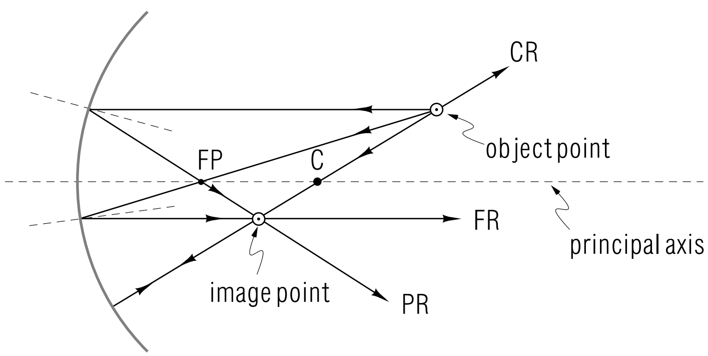
MISN-0-221

SPHERICAL MIRRORS



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

SPHERICAL MIRRORS
by
Kirby Morgan

Title: **Spherical Mirrors**

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Input Skills:

1. Vocabulary: geometrical optics, image, light ray, ray diagram, reflection (MISN-0-220); planar mirror, magnification MISN-0-260).
2. Define real and virtual images and illustrate with a sketch how they are different (MISN-0-260).
3. Determine the image position and magnification of an object placed in front of a plane mirror, using the ray-tracing method (MISN-0-260).

Output Skills (Knowledge):

K1. Vocabulary: concave, convex, principal axis, center of curvature, radius of curvature, paraxial rays, focal point, spherical mirrors, spherical aberration.

K2. State the definitions of the three principal rays and illustrate with a sketch.

K3. State Descartes' formula: define all symbols.

K4. State the focal length in terms of the radius of curvature of a spherical mirror, and illustrate with a sketch how the focal point is found experimentally.

Output Skills (Problem Solving):

S1. Apply Descartes' formula to spherical mirrors to determine image position and magnification, and also to determine whether an image is real or virtual, upright or inverted.

S2. Determine the positions, magnification, and types of images formed by a spherical mirror using the ray-tracing technique.

Post-Options:

1. "Refraction at Spherical Surfaces" (MISN-0-222).
2. "Thin Spherical Lenses" (MISN-0-223).

SPHERICAL MIRRORS

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1. Overview

1a. Why Non-Planar, Spherical. Non-planar mirrors are needed in situations such as these: (1.) when an image needs to be expanded, as with shaving, dental, and surgical mirrors; (2.) when an image needs to be compressed, as with wide angle automotive mirrors; and (3.) when we need a nearby image of a far-away object, as with astronomical telescope mirrors. In contrast to the planar mirror image, which is always virtual, upright, unmagnified, and equidistant from the mirror,[1] the non-planar mirror image may be real or virtual, upright or inverted, of different size than the object, and at a different distance from the mirror. Such non-planar mirrors are usually spherical, both because they are then easy to manufacture and because they usually produce a reasonable compromise between competing desirable image qualities.

1b. Four Design Methods. The designing of an object-mirror-image system is carried out either through ray tracing or through formal mathematics and, within each of those two methods, either approximately or exactly. Altogether, then, there are four methods. The two "exact" methods are used in all cases requiring high precision, but they are laborious unless sophisticated "computer assisted design" programs are used. The approximate formal method is generally used to study the approximate effect of varying the design parameters. The approximate ray tracing method provides a picture of the design and serves as a check on the formal design. All four methods use the law of reflection plus the knowledge that an image point is where various rays emanating from an object point either all intersect or appear to all intersect. Repeating this procedure for various points on the object allows one to build up the image, point by point.

1c. Concave and Convex Mirrors. A spherical mirror (i.e. a mirror in the shape of a partial spherical shell) can be either "concave" or "convex." If the reflecting surface is on the same side of the partial shell as the center of curvature, the mirror is said to be "concave." If the reflecting surface is on the other side, the mirror is said to be "convex."

[1] See "Reflection and Planar Mirrors" (MISN-0-260).

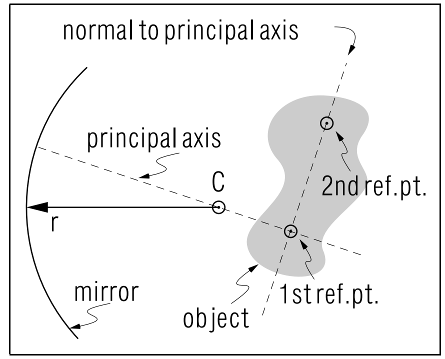


Figure 1. Relationship of principal axis to mirror's center of curvature (C) and to object's reference points.

2. Exact Methods: Principal Rays

2a. The Principal Axis. A line drawn through both the "center of curvature" of a spherical mirror and a point on an object placed in front of the mirror is called the "principal axis." This is shown in Fig. 1 for an object of arbitrary shape. A line is drawn normal to the principal axis and a second reference point is chosen that lies on the object and on this line. The whole diagram is then rotated so that the principal axis is horizontal (see Fig. 2). It is traditional to replace the two reference points by that simplest of oriented line segments, an arrow, (see Fig. 2). After constructing the image points corresponding to the head and tail of the arrow, we will be able to sketch the rest of the object's image without further optical construction.

2b. The Three Principal Rays. An infinite number of light rays emanate from an object point in front of a spherical mirror, but we use

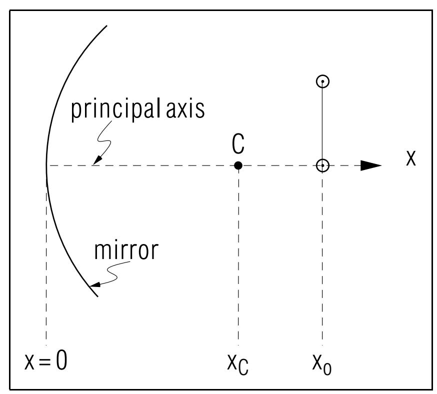


Figure 2. Illustration of object distance x_o and center-of-curvature distance x_C.

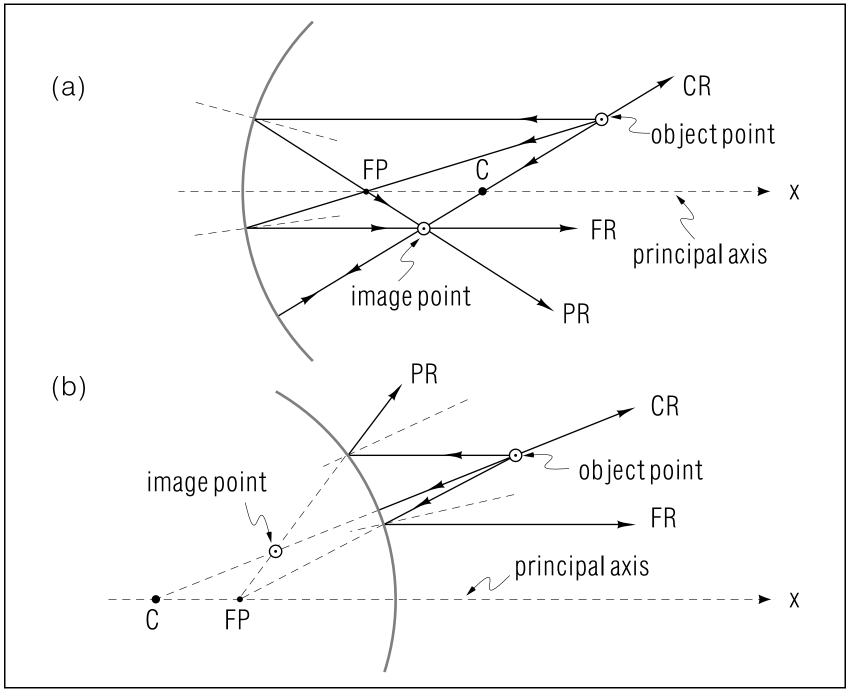


Figure 3. Principal rays, illustrated for (a) concave mirror and (b) convex mirror. The dashed lines at the mirror surfaces are surface normals (radial lines), used for constructing the reflecting angles.

just three of the rays to construct the approximate image point. These three rays are called the "principal rays" for the system of object point and mirror. Fig. 3 shows the three principal rays, each of which reflects from the mirror's surface according to the basic rule of reflection: the angle of incidence equals the angle of reflection. The three rays are:

1. The "Parallel Ray" (PR in Fig. 3), which leaves the object point and heads toward the mirror on a path parallel to the principal axis.
2. The Focal Ray (FR), which leaves the object point and heads toward the mirror in a direction such that after reflection it travels parallel to the principal axis.
3. The Central Ray (CR), which leaves the object point and heads toward the mirror along a mirror radius. Upon reflection, this ray

travels directly back along its incident path because it strikes the mirror surface at normal incidence.

3. Constructing the Image

3a. Finding an Image Point. An object point's "real" image point is found by constructing the three principal rays, as shown in Fig. 3, and finding the after-reflection point at which they intersect. However, if the rays are diverging from one another after reflection from the mirror's surface, there is no place where they intersect and hence the image is not "real." For this case the rays must be extended backward in order to find their after-reflection point of apparent intersection: in this case the image is "virtual" rather than "real."

3b. Paraxial Rays and "Focus". In general, the three principal rays from an object point will not intersect in the neighborhood of a single point unless they travel nearly parallel to the principal axis. Rays that are indeed nearly parallel to the principal axis are called paraxial rays and are important since they produce "sharp" images. Rays which are not paraxial produce an object-point image that is spread out, a "fuzzy" image. This effect is called "spherical aberration" and it is present to some extent in the images of all objects except those an infinite distance from the mirror (an infinite distance makes such rays paraxial).

3c. Constructing an Arrow's Image. The image of a vertical object-arrow situated in front of a spherical mirror is found by tracing the principal rays from the object's head point to its image point and

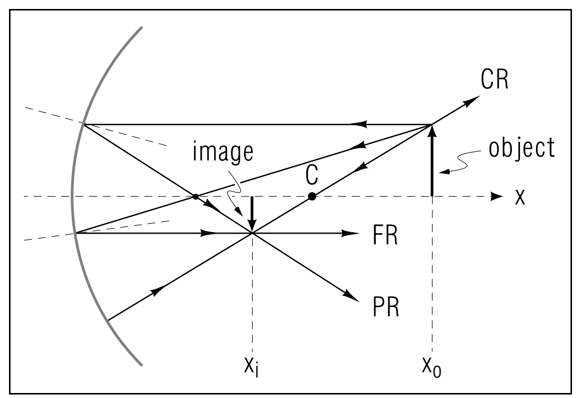


Figure 4. The tail of the image arrow is on the principal axis, at the same x position as the head.

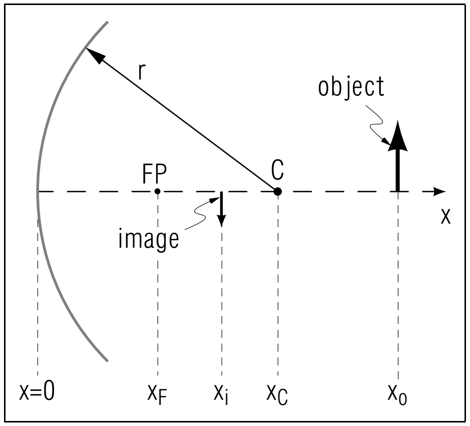


Figure 5. Definitions of quantities used in Descartes' formula. The center of curvature is C, the focal point is FP.

then sketching in the rest of the image arrow. Since the object arrow's tail is located on the principal axis, as in Fig. 4, the image tail-point will also lie on the principal axis. This is because its three principal rays entirely coincide with that axis. The image head-point is at the intersection of the three principal rays coming from the object head-point, as in Fig. 4. For paraxial rays, construction of any two of the three rays is sufficient to determine the image point. The third ray can be used to check the intersection of the other two.

3d. Classifying the Image. Images may be classified as real or virtual, upright or inverted, enlarged or reduced. For example, in Fig. 4 the image is real, inverted, and smaller than the object (i.e. its magnification or ratio of image size to object size is less than one). For highly paraxial rays, the magnification M can also be quite accurately expressed in terms of the object and image heights or in terms of the object and image distances:

$$M \equiv \frac{h_i}{h_o} = -\frac{x_i}{x_o} \qquad \text{approximately.} \tag{1}$$

A negative magnification will mean that the image is inverted. For an object in front of a spherical mirror, all real images are inverted and all virtual ones are upright. *Help: [S-1]* Also notice that comparing different mirrors, using a fixed object distance, the magnification is proportional to the image distance.

4. Approximate Formal Method

4a. Descartes' Formula. Descartes'[2] formula for the position of an image formed by highly paraxial rays from a spherical mirror is usually quite accurately given by:

$$\frac{1}{x_o} + \frac{1}{x_i} = \frac{2}{x_C} \qquad \text{approximately,} \tag{2}$$

where x_o is the distance along the principal axis from the mirror to the object, x_i is the distance from mirror to image, and x_C is the distance from the mirror to its center of curvature (see Fig. 5). Any distance measured on the same side of the mirror as the reflecting surface is positive; on the other side, negative. Thus for a concave mirror x_C is positive; for a convex mirror, negative.

4b. Focal Point, Focal Length. Descartes' formula can also be written in terms of the focal length of a spherical mirror. When light is incident parallel to the principal axis of a spherical mirror (as it would from a very distant object on the axis), the point at which the reflected paraxial rays (or their backward extensions) intersect is called the focal point. This is illustrated in Fig. 6. The distance x_F, measured along the principal axis from the mirror to the focal point F, is known as the focal length of the mirror, and can be determined by letting $x_o \to \infty$ in Descartes' formula, Eq. (2). This immediately gives for the location of the focal point:

$$x_F = x_C/2 \qquad \text{approximately.} \tag{3}$$

Descartes' formula can thus be rewritten in terms of the focal length as:

$$\frac{1}{x_0} + \frac{1}{x_i} = \frac{1}{x_F} \qquad \text{approximately.} \tag{4}$$

Since $x_F = x_C/2$, it is positive for concave mirrors and negative for convex ones, as is x_C.

4c. Example of Use of Descartes' Formula. Given an object 2 cm high placed 3 cm from a convex mirror that has a radius of curvature of 12 cm, you can find the position and size of the image and the focal length of the mirror. Using Descartes' formula, we see at once that the image is 2 cm from the mirror on the side opposite that of the object. *Help: [S-2]* The focal point is calculated, using Eq. (3), to be 4 cm beyond the image. *Help: [S-3]* Finally, Eq. (1) gives the image as 4/3 cm high, upright, and virtual. *Help: [S-4]*

[2] Descartes is pronounced "day-cart′."

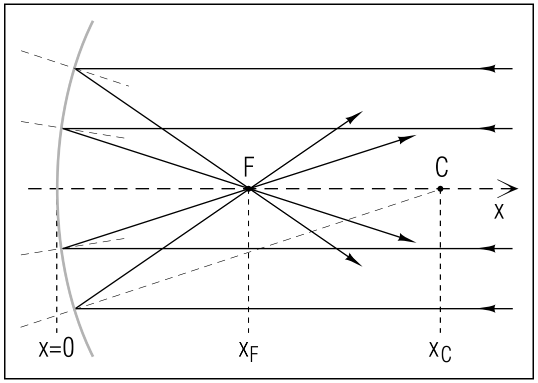


Figure 6. Parallel incident rays are focused at a single point, the focal point. The dashed radial lines all go through C.

4d. Concave → Planar → Convex as Radius Decreases. It would be amusing to watch the change in an object's mirror-formed image if the radius of curvature of the mirror could be gradually changed from highly concave to planar to highly convex while keeping the mirror and the object position fixed. Let us carry out the experiment mathematically with the aid of Descartes' formula. First, with the mirror highly concave, the radius will be small and positive so the image will be located just beyond the focal point: it will be small, real and inverted. *Help: [S-5]* As R is increased, one can follow the image's four characteristics (position, size, image type and orientation) by mentally working Descartes' formula. There are interesting regions marked by sharp boundaries, and the characteristics one finds for the planar mirror ($R \to \infty$) are in agreement with everyday observation. In order for Descartes' formula to change continuously, after $R \to \infty$ it should be increased away from negative infinity (toward zero), and the mirror will finally end up being highly convex. One finds that, for convex mirrors, the image is always virtual, while for concave mirrors the image is virtual only if the object is inside the focal point ($x_o < x_F$) and real otherwise ($x_o > x_F$). It is possible to construct a graph representing these characteristics of the image as the radius of curvature is varied: you may find it interesting to make such a graph yourself, plotting each of the image properties versus the inverse of the radius. We suggest labeling the regions of image and mirror type.

5. Approximate Graphical Method

If the principal rays are paraxial, the process of plotting their paths can be simplified. First, the spherical mirror is replaced by a plane surface

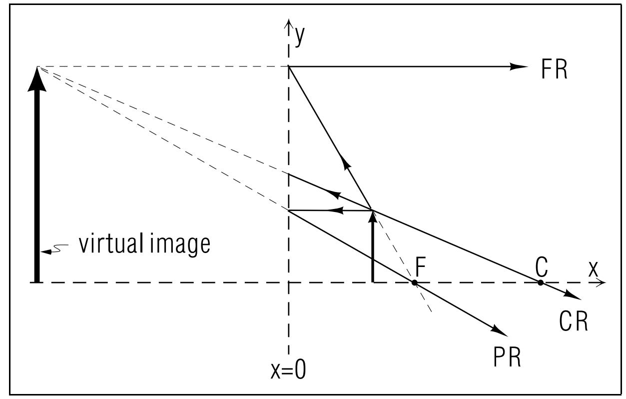


Figure 7. Illustration of "paraxial ray construction" for spherical mirrors (see text). Here the object is inside the focal point so the image is virtual.

at the same position. The principal ray paths are then determined solely by the positions of the object point, focal point, and center of curvature rather than by apparent angles of incidence and reflection. Fig. 7 gives an illustration of this "ray-tracing" technique. To enhance the accuracy of the method, the x and y directions are plotted to different scales. This is permissible just as long as the three principal rays are drawn as "reflecting from" the surface at $x = 0$ in such a way as to pass through the F and C points properly.

Acknowledgments

This module is based on one written by Peter Signell. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

Glossary

- **center of curvature**: for a lens or mirror, the center of the (imaginary) circle of which the lens or mirror's arc is a part.
- **central ray, mirror**: a light ray that leaves the object point and heads toward the mirror along a mirror radius. Upon reflection, this ray travels directly back along its incident path because it strikes the mirror surface at normal incidence.
- **concave**: a curved surface that looks like the inside of a spherical shell.

- **convex**: a curved surface that looks like the outside of a spherical shell.
- **Descartes' formula**: is used to calculate the position of an image formed by paraxial rays striking a spherical mirror:

$$\frac{1}{x_o} + \frac{1}{x_i} = \frac{1}{x_F} \,,$$

  where x_o is object distance, x_i is image distance, x_F is mirror's focal length.
- **focal length**: the distance from a lens or mirror to its focal point along its principal axis.
- **focal point**: point of intersection of reflected rays that approached the lens or mirror parallel to its principal axis.
- **focal ray, mirror**: a light ray that leaves the object point and heads toward the mirror in a direction such that after reflection it travels parallel to the principal axis. Either the ray or its backward extension goes through the focal point,
- **parallel ray**: a ray that leaves the object point and heads toward the mirror on a path parallel to the principal axis.
- **paraxial rays**: rays that travel nearly parallel to the principal axis of a lens or mirror. The closer the rays are to being parallel to the principal axis, the better the focus but the less they are useful.
- **principal axis**: a line drawn through a lens' or spherical mirror's center of curvature and through a reference point on the object being observed.
- **principal rays**: the three rays (central, focal, and parallel) used to construct the image of an object. Only two rays are necessary for construction of the image.
- **radius of curvature**: distance from a lens or mirror surface to its center of curvature.
- **spherical aberration**: A defect of spherical mirrors that arises because the light rays from a single point on the object do not form a single point on the image, thus producing a blurred or "fuzzy" image.
- **spherical mirror**: A mirror in shape of a partial spherical shell.

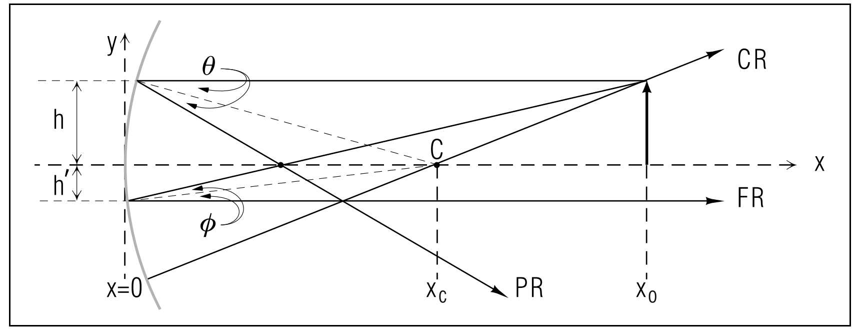


Figure 8. Definition of the symbols θ, ϕ, h, h' occurring in the ray equations.

A. Exact Formal Method

(for those interested)

Aa. The Exact Ray Equations. Using a little trigonometry, and the angle-distance definitions shown in Fig. 8, we find for the y-coordinates of the three rays:

$$y_{CR}(x) = \frac{h(x - x_C)}{x_0 - x_C};$$

$$y_{PR}(x) = h + (\Delta - x)\tan 2\theta,$$

where:

$$\Delta \equiv x_C(1 - \cos\theta), \qquad \theta \equiv \sin^{-1}(h/x_C);$$

$$y_{FR}(x) = -h' = -x_C \sin\theta,$$

and where ϕ is the solution to:

$$x_C \sin\theta = x_o \tan(2\theta) - h - x_C(1 - \cos\theta)\tan(2\theta).$$

Note that each of the ray equations has the form: $y = mx + b$. Also note that $y_{PR}(0) > h$.

The point of intersection of any two of these rays can be found by equating their y-values and solving for the resulting x-value and its corresponding y-value.

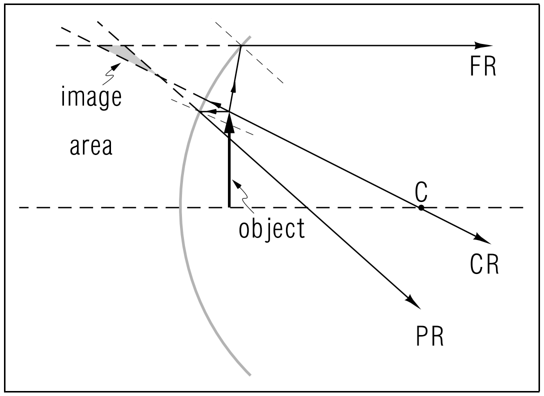


Figure 9. Illustration of spherical aberration. The shaded area is bounded by the principal rays.

Ab. Proof of Spherical Aberration by Ray Tracing. If one makes an exact trace of the three principal rays from a point object, it quickly becomes apparent that the three rays do not intersect in a single point. This is illustrated in Fig. 9. Note that the image of the object arrow-point is spread out over a large region. This "spherical aberration" means that one can not really specify a single x-value for the image position. This is the origin of the "approximate" labels on the equations in the text of this module.

Ac. Paraxial Rays, Aperture, Spherical Aberration. Suppose we continuously shrink the size of the object in Fig. 9 so that the angles between the principal rays and the principal axis become smaller and smaller (Fig. 10). When these angles have become much smaller than one radian, we find that the amount of spherical aberration then decreases as

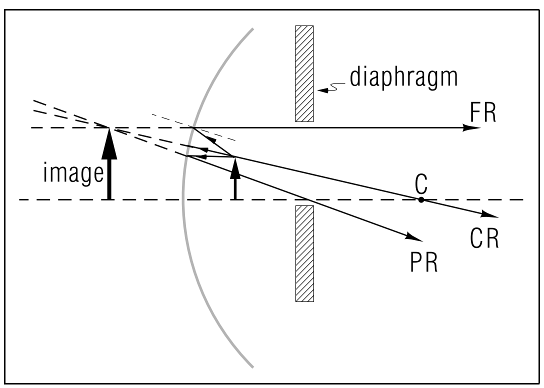


Figure 10. Same set-up as in Fig. 9 but with wide-angle rays blocked off by a diaphragm. Note the precise image. The diaphragm's opening is then said to be a "small aperture."

the squares of those small angles. Thus a small enough aperture will produce an aberration that is negligible, as is the case shown in Fig. 10. Put another way, one can avoid spherical aberration by using spherical mirrors in situations where only paraxial rays contribute to the image.

B. The Paraxial Approximation

Ba. Paraxial Rays and Descartes' Formula[]. In order for a point object to have a point image, we must require there to be a single intersection point for all reflected rays. The x-position of that intersection is the image position x_i, so the principal rays in Fig. 8 must obey the condition:

$$y_{CR}(x_i) = y_{PR}(x_i) = y_F R(x_i).$$

This condition is not satisfied unless the rays are paraxial so that we can make the approximations (assuming angles are expressed in radians): $\tan(2\theta) \approx 2\theta$, $\cos\theta \approx 1$, $\sin\theta \approx \theta$, $\tan(2\phi) \approx 2\phi$, $\cos\phi \approx 1$, and $\sin\phi \approx \phi$. Of course these approximations are only valid for angles much smaller than one radian. With these replacements the ray equations reduce to:

$$y_{CR}(x) = \frac{h(x - x_C)}{x_o - x_C} \tag{5}$$

$$y_{PR}(x) = h\left(1 - \frac{2x}{x_C}\right) \tag{6}$$

$$y_{FR}(x) \approx \frac{hx_C}{2x_o - x_C} \tag{7}$$

These three paths do in fact intersect at the position x_i given by Descartes' formula, as can be easily verified.

Bb. Fractional Error. The fractional error incurred in making a small angle replacement, to produce a paraxial ray, is approximately the square of the ray's angle to the axis in radians.[3] If, for example, θ and ϕ in Fig. 8 are each 0.1 radian, the error will be about 1%. This assumes that non-paraxial rays are not permitted to contribute to the image, either because of the limited size of the mirror or because such rays are blocked by a shield. Descartes' formula is quite accurate, and the image is quite sharp if the rays are paraxial:

$$\theta^2, \phi^2 \ll 1\,(\text{radian})^2\,.$$

[3] For example, if θ is expressed in radians then $\cos\theta = 1 - \theta^2/2 + \ldots$ so the fractional error is $\theta^2/2$. See "Taylor's Series for the Expansion of a Function About a Point" (MISN-0-4).

PROBLEM SUPPLEMENT

1. An object is placed 15 cm in front of a concave spherical mirror which has a radius of curvature of 60 cm.

   a. Determine the distance from the mirror to the focal point and to the image.

   b. If the object is 1 cm high, determine the height and orientation of the image, and whether it is real or virtual.

   c. Draw a ray-tracing diagram which graphically demonstrates that your solutions to parts (a) and (b) are correct. [B]

2. As above, but given that the mirror is convex with a radius of 25 cm and that a 1 cm high object is 15 cm from the mirror. [G]

3. Draw a sketch demonstrating that paraxial rays incident on a spherical mirror can produce significant spherical aberration, hence an indistinct image, if the mirror has too much curvature. [E]

4. As in Problem 1, but given that the mirror is concave with a radius of 1.00 m. The object distance is 0.75 m and its height is 3 cm. [A]

5. As in Problem 1, but given that there is an upright virtual image the same size as the object and that the 1/2 inch high object is located 1 foot from the mirror.

   d. Determine the radius of curvature of the mirror. NOTE: This problem involves using the law of reflection directly. [D]

6. (just for those interested) As in Problem 1, but the mirror is convex with a radius of 30 cm and a 0.5 cm high virtual object is at a distance 24 cm from the mirror ("on" the unsilvered side, being an image produced by another mirror or lens).

   d. Determine the magnification. [F]

7. An object one inch high and 18 inches from a spherical mirror produces an upright image two inches high.

   a. Use Descartes' formula to determine the mirror's focal point and radius of curvature, and the position of the image.

   b. Determine whether the mirror is concave or convex, the image real or virtual.

   c. Sketch the situation, labeling the principal rays, to help demonstrate that your answers to (a) are correct. [C]

Brief Answers:

A. a. $x_F = x_C/2 = 1.00\,\text{m}/2 = 0.50\,\text{m}$ (Ans., Prob. 4)

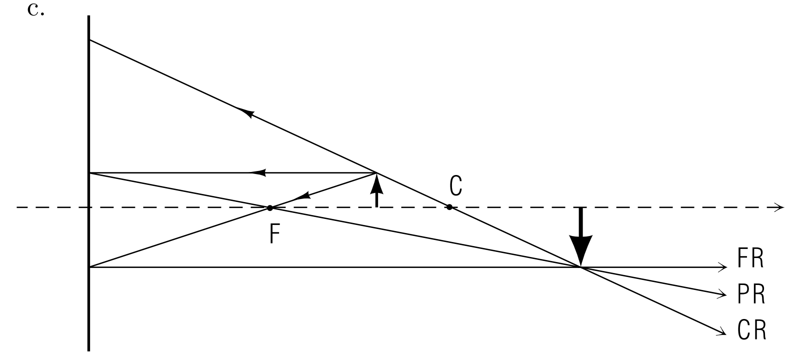
$$\frac{1}{x_i} = \frac{1}{x_F} - \frac{1}{x_o} = \frac{1}{0.50\,\text{m}} - \frac{1}{0.75\,\text{m}} = \frac{1}{1.50\,\text{m}}$$

$x_i = 1.50\,\text{m}$ (Answer to Problem 4)

b. $y_i = My_o = -\dfrac{x_i y_o}{x_o} = -\dfrac{(1.50\,\text{m})(3\,cm)}{0.75\,\text{m}} = -6\,\text{cm};$

inverted because y_i is negative, real because x_i is positive.

c.



B. a. $x_F = x_C/2 = 30\,\text{cm}$

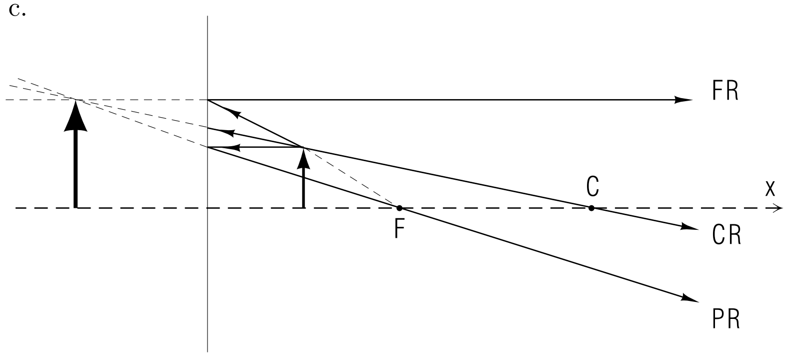
$$\frac{1}{x_i} = \frac{1}{x_F} - \frac{1}{x_o} = \frac{1}{30\,\text{cm}} - \frac{1}{15\,\text{cm}} = -\frac{1}{30\,\text{cm}}$$

$x_i = -30\,\text{cm}$

b. $y_i = My_o = -\dfrac{x_i y_o}{x_o} = -\dfrac{(-30\,\text{cm})(1\,\text{cm})}{15\,\text{cm}} = 2\,\text{cm}$

upright because y_i is positive, virtual because x_i is negative.

c.



C. a. $x_i = Mx_o = -\dfrac{y_i x_o}{y_o} = -\dfrac{(2\,\text{in})(18\,\text{in})}{1\,\text{in}}$

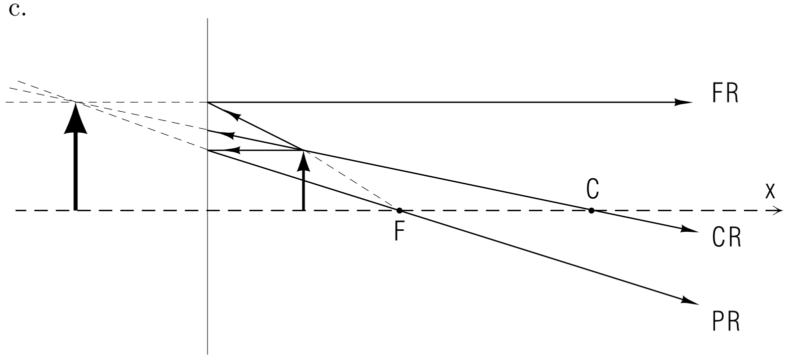
$x_i = -36\,\text{in}$

$\dfrac{1}{x_F} = \dfrac{1}{x_i} + \dfrac{1}{x_o} = \dfrac{1}{-36\,\text{in}} + \dfrac{1}{18\,\text{in}} = \dfrac{1}{36\,\text{in}}$

$x_F = 36\,\text{in}$; $x_C = 2x_F = 72\,\text{in}$

b. x_i is negative so virtual; x_C is positive so concave

c.



D. Given: $M = +1$, $y_o = 0.5\,\text{in}$, $x_o = 12\,\text{in}$

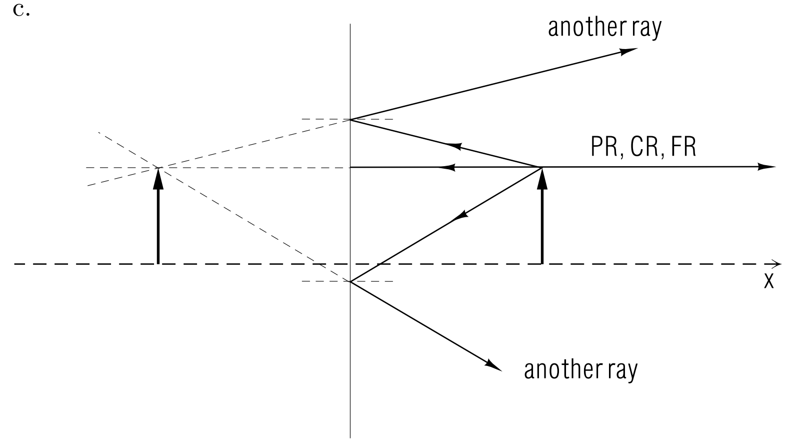
a. $x_i = -Mx_o = -(+1)12\,\text{in} = -12\,\text{in}$; virtual

$\dfrac{1}{x_F} = \dfrac{1}{x_i} + \dfrac{1}{x_o} = \dfrac{1}{-12\,\text{in}} + \dfrac{1}{12\,\text{in}} = 0$

$x_F = \infty$, a plane mirror

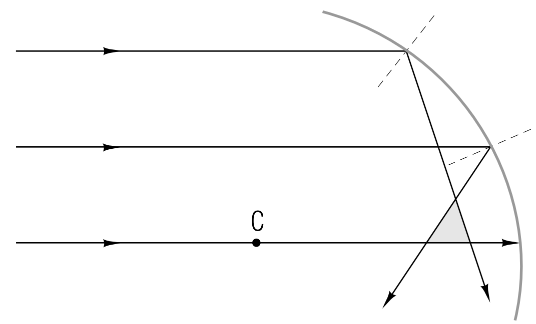
b. $y_i = My_o = 0.5\,\text{in}$, upright because $y_i > 0$

c.



d. $x_C = 2x_F = \infty$

E.



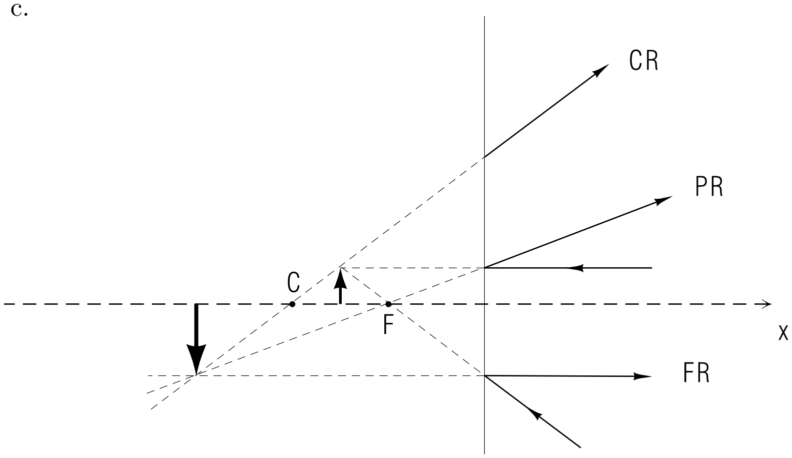
F. a. $x_F = x_C/2 = -30\,\text{cm}/2 = -15\,\text{cm}$

$\dfrac{1}{x_i} = \dfrac{1}{x_F} - \dfrac{1}{x_o} = \dfrac{1}{-15\,\text{cm}} - \dfrac{1}{-24\,\text{cm}} = \dfrac{1}{-40\,\text{cm}}$

$x_i = -40\,\text{cm}$

b. $y_i = My_o = -\dfrac{x_i y_o}{x_o} = -\dfrac{(-40\,\text{cm})(0.5\,\text{cm})}{-24\,\text{cm}} = -0.83\,\text{cm}$, inverted because $y_i < 0$, virtual because x_i is negative.

c.



d. $M = -\frac{x_i}{x_o} = -\frac{-40\,\text{cm}}{-24\,\text{cm}} = -1.67$

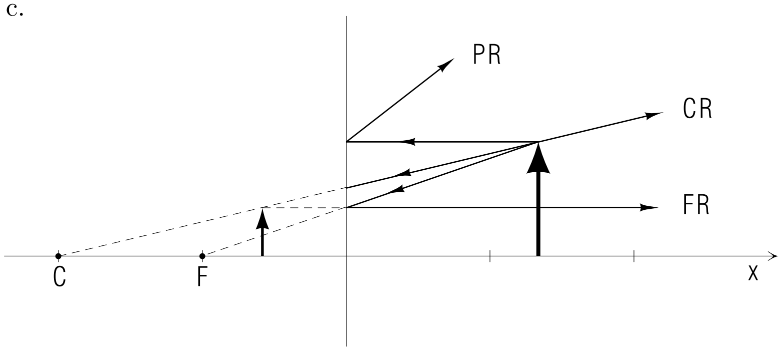
G. a. $x_F = x_C/2 = (-25\,\text{cm})/2 = -12.5\,\text{cm}$

$$\frac{1}{x_i} = \frac{1}{x_F} - \frac{1}{x_0} = \frac{1}{-12.5\,\text{cm}} - \frac{1}{15\,\text{cm}} = \frac{1}{-6.82\,\text{cm}}$$

$x_i = -6.82\,\text{cm}$

b. $y_i = My_o = -\frac{x_i y_o}{x_o} = -\frac{(-6.82\,\text{cm})(1\,\text{cm})}{15\,\text{cm}} = 0.45\,\text{cm}$, upright because y_i is positive, virtual because x_i is negative

c.



SPECIAL ASSISTANCE SUPPLEMENT

S-1 *(from TX-3d)*

Taking x_o as positive:

a. For a real image, $x_i > 0$.

$M = -\frac{x_i}{x_o}$ is negative and thus the image is inverted.

b. For a virtual image, $x_i < 0$.

M is positive and image is upright.

S-2 *(from TX-4c)*

$$\frac{1}{x_o} + \frac{1}{x_i} = \frac{2}{x_C}$$

$$\frac{1}{3\,\text{cm}} + \frac{1}{x_i} = \frac{2}{-12\,\text{cm}}$$

$x_i = -2\,\text{cm}$; i.e. image is 2 cm behind the mirror.

S-3 *(from TX-4c)*

$x_F = \frac{x_C}{2} = \frac{-12\,\text{cm}}{2} = -6\,\text{cm}$, i.e. 4 cm beyond image

S-4 *(from TX-4c)*

$$M = \frac{h_i}{h_o} = -\frac{x_i}{x_o} = -\frac{-2\,\text{cm}}{3\,\text{cm}} = \frac{2}{3}$$

$$h_i = \frac{2h_o}{3} = \frac{4}{3}\,\text{cm}$$

M is positive, so image is virtual and upright.

S-5 *(from TX-4d)*

$$\frac{1}{x_o} + \frac{1}{x_i} = \frac{2}{x_C} = \frac{1}{x_F}$$

Solving for x_i: $x_i = \dfrac{x_o x_F}{x_o - x_F}$.

When $R \to 0$, that implies $x_F \to 0$, and x_i will approach x_F and be larger than x_F. The image is small ($M \approx 0$), inverted ($M < 0$) and real ($x_i > 0$).

MODEL EXAM

1. See Output Skills K1-K4 in this module's *ID Sheet*. One or more of these skills, or none, may be on the actual exam.

2. An object one inch high and 18 inches from a spherical mirror produces an upright image two inches high.

   a. Use Descartes' formula to determine the mirror's focal point and radius of curvature, and the position of the image.

   b. Determine whether the mirror is concave or convex, the image real or virtual.

   c. Sketch the situation, labeling the principal rays, to help demonstrate that your answers to (a) are correct.

Brief Answers:

1. See this module's *text*.

2. See this module's *Problem Supplement*, problem 7.