual wave displacements.
- **destructive interference**: a combination of two or more wave displacements at a particular point in space where the net wave intensity is less than the intensity of any of the individual wave displacements.
- **interference pattern**: a spatial variation in the net intensity of the combined wave disturbance due to two or more sources of waves that have constant phase differences.
- **interference order**: the number of complete wavelengths in the path difference between waves from two successive synchronized wave sources that combine at a point in space to form an interference maximum. This number is used to count the maxima, e.g. the third order maxima are the third intensity peaks in the interference pattern on either side of the central maximum.
- **maxima**: in interference patterns, points of total constructive interference, where the intensity of the interference pattern is at its greatest. These points occur when the wave displacements from the various sources present are completely in phase.
- **minima**: in interference patterns, points of destructive interference, where the intensity of the interference pattern is at its weakest. These points occur when the phase difference of the individual wave displacements results in a net disturbance of minimal amplitude.
- **path difference**: in interference patterns, the difference in the distance that two waves from successive synchronized wave sources travel to reach a given point in space where the net wave displacements are being examined.
- **phase difference**: in interference patterns, the difference in the phase of two wave displacements that combine at a given point in space where the net wave displacements are being examined. The phase difference is related to the path difference and the wavelength of the wave.
- **synchronized sources**: of waves emit waves of the same frequency and phase.

A. Time Averaging of Intensities

The intensity of the net wave disturbance produced by two or more synchronized wave sources at point P may be written as:

$$I(P,t) = I_0(P)\cos^2(\omega t + \phi)\,,$$

where all spatial coordinate dependencies are included in the *time-independent* factor $I_0(P)$. Since the maximum value of $\cos^2\theta$ is 1 and its minimum value is zero, the intensity at a fixed point in space oscillates back and forth between its peak value of $I_0(P)$ and its minimum value of zero. The angular frequency of this oscillation is ω. These fluctuations are usually too rapid to be detected, so measurements are made of the time-averaged intensity.

The long term time-average is the same as the average over one wave period, since all wave periods are duplicates of each other, so we need only calculate the average over one period. The time-average of the intensity is denoted $I_{av}(P)$ and is defined by:

$$I_{av}(P) = \frac{1}{T}\int_0^T I_0(P,t)\,dt = \frac{1}{T}\int_0^T I_0(P)\,\cos^2(\omega t + \phi)\,dt\,,$$

where $T = 2\pi/\omega$, which is one period of the wave oscillation. For radio waves, one period is in the region of microseconds (10^{-6} s).

If we let $\omega t + \phi = x$ so $\omega dt = dx$, and use $\omega T = 2\pi$, the integral assumes this form:

$$I_{av}(P) = \frac{I_0(P)}{2\pi}\int_\phi^{2\pi+\phi}\cos^2 x\,dx = \frac{I_0(P)}{2\pi}\int_0^{2\pi}\cos^2 x\,dx\,,$$

where we have used the fact that the integral over any one period of extent T is the same as the integral over any other period of extent T.

We now use the fact that the integral of $\cos^2$ over a complete period is exactly the same as the integral of $\sin^2$ over a complete period (one looks just like the other, on a graph, but shifted.) Then we can rewrite the integral over $\cos^2$ as half the integral over the sum of the $\sin^2$ plus the $\cos^2$. However, the sum of those two quantities is just 1, so we get:

$$I_{av}(P) = \frac{I(P)}{2\pi}\,\frac{1}{2}\int_0^{2\pi} 1\,dx = \frac{1}{2}\,I(P)\,.$$

Thus the time-average of an intensity varying as $cos^2(\omega t + \phi)$ is one-half the peak intensity. Of course the same result holds if the time dependence is $\sin^2(\omega t + \phi)$.

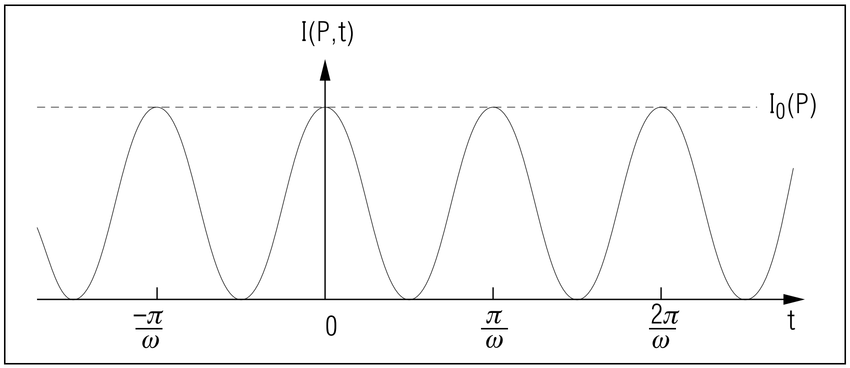
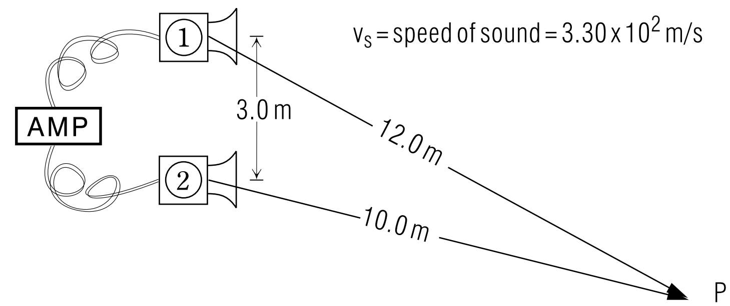


Figure 9. The variation in wave intensity as a function of time at a given space point P at which we observe interference. The time origin was arbitrarily chosen to coincide with an intensity peak.

PROBLEM SUPPLEMENT

Problem 5 also occurs in this module's *Model Exam*.

1.



Two identical speakers are connected to the left and right channel output of a stereo amplifier. A single-frequency tone, $\nu = 660\,\text{Hz}$, is played over the system. A point P is 12.0 m from speaker #1 and 10.0 m from speaker #2.

a. Assuming the speakers are properly connected to the amplifier, i.e. are "in phase," they constitute two synchronized sources of sound waves. Determine wherhet the interference occurring at point P is constructive or destructive.

b. By reversing the connections on one of the speakers, the speakers are 180° "out of phase": when speaker #1 is producing a wave "crest," speaker #2 is producing a wave "trough." Describe the type of interference now taking place at point P.

2. The wave disturbances at a particular point in space P, produced by two wave sources, are given by the equations:

$$\xi_1(P,t) = \xi_0 \sin(kd_1 + \omega t + \phi_1)$$
$$\xi_2(P,t) = \xi_0 \sin(kd_2 + \omega t + \phi_2)$$

The sources are synchronized if the frequencies are the same (they are) and if the phase constants are the same, i.e. $\phi_1 = \phi_2$. Suppose that $\phi_1 = \phi_2 = -\pi/2$. This means that, at $t = 0$ and $d_1 = d_2 = 0$:

$$\xi_1(0,0) = \xi_2(0,0) = \xi_0 \sin(-\pi/2) = -\xi_0$$

which means that a wave "trough" is leaving each source. Sketch ξ_1, ξ_2 and the net wave disturbance, $\xi = \xi_1 + \xi_2$, as functions of ωt when:

a. $d_1 = \lambda/4,\ d_2 = 3\lambda/4$

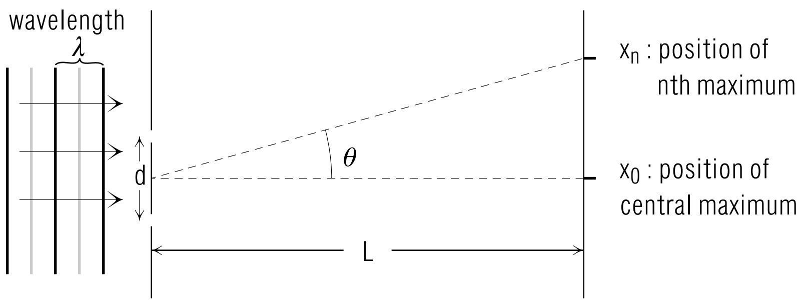
b. $d_1 = \lambda/4,\ d_2 = 5\lambda/4$

c. $d_1 = \lambda/4,\ d_2 = \lambda$.

Characterize the type of interference for each situation.

3. A point in space is 45.0 meters from one wave source and 45.4 meters from another wave source. The two wave sources are synchronized, of equal intensity and produce waves of frequency 165 Hz and wavelength 2.00 meters. Determine the average wave intensity at the given point and then write down your answer in terms of the average intensity either wave would show if the other wave was not present.

4. In Young's 2-slit interference pattern, determine the spacing between adjacent maxima and between adjacent minima (see the figure below).



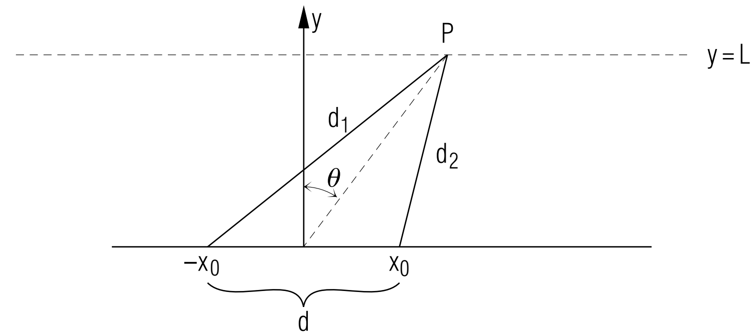
5. Two identical synchronized wave sources emit wavelength $\lambda = 5.86$ m, and are located at $x = \pm 4.00$ m, $y = 0$.

   a. Determine whether the amount of wave disturbance at the point $x_0 = 8.00$ m, $y_0 = 6.93$ m is a minimum or a maximum. Draw a rough sketch of the geometrical layout and show all the steps involved in obtaining the answer.

   b. Sketch a rough graph of the wave disturbance at the point $x_1 = 1.00$ m, $y_1 = 3.00$ m as a function of time. Let $t = 0$ on your graph be the time at which a wave crest from the source at $x = -4.00$ m, $y = 0$ arrives at the point (x_1, y_1). Also show on the graph the disturbance which each source alone would have produced. *Help: [S-3]*

   c. Using graph paper, a pen or pencil, and a ruler, show that $x = 0.63$ m must be a point of minimum intensity (within graphical accuracy) along the line $y_2 = 3.00$ m if the two sources are located as above but have wavelengths of $\lambda = 2.00$ m. Describe all necessary reasoning.

6. (**only for those interested**; must use a significant amount of algebraic manipulation):



Two synchronized wave sources are located at $(d/2,0)$ and $(-d/2,0)$. Using geometry and the condition for constructive interference, derive a formula for the exact positions, x, of interference maxima along the line $y = L$. Use no approximations.

7. (**only for those interested**; must use a significant amount of algebraic manipulation): Derive the result of Problem 4 a different way by noting that the constructive interference condition, $d_1 - d_2 = n\lambda$, where $n\lambda$ is a constant for a given n, is the equation of an hyperbola (actually a family of hyperbolas, since $n = 0, 1, 2, \ldots$). The Cartesian coordinate equation of an hyperbola, centered at the origin with foci at x_0 and $-x_0$, is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1,$$

where a and b are the semi-major and semi-minor axes of the hyperbola. In terms of d_1 and d_2 an hyperbola can be written:

$$d_1 - d_2 = 2a$$

so $a = n\lambda/2$. Finally the relation $x_0 = (a^2 + b^2)^{1/2}$ defines b, given a and x_0. Use the above information to solve for the positions x where the hyperbolas intersect the line $y = L$. These are the locations of interference maxima along $y = L$. Note that the same type of derivation could be carried out to locate interference minima using the relation:

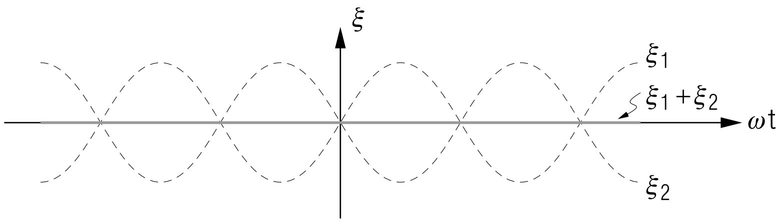
$$d_1 - d_2 = \left(n + \frac{1}{2}\right)\lambda,\ n\ =\ 0,\, 1,\, 2\ldots$$

Brief Answers:

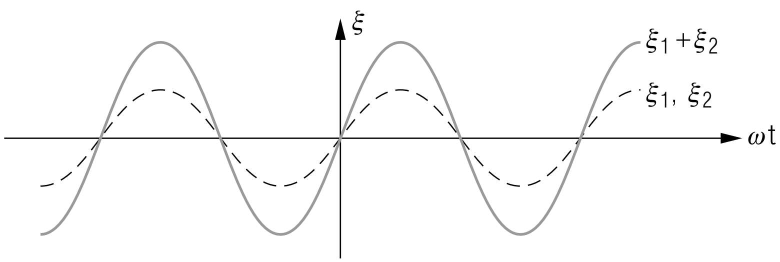
1. a. constructive interference

   b. destructive interference

2. a. Total Destructive Interference:



   b. Total Constructive Interference:



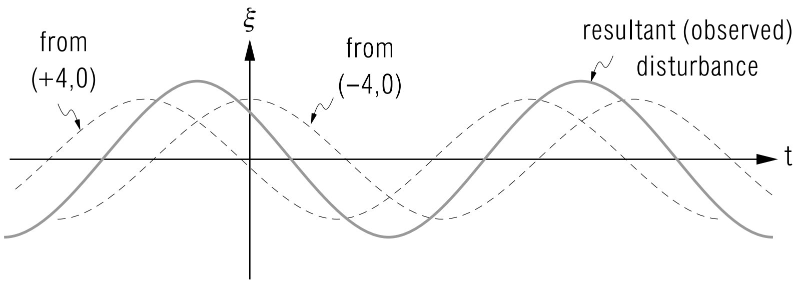
   c. "Intermediate" Interference *Help: [S-3]*

3. $I_{av}(both\ sources) = 2.62 I_{av}(one\ source)$.

4. $\Delta x = \lambda L/d$ for adjacent maxima; $\Delta x = \lambda L/d$ for adjacent minima.

5. a. Maximum *Help: [S-3]*

   b. The wave from $(+4.00\,\mathrm{m},0)$ will arrive 0.271 of a period ahead of the wave from $(-4.00\,\mathrm{m},0)$ hence: *Help: [S-2]*



c. (Construct the graph).

6.

$$x_n = \pm\left(\frac{n\lambda}{2}\right)\sqrt{1+\frac{L^2}{(d/2)^2-(n\lambda/2)^2}}$$ *Help: [S-1]*

7. Same as Answer 6.

SPECIAL ASSISTANCE SUPPLEMENT

S-1 *(from PS, problem 6)*

$d_1 - d_2 = n\lambda$, where d_1 and d_2 are defined in terms of somewhat unwieldy square roots. Be sure to move d_2 to the other side of the equation before squaring to solve for x, i.e.

$$d_1^2 = (d_2 + n\lambda)^2 = d_2^2 + 2n\lambda d_2 + (n\lambda)^2.$$

Substituting the expressions for d_1^2 and d_2^2 (leaving d_2 as is, temporarily) several terms cancel. You can then isolate d_2 on one side and square again, then solve for x.

S-2 *(from PS-problem 5b)*

Most parts of this problem are contained in problems 1-4 and this module's *text* and *Glossary*. Do problems 1-4 first. Do not merely copy their answers: make sure you work out their complete solutions yourself.

Apart from the above, be aware that the sources are both on the x-axis and that the point at which you are asked to find the degree of interference is above the x-axis (a positive y value).

Also, the path-difference-to-wavelength ratio equals the time-difference-to-period ratio. Here the time difference is the difference in the amount of time it takes simultaneously-emitted waves to get from their sources to the point in question. That is, one can clock the amount of time it takes a wave peak, say, to get from one source to the point. One can also clock the time it takes for the simultaneously-emitted peak from the other source to get to the point: the time difference is the difference in those two times.

Finally, note that the crest from the source at (+4.00 m,0) arrives before the crest from the source at (−4.00 m,0) because it has a shorter distance to travel. That means it must arrive at a time before time zero so it must arrive at a negative time (see the figure in the answer).

S-3 *(from PS-problem 2c)*

The curves plotted represent:

$$\begin{aligned}\xi_1 &= \xi_0 \sin(\omega t)\\ \xi_2 &= \xi_0 \sin(\omega t + 3\pi/2)\\ &= \xi_0 \sin(\omega t - \pi/2).\end{aligned}$$

Note that one must use: $k = 2\pi/\lambda$ and $(3/4)(2\pi) = 3\pi/2$.

MODEL EXAM

Note: You must bring a ruler and a pen or pencil to the exam. Ask the exam manager for a piece of graph paper if you need it and it is not given on the exam.

1. See Output Skills K1-K3 in this module's *ID Sheet*. One or more of these skills, or none, may be on the actual exam.

2. Consider two synchronized wave sources with wavelength $\lambda = 5.86\,\mathrm{m}$, located at $x = \pm 4.00\,\mathrm{m}$, $y = 0$.

   a. Determine whether the amount of wave disturbance at the point $x_0 = 8.00\,\mathrm{m}$, $y_0 = 6.93\,\mathrm{m}$ is a minimum or a maximum. Draw a rough sketch of the geometrical layout and show all the steps involved in obtaining the answer.

   b. Sketch a rough graph of the wave disturbance at the point $x_1 = 1.00\,\mathrm{m}$, $y_1 = 3.00\,\mathrm{m}$ as a function of time. Make $t = 0$ be the time at which a wave crest from the source at $x = -4.00\,\mathrm{m}$, $y = 0$ arrives at the point (x_1, y_1). Also show on the graph the disturbance which each source alone would have produced.

   c. Using graph paper, a pen or pencil, and a ruler, show that $x = 0.63\,\mathrm{m}$ must be a point of minimum intensity (within graphical accuracy) along the line $y_2 = 3.00\,\mathrm{m}$ if the two sources are located as above but have wavelengths of $\lambda = 2.00\,\mathrm{m}$. Describe all necessary reasoning.

Brief Answers:

1. See this module's *text*.

2. See this module's *Problem Supplement*, problem 5.