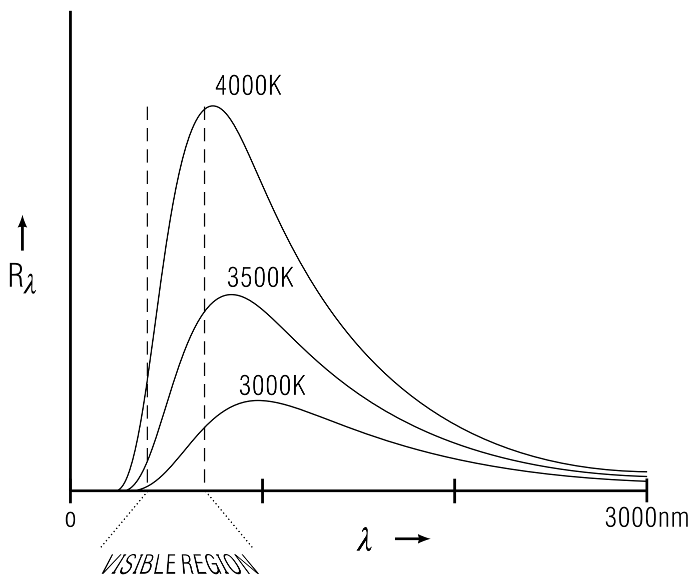
PHYSNET

MISN-0-218

CONTINUOUS SPECTRA: PLANCK'S LAW



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

CONTINUOUS SPECTRA: PLANCK'S LAW

by

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Title: **Continuous Spectra: Planck's Law**

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Input Skills:

1. Vocabulary: spectrum, wavelength, radiant energy (MISN-0-212), tristimulus values (MISN-0-227), chromaticity coordinates (MISN-0-229), absolute temperature (MISN-157), angstrom (Å) (MISN-0-405), nanometer (nm) (MISN-0-212), watt (MISN-0-20).
2. Given the wavelength distribution of light energy coming from an object, and a table of tristimulus values, compute the light's chromaticity coordinates (MISN-0-229).

Output Skills (Knowledge):

K1. Vocabulary: black body or ideal absorber or emitter; black body spectrum; spectral and total emittance; Planck's Law, optical pyrometer.

K2. Sketch the black body spectrum for a progression of temperatures, paying particular attention to the peak shift and to the general height shift.

K3. Describe in detail how one computes the locus of black body points on the Chromaticity Diagram.

K4. Describe how an optical pyrometer works.

Output Skills (Rule Application):

R1. Given the integrated form of Planck's law, determine temperature from total emittance and vice versa.

Output Skills (Project):

P1. Construct an appropriate computer or spreadsheet program and use it to calculate the chromaticity coordinates for a black body a given temperature.

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1. Introduction

A solid begins to glow visibly when its temperature approaches 800 K. Why does its color appear dull red, then orange, yellow, and finally "white hot" as its temperature is raised? What does this tell us about the temperature of the sun's surface and the night sky? To the extent that objects approximate "black body" emission, this module answers those kinds of questions.

We also note that the failure of the laws of classical mechanics to describe this kind of emission and the subsequent discovery of Planck's Law constituted one of the foundations of modern Quantum Mechanics.

2. Temperature and Color

2a. The Temperature-Color Correlation. As a solid is heated higher and higher above room temperature, it eventually begins to glow visibly. For common metals and other "black" bodies this occurs at about 800 K. As the temperature is raised higher and higher, the object turns from dull red to, successively: red-orange, "straw yellow," white, then blue-white. There is a one-to-one correspondence between temperature and color so the temperature of the object can be estimated from its glow color. For example, a blacksmith wishing to harden a piece of iron will heat it to "white-hot," then wait until it has cooled to "straw yellow" and at that precise moment will plunge it into a water bath. As another example of the correspondence of color and temperature, incandescent light bulbs are often rated in "degrees kelvin," written K, in order to specify the colors of their glowing filaments. Home light bulbs are usually stated to have colors in the 2600 K-3000 K range. The quoted temperature of a bulb is of course the temperature of its glowing filament. Such temperatures are not far from the melting point of the bulbs' tungsten filaments, 3500 K.

2b. The Ideal Emitter. Throughout this module we will be discussing the attributes of an "ideal emitter." In the literature an ideal emitter is often called a "black body" because the ideal emitter is also an ideal absorber, one that absorbs all light that reaches its surface.[1] Real metals are quite good absorbers, or at least their graphs of absorption versus wavelength are fairly constant throughout the visible region at "glowing" temperatures, so metals are mainly what we will be thinking of as we study emission in this module. That means that although they may not absorb and emit with as much intensity as a true ideal emitter, the *shapes* of their emission curves are still close to those of an ideal emitter at those temperatures. Thus for those temperatures their perceived colors are close to being those of an ideal emitter. Thus we will be thinking mainly of metals as we study emission in this module.

2c. The Emission Mechanism. The correlation between the color of light from a glowing material and its temperature is due to the fact that the conduction electrons in the material are at the "temperature" of the material. This means that their distributions of energy are characteristic of that temperature. If we plot the distribution of the electron or ion energies (number of particles versus particle energy) versus energy, it is very close to the distribution one can calculate for a collection of independent particles at that temperature.[2] The photons are constantly being emitted and absorbed as the charged particles jump up and down between essentially continuous energy levels, and the more charged particles there are jumping between two particular levels the more photons there will be at the corresponding wavelength. Thus the photon distribution in wavelength is a direct product of the charged-particle distribution in energy, and the latter is a known function of the temperature of the material. This means the photon distribution in wavelength, its "spectrum," is a known function of the temperature of the material.

3. The Spectrum

3a. Emittance and Spectral Emittance. A body that is glowing, because of its elevated temperature, is emitting light of all wavelengths and of course these can be displayed as a spectrum.[3] We can produce a

[1] The radiation from an ideal emitter is usually called "black-body radiation." []
[2] See "The Fermi-Dirac Distribution," MISN-0-168.
[3] The word "spectrum" implies that we have broken a quantity (here the emitted electromagnetic energy) into its single-wavelength components and have probably made a graph showing intensity versus wavelength.

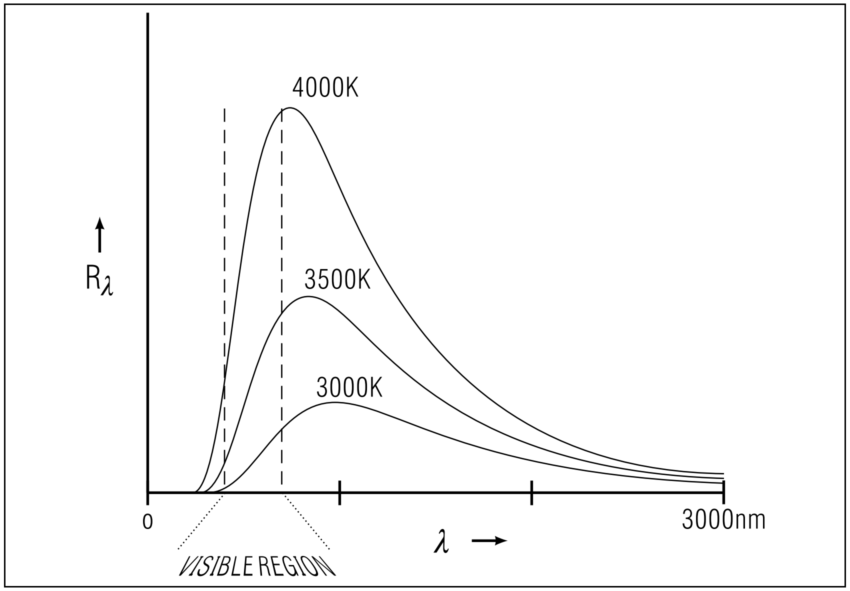

Figure 1. The spectral emittance of an ideal emitter at three different temperatures. The visible region is expanded in Fig. 2.

table of the amount of power emitted into various spectral regions or we can plot a graph of emitted power versus wavelength.

The power emitted by the body per unit radiating area is called the body's "emittance" R and it varies with the temperature of the body. If we break that emitted radiation down into its spectral components, we get the body's "spectral emittance" R_λ which can be plotted as a spectrum. If we integrate over the spectral emittance we get the emittance:

$$R = \int_0^\infty R_\lambda \, d\lambda \,.$$

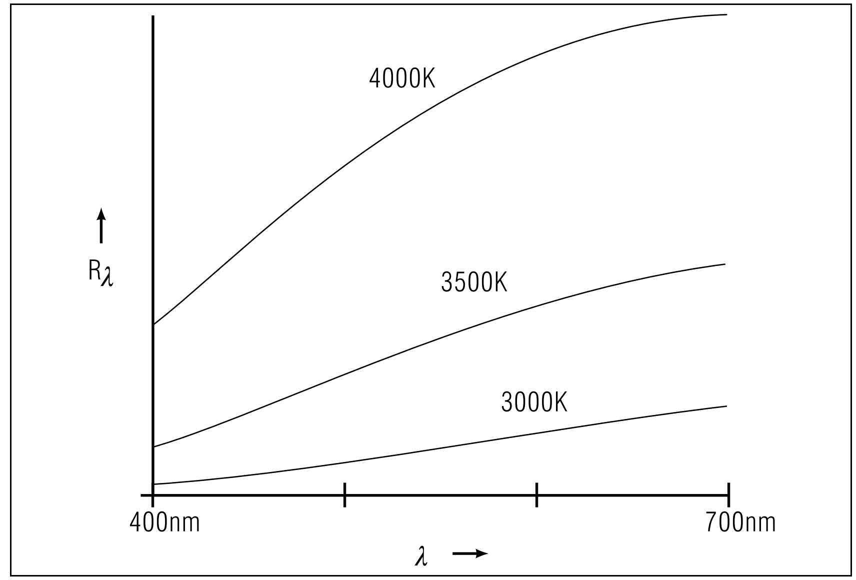

Figure 2. Same as Fig. 1, but showing the visible region expanded.

3b. Planck's Law. If we plot a glowing body's spectral emittance versus wavelength we find that the graph's curve follows Planck's law:

$$R_\lambda = \frac{C_1}{\lambda^5 \left(e^{C_2/\lambda T} - 1\right)} \,. \tag{1}$$

where R is the radiant power emitted per unit radiating surface area per unit wavelength (λ is wavelength), and the two constants are:

$$C_1 = 3.7402 \times 10^{-16}\,\mathrm{W/m^2}\,,$$
$$C_2 = 1.43848 \times 10^{-2}\,\mathrm{m\,K}\,.$$

The spectral emittance of Eq. (1) is plotted in Fig. 1 for three temperatures.

If we integrate over all wavelengths we get the total power radiated per unit radiating surface area:[4]

$$R = \sigma T^4 \,,$$

where:

$$\sigma = 5.672 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}\,.$$

Finally, the wavelength position of the peak of the curve in Eq. (1) is obtained by setting the function's derivative with respect to λ equal to zero.[5] This produces:

$$\lambda_{\max} = \frac{C_3}{T}\,,$$

where:

$$C_3 = 0.28978 \times 10^{-2}\,\mathrm{m\,K}\,.$$

3c. The Optical Pyrometer. When a fairly precise measurement of temperature is needed, as in steel making, an optical pyrometer is used. This device obtains the temperature of an incandescent body by measuring its emittance in a narrow band of wavelengths (in the red, using a filter). This is accomplished by means of a comparison filament, in the instrument, which is heated by an electric current passing through it. The filament is viewed, in the instrument, against the background of the object whose temperature is being measured. The current is varied and when the filament appears neither darker nor brighter than the background, so it seems to disappear, the filament is at the same temperature as the object. The temperature of the filament is a known function of the current going through it. An optical pyrometer for "close up" work relies on the source being, in effect, a semi-infinite plane so the intensity has not yet started falling off with distance. For further-away work, a telescope in the instrument is focused on the source in order to compensate for the falling off of the intensity with distance.

[4]Defining $x \equiv \lambda T/C_2$, the integral becomes:

$$R = T^4 \frac{C_1}{C_2^4} \int_0^\infty \frac{1}{exp^{(1/x)} - 1}\, dx\,,$$

and then the integration on x is performed numerically.

[5]Defining $x \equiv C_2/(\lambda_{\max} T)$, and setting the derivative of R_λ with respect to λ equal to zero, results in this transcendental equation:

$$5 - x = 5e^{-x}\,,$$

It has the numerical solution: $x = 4.965$.

4. Color as a Function of Temperature

The usual techniques for finding the color of a spectral distribution entering the eye can be used to find the color of a glowing ideal emitter as a function of temperature.[6] All one need do is produce the curve implied by Eq. (1), at some specific temperature, and then multiply it by each of the CIE tristimulus curves, wavelength-by-wavelength (the CIE values are given in Appendix A of this module).

Integrating each of the three responses and converting them to relative responses then gives us the Chromaticity coordinates, x and y, of the color of an ideal emitter at that temperature. One can repeat the process for a series of temperatures and thereby produce the "black-body" trajectory usually shown in Chromaticity Diagrams (see Appendix B of this module).

Acknowledgments

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Glossary

- **black body**: an ideal emitter/absorber. It is called a "black body" because a body that truly is "black" must absorb all light reaching it; it must thereby also be an ideal emitter/absorber, one that obeys Planck's law.
- **ideal absorber**: an object which absorbs all electromagnetic energy that falls upon it, reflecting none of it.
- **ideal emitter**: an object that emits electromagnetic energy according to Planck's law.
- **black body spectrum**: the spectral distribution of electromagnetic energy emitted by a black body. The spectral the distribution is given by Planck's law.
- **spectral emittance**: the spectral distribution of electromagnetic energy being emitted by an object. The SI unit is watts per square

[6]See "Colors From Spectral Distributions," MISN-0-229.

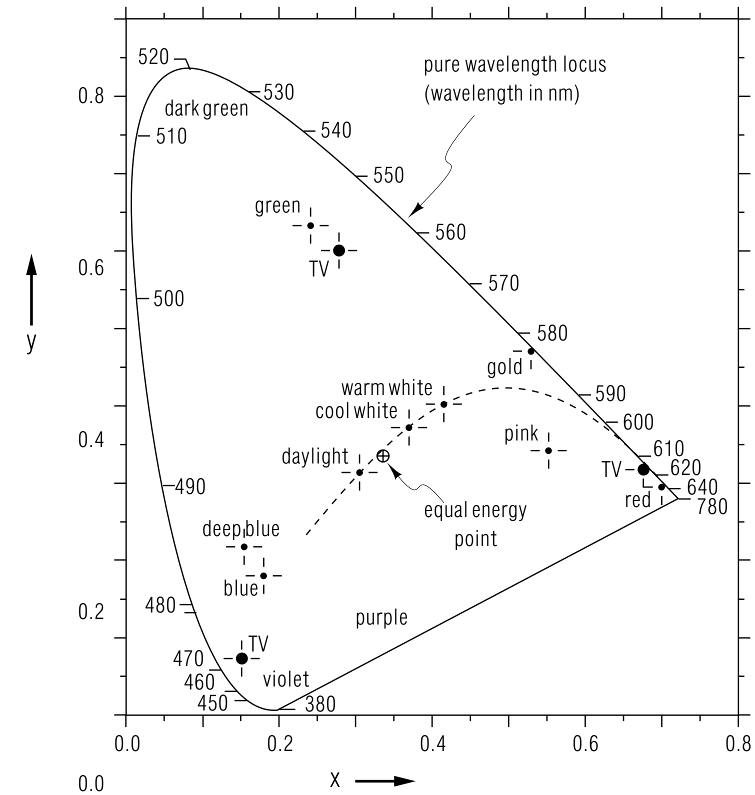
meter per meter (of wavelength).

- **emittance**: the electromagnetic energy being emitted by an object, with units of power per unit area; it is the integral over wavelength of the spectral emittance. The SI unit is watts per square meter.
- **Planck's law**: the equation giving the spectral emittance of a black body, it is derived from quantum mechanics.
- **optical pyrometer**: a device that obtains the temperature of an incandescent body by measuring its emittance in a narrow band of wavelengths (in the red, using a filter).

A. CIE Tristimulus Values

	tri-stimulus values		
λ(nm)	x_λ	y_λ	z_λ
400	0.014	0.000	0.068
420	0.134	0.004	0.646
440	0.348	0.023	1.747
460	0.291	0.060	1.669
480	0.096	0.139	0.813
500	0.005	0.323	0.272
520	0.063	0.710	0.078
540	0.290	0.954	0.020
560	0.595	0.995	0.004
580	0.916	0.870	0.002
600	1.062	0.631	0.001
620	0.854	0.381	0.000
640	0.448	0.175	0.000
660	0.165	0.061	0.000
680	0.047	0.017	0.000
700	0.011	0.004	0.000

B. The Chromaticity Diagram



PROBLEM SUPPLEMENT

Problems 2 and 3 also occur on this module's *Model Exam.*

1. On a surface perpendicular to the incoming rays the earth receives solar radiation at the rate of about $1.4\,\mathrm{KW/m^2}$. Assume the earth is an ideal radiator so the total energy received is radiated back into space. However, the total surface of the earth radiates and it is four times the earth's cross-sectional area that receives the radiation. Determine the average surface temperature of the earth.

2. A red hot burner on a stove is consuming 2.8 KW of power. Its element has a rectangular cross section 3.0 mm by 8.0 mm and its length is 60.0 cm. Determine its temperature, making the quite good assumption that it radiates as a black body.

3. Construct an appropriate computer or spreadsheet program and use it to calculate the chromaticity coordinates for a black body at this temperature:
$$T = 1400\,\mathrm{K} + (CBI\ ID) \times 6.0\,\mathrm{K}\,,$$
where $(CBI\ ID)$ is **your** CBI ID number (which is somewhere in the range 1-999). For computer or spreadsheet assistance, ask for MISN-8-100 and MISN-8-218 in the CBI Consulting Room during your class's regularly scheduled hours.

Brief Answers:

1. 45 °F

2. $1.4 \times 10^3\,\mathrm{K}$

MODEL EXAM

$$\begin{aligned} R &= \sigma T^4 \\ R_\lambda &= \frac{C_1}{\lambda^5\left(e^{C_2/\lambda T}-1\right)} \\ \sigma &= 5.672\times10^{-8}\,\mathrm{Watts\,m^{-2}\,K^{-4}} \\ C_1 &= 3.7402\times10^{20}\,\mathrm{nm^4\,W/cm^2} \\ C_2 &= 1.43848\times10^{7}\,\mathrm{nm\,K} \end{aligned}$$

1. See Output Skills K1-K4 in this module's *ID Sheet*. One or more, or none of these skills may be on the actual exam.

2. A red hot burner on a stove is consuming 2.8 KW of power. Its element has a rectangular cross section 3.0 mm by 8.0 mm and its length is 60.0 cm. Determine its temperature, making the quite good assumption that it radiates as a black body.

3. Staple your project report for problem 3 in the *Problem Supplement* to your exam pack, between the Exam Answer Sheet(s) and the scratch paper. Include the annotated original of your computer output and a copy of the program or spreadsheet function you used. The annotations should be in sufficient detail so another person could exactly repeat your calculation from them alone.

Brief Answers:

1. See this module's *text*.

2. See this module's *Problem Supplement*, problem 2.

3. See this module's *Problem Supplement*, problem 3.