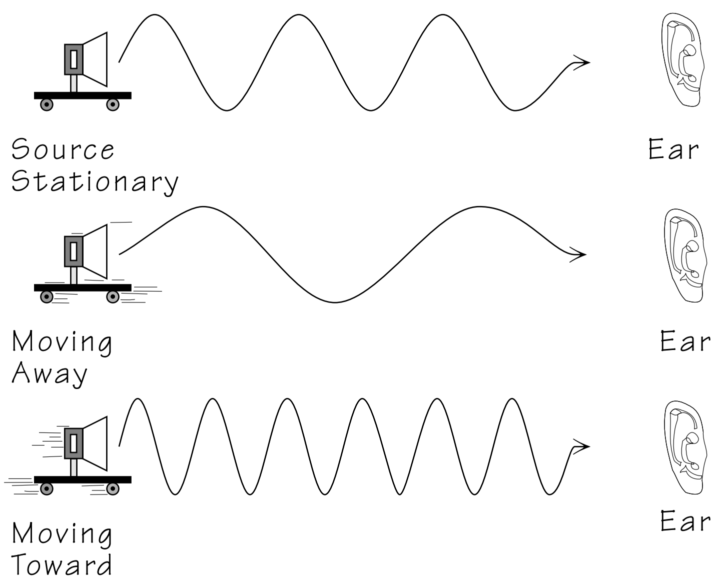
MISN-0-204

THE DOPPLER EFFECT



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

THE DOPPLER EFFECT

by

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Title: **The Doppler Effect**

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Input Skills:

1. Vocabulary: wavefront (MISN-0-203) or (MISN-0-430).
2. State the relationships between the wavelength, frequency, period, and speed of a wave (MISN-0-201) or (MISN-0-430).

Output Skills (Knowledge):

K1. Vocabulary: relative speed, Doppler broadening, Doppler effect, Doppler shift.

K2. Describe how the Doppler effect is used by astronomers and cosmologists to justify the "expanding universe" model.

Output Skills (Problem Solving):

S1. Solve any Doppler shift problem by *deriving the shift for that particular case alone* (not by using the general Doppler shift formula and not by deriving the general case and then using it).

S2. Use the Doppler shift formula to determine the Doppler shift for given motions of a sound wave source and a receiver relative to each other as well as to the acoustic medium.

S3. Given a value for the Doppler shift, calculate the relative speed between receiver and source.

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1. Introduction

1a. The Effect. You have probably had the experience of hearing an ambulance approaching with its siren blaring, and you may have noticed that there was a sharp drop in the pitch of the siren as it passed you. That was because, as it passed you, the siren changed from coming toward you to going away from you. To a person in the ambulance the siren stayed at a frequency that was between the "coming toward" you (higher) one and the "going away from" you (lower) one. The single frequency heard by the person in the ambulance was, in fact, the frequency you would have heard if the ambulance had been stationary. This is true in general: an approaching source of sound shows a higher frequency than a stationary sound, and a receding source shows a lower frequency. This change in the observed frequency of the sound, due to the motion of the source, is a consequence of the wave nature of sound and this phenomenon is called the "Doppler effect." The change in the observed frequency is called the "Doppler shift."

The Doppler effect is useful for measuring the velocities of wave sources. For example, it is used for measuring blood flow non-invasively, for measuring the speed of cars, and, since light is also a wave, for determining the velocities of stars.

1b. Questions to be Answered. In the search for an understanding of the Doppler effect, several interesting questions present themselves:

1. Does the Doppler shift depend only on whether the source is moving, or on whether both the source and observer are moving, or does it perhaps depend only on their relative velocity?
2. Does the Doppler shift depend on whether the air, the transporter of the sound, is moving?
3. Can one derive an exact formula, good for *all* situations, wherein one can plug in all the relevant velocities, plus the frequency of the source in its own rest frame, and get out the observed frequency?

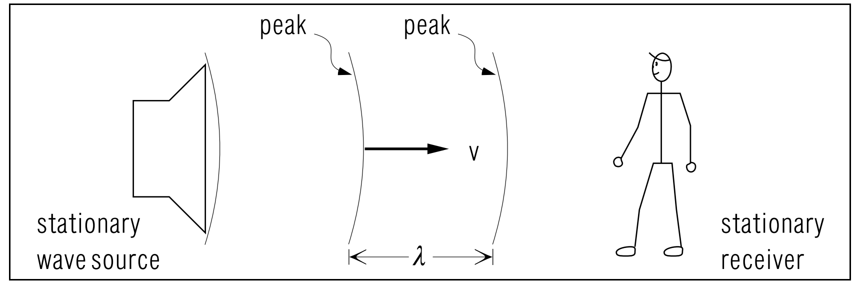

Figure 1. A stationary source producing sound waves that reach a stationary receiver.

2. The Doppler Effect for Sound

2a. Wave Source and Receiver Both Stationary. Here we review the wavelength-frequency-speed equation for the non-Doppler case, where the sound wave source and receiver are both at rest with respect to the air (the medium in which the sound propagates). The source produces waves of wavelength λ and frequency ν that travel at wave speed v, the speed of sound in air (see Fig. 1). The equation is:

$$\lambda\nu = v\,. \tag{1}$$

2b. Wave Source Approaching Stationary Receiver. Now we find the equation for the shift in frequency, the Doppler shift, when the wave *source* is moving at speed v_{src} toward a stationary receiver.[1] In Fig. 2 we show two frames of a "movie" of what is happening. At a certain instant of time, wave peak #1 is emitted by the source (see Fig. 2a).[2] Exactly one wave period T later, peak #1 has moved a distance λ (one wavelength) and peak #2 is emitted (Fig. 2b). However, peak #2 is not emitted at the same position in space as peak #1 was emitted because in the time T the wave source has moved a distance $v_{src}T$ (assume that $v_{src} < v$). Thus it is with each successive wave peak; the source is constantly advancing so that each wavelength is diminished by the distance

[1] By "stationary," we mean with respect to the air through which the wave propagates. Thus to a "stationary observer" there can be no wind blowing.

[2] The lines denoting wave peaks are drawn along the individual crests (peaks) of the sound wave. If the wave could be shown in three dimensions, the peak lines would be seen to be parts of spherical surfaces spreading out from the source. In some commercial textbooks the wave peak lines (or surfaces) are erroneously referred to as "wave fronts."

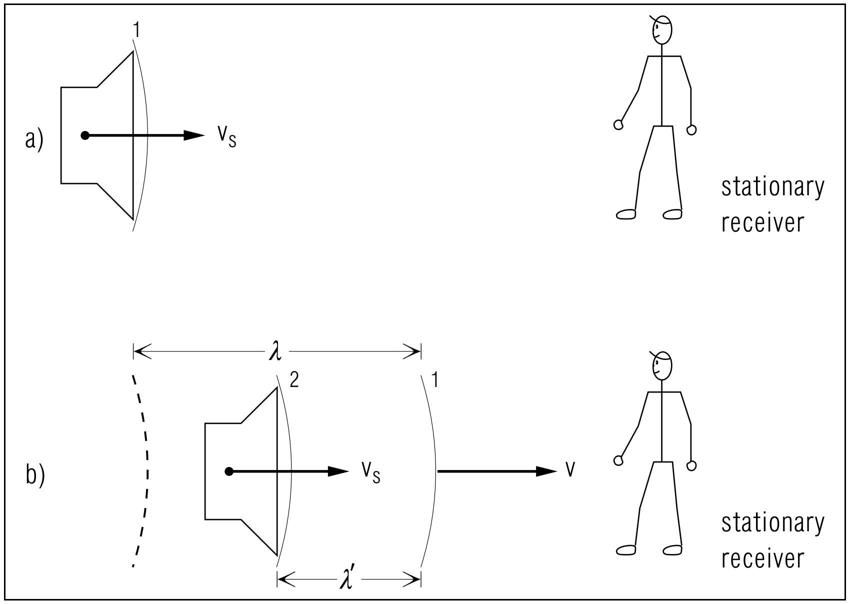

Figure 2. A moving wave source emitting wave peaks that reach a stationary receiver: (a) wave peak #1 is emitted; (b) wave peak #2 is emitted.

$v_{src}T$. The receiver perceives waves of wavelength λ', where

$$\lambda' = \lambda - v_{src}T\,. \tag{2}$$

Now, what about the other wave characteristics, such as wave speed and frequency? Once the wave is introduced into the air, its speed is determined by the properties of the medium (density, elastic response, temperature, etc.) and not by the way the wave was produced. The wave "forgets" its history, i.e. whether its source was moving or not, hence the wave speed is still v. Although the source is producing waves of frequency ν, it cannot be this frequency that the receiver perceives, but a frequency ν' which satisfies the equation:

$$\nu'\lambda' = \nu'(\lambda - v_{src}T) = v\,. \tag{3}$$

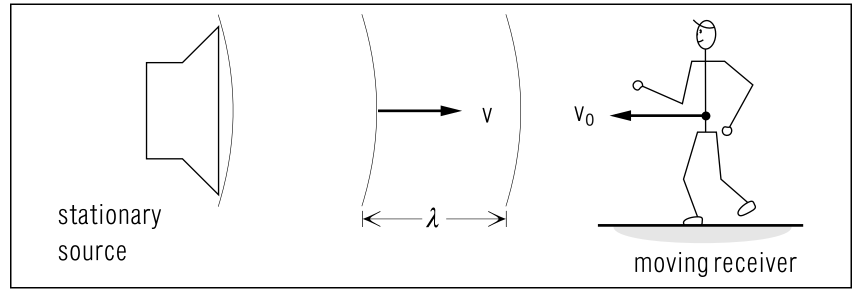

Figure 3. A receiver approaching a stationary source.

This frequency, ν', determines the pitch heard by the receiver. By exploiting the relationship between frequency and period,

$$T = \frac{1}{\nu}\,, \tag{4}$$

and using Eqs. (1) and (4) to eliminate λ, Eq. (3) can be rearranged as:

$$\frac{v_{src}}{v} = \frac{\nu' - \nu}{\nu'}\,.\ \textit{Help: [S-3]} \tag{5}$$

We now write $\Delta\nu$ as the Doppler Shift, the change in frequency:

$$\Delta\nu \equiv \nu' - \nu\,. \tag{6}$$

Then using it to eliminate ν', we get:

$$\frac{\Delta\nu}{\nu + \Delta\nu} = \frac{v_{src}}{v}\,. \tag{7}$$

If $\Delta\nu$ is small compared to ν, this can be expressed as

$$\frac{\Delta\nu}{\nu} \approx \frac{v_{src}}{v}\,. \qquad (\Delta\nu \ll \nu)\,. \tag{8}$$

In words, the fractional change in frequency is approximately equal to the ratio of the speed of the source to the speed of sound.

2c. Receiver Approaching Stationary Source. Suppose the receiver is approaching a stationary source with speed v_{rcv} as depicted in Fig. 3. The receiver would measure the wavelength of the waves as λ, just as in the case of both source and receiver at rest. However, since the receiver is in motion relative to the air in which the waves have speed v, the receiver perceives a higher speed for the waves relative to the receiver itself:

$$v_{\rm rel} = v + v_{rcv} \, . \tag{9}$$

[3] The frequency ν' perceived by the receiver is determined by this relative speed and the wavelength λ by the following modified version of Eq. (1):

$$\nu' \lambda = v_{\rm rel} = v + v_{rcv} \, . \tag{10}$$

Eliminating λ using $\lambda = v/\nu$, and rearranging as before, we obtain:

$$\frac{\nu' - \nu}{\nu} = \frac{v_{rcv}}{v} \, . \tag{11}$$

Substituting $\Delta\nu$ into Eq. (11), we obtain:

$$\frac{\Delta\nu}{\nu} = \frac{v_{rcv}}{v} \, . \tag{12}$$

In this situation the fractional change in frequency, $\Delta\nu/\nu$, is exactly equal to the ratio of the speed of the receiver to the speed of sound.

2d. Source and Receiver Approaching Each Other. By combining the arguments of the previous two situations, the reader should verify that the following equation must be satisfied for waves emitted by a moving source and perceived by a moving receiver, approaching one another:

$$\nu' \left(\lambda - v_{src} T\right) = v + v_{rcv} \, , \qquad \textit{Help: [S-4]} \tag{13}$$

which can be transformed into:

$$\frac{\Delta\nu}{\nu} = \frac{v_{src} + v_{rcv}}{v - v_{src}} \, . \qquad \textit{Help: [S-8]} \tag{14}$$

▷ When calculating the Doppler shift, do it without blindly plugging into a prederived formula. Use the fact that sound travels with a characteristic speed relative to the medium through which it propagates, plus the definition of wavelength and frequency, to calculate the frequency shift. Count the numbers of waves emitted or received in some appropriate period of time by an appropriate source or receiver. Finally, check your answer with a remembered formula.[4]

[3] This is one-dimensional relative motion. For those interested, two-dimensional (vector) relative speeds are examined in "Relative Linear Motion and Frames of Reference ," MISN-0-11.

[4] This process is illustrated in Problems 3 and 4 in the Problem Supplement.

2e. Relative Linear Motion: Three Cases. In the three previously discussed situations, the final expression was determined by assuming that the source and the receiver were approaching each other along a straight line.

▷ Reconstruct the arguments for relative motion along a straight line for the source and the receiver moving away from each other. The following general relationships may thus be established:

Case 1. Moving source, receiver at rest.

$$\frac{\Delta\nu}{\nu + \Delta\nu} = \pm \frac{v_{src}}{v} \, , \tag{15}$$

so, for cases where $\Delta\nu$ is very small compared to ν,

$$\frac{\Delta\nu}{\nu} \approx \pm \frac{v_{src}}{v} \, . \qquad (\Delta\nu << \nu) \tag{16}$$

The plus sign is used for an approaching source, the negative sign for a receding source.

Case 2. Source at rest, receiver moving.

$$\frac{\Delta\nu}{\nu} = \pm \frac{v_{rcv}}{v} \, . \tag{17}$$

The plus sign is used for an approaching receiver, the negative sign for a receding one.

Case 3. Source and receiver both moving.

$$\frac{\Delta\nu}{\nu} = \frac{\pm v_{src} \pm v_{rcv}}{v \mp v_{src}} \, . \tag{18}$$

The upper sign on v_{src} is used for an approaching source, the lower sign for a receding one. Similarly, the upper sign on v_{rcv} is used for an approaching receiver, the lower sign for a receding one.

▷ How fast would a piano have to be approaching a receiver for the note A (440 Hz) to sound like $B^\flat$ (466 Hz), one-half step higher on the musical scale? Take the speed of sound to be 3.30×10^2 m/s. *Help: [S-5]*

2f. Moving Source Not Equivalent to Moving Receiver. Eqs. (15) and (17) are different because the Doppler shift produced by motion of the sound wave *source* is different from that produced by an identical relative motion of a sound wave *receiver*. To see the significance of this, imagine two receivers on different ships. Suppose the observers know that only one ship is moving, but don't know which one it is. If one ship emits a blast from its horn and the receiver on the other ship measures the sound frequency, the person operating the receiver can use the known rest frequency of the horn, the relative speed of the ships, and the speed of sound to see whether the measurements are consistent with Eq. (15) or Eq. (17) and thus determine which of the ships is moving.

2g. The Medium is the Preferred Reference Frame. The non-symmetry of source and receiver is a direct consequence of the fact that sound requires an elastic medium for its propagation. In the cases considered so far, air has been that medium. A coordinate system fixed in this *medium* is called a "preferred" reference frame for the phenomena: all velocities in a sound-wave problem are to be measured with respect to a coordinate system fixed in this frame.

3. The Doppler Effect for Light

3a. Introduction. There is a remarkable difference between the Doppler effect in sound waves and the Doppler effect in light waves. For light waves there is no preferred frame of reference, no material "medium" in which the waves travel. In fact, light waves travel at maximum speed through a complete vacuum where sound waves cannot travel at all! One consequence is that, unlike a sound wave, the speed of a light wave is the same to all receivers regardless of their velocities. The details are left to a careful derivation elsewhere.[5] Nevertheless, when the speed of the *source* of the light is small compared to the speed of light, the Doppler (frequency) shift is given, to a good approximation, by the same equation as for the case of sound with a moving source and stationary receiver):

$$\frac{\Delta\nu}{\nu} = \pm\frac{v}{c}\,. \qquad (v \ll c) \tag{19}$$

Here c is the speed of light in a vacuum and v is the *relative* speed of the light source and the receiver, irrespective of which one is taken to be at rest. The upper sign is used if the source and the receiver are approaching each other; the lower sign if they are receding from each other.

3b. Doppler Broadening of Spectral Lines. An important example of the Doppler effect for light is the "broadening" of spectral lines due to the thermal motion of the atomic or molecular sources of light.

[5]See "Topics in Relativity: Doppler Shift and Pair Production" (MISN-0-308).

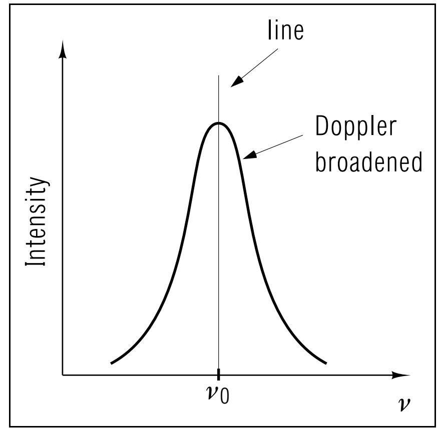

Figure 4. Doppler broadening of the natural spectral line shape.

These sources emit or absorb electromagnetic radiation in a spectrum of discrete frequencies called "spectral lines." Each spectral "line" may be represented on a graph of radiation intensity as a function of frequency and looks as shown in Fig. 4. The spectral line has a peak at some frequency ν_0 and a "width" measured at half the peak intensity. This width has a minimum value called the "natural line width," representing the fact that frequencies other than ν_0 may be emitted or absorbed, although such frequencies occur with lower intensities. However, since the atomic or molecular sources are in random thermal motion[6] with a Gaussian distribution of velocities, the observed frequencies are a similar distribution about the peak frequency ν_0. This effect is called the "broadening" of the spectral line (beyond its " natural" width). Such a spectral "line" has a width of *roughly* $2\Delta\nu$, where $\Delta\nu$ is the Doppler shift for a source moving at the average speed of the thermal distribution.

▷ If hydrogen atoms on the surface of a blue-white star have an average thermal speed of 2.5×10^4 m/s, show that the Doppler broadening of the H_α line, $\lambda = 656.5$ nm, is 7.6×10^{10} Hz. *Help: [S-6]*

3c. Receding Galaxies Emit Doppler Shifted Light. One of the most important pieces of evidence supporting the expanding universe model is the Doppler shift of spectral line frequencies of light emitted from stars in distant galaxies. These spectral lines are easily identified by comparison with laboratory spectra, but are consistently shifted to lower frequencies. Such shifts, called "red shifts," indicate that the star as a whole is receding from the earth at a speed determined from Eq. (19). No

[6]For further discussion of thermal motion see this module's *Glossary*.

such stellar spectra have been observed to be Doppler shifted to higher frequencies ("blue-shifted"), leading to the inescapable conclusion that the galaxies are receding from each other and that the universe is expanding.

4. Limitations of the Results

Experience shows that the assumptions about wave motion on which our Doppler shift results were derived are not always valid. For one thing, the elastic restoring force by which particles in an acoustic medium are returned to their equilibrium positions are not always close to being linear. This is almost always true for large enough displacements (high enough "volume" of sound). The result is a wave equation with spatial derivatives of higher order than $\partial^2/\partial x^2$. The wave speed is then no longer $\lambda\nu$ and the wave changes shape; the wave "disperses."

Another limitation to our Doppler shift equations arises from the assumption that the speed of the source is less than the speed of the wave. However, if the receiver is receding from the source at a speed exceeding the wave speed, the wave will never catch up and hence will never be observed at all!

Acknowledgments

Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

Glossary

- **Doppler broadening**: the broadening of the natural linewidth of a spectral line, due to the Doppler shift produced by the random thermal motion of the atomic or molecular sources of radiation.
- **Doppler effect**: the phenomenon whereby the observed frequency of a wave depends on the motions of the wave source and the receiver.
- **Doppler shift**: the change in the observed frequency of a wave due to the motion of the wave source and/or the receiver.

- **thermal motion**: the motions of objects that produce the measurable quantity called "temperature." For a gas, such as air, the velocities of the gas molecules have random directions and their magnitudes that are random within a Gaussian distribution function (for further discussion see "Energy and Boltzmann Distributions," MISN-0-159). The temperature of a gas is a simple function of the mean value of that distribution. Thus an increase of temperature is a signal that the mean speed of the gas molecules has shifted upward.
- **relative speed**: the speed of an object as measured by an observer who is moving with respect to another observer who has also measured the object's speed. Consider a road barrier: a "stationary" observer will measure its speed as zero, but if its speed is measured by an observer traveling toward it at a high speed, that "relative speed" will not be zero but will be the speed of that observer. In general, fhe speed of an object, as measured by two observers, will differ by the speed of either one of the observers itself as measured by the other observer. In the case of sound, the speed of a wave crest as measured by an observer at rest with respect to the air, a "stationary" observer, is the normally-quoted speed of sound in air, v. Suppose now that the "stationary" observer sees another observer moving at speed v_{obs} toward the oncoming wave crests: this second, "non-stationary," observer will measure the speed of the wave crests as $v + v_{obs}$.

PROBLEM SUPPLEMENT

Note: Problems 6 and 7 also occur in this module's *Model Exam.*

1. Express Eqs. (15), (17), and (18) in terms of ν, ν' and the speeds of the source and the receiver, without $\Delta\nu$.

2. Taking the speed of sound as 3.40×10^2 m/s, consider the following six situations:

   a. a receiver moves toward a stationary source at 1.70×10^2 m/s,

   b. a receiver moves away from a stationary source at 1.70×10^2 m/s,

   c. a source moves toward a stationary receiver at 1.70×10^2 m/s,

   d. a source moves away from a stationary receiver at 1.70×10^2 m/s,

   e. a source and a receiver move away from each other, each moving relative to the stationary acoustic medium at 85 m/s,

   f. a source and a receiver are stationary relative to each other but move with speed $v = 1.0 \times 10^1$ m/s relative to the acoustic medium (the air) in the direction from the source to the receiver.

   In each case the source emits sound waves of frequency $\nu = 3.00 \times 10^2$ Hz. Determine the frequency the receiver detects in each case. Also, in part (f), determine the wavelength of the waves. *Help: [S-1]*

3. A stationary source emits sound waves of frequency 6.00×10^2 Hz. A receiver at rest detects these waves. In solving the following parts use numbers, not symbols.

   a. How many "wavelets" (complete cycles of the traveling wave) does this source emit in 5 seconds?

   b. Over what distance in space is this wave train spread out if the speed of sound is 3.40×10^2 m/s?

   c. What is the distance between adjacent "crests" of this wave? Get this result by using the number of wavelets in the wave train of part (a) and the length of the wave train. This is the wavelength of the waves. Does this agree with the standard way you know for calculating the wavelength (given the speed and frequency of the wave)?

   d. With what speed does this wave train move by the receiver?

   e. What length of wave train moves by the receiver in 1 second?

   f. How many wavelets are contained in this wave train (of 1 second duration)?

   g. How many wavelets move by the receiver in 1 second?

   h. What frequency of sound does the receiver detect?

4. The 6.00×10^2 Hz source of Problem 3 is moving toward a receiver at a speed of 4.0×10^1 m/s while the receiver moves toward the source at 6.0×10^1 m/s with respect to the air. In solving the following parts use numbers, not symbols.

   a. The source emits sound waves as it moves. In 5 seconds, how many wavelets (complete cycles of the traveling wave) has it emitted?

   b. How far from its original source location has the front of the wave train gone (toward the receiver) in this 5 seconds?

   c. How far from the initial source location was the last of the 5 second wave train emitted?

   d. From parts (b) and (c), over what distance in the air is this wave train extended?

   e. To a receiver at rest with respect to the air, what is the distance between the adjacent wavelets? Get this from parts (a) and (d). What wavelength does the receiver measure for this sound wave?

   f. Compare this result with the wavelength that would be observed if the source were at rest.

   g. How fast does this wave train go by the moving receiver?

   h. What distance of wave train moves by the receiver in one second?

   i. How many wavelets go by the receiver in this one second?

   j. What frequency does the receiver measure?

   k. Compare the answer to part (j) with the frequency calculated using the Doppler shift formula stated in the module text.

5. A physicist receives a traffic citation for running a red light ($\lambda = 6.20 \times 10^2$ nm). She claims that, because she was approaching the light, the Doppler effect made the light look green ($\lambda = 5.40 \times 10^2$ nm).

   a. Calculate the speed the physicist would have had to be traveling for her assertion to be true.

b. Would she have been exceeding the speed limit?

6. In this problem, derive the Doppler shift for each part separately, using numbers rather than symbols throughout the derivation, and justifying each basic step. This means following exactly the types of procedures you were led through in Problems 3 and 4, where you were always asked for numbers, never for symbols. For each part below, determine the frequency of sound as measured by the specified receiver. The source frequency is $\nu = 1.00 \times 10^2$ Hz in its own rest frame.

   a. The receiver is stationary and the source moves away from it at 7.0×10^1 m/s.

   b. The source is stationary and the receiver moves away from it at 7.0×10^1 m/s.

   c. The receiver and the source move away from each other, each moving at 1.00×10^2 m/s relative to the air.

   d. The receiver and the source move toward each other, each moving at 1.00×10^2 m/s relative to the air.

7. An astronomer measures the frequencies of the spectral lines in a star's spectrum and finds them to be red-shifted by 2%. This measurement is made at a time when the earth's velocity due to its motion about the sun makes a right angle with the direction to the star. Calculate the speed with which the star appears to be receding from the earth. Assume the earth is not moving toward or away from the star at the time of measurement. *Help: [S-7]*

Brief Answers:

1. Case1: $$\nu' = \nu \frac{v_{\rm sound}}{v_{\rm sound} \mp v_{\rm source}}. \quad (v_{\rm receiver} = 0) \qquad (15')$$

   Case2: $$\nu' = \nu \frac{v_{\rm sound} \pm v_{\rm receiver}}{v_{\rm sound}}. \quad (v_{\rm source} = 0) \qquad (17')$$

   Case3: $$\nu' = \nu \frac{v_{\rm sound} \pm v_{\rm receiver}}{v_{\rm sound} \mp v_{\rm source}}. \qquad (18')$$

2. a. $\nu' = 450$ Hz.

   b. $\nu' = 150$ Hz.

   c. $\nu' = 600$ Hz.

   d. $\nu' = 200$ Hz.

   e. $\nu' = 180$ Hz.

   f. $\nu' = 300$ Hz, $\lambda' = 1.10$ m.

3. a. 3000.

   b. 1700 m.

   c. 0.567 m, yes.

   d. 340 m/s.

   e. 340 m.

   f. 600 wavelets.

   g. 600.

   h. 600 Hz.

4. a. 3000.

   b. 1700 m.

   c. 200 m.

   d. 1500 m.

   e. 0.500 m, 0.500 m.

   f. 0.567 m.

   g. 400 m/s.

   h. 400 m.

   i. 800.

   j. 800 Hz.

   k. 800 Hz.

5. a. 4×10^7 m/s (approximately, since $v \ll c$ does not hold).

   b. Obviously yes.

6. a. 83 Hz.

   b. 79 Hz.

   c. 55 Hz.

   d. 183 Hz.

7. $v = 6 \times 10^6$ m/s. *Help: [S-7]*

SPECIAL ASSISTANCE SUPPLEMENT

S-1 *(from PS-Problem 2)*

Notice that the answers to parts (a) and (c) are not the same, even though in both cases the relative speed of the source and the observer are the same. Similarly, the relative speed of the source and the observer in parts (b), (d) and (e) are the same but the Doppler shifts are all different. Also note that (b) of Text Eq. (15) is not applicable because it is an approximation valid only if v_s is small compared to v. Try parts (c) and (d) a source speed having the value 34 m/s. Subequation (b) of Text Eq. (15) is now a better approximation, especially if you retain only one digit beyond the decimal point.

S-3 *(from TX-2b)*

Direct substitution gives:

$$\nu' \left(\frac{v}{\nu} - v_s \frac{1}{\nu}\right) = v$$

and then simple algebraic manipulations give:

$$\frac{\nu'}{\nu}\,(v - v_s) = v$$

$$\frac{\nu'}{\nu}\left(1 - \frac{v_s}{v}\right) = 1$$

$$1 - \frac{v_s}{v} = \frac{\nu}{\nu'}$$

$$\frac{v_s}{v} = 1 - \frac{\nu}{\nu'}$$

and the result follows immediately.

S-4 *(from TX-2d)*

Treat the output frequency of Sect. 2b as the input frequency for Sect. 2c.

S-5 *(from TX-2e)*

Solution: Since the source of the sound is moving, we use Case 1. Let us calculate the speed v_{src} from the exact formula, Text Eq. (15):

$$\frac{\Delta\nu}{\nu + \Delta\nu} = \frac{v_{src}}{v} \Longrightarrow v_{src} = v\left(\frac{\Delta\nu}{\nu + \Delta\nu}\right)$$

Now $\nu = 440\,\mathrm{Hz}$, $\nu + \Delta\nu = 466\,\mathrm{Hz}$, and $\Delta\nu = 26\,\mathrm{Hz}$ so:

$$v_{src} = (330\,\mathrm{m/s})\left(\frac{26\,\mathrm{Hz}}{466\,\mathrm{Hz}}\right) = 18.4\,\mathrm{m/s}.$$

Now let us calculate v_{src}, using the approximation of Text Eq. (17):

$$\frac{\Delta\nu}{\nu} \approx \frac{v_{src}}{v}$$

$$v_{src} \approx v\frac{\Delta\nu}{\nu} = (330\,\mathrm{m/s})\left(\frac{26\,\mathrm{Hz}}{440\,\mathrm{Hz}}\right) \approx 19.5\,\mathrm{m/s}$$

The fractional error introduced by the approximation is 1.1/18.4, about 6.0%.

S-6 *(from TX-3b)*

$$\frac{\Delta\nu}{\nu_0} = +\frac{v}{c} = \frac{2.5\times10^4\,\mathrm{m/s}}{3.0\times10^8\,\mathrm{m/s}} = 8.33\times10^{-5}\,,$$

which is the shift from ν_0 for the sources that are approaching the receiver with speed v. At any given instant, about the same number are receding from the receiver and for these the shift would have a minus sign. Hence the *full* width at half maximum would be:

$$2\Delta\nu = 2\left(\frac{\Delta\nu}{\nu_0}\right)\nu_0 = 2\,(8.33\times10^{-5})\frac{3.00\times10^8\,\mathrm{m/s}}{656.5\,\mathrm{nm}} = 7.6\times10^{10}\,\mathrm{Hz}.$$

S-7 *(from PS-7)*

1. We have found that most students who seek help on this problem simply did not really try to learn the relevant material in the text so they have no idea what the words in the problem mean. One of the most important skills to learn in college is to properly decide when you cannot solve a problem unless you understand the terms involved. This is one of those cases where you need to find the section where the problem's words are used and really understand all of the processes that the terms describe. That may force you to go back to earlier sections and really learn that material as well. You will know that you have succeeded in understanding the subject when the meaning of the phrase "red-shifted by 2.00%" is clear to you and you realize that you know how to use the phrase mathematically.
2. Another aspect that has stymied some students is that they do not know that they should actually start working on a problem even if they cannot find a formula which contains, on its right-hand-side, the exact list of quantities that are given in the problem. In real life, one often has to just start working on a problem, using one's general understanding of the phenomenon and see where one *can* start making correct inferences. Sometimes quantities whose values are not known simply disappear from the problem due to unforeseen cancellations.
3. Finally, there have been a few students who said they understood how to use percentages in describing an increase in a number of apples or an increase in their pay, but who say that applying a percentage increase to a number of cycles per second is mathematically different. When we have told them that there is no difference mathematically, they have generally gone away satisfied.

S-8 *(from TX-2d)*

Notice which quantities must disappear in going from the first equation to the second. Get rid of those quantities by substituting for them in terms of quantities that do occur in the second equation. After that it is just a matter of simple rearrangement to get the second equation in the form shown.

MODEL EXAM

1. See Output Skills K1-K2.
2. In this problem, derive the Doppler shift for each part separately, using numbers rather than symbols throughout the derivation, and justifying each basic step. For each part below, determine the frequency of sound as measured by the specified receiver. The source frequency is $\nu = 100\,\text{Hz}$ in its own rest frame.
   a. The receiver is stationary and the source moves away from it at $70\,\text{m/s}$.
   b. The source is stationary and the receiver moves away from it at $70\,\text{m/s}$.
   c. The receiver and the source move away from each other, each moving at $100\,\text{m/s}$ relative to the air.
   d. The receiver and the source move toward each other, each moving at $100\,\text{m/s}$ relative to the air.
3. An astronomer measures the frequencies of the spectral lines in a star's spectrum and finds them to be red-shifted by 2.00%. This measurement is made at a time when the earth's velocity due to its motion about the sun makes a right angle with the direction to the star. Calculate the speed with which the star appears to be receding from the earth. Assume the earth is not moving toward or away from the star at the time of measurement.

Brief Answers:

1. See this module's *text*.
2. See this module's *Problem Supplement*, problem 6.
3. See this module's *Problem Supplement*, problem 7.