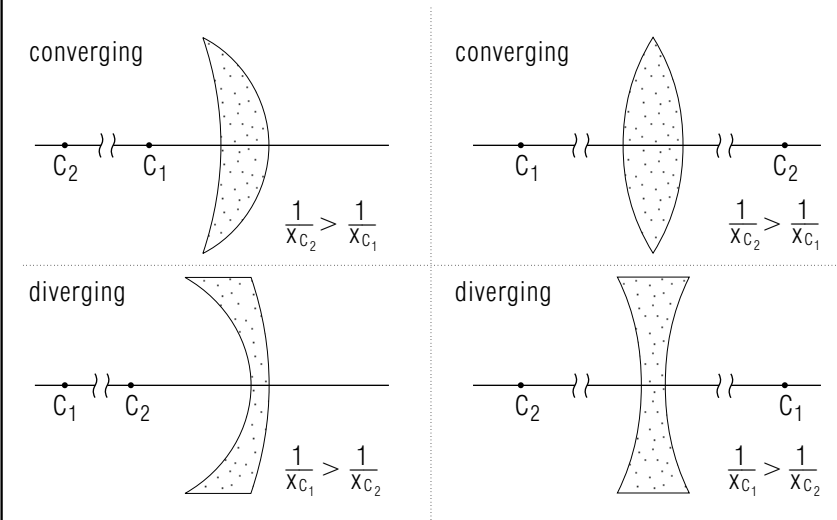


THIN SPHERICAL LENSES



THIN SPHERICAL LENSES

by
M. Brandl

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Input Skills:

1. Vocabulary: Descartes' formula for refraction, image focus, concave surface, convex surface (MISN-0-222), paraxial rays (MISN-0-221).
2. Trace rays across spherical boundaries using Snell's law (MISN-0-222).
3. Describe the types of object-image relationships found in geometrical optics. (MISN-0-221, MISN-0-222).
4. Describe the conditions under which Descartes' formula is a good approximation to reality.

Output Skills (Knowledge):

- K1. Vocabulary: converging lens, Descartes' formula for a thin lens, diverging lens, focal length, image focal point, lens-maker's formula, object focal point, real image point, thin lens equation, virtual image point, virtual object point.
- K2. Given Descartes' formula for a thin lens, derive the lens maker's formula and the thin lens equation.

Output Skills (Problem Solving):

- S1. Solve problems using the thin lens equation and the lens-maker's formula. Determine the locations of the focal points and image, determine whether the image is real or virtual, erect or inverted, and determine the magnification and image size.

Post-Options:

1. "Camera, Eye, Magnifier, Microscope" (MISN-0-263).

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THIN SPHERICAL LENSES

by
M. Brandl

1. Introduction

A lens is simply a piece of transparent material whose sides have been ground into curved surfaces. The most common materials are glass and plastic. Usually both surfaces of a lens are spherical, or one spherical and the other planar. Light passing through a lens undergoes refraction (it changes direction) at each of the two surfaces. We can always construct a lens-produced image by applying Descartes' refraction formula at each of the two surfaces. This module¹ treats the special case of the "thin" lens, one whose thickness is much smaller than the object and image distances involved. Then the laborious ray-tracing procedure can be replaced by a simple formula, also due to Descartes.

2. Descartes' Formula for a Thin Lens

2a. The Formula. The object and image positions of paraxial rays refracted by any thin lens (see Figs. 1 and 2) are related to the shape and

¹See "Refraction at Spherical Surfaces" (MISN-0-222).

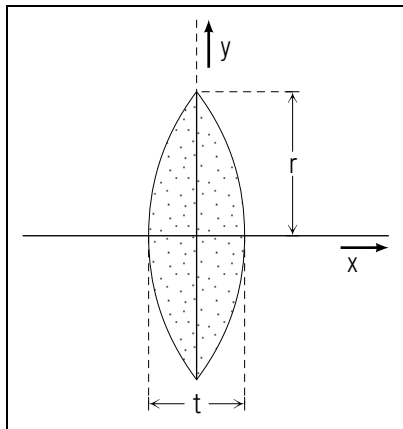


Figure 1. A typical thin lens and a coordinate system for measuring positions.

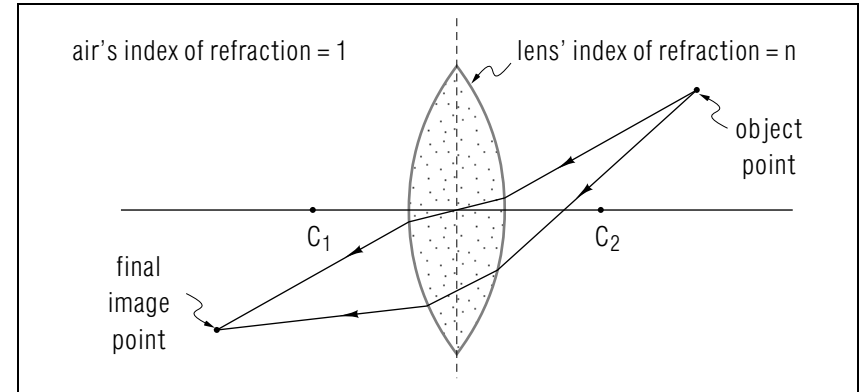


Figure 2. Quantities occurring in Descartes' formula. Here C_1 denotes the Center of Curvature of the right lens surface, the first surface encountered by the light, and C_2 the Center of Curvature of the left surface.

material of the lens by Descartes' formula:

$$\frac{1}{x_o} - \frac{1}{x_i} = (n - 1) \left(\frac{1}{x_{C_2}} - \frac{1}{x_{C_1}} \right). \quad (1)$$

where n is the index of refraction of the lens. The positions labeled "x" are measured along the principle axis of the lens with the coordinate origin being at the center of the lens:

$x_o = x$ -position of object,

$x_i = x$ -position of image,

$x_{C_1} = x$ -position of CC of first lens surface the light encounters,

$x_{C_2} = x$ -position of CC of second lens surface the light encounters.

where we have used "CC" as an abbreviation for "Center of Curvature." Descartes' formula, Eq. (1), is derived elsewhere and the symbols used in the formula are discussed further there.²

2b. Conditions for Validity of Descartes' Formula. Descartes' formula relating lens-produced objects and images is an accurate representation of reality only if the lens involved is thin and the light rays

²See the Appendix.

involved are paraxial. A typical thin lens is shown in Fig. 1. It is a disk of radius r and thickness t whose sides have been ground into spherical surfaces. If we wish to apply Descartes' formula to such a lens we must be sure that the lens radius r is much smaller than any of the four “ x ” distances in Eq. (1), so that only paraxial rays are used, and we must also be sure that the lens thickness t is much smaller than the “ x ” distances so the lens is indeed “thin.”

3. Focal Points

3a. Definition of Focal Points. The two focal points of a lens are critical components of optical formulas and of ray tracing techniques. They are defined in the same way as the focal points of a single refracting surface. The “object focal point” F is that object point which produces an image at infinity. This means that any ray passing through the point F before entering the lens will leave the lens traveling parallel to the principal axis (see Fig. 3). The “image focal point” F' is that image point which is produced by an object at infinity. This means that any ray traveling parallel to the axis before entering the lens will pass through the point F' after leaving the lens (see Fig. 4). Thus for rays that are in some region traveling parallel to the axis of a lens, there is either a point object at the lens' object focal point or a point image at its image focal point.

3b. Positions of the Focal Points: Focal Length. The position of a lens's object focal point, x_F , is obtained by setting $x_i = \infty$ in Eq. (1):

$$\frac{1}{x_F} = (n - 1) \left(\frac{1}{x_{C_2}} - \frac{1}{x_{C_1}} \right). \quad (2)$$

This is called “the lens-maker's formula.” Similarly, the position of the image focal point, $x_{F'}$, is obtained by setting $x_o = \infty$ in Eq. (1):

$$\frac{1}{x_{F'}} = -(n - 1) \left(\frac{1}{x_{C_2}} - \frac{1}{x_{C_1}} \right). \quad (3)$$

Notice that $x_{F'} = -x_F$, so the object and image foci are on opposite sides of the lens and are equidistant from it. The numerical value of this distance is called the “focal length” of the lens:

$$x_F \equiv \text{“focal length.”}$$

For example, a lens with a quoted focal length of +10 cm has its image focal point located on the exit side of the lens, 10 cm away from it.

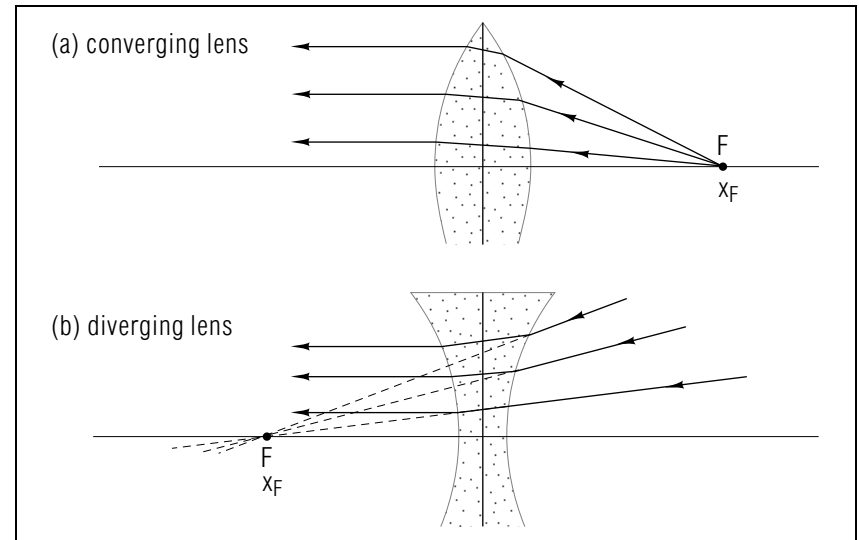


Figure 3. Object focal points for two types of lenses.

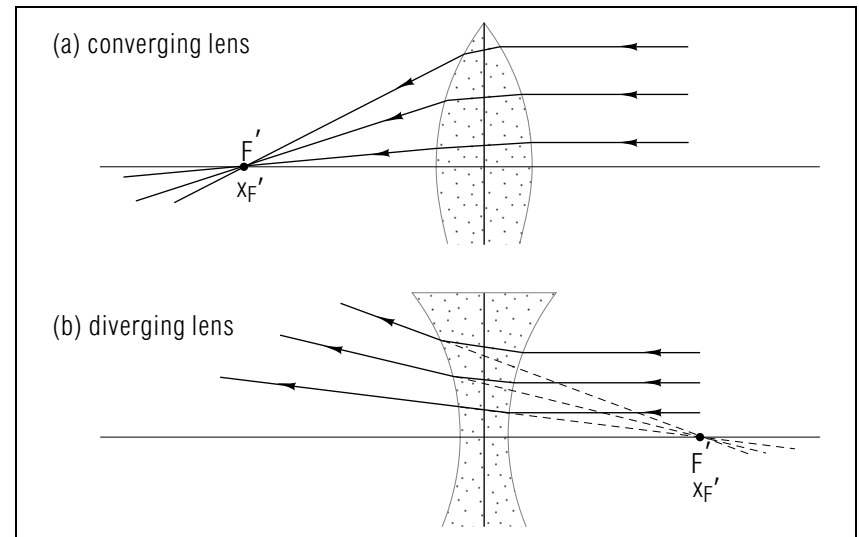


Figure 4. Image focal points for two types of lenses.

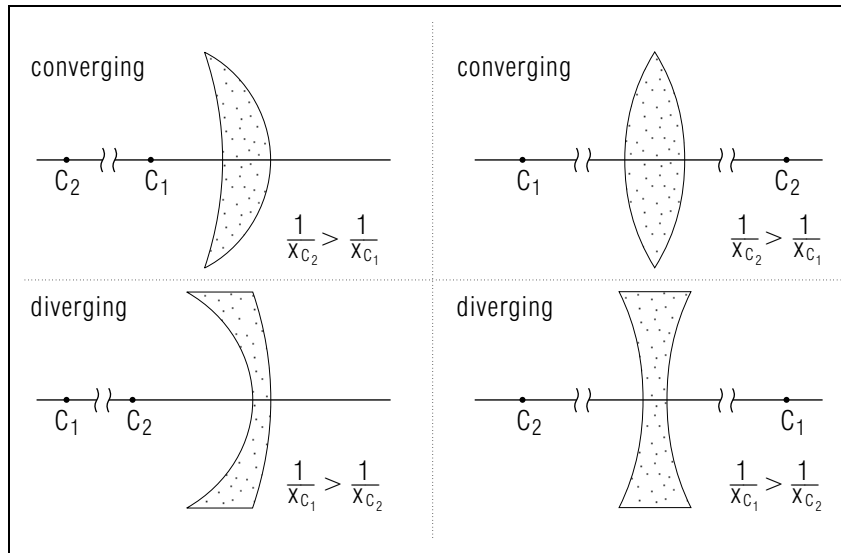


Figure 5. Converging and diverging lenses.

3c. Converging and Diverging Lenses. Figs. 3 and 4 show two different types of lenses, called “converging” and “diverging” (see Fig. 4). A converging lens is one which causes rays that enter it traveling parallel to the axis to converge toward the axis after refraction. A diverging lens is one which causes rays that enter it traveling parallel to the axis to diverge away from the axis after refraction. As can be seen from Figs. 3 and 4, a lens is converging if the image focal point is on the side of the lens where the light exits the lens, and diverging if on the side of the lens where the light enters it. Thus a thin lens is converging if its focal length is positive, diverging if it is negative.

3d. Determining “Converging” or “Diverging”. In Fig. 5 we show four combinations of lens surfaces, with the surface to the right labeled “1” and the surface to the left labeled “2.” Comparing the signs of the positions of the centers of curvature in that figure to Eq. (3), one sees that a lens is:

$$\begin{aligned} \text{converging if: } & \frac{1}{x_{C_2}} > \frac{1}{x_{C_1}}, \text{ hence } x_{C_1} > x_{C_2}, \\ \text{diverging if: } & \frac{1}{x_{C_1}} > \frac{1}{x_{C_2}}, \text{ hence } x_{C_2} > x_{C_1}. \end{aligned}$$

Notice that converging lenses are thicker in the center than at the perimeter while diverging lenses are thicker at the perimeter than in the center.

3e. The Thin Lens Equation. We can combine Eqs. (1) and (2) to give the “thin lens equation”:

$$\frac{1}{x_o} - \frac{1}{x_i} = \frac{1}{x_F}, \quad (4)$$

which is also called the “Gaussian form of the thin lens equation.” This is probably the most widely-used equation in all of introductory optics.

4. Ray Diagrams

4a. Overview. Given a lens and object, we pick a point on the object and trace several rays from it through the lens and beyond. The place where the rays intersect after refraction locates the image of that object point. We need only trace rays from two points on the object, since the positions of the two corresponding image points will tell us where the image is, how big it is, and its orientation. The first object point chosen is usually the “head” of the object. Its image is found by tracing the three “principal rays,” shown in Fig. 6 for the case of a converging lens, in Fig. 7 for a diverging lens. A second image point can be found by picking another object point and tracing the principal rays from it. In general, any two object points having the same x -coordinate will give rise to image points which have the same x -coordinate, so, if the object is perpendicular to the principal axis, the image will be also.

4b. Principal Rays for a Converging Lens. The first principal ray, the “parallel ray,” PR in Fig. 6, starts at the object point and travels parallel to the axis. After refraction by the lens it passes through the image focal point F' .

The second principal ray, the “focal ray,” FR in Fig. 6, starts at the object point and passes through the object focal point F . After refraction at the lens, it travels parallel to the axis.

The third principal ray, the “central ray,” CR in Fig. 6, starts at the object point and passes through the center of the lens, C_L . It is not refracted and continues on in the same direction.

The image point is the point at which all three rays intersect after refraction. We really need only two of the rays to find the image point;

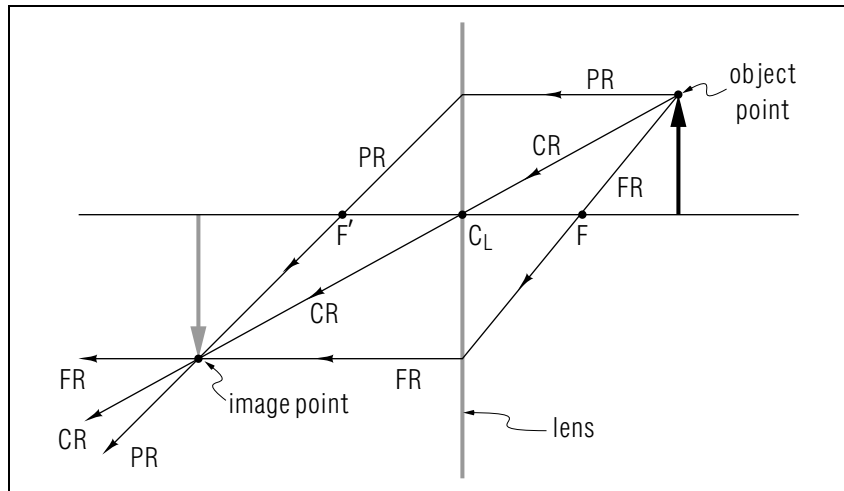


Figure 6. Image formation by converging lens.

the third serves as a check.

4c. Principal Rays for a Diverging Lens. Fig. 7 is a ray diagram for a diverging lens. The principal rays are drawn using precisely the same rules as for a converging lens, only now the end result looks a bit different, since the object focus is on the “image” side of the lens, and the image focus is on the object side. Therefore, the parallel ray does not “really”

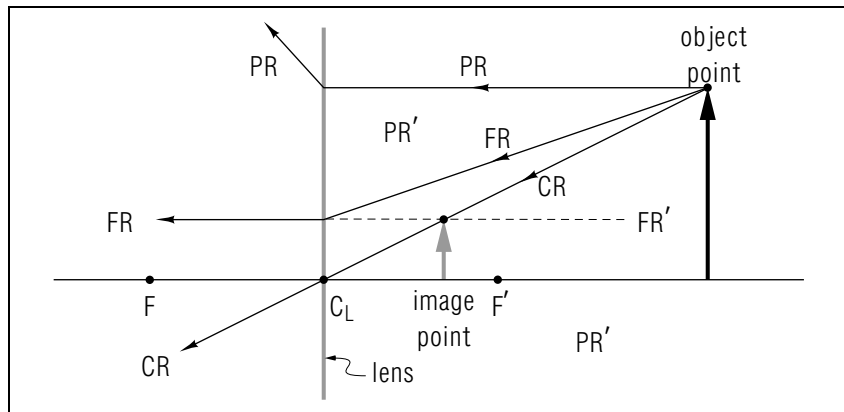


Figure 7. The principal rays locate the image formed by a diverging lens.

pass through the image focus after refraction, or, rather, a physical light ray which strikes the lens traveling parallel to the axis would not pass through the image focus. However, it is traveling as though it came from that point, so we draw its “virtual extension” PR' as passing through the image focus F' . Likewise, the focal ray does not really pass through the object focus F before refraction. It is just heading toward that point.

The parallel and focal rays never “really” intersect after refraction, so we must trace their virtual extensions PR' and FR' back to the point at which they intersect, and call that the image point. The central ray CR is the only one that physically passes through the image point, since it is not refracted by the lens.

The image formed in this case is a virtual one, since the light rays involved do not physically intersect at the image points, but only appear to have intersected there.

4d. The Central Ray is Unrefracted. An object-lens system’s “central” ray can be drawn as a straight line from object to lens to image: it is unrefracted by the lens. As this ray passes through the center of the lens, it “sees” the two surfaces of the lens as being essentially two planes parallel to each other. The ray therefore leaves the lens traveling parallel to its original direction. Furthermore, since the thickness of the lens is assumed to be negligible, the amount by which the ray’s path is shifted up or down by refraction at the two surfaces is also negligible. Therefore, we think of the central ray as being unaffected by the lens.

5. Objects and Images

5a. Real and Virtual Object Points. Most objects used in thin-lens-equation applications are physically real—that is, they are normal, every-day type physical objects. A lens simply catches the light rays diverging from an object and bends them to form the image. A lens doesn’t “know” whether there’s a physical object at the place from which the rays are coming. All it does is alter directions of rays that pass through it. Thus an object point can be either a point from which rays are diverging or a point toward which rays are converging (see Fig. 8):

A real object point is a point *from* which the rays approaching the lens appear to be diverging (see Fig. 8). Since we assume that the light always comes from the right of the lens, a real object point must be to the right of the lens. Therefore, a real object’s position enters into the

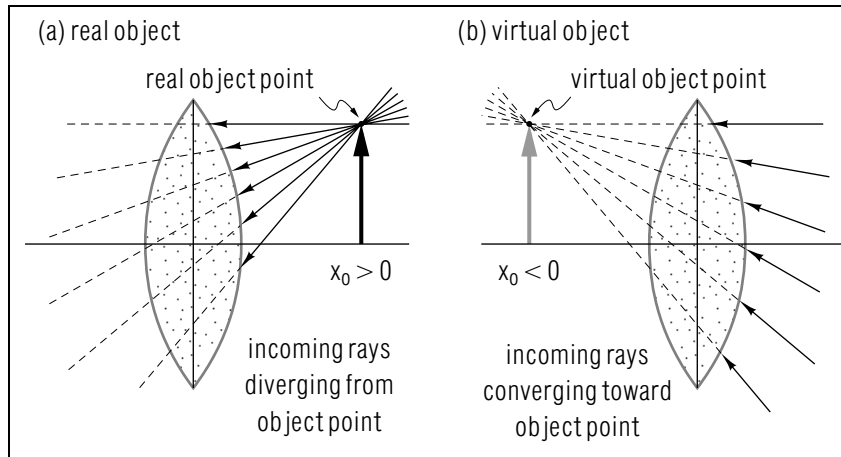


Figure 8. The light entering a lens can come from either a real or virtual object.

thin lens equation as a positive number: $x_o > 0$.

A virtual object point is a point *toward* which the rays approaching the lens appear to be converging (see Fig. 8). A virtual object point must therefore be to the left of the lens and it would enter the thin lens equation as a negative number: $x_o < 0$.

If the rays approaching the lens are diverging, we have a real object.

If the rays approaching the lens are converging, we have a virtual object.

A real object can be either a physical object or an image (either real or virtual) formed by another lens which is placed to the right of the lens under consideration. A virtual object can only be a real image formed by another lens which is placed to the right of the lens under consideration.

5b. Real and Virtual Image Points. Lenses, just like spherical mirrors or single refracting surfaces, can produce either real or virtual images.³

A real image point is a point at which the light rays actually intersect after refraction at the lens, (see Fig. 9). The rays leave the left side of the lens converging toward the image point, so a real image point must be to the left of the lens. Therefore, a real image has a position $x_i < 0$.

³See “Spherical and Planar Mirrors” (MISN-0-221).

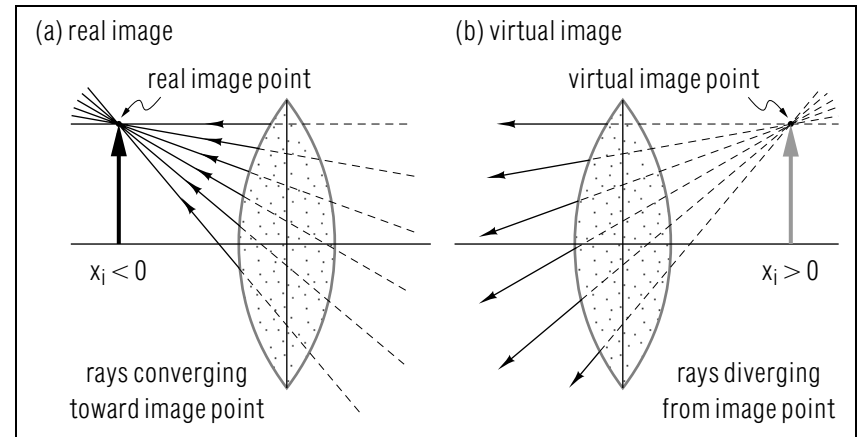


Figure 9. The light leaving a lens can form either real or virtual images.

A virtual image point is a point from which the light rays seem to be diverging after refraction at the lens, although they did not physically intersect there (see Fig. 9). Thus a virtual image point must be at the right of the lens (on the side the light comes from), and so a virtual image has a position $x_i > 0$.

5c. Image Size and Magnification. For the case of a thin lens and paraxial rays, triangulation on the central ray (see Fig. 10) produces the image height in terms of the object height and the object and image distances:

$$h' = \left(\frac{x_i}{x_o} \right) h. \quad (5)$$

The magnification is the ratio of image size to object size:

$$M \equiv \frac{h'}{h} = \frac{x_i}{x_o}. \quad (6)$$

If the image is erect, M is positive; if the image is inverted, M is negative.

5d. Turn the Lens Around: No Change. If we turn a thin lens around so the entry and exit surfaces are exchanged, Eq. (2) shows that when the lens is turned around, so

$$x_{C_2} \rightarrow -x_{C_1} \quad \text{while} \quad x_{C_1} \rightarrow -x_{C_2},$$

the equation is unchanged so optical properties of the lens are unchanged. Thus if we have a thin lens whose two surfaces have different radii, turning

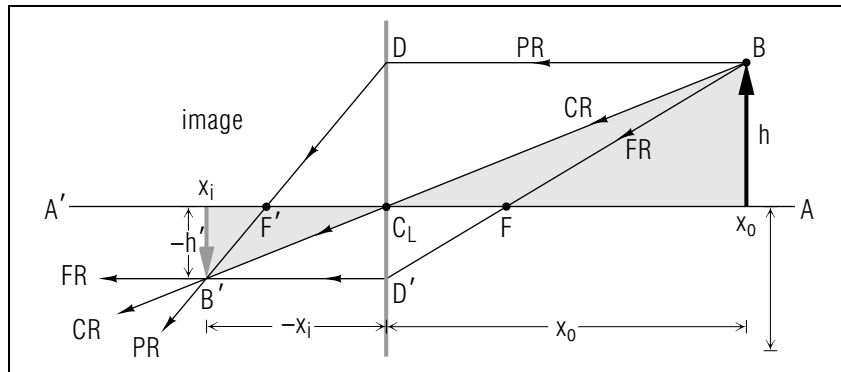


Figure 10. Triangulation on the central ray produces image height magnification. The positive distances $(-x_i)$ and $(-h')$ are the lengths of sides of triangles.

the lens around will have no effect whatever on an object's image!

5e. Two Successive Lenses. In many devices employing lenses the light must go through a number of lenses in succession: we will deal with the case of two lenses but the technique can be extended to any number. We will imagine a second lens to the left of the lens shown in Fig. 10: we will trace rays that go out from a point on the object, go through the first lens, go through the space between the lenses, go through the second lens, and finally come together at a point to form the two-lens-system image. Of course one can always trace rays through any optical system by drawing them as straight lines except where their directions change abruptly at air-glass interfaces in accord with Snell's Law. However, that is a tedious process so we will outline how one uses paraxial ray equations for the two-lens case.

The basic idea used in solving two-lens systems is to find the image that would be produced by the first lens alone, then use that image as the object for the second lens (treating it as being alone). For the case in Fig. 10, imagine a second lens located at the left edge of the sketch. Notice that the first lens's focal ray becomes the parallel ray for the second lens. However, the second lens's other two principal rays are not principal rays for the first lens so they are not shown in Fig. 10. Assuming the focal point of the second lens is between x_i and the left edge of the drawing, the second lens's *focal ray* goes from B to a point below D' , up through B' to F_2 (not shown), on to the second lens where it is bent again, and on to the final image.

▷ Draw a second lens in Fig. 10, give it a focal point, and sketch its principal rays all the way from the original object (to the right of the first lens) to the two-lens-system's final image.

6. Lens and Object Size Effects

6a. Effect on the Image. If, in a standard object-lens-image arrangement, the radii of curvature of the lens are kept constant while the size of the lens is decreased, the brightness of the image will decrease but the size and position of the image will remain unchanged. The image will become dimmer because fewer rays of light from the object are bent by the smaller lens to now form the image (see Fig. 11). The size and position of the image will remain unchanged because the rays still bent by the lens to form the image are following the very same paths they would have when the lens was larger, and that is because Snell's law bends them at the lens surfaces the same amount as when the lens was larger. Of course the rays that miss the lens are not bent by the lens: they just keep traveling in straight lines and do not participate in forming the image. If one takes the opposite tack and increases the size of the lens, thus increasing its "light gathering power," the image will become brighter.

6b. Effect on the Analysis. Since the position and size of an image are unaffected by the size of the lens, the usual analysis in terms of the three principal rays gives the correct answers even if two of those rays never actually hit the lens and thus do not participate in forming the image (see Fig. 11). One need only pretend that those two rays that go through the focal points are bent by the lens so they help form the image. This works because the size and position of the image would be unaffected by increasing the size of the lens until the two focal-point rays would indeed be bent by the lens. Thus one can mentally increase the size of the lens until the rays are bent, do the analysis, and then mentally decrease the size of the lens back to its given value. If the size of the lens was actually increased, the image would only become brighter: when the lens size was decreased back to its given value the image would only become dimmer.

Acknowledgments

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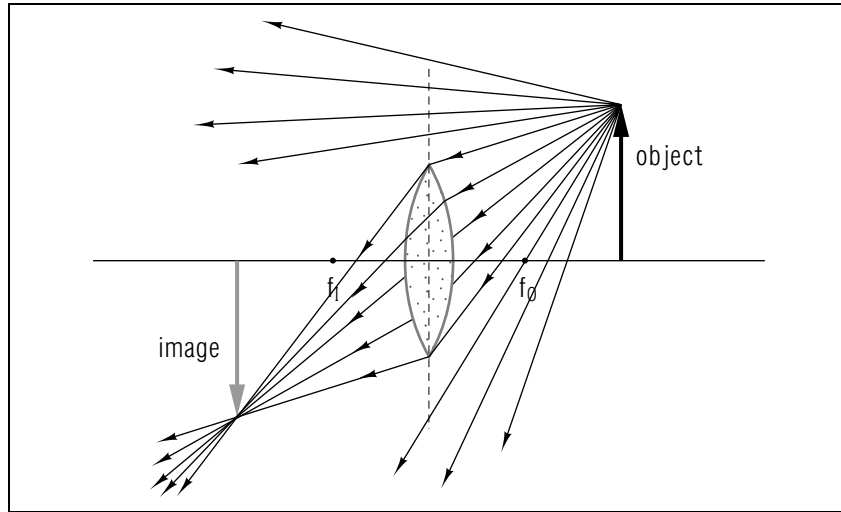


Figure 11. Rays from an object form an image even when the lens is smaller than the object.

sity.

Glossary

- **converging lens:** a lens which causes rays that enter it traveling parallel to the axis to converge toward the axis after refraction. A lens is converging if the image focal point is on the exit side. A thin lens is converging if its focal length is positive.
- **Descartes' formula for a thin lens:** a relationship between: (1) the object and image positions of paraxial rays refracted by a thin lens; (2) the index of refraction of the lens; and (3) the radii of curvatures of the two lens surfaces:

$$\frac{1}{x_o} - \frac{1}{x_i} = (n - 1) \left(\frac{1}{x_{C_2}} - \frac{1}{x_{C_1}} \right).$$

where “1” denotes the lens surface through which light enters the lens, and “2” denotes the surface through which the light leaves the lens

- **diverging lens:** A diverging lens is one which causes rays that enter it traveling parallel to the axis to diverge away from the axis after refraction. A lens is diverging if the image focal point is on the entry side. A thin lens is diverging if its focal length is negative.

- **focal length:** the distance from a lens to the point where rays from infinity are focused.
- **object focal point:** an object placed at this point produces an image at infinity, meaning the outgoing rays are parallel to the principal axis.
- **image focal point:** an image is produced at this point if an object is placed at infinity, meaning that the incoming rays are parallel to the principal axis.
- **lens-maker's formula:** The relationship between the position of a lens's object focal point, its index of refraction, and its two radii of curvature:

$$\frac{1}{x_F} = (n - 1) \left(\frac{1}{x_{C_2}} - \frac{1}{x_{C_1}} \right).$$

- **real object point:** a point from which the rays striking the lens appear to be coming. Since we assume that the light always comes from the right of a lens, a real object point must be to the right of the lens. Therefore, a real object's point enters into the thin lens equation as a positive number.
- **real image point:** a point at which the light rays coming from an object actually intersect after refraction by a lens.
- **thin lens equation:** a relation between the object, image, and focal-point distances for a thin lens:

$$\frac{1}{x_o} - \frac{1}{x_i} = \frac{1}{x_F}.$$

- **virtual image point:** a point from which the light rays seem to be diverging after refraction by a lens, although they do not physically intersect at that point. A virtual image point is always on the same side of a lens as the rays that are approaching the lens.
- **virtual object point:** an object point that is on the opposite side of a lens from the usual side for “real” objects. A virtual object point is usually the result of a prior lens in the path of the light rays approaching a second lens.

A. Derivation of Descartes' Formula

(for those interested)

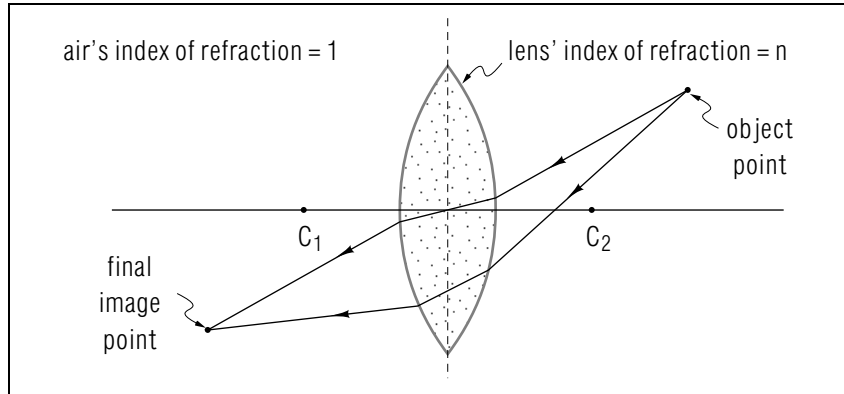


Figure 12. Quantities occurring in Descartes' formula.

Aa. Notation. We will use a notation involving the positions of points relative to a given origin rather than distances, lengths, and radii, thereby eliminating the problem of sign conventions. In deriving Descartes' formula for a thin lens, we will be using two separate coordinate systems, one for each surface of the lens. This situation is shown in Fig. 12. Both x -axes lie along the principal axis of the lens, which is defined as the line joining the centers of curvature of the two surfaces.

The origin of the unprimed (x) system is the point at which the first (right) surface intersects the principal axis, and the origin of the primed (x') system is the point at which the second (left) surface intersects the principal axis. If the thickness of the lens is t , then the relationship between the position of any point as measured in the primed and unprimed systems is, obviously,

$$x' = x + t. \quad (7)$$

Ab. Derivation. Consider now the situation shown in Fig. 12. Our lens is a slab of material of index of refraction n and of thickness t , whose sides are spherical surfaces. In the figure, x_{C_1} is the x -axis position of the center of curvature of the first lens surface that the light meets as it travels from right to left. Similarly, x_{C_2} is the x -axis position of the center of curvature of the second lens surface that the light meets as it travels from right to left. The lens is surrounded on all sides by air (index of refraction = 1).

An object is placed somewhere to the right of the lens. Light rays leaving an object point are refracted at the first surface. The intermediate

image point is the point at which these rays would intersect if they were not refracted at the second surface. The rays are refracted at the second surface, however, and now intersect at the final image point.

Descartes' formula for refraction at a spherical surface allows us to find the position of the intermediate image relative to the first surface:

$$\frac{1}{x_o} - \frac{n}{x_{ii}} = \frac{(1-n)}{x_{C_1}},$$

so

$$\frac{n}{x_{ii}} = \frac{1}{x_o} + \frac{(n-1)}{x_{C_1}}, \quad (8)$$

where these positions are all taken relative to the position of the first surface:

x_o = position of object,

x_{ii} = position of intermediate image,

x_{C_1} = position of center of curvature of first surface.

We can now apply Descartes' formula again at the second surface to find the final image position, using the intermediate image as the object seen by that surface. The intermediate image's position relative to the second surface is just

$$x'_{ii} = x_{ii} + t, \quad (9)$$

and Descartes' formula gives

$$\frac{n}{x'_{ii}} - \frac{1}{x'_i} = \frac{(n-1)}{x'_{C_2}}, \quad (10)$$

where these positions are all taken relative to the position of the second surface:

x'_{ii} = position of intermediate image,

x'_i = position of final image,

x'_{C_2} = position of center of curvature of second surface.

Equations (8), (9), and (10), taken together, can be solved to yield the final image position no matter how thick the lens is. The final expression would, however, be quite complicated. We can simplify things by assuming that the thickness of the lens is much smaller than any of the other distances involved in the problem. If we let $t \rightarrow 0$, the origins of the two coordinate systems will coincide, so $x' = x$ for any point. In particular, $x'_{ii} = x_{ii}$, $x'_i = x_i$, and $x'_{C_2} = x_{C_2}$, so Eqs. (8) and (10) become

$$\frac{n}{x_{ii}} = \frac{1}{x_o} + \frac{n-1}{x_{C_1}}, \quad (11)$$

and

$$\frac{n}{x_{ii}} - \frac{1}{x_i} = \frac{n-1}{x_{C_2}}. \quad (12)$$

Inserting Eq. (11) into Eq. (12) gives

$$\frac{1}{x_o} - \frac{1}{x_i} = (n-1) \left(\frac{1}{x_{C_2}} - \frac{1}{x_{C_1}} \right). \quad (13)$$

which is Descartes' formula.

PROBLEM SUPPLEMENT

Note: Problem 4 also occurs in this module's *Model Exam*.

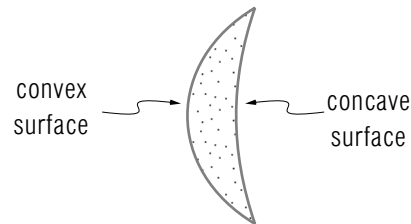
- A double-convex lens (i.e., both surfaces bulge out) has an index of refraction of 1.33. The radii of curvature of its surfaces are 0.10 m and 0.20 m.
 - Find the positions of the focal points. Is this lens converging or diverging?

Find the image position and magnification for an object placed at each of the positions given below. Is the image real or virtual, erect or inverted?

 - $x_o = +0.40$ m
 - $x_o = +0.30$ m
 - $x_o = +0.10$ m
 - $x_o = +4.0$ cm
 - Draw ray diagrams approximately to scale for cases (c) and (d), using two of the principal rays. Show the positions of the focal points and the image.
- When a real object is placed at a certain position ($x_o > 0$) to the right of a certain lens, a real image is formed 0.30 m to the left of the lens. When the object is moved 5.0 cm further to the right, to position $x_o + 5.0$ cm, the real image moves to a new position 0.20 m to the left of the lens. Using the thin lens equation for these two situations, find: *Help: [S-1]*
 - the original real object position x_o . Note there are two solutions for x_o , but only one corresponds to a real object.
 - the focal length of the lens (that is, the position of the object focus, x_F).
- A system is constructed of two converging lenses with their principal axes coinciding. The first (right-hand) lens has a focal length of 0.20 m, (i.e., $x_F = +0.20$ m is the first lens' object focal point position relative to first lens), and the second (left-hand) lens has a focal length of

+0.10 m. Find the position of the final image of an object at infinity produced by the whole system for the following separations between the two lenses, by using the image produced by the first lens as the “object” seen by the second. *Help: [S-2]* Is that second-lens “object” real or virtual? Is the final image real or virtual?

- 0.40 m
 - 0.30 m
 - 0.25 m
 - 0.10 m
 - Draw a ray diagram for the second lens in case (d) using the image from the first lens as the “object.”
4. A thin lens with an index of refraction equal to 1.50 has one concave surface and one convex surface:



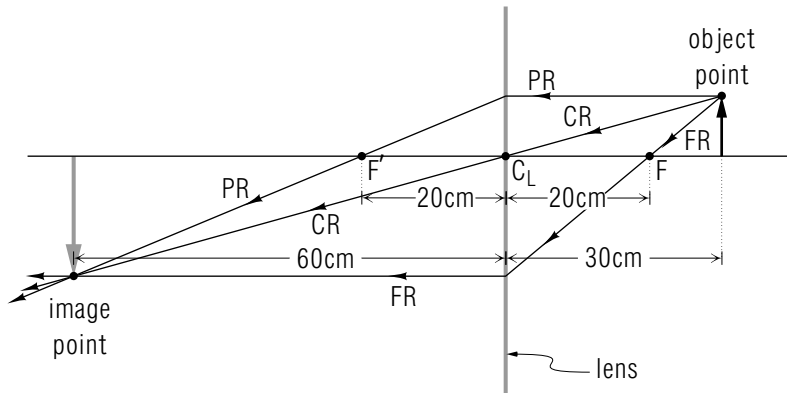
The concave surface has a radius of curvature of 0.60 m, and the convex surface has a radius of curvature of 0.20 m. An object is placed 0.40 m to the right of the lens.

- Find the positions of the focal points of the lens. Is the lens converging or diverging?
- Find the position of the image.
- Is the image real or virtual? Explain.
- Find the magnification of the image. Is the image erect or inverted?
- Draw a ray diagram (as close to scale as possible) showing how the image is formed by tracing two of the principal rays from the object through the lens. Explain what happens to each ray after refraction, and label the position clearly.
- Draw a diagram similar to that in part (e) above, but this time having the rays be only those that actually enter an eye looking at

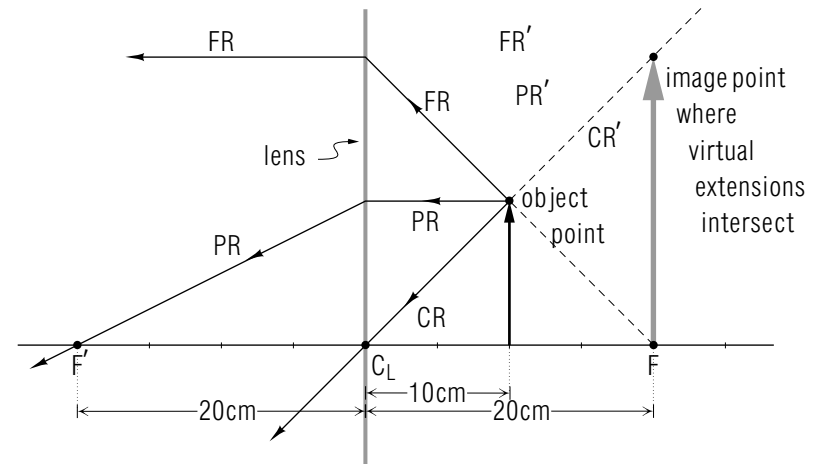
the object through the lens. Be sure to show the uppermost and lowermost rays that enter the eye (all rays outside that range miss the eye and hit the face, the body, or various parts of the room).

Brief Answers:

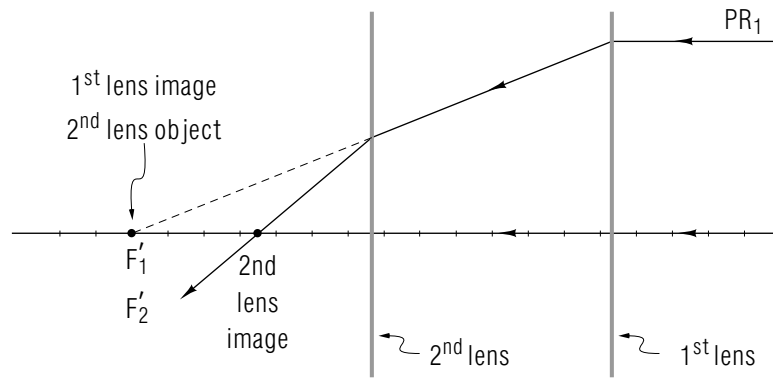
- All positions are relative to the lens, with x increasing to the right. Since the lens is double-convex, the center of curvature of the right surface must be to the left of the lens ($x_{C1} < 0$), and the center of curvature of the left surface must be to the right of the lens ($x_{C2} > 0$). The ambiguity of the problem lets us choose either $x_{C1} = -0.10$ m and $x_{C2} = +0.20$ m, or $x_{C1} = -0.20$ m and $x_{C2} = +0.10$ m. *Help: [S-3]*
 - $x_F = +0.20$ m is the object focus position (i.e., the focal length is $+0.20$ m) and $x_{F'} = -0.20$ m is the image focus position. The lens is converging.
 - $x_i = -0.40$ m. The image is 0.40 m to the left of the lens. $M = -1.0$. The image is real and inverted.
 - $x_i = -0.60$ m. The image is 0.60 m to the left of the lens. $M = -2.0$. The image is real and inverted.
 - $x_i = +0.20$ m. The image is 0.20 m to the right of the lens. $M = +2.0$. The image is virtual and erect.
 - $x_i = +5.0$ cm. Image is 5 cm to right of the lens. $M = +1.3$. The image is virtual and erect.
 - Ray diagram for part (c):



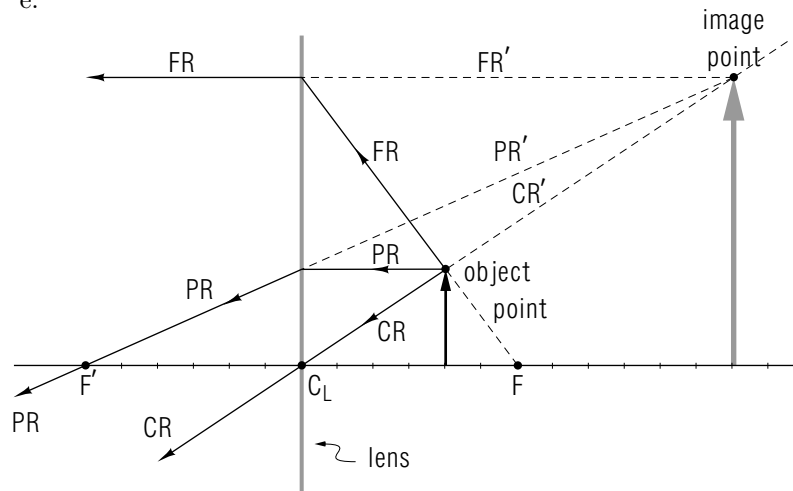
Ray diagram for part (d):



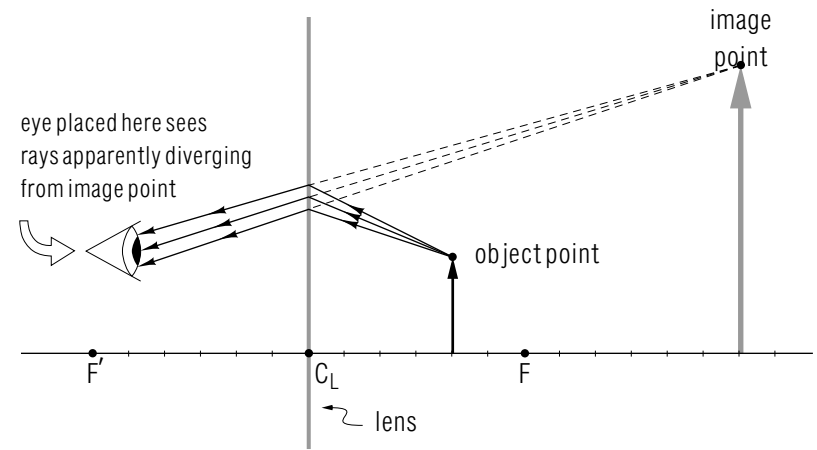
- $x_o = +15$ cm; that is, real object is 15 cm to the right of the lens. (The other solution is $x_o = -0.20$ m, which would mean we had a virtual object).
 - $x_F = +0.10$ m
(Using the other solution for x_o would have given us $x_F = -0.60$ m, which means that if we had not specified that we were using a real object, then we would have had two valid solutions for the object position and focal length.)
- The final image is 0.20 m to the left of the second lens. The second lens sees a real object; the final image is real.
 - The final image is at infinity (either $+$ or $-$, it doesn't matter which one). The second lens sees a real object; the final image is either real or virtual, since you can't make the distinction between real and virtual for an image at infinity.
 - The final image is 0.10 m to the right of the second lens; the second lens sees a real object; the final image is virtual.
 - The final image is 5.0 cm to the left of the second lens; the second lens sees a virtual object; the final image is real.
 - Note that there is no Principal Ray for the second lens.



4. a. $x_F = +0.60$ m, object focus is 0.60 m to right of lens (since the light comes from the right side) $x'_F = -0.60$ m; left of lens. Lens is converging, since object focus is on object side (or focal length is positive).
- b. $x_i = +1.20$ m; image is 1.20 m to right of lens.
- c. Image is virtual since light rays do not really diverge from image position, but only seem to come from there (see ray diagram following).
- d. $M = +3.0$. Image is erect.
- e.



f. Rays leaving the object point above or below those shown will miss the eye:



SPECIAL ASSISTANCE SUPPLEMENT

S-1

(from PS-2)

Hints:

$$\frac{1}{x_o} - \frac{1}{-30 \text{ cm}} = \frac{1}{x_o + 5 \text{ cm}} - \frac{1}{-20 \text{ cm}},$$

$$x_o^2 + (5 \text{ cm}) x_o - 300 \text{ cm}^2 = 0,$$

$$x_o = \frac{-5 \text{ cm} \pm \sqrt{25 \text{ cm}^2 + 1200 \text{ cm}^2}}{2}.$$

S-2

(from PS-3)

1. Solve for the image produced by the first lens alone, as though the second lens did not exist.
2. Erase the original object and first lens from your mind and sketch, leaving only the first-lens image.
3. In your mind and sketch, relabel the first-lens image, now calling it the second lens's object.
4. All you have left now is one object and one lens. Solve for the image.

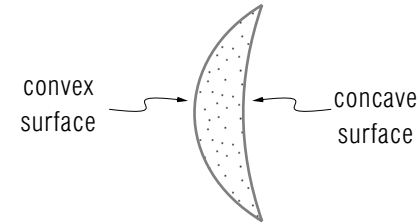
S-3

(from PS-1)

The text states that it doesn't matter which lens surface is called "left," which "right." You can verify this statement by solving the problem both ways.

MODEL EXAM

1. See Output Skills (Knowledge) in this module's *ID Sheet*.
2. A thin lens with an index of refraction equal to 1.5 has one concave surface and one convex surface:



The concave surface has a radius of curvature of 0.60 m, and the convex surface has a radius of curvature of 0.20 m. An object is placed 0.40 m to the right of the lens.

- a. Find the positions of the focal points of the lens. Is the lens converging or diverging?
- b. Find the position of the image.
- c. Is the image real or virtual? Explain.
- d. Find the magnification of the image. Is the image erect or inverted?
- e. Draw a ray diagram (as close to scale as possible) showing how the image is formed by tracing two of the principal rays from the object through the lens. Explain what happens to each ray after refraction, and label the position clearly.
- f. Draw a diagram similar to that in part (e) above, but this time having the rays be only those that actually enter an eye looking at the object through the lens. Be sure to show the uppermost and lowermost rays that enter the eye (all rays outside that range miss the eye and hit the face, the body, or various parts of the room).

Brief Answers:

1. See this module's *text*.

2. See Problem 4 in this module's *Problem Supplement*.