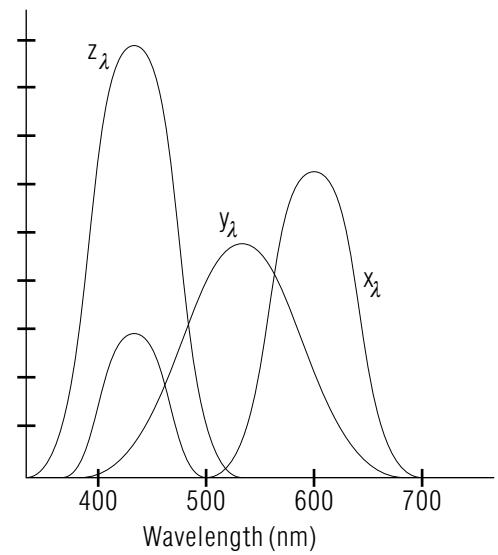


## SPECIFICATION OF COLOR: CHROMATICITY



## SPECIFICATION OF COLOR: CHROMATICITY

by

Peter Signell, Michigan State University

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**Input Skills:**

1. Vocabulary: wavelength (MISN-0-201), spectrum (MISN-0-212).

**Output Skills (Knowledge):**

- K1. Vocabulary: monochromatic, hue, saturation, brightness, cones, chromaticity coordinates, integrated response values, fractional integrated response values, tri-stimulus response values, Chromaticity Diagram, RGB, white, black, spectral components.
- K2. Describe how you could generate the Chromaticity Diagram, complete with color labels scattered around it, using only the 1931 CIE Tri-stimulus values, graph paper, a calculator, and two variable-wavelength light beams.
- K3. Explain how two light beams can have different spectral components and yet have exactly the same color.
- K4. Explain how a color TV set produces colors and why the set cannot produce all the colors that can be seen by the eye.

**Output Skills (Rule Application):**

- R1. Given a beam of light of a specified wavelength, use a Tri-Stimulus Table to determine the beam's chromaticity coordinates.
- R2. Given the chromaticity coordinates of a color, determine its hue and saturation, and vice versa.
- R3. Given a color that is to be produced by mixing two fully saturated beams, determine the proper fraction of each beam in the mixture, and vice versa (given the component beams, determine the resulting color). See this module's *Local Guide*.

**External Resources (Required):**

1. A ruler, preferably a clear one.

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# SPECIFICATION OF COLOR: CHROMATICITY

by

Peter Signell, Michigan State University

## 1. Introduction

**1a. Purpose.** In this module we show you how to make use of an international standard for the quantitative description of color. According to this standard, each color is specified by a unique combination of three numbers: two numbers specifying a color's "chromaticity" and one number specifying its "brightness." In this module we deal mainly with a color's chromaticity and only briefly with its brightness.

A modern Tri-Stimulus Colorimeter containing a microprocessor can be used to do the whole job of color measurement: light of the desired color enters the device, there is a short period of measurement and computation, then its readout displays the three numbers. Here we show you the physics that is basic to the processes going on in the device and we show you how to interpret the results. In addition, the understandings developed along the way will provide a basis for developing an understanding of the colors that we perceive for various objects under various conditions.<sup>1</sup>

The subject of color specification is of considerable importance to industry in both designing the perceived colors of objects under various lighting conditions and in maintaining quality control, over extended periods of time, for paints, textiles, plastics, foods, etc.

**1b. The "1931 CIE" Measure: a Standard.** The "1931 CIE" measure is a basic industry standard for specifying a color. The measure was fixed by an international congress (the "CIE") in 1931, with the intent of mimicking the processing that takes place in the eye-brain system of a "standard observer."

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<sup>1</sup>See "Colors from Spectral Distributions" (MISN-0-229), which extends the treatment of the present module to cover light containing all visible wavelengths, the kind of light normally entering the human eye. Then "Color Effects from Light Source and Reflection" (MISN-0-265) follows changes in the spectral distribution of a ray of light as it emerges from its source, travels to an object, reflects off the object, travels to the eye, then produces in the brain a perception of "the color of the object." Finally, "Land's Observations on Color Perception" (MISN-0-228) shows the striking effects that the *content* of an entire scene can have on the colors perceived for various objects in the scene.

Since 1931 a lot more has been learned about how the eye-brain systems of most people produce color perception and this is currently an active field of research. Current theories of color perception greatly refine and expand on the ideas present in the 1931 CIE standard, but the resulting revisions to the numbers used in the standard have only been small and only need to be taken into account where great precision is required. Thus the 1931 standard is still widely used in specifying colors and it makes a good beginning point for study of the eye-brain color perception system.

## 2. The Young-Helmholtz Basic Model

**2a. Overview of the Theory.** The Young-Helmholtz theory, which is a main basis for the 1931 CIE standard, is an "additive three-component" description.<sup>2</sup> From many experiments Young deduced that there are three types of receptors in the eye, and that each type of receptor sends separate information to the brain for building a perception of color. Helmholtz later made an important addition by saying that each type of receptor responds to a wide range of wavelengths. Young gave a color name to each of the three types of receptors, but we will simply call them  $x$ ,  $y$ , and  $z$  (this is a universal designation and professionals generally consider Young's color names to be interchangeable with these labels).

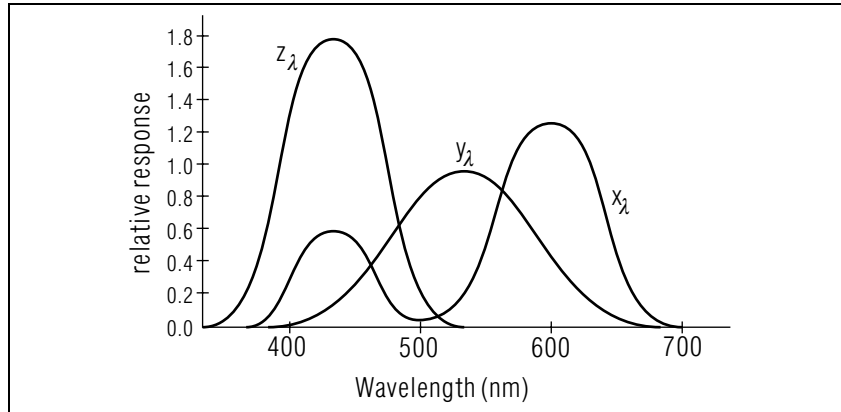
**2b. The Three Types of Cones.** The retina<sup>3</sup> of a human eye contains about seven million color-detecting nerves, called cones, each of which belongs to one of three distinct cone types. All cones of the same type respond similarly to light from a particular source. However, cones of differing type respond differently to light from the same source (except for the color "white," as we shall discuss later). Thus any " $x$ " cone responds similarly to any other " $x$ " cone but differently from a " $y$ " cone or a " $z$ " cone. The responses of the " $x$ ," " $y$ ," and " $z$ " cones to light of individual wavelengths are plotted in Fig. 1 and are listed in Table A-1, Appendix A, this module. These responses are called the "CIE 1931 Tri-Stimulus Values" and are written  $x_\lambda$ ,  $y_\lambda$ , and  $z_\lambda$ . The  $\lambda$  subscripts show that they are functions of wavelength.<sup>4</sup> Notice that in Fig. 1 the  $x$  cones

---

<sup>2</sup>Note that this is *not* a three-color description.

<sup>3</sup>The eye's "retina" is a surface, in the back of the eye, composed of nerve endings. Light absorbed on a retinal nerve ending causes a chemical reaction that starts a signal down that particular nerve, heading toward the brain, whereupon the nerve "resets" and is ready to fire again.

<sup>4</sup>In producing the responses shown in Fig. 1 and Table A-1, the light incident on the cones is kept at the same intensity at all wavelengths so the responses can be said to all



**Figure 1.** The 1931 CIE presumed response curves (“tri-stimulus values”) for the three types of retinal cones.

are generally more sensitive on the right side (“red”) of the graph, the  $y$  cones in the middle (“green”) part, and the  $z$  cones on the left (“blue”). Note also that the “ $x$ ” cones have two peaks of sensitivity.

**2c. Signals from the Cones.** When light falls on cones in the retina of the eye, each cone sends a message that communicates the intensity of its response. This signal goes out over an optic nerve<sup>5</sup> to a preprocessing center back of the eye. The preprocessing center combines the signals from three adjacent  $x$ ,  $y$ , and  $z$  cones and sends three signals, containing the information in a somewhat altered form, on to the brain. The brain, in turn, converts the three numbers to a perception of colored light at a point corresponding to the location, in the eye, of the three adjacent cones. The collection of all such sets of three signals from all over the back of the eye allow the brain to construct a complete colored scene.<sup>6</sup>

**2d. Three Examples.** As a first example, consider what happens when you look at a light source that emits light at the single wavelength

have the same cone stimulus. If equal-intensity light at all these wavelengths are mixed, the resultant color is said to be “idealized white light.” There is no natural source that produces such light, but overhead sunlight in the tropics is a modest approximation.

<sup>5</sup>The eye’s “optic nerve” is a bundle of nerve cells leaving the retina of the eye, heading toward the brain. These cells transport the visual information received at the retina.

<sup>6</sup>For more information see *Color: A Guide to Basic Facts and Concepts*, Burnham, Hanes, and Bartleson, John Wiley and Sons, 1963, pp. 56. For access, see this module’s *Local Guide*.

of 600 nm.<sup>7</sup> When the retina of your eye absorbs that light, the  $x$  cones are highly stimulated, the  $y$  cones are moderately stimulated, and the  $z$  cones are negligibly stimulated (see Fig. 1). We say there is a large  $x$ -cone response, a modest  $y$  one, and a negligible  $z$  one. The  $x$  cones send out large signals, the  $y$  ones modest signals, and the  $z$  ones negligible ones. As a result of this combination of signals from the cones, the brain constructs a perception of the color “yellow-orange.”

As a second example, suppose you look at a light source that emits light at the single wavelength of 570 nm, down from the 600 nm used in the previous example. Now the  $x$  and  $y$  cones send out equal-strength signals but the  $z$  cones send out only a small signal. As a result of this combination of signals from the cones, the brain produces a perception of the color “yellow.”

Finally, suppose the light entering the eye is such that all three cone types send out equal-strength signals. As a result of this combination of signals from the cones, the brain produces a perception of the color “white” (or “grey” if the brightness is low).<sup>8</sup>

### 3. The Chromaticity Diagram

**3a. Overview.** The Tri-Stimulus Values can be used to make a diagram that numerically specifies two non-brightness parts of a color’s description: this is called the CIE Chromaticity Diagram. We will first discuss the procedure for measuring the various quantities that describe the response of the eye to specific light beams. Then we will construct the periphery of the Chromaticity Diagram by plotting the positions (on the diagram) of single-wavelength light beams. Next we will plot the positions of colors corresponding to mixtures of single-wavelength light beams. Finally, we will use the diagram to relate colors to the color-technology terms “hue,” “saturation” and “brightness.”

**3b. Integrated Responses.** If a light beam having a single wavelength falls on, say, an  $x$  cone, that cone responds, as shown in Fig. 1, for that type of cone and that wavelength. If we now add, to the beam, a second ray having the same intensity as the first, but a different wavelength, the response of the cone will increase and the amount of increase will be just the amount shown in Fig. 1 for the second wavelength. In general,

<sup>7</sup>Such a single-wavelength light beam is said to be “monochromatic” and is routinely called a “monochromatic beam.”

<sup>8</sup>See “The Measure of Perceived Color” (MISN-0-229), a systematic presentation.

light has a continuous distribution of wavelengths and thus each type of cone has an “Integrated Response” to the light. The three Integrated Responses, one for each type of cone, are denoted  $X$ ,  $Y$ , and  $Z$ . The apparent brightness of a light beam is just the total integrated response,  $X + Y + Z$ .

### 3c. Fractional Integr’d Responses, Chromaticity Coordinates.

The apparent brightness of a light beam is normally eliminated from the discussion by dividing each of the three integrated responses by the sum of the three. The resulting fractional integrated responses are labeled  $x$ ,  $y$ , and  $z$ :  $x = X/(X+Y+Z)$ ,  $y = Y/(X+Y+Z)$ ,  $z = Z/(X+Y+Z)$ . Then if  $x = 0.3$  for some light beam, then we can say that 30% of the response of the eye to that beam is from  $x$  cones. Now of course  $x + y + z = 1$ , so knowing any two of  $x$ ,  $y$ , and  $z$  immediately gives us the third. Thus a knowledge of all three is superfluous. When describing the color of a light beam entering the eye, the convention is to just specify  $x$  and  $y$  and these are called the “chromaticity coordinates” of the color.

**3d. Example.** Suppose that, for a particular light beam entering the eye, the integrated cone responses have the values  $X = 40$ ,  $Y = 80$ , and  $Z = 120$ . Then the total response is  $X + Y + Z = 240$  and the fractional responses are  $x = 40/240 = 0.167$ ,  $y = 80/240 = 0.333$ , and  $z = 120/240 = 0.500$ .

▷ Show that, for the beam quoted above, the “chromaticity coordinates” for our Example color are:  $x = 0.167$ ,  $y = 0.333$ .

**3e. Single-Wavelength Beams and the Diagram Boundary.** To obtain the chromaticity coordinates of a single-wavelength light beam, we need merely calculate the fractional values  $x$  and  $y$  for that beam. For example, one sees from Fig. 1 and Table A-1 (Appendix A, this module), that a beam of 540 nm light has the integrated response values  $X = 0.290$ ,  $Y = 0.954$ ,  $Z = 0.020$ . For this beam  $X + Y + Z = 1.264$  so  $x = 0.229$  and  $y = 0.755$ . These latter two numbers are given in the last two columns of the table, but to greater accuracy.

If you physically look at a 540 nm beam you will see that its color is green, so you could plot the point  $x = 0.229$ ,  $y = 0.755$  on the diagram and label it “540 nm” and “green.” After doing this for many different single-wavelength light beams you would have constructed the curved boundary in the Chromaticity Diagram, as shown in Fig. 2.

There are two distinct parts to the boundary: the curved part where each different point corresponds to a different wavelength, and the straight

part at the bottom where each different point corresponds to a different combination of the violet and red end-point wavelengths.

▷ Show that the Chromaticity coordinates in Fig. 2 for a 600 nm gold light beam are as given in the Table A-1.

**3f. Mixtures of Single-Wavelength Beams.** Given the single wavelengths on the boundary of the Chromaticity Diagram, one can move on to colors that correspond to mixtures of such pure wavelengths. Any beam of visible light can be broken up into its single-wavelength components, so all colors must be either on the periphery or in the interior of the Chromaticity Diagram.

For example, mixing equal intensities of 625 nm red and 470 nm violet light beams produces a perception of a light purple beam so we could locate the point that is half way between the 625 nm and 470 nm points on the Chromaticity Diagram and write in the word “light purple.” Similarly, a mixture having equal components of 489 nm blue light and 595 nm orange light produces a perception of a “white” light beam so we locate the point that is half way between the 489 nm and 595 nm points on the Diagram and label it “white.” Of course the color “pink” turns out to be between the colors “red” and “white” on the diagram.

## 4. Mixing Wavelengths

**4a. A Color as a Mixture of Two Pure Colors.** Given a position on the Chromaticity Diagram, corresponding to a particular color, one can measure on the diagram and determine a combination of two pure-wavelength light beams which will produce that color.

▷ Show that a straight line from the 450 nm violet point to the 550 nm green point passes through the  $x = 0.277$ ,  $y = 0.580$  point in Fig. 2, a shade of green. We will hereafter refer to this as the (0.277,0.580) point on the Diagram. Since the point is closer to the 550 nm end of the line, that must be the dominant partner in this “450 nm plus 550 nm” combination.

To find the actual fractions of each member of the combination, one can either do a calculation based on the values in the Table A-1, or one can measure distances among the three points on the Chromaticity Diagram. Of course the point corresponding to the target color must lie on the straight line connecting the two proposed components, or the target color cannot be produced by any combination of those two components. The equivalence of the calculational and measurement methods can be easily

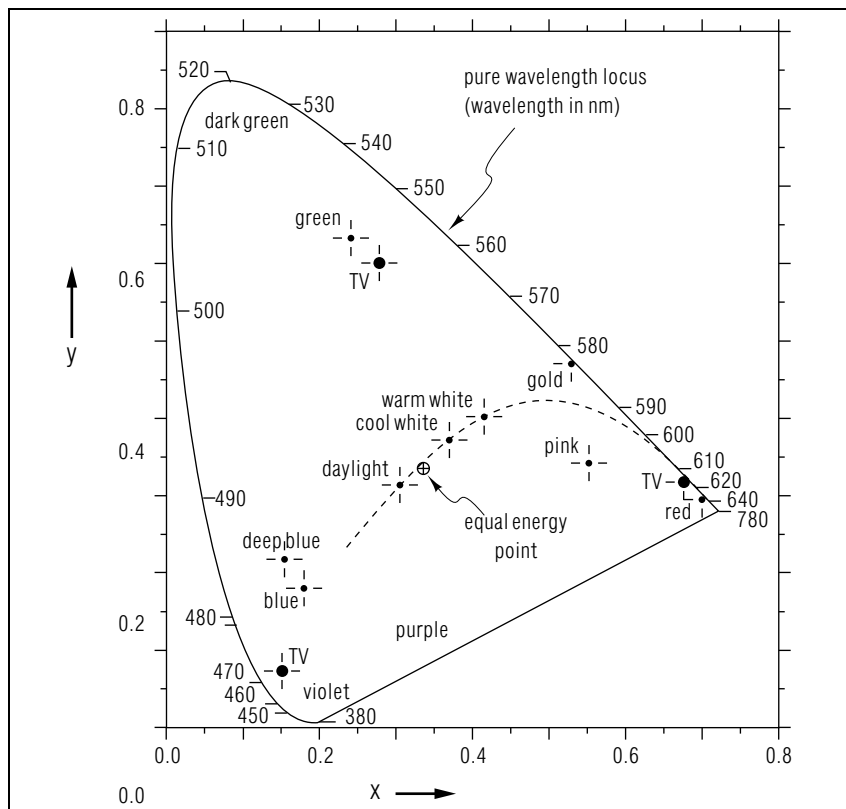


Figure 2. The “1931 CIE” Chromaticity Diagram

shown (see below).

**4b. Algebraic/Computational Method.** To find the combination of two rays that produces a given color,  $(x,y)$ , assume that the combination has a fraction “ $f_2$ ” of component ray #2 with color  $(x_2,y_2)$ , and hence a fraction  $1 - f_2$  of component ray #1 with color  $(x_1,y_1)$ . Then you can write for the color  $(x,y)$ :

$$\begin{aligned} x &= (1 - f_2)x_1 + f_2x_2 \\ y &= (1 - f_2)y_1 + f_2y_2 \end{aligned} \quad (1)$$

where you should remember that  $f_2$  is defined as the fraction in the mixture that is color #2 and that  $(x_2,y_2)$  is the position of color #2 on the Chromaticity Diagram. If it is a pure wavelength color, its  $(x_2,y_2)$  are

also in Table A-1. You need only solve one of the two equations labeled Eq. (1): the two are redundant if the three points actually do lie on a straight line as assumed in Eq. (1). For an example, see Sect. 4d.

▷ Suppose you interchange the ray labels #1 and #2 and then redo the calculation of Eqns. (1). Show that in all such cases you will get exactly the same values for  $x$  and  $y$  that you got before the interchange.

*Help: [S-4]* Given this result, does it matter which ray you label #1 and which one you label #2 in order to do the calculation?

**4c. Graphical/Measurement Method.** To use graphical measurements to find the combination of two rays that produces a given color, first draw the straight line that connects the three points on the Chromaticity Diagram, then measure the distances between the points. Call  $d_1$  the distance from the point for component #1 to the point for the target point, and  $d_2$  the distance from the target point to the point for component #2. Then the fraction of component #2 in the mixture is:

$$f_2 = d_1 / (d_1 + d_2), \quad (2)$$

where this equation comes from solution of Eq. (1) for  $f_2$  plus a little trig work. *Help: [S-3]*

**4d. An Example.** In Sect. 4a it was proposed that a 450 nm light ray and a 550 nm light ray be combined to produce the color at point  $(0.277, 0.580)$  on the Chromaticity Diagram.

▷ Solve this problem by the algebraic method, showing that the solution is 83% 550 nm light, 17% 450 nm light. *Help: [S-1]*

▷ Solve the problem again, now by the graphical method. Show that the answer so obtained is the same as that found in the algebraic method, within the accuracy of graphical work.

**4e. Different Ways of Producing the Same Color.** Any color that is in the interior of the Chromaticity Diagram (not on its boundary) can be produced in an infinite number of different ways. For the example, there are an infinite number of straight lines that can be drawn through the point at  $(0.277, 0.580)$ , all at different angles. For each of these lines one can determine the percentages of the two boundary components that will produce the color at  $(0.277, 0.580)$ .

▷ Show that a (60% 510 nm, 40% 610 nm) combination will produce the same color as a (16% 450 nm, 84% 550 nm) combination (within the 2-digit accuracy of the quoted percentages).

## 5. Other Measures

**5a. Hue and Saturation.** Instead of  $x$  and  $y$ , the two chromaticity coordinates, one can specify position on the Chromaticity Diagram by considering the color to be a mixture of “white” and a color on the boundary of the Diagram. The color on the boundary is called the target color’s “hue” while the fraction of the boundary color in the mixture is called the color’s “saturation.” Thus both light and dark pink would be described as having a red hue, but the light color would be said to be less saturated than the dark one.

As an example, consider a color described as “86% saturated 573 nm light.” On the Chromaticity Diagram, the color of this light would be 86% of the way from the “white” point at (.333,.333) to the 573 nm point on the boundary. Another way of saying it is that the color could be produced by mixing 573 nm yellow-green light and white light, with the yellow-green light being 86% of the mixture. To get that color’s  $x$  and  $y$  values, its Chromaticity Coordinates, you simply draw, right on a copy of the diagram, a straight line that goes from the “white” point to the 573 nm point on the boundary. Finally, locate the target color as the point that is 86% of the way from the white point to the boundary.

▷ Show that, for 86% saturated 573 nm light,  $x = 0.45$ ,  $y = 0.51$ .

A slight complication arises when the line drawn from the “white” point to the target point, and on to the boundary, hits the boundary at a point on the straight line section along the bottom of the Diagram. There the boundary points do not correspond to pure wavelengths but rather to mixtures of pure-wavelength light. One solution, good for anywhere on the boundary, is to specify the hue as the polar angle of the line that goes from “white” to the point in question and then on to the boundary.<sup>9</sup> For example, a  $0^\circ$  hue is “red,” a  $90^\circ$  hue is “green,” a  $180^\circ$  hue is “blue,” and a  $270^\circ$  hue is “purple.”

▷ Show that the color at (0.277,0.580) can be described as 61% saturated 541 nm light.

**5b. RGB.** The RGB method of specifying a color is useful when dealing with colors on a TV screen or on a computer’s color monitor. Color monitors (including the screens in ordinary color TV sets) have three phosphor dots next to each other at each location on the screen, and each

<sup>9</sup>This method is used in the color-specification part of the computer programming language CT (© Carnegie-Mellon University).

dot gives out a certain intensity of light at any particular instant. The dots are sufficiently small so the human eye does not resolve them individually, but only responds to the three as a group. The colors produced individually by each of the three types of phosphor dots are labeled “TV” on Fig. 2, and they are known by the names “Red,” “Green,” and “Blue.” Collectively, the three are referred to by the letters RGB.

All colors produced by a TV set or color monitor are within the triangle formed by connecting the three RGB points marked “TV” on Fig. 2. Any color within the triangle can be specified by the relative amounts of R, G, and B light which will produce that color. Any color outside the triangle cannot be produced by a TV set or color monitor using those standard phosphors. In practical terms, one can determine the amounts of R, G, and B necessary to produce a particular color by first combining any two of R, G, and B, then combining the resulting color with the other one of the R, G, and B set.

**5c. Brightness.** As one varies the total intensity of a light beam, its chromaticity coordinates stay fixed but its “brightness” changes. One can do this for all the colors on the Diagram at once by watching the Diagram as its brightness is changed continuously. The diagrams would all have identical shapes and boundaries but the labels we place on the diagram would vary from diagram to diagram. Thus, for example, as intensity decreased the white point on the Diagram would turn from white to light grey to dark grey and finally to black. The latter color, black, is the total absence of light, so it corresponds to zero intensity and a totally black diagram. Color charts are available that show the progression of color changes perceived as brightness alone is changed.

## Acknowledgments

We thank K. Franklin and J. Kovacs for their contributions to a precursor of this module. Professor Jim Linneman made a useful suggestion. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

## Glossary

- **black:** this is the “color” sensation produced by the absence of light.

- **brightness (of a particular color):** overall energy intensity of a particular light; e.g., “60 watts/square meter.” Brightness can be varied independently of hue and saturation.
- **chromaticity coordinates:** the *fractional* integrated responses of the  $x$  and  $y$  cones to a particular light beam. The numbers describe the “color” of a light beam but omit variations in perceived color produced by variations in intensity.
- **cones:** one of two types of nerve cells in the retina of the eye that act as light receptors. There are three varieties of cones, each with a different sensitivity to various wavelengths of the visible spectrum. The cones are responsible for color vision.
- **hue (of a particular color):** the *single-wavelength* color that can be mixed with white to produce the color being specified; e.g., “380 nm violet” is a “hue.” Hue can be varied independently of saturation and brightness.
- **monochromatic:** an adjective applied to a light beam, indicating that the beam contains only a single wavelength. In reality, any light beam has a range of wavelengths but that range can often be made so small that it can be considered to be a single wavelength for the purpose at hand. A device that emits essentially monochromatic light whose wavelength can be changed at will is called a “monochromator.”
- **RGB:** the three color beams used to excite the phosphors in television and computer screens; so-called because the three beams excite screen phosphors whose colors are red, green, and blue, respectively. Such screens can only produce colors whose chromaticity coordinates are within the chromaticity-diagram triangle defined by the chromaticity coordinates of the three screen phosphors.
- **saturation (of a particular color):** the fractional amount of the hue, in the mixture of hue plus white, needed to produce the color being specified; e.g., “15% saturated 650 nm red” is a mixture of 15 parts 650 nm red to 85 parts white: it is perceived in the brain as “light pink.” Saturation can be varied independently of hue and brightness.
- **spectral components:** the single-wavelength light beams that are the constituents of a multi-component beam. In practice, to analyze a spectrum that is continuous, we break it down into very narrow wavelength bands, called “bins,” and the amount of light energy in each bin

is measured. The bin light-energy values are plotted at their respective wavelength center-points and a smooth curve is drawn. The narrower the bins, the greater the accuracy of the resulting curve. A bin width greater than 10 nm will generally give only very crude results.

- **tri-stimulus response curves/values:** numbers that represent the responses, as functions of wavelength, of the three types of cones of a “standard observer,” when those cones are successively subjected to equal intensities at each visible wavelength.
- **white:** The color sensation produced by light that has chromaticity coordinates (0.333,0.333). This color also includes all shades of gray, obtained by keeping the “color” fixed and varying the intensity.



## A. Pure-Color Response and Chromaticity Values

$\lambda(\text{nm})$	tri-stimulus values			single-wavelength chromaticity coordinates	
	$x_\lambda$	$y_\lambda$	$z_\lambda$	$x$	$y$
400	0.014	0.000	0.068	0.1733	0.0048
410	0.044	0.001	0.207	0.1726	0.0048
420	0.134	0.004	0.646	0.1714	0.0051
430	0.284	0.012	1.386	0.1689	0.0069
440	0.348	0.023	1.747	0.1644	0.0109
450	0.336	0.038	1.772	0.1566	0.0177
460	0.291	0.060	1.669	0.1440	0.0297
470	0.195	0.091	1.288	0.1241	0.0578
480	0.096	0.139	0.813	0.0913	0.1327
490	0.032	0.208	0.465	0.0454	0.2950
500	0.005	0.323	0.272	0.0082	0.5384
510	0.009	0.503	0.158	0.0139	0.7502
520	0.063	0.710	0.078	0.0743	0.8338
530	0.166	0.862	0.042	0.1547	0.8059
540	0.290	0.954	0.020	0.2296	0.7543
550	0.433	0.995	0.009	0.3016	0.6923
560	0.595	0.995	0.004	0.3731	0.6245
570	0.762	0.952	0.002	0.4441	0.5547
580	0.916	0.870	0.002	0.5125	0.4866
590	1.026	0.757	0.001	0.5752	0.4242
600	1.062	0.631	0.001	0.6270	0.3725
610	1.003	0.503	0.000	0.6658	0.3340
620	0.854	0.381	0.000	0.6915	0.3083
630	0.642	0.265	0.000	0.7079	0.2920
640	0.448	0.175	0.000	0.7190	0.2809
650	0.284	0.107	0.000	0.7260	0.2740
660	0.165	0.061	0.000	0.7300	0.2700
670	0.087	0.032	0.000	0.7320	0.2680
680	0.047	0.017	0.000	0.7334	0.2666
690	0.023	0.008	0.000	0.7344	0.2656
700	0.011	0.004	0.000	0.7347	0.2653

## LOCAL GUIDE

Note: The graphical method described in Section 4c may be required on the exam you take so you should bring a ruler (preferably a clear one) to the Exam Room.

The book listed in this module's *ID Sheet* is on reserve for you in the Physics-Astronomy Library, Room 230 in the Physics-Astronomy Building. Tell the person at the desk that you want a book that is on reserve for CBI (a BOOK, not a reading). Then tell the person the name of the book you want.

## PROBLEM SUPPLEMENT

▷ **Warning:** this symbol marks problems scattered throughout the text. Those problems are part of this module's problem set: do all of them properly before attempting the problems below.

*Note:* The problems below also occur on this module's *Model Exam*.

- Starting with the 1931 CIE Tri-Stimulus Values in Appendix A, determine the Chromaticity Coordinates of: (a) 100% saturated 500 nm light and (b) 100% saturated 600 nm light.
- Using the results of Problem 1, determine the Chromaticity Coordinates of a 2:1 mixture of 500 nm and 600 nm light.

### Brief Answers:

- $(0.008, 0.538)$  *Help: [S-1]*
  - $(0.627, 0.372)$  *Help: [S-1]*
- $x = 0.21, y = 0.48.$  *Help: [S-2]*

## SPECIAL ASSISTANCE SUPPLEMENT

**S-1** (from TX-4d, PS-Problem 1, (a) and (b))

*Consultant:* do not help this student unless all of the relevant work indicated below is shown to you in the manner described.

*Student:* Go back to this module's *text*. Start at the beginning of the module and go through the text, understanding each paragraph before going on to the next. Work each "try-it" as you come to it, writing out all the steps explicitly on paper. If you get stuck on a "try-it," bring what you have done, up to the point where you became stuck, to a Consultant and get help on getting going again. If you finish all of the try-it's and still can't work this problem, bring your explicitly worked-out try-it's and your work on this problem, up to the point where you became stuck, to a Consultant.

**S-2** (from PS-Problem 2)

First, work on Problem 1, above, until you get the right answers. Write out every step you used in solving the try-it's and the problem. Do the same for this problem. If you still can't get this problem, bring all of your work to a Consultant for help.

**S-3** (from TX-4a,4c)

To derive Eq. 2 from Eq. 1, first formally solve Eq. 1 for  $f$ . Then, on a sketch imitating part of the Chromaticity Diagram, draw a line connecting point  $(x_1, y_1)$  and point  $(x_2, y_2)$  and going through the target point  $(x, y)$ . Draw straight lines connecting points  $(x_1, y_1)$ ,  $(x_2, y_2)$ , and  $(x_2, y_1)$ , making a triangle. Then connect points  $(x, y)$  and  $(x, y_1)$ . Now look at the sketch and see that the ratio  $d_1/d$  equals the solution of Eq. 1 that you just found for  $f$ .

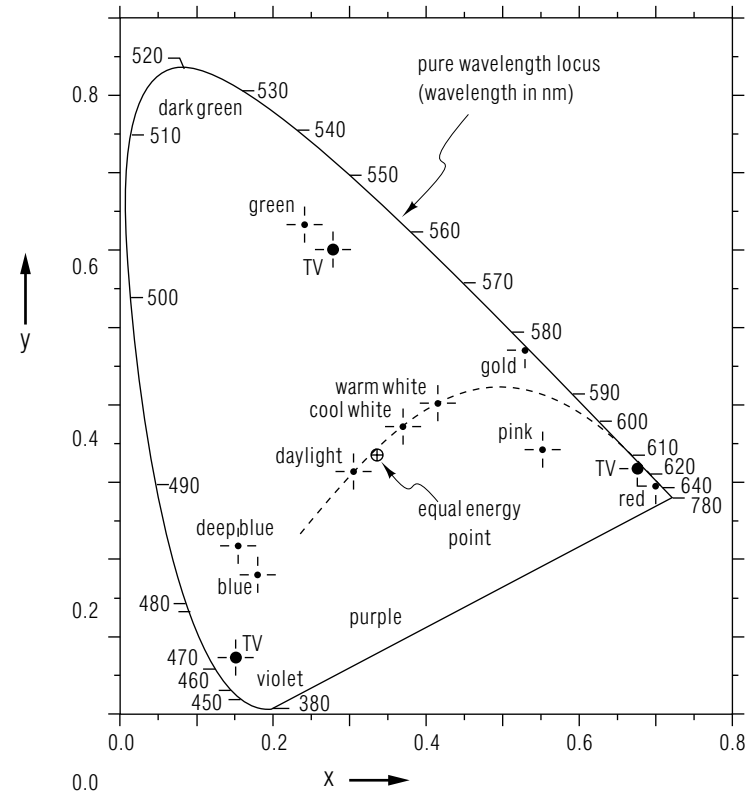
**S-4** (from TX-4b)

For  $(1 - f_1)$  substitute its equivalent,  $f_2$ . Then notice that, if you everywhere exchange the labels #1 and #2, the equations themselves do not change.

## MODEL EXAM

1. See Output Skills K1-K4 on this module's *ID Sheet*. One or more of these skills, or none, may be on the actual exam.
2. Starting with the 1931 CIE Tri-Stimulus Values, determine the Chromaticity Coordinates of: (a) 100% saturated 500 nm light and (b) 100% saturated 600 nm light.
3. Using the results of Problem 2, determine the Chromaticity Coordinates of a 2:1 mixture of 500 nm and 600 nm light.

$\lambda(\text{nm})$	tri-stimulus values		
	$x_\lambda$	$y_\lambda$	$z_\lambda$
400	0.014	0.000	0.068
420	0.134	0.004	0.646
440	0.348	0.023	1.747
460	0.291	0.060	1.669
480	0.096	0.139	0.813
500	0.005	0.323	0.272
520	0.063	0.710	0.078
540	0.290	0.954	0.020
560	0.595	0.995	0.004
580	0.916	0.870	0.002
600	1.062	0.631	0.001
620	0.854	0.381	0.000
640	0.448	0.175	0.000
660	0.165	0.061	0.000
680	0.047	0.017	0.000
700	0.011	0.004	0.000



### Brief Answers:

1. See this module's *text*.
2. See this module's *Problem Supplement*, problem 1.
3. See this module's *Problem Supplement*, problem 2.

