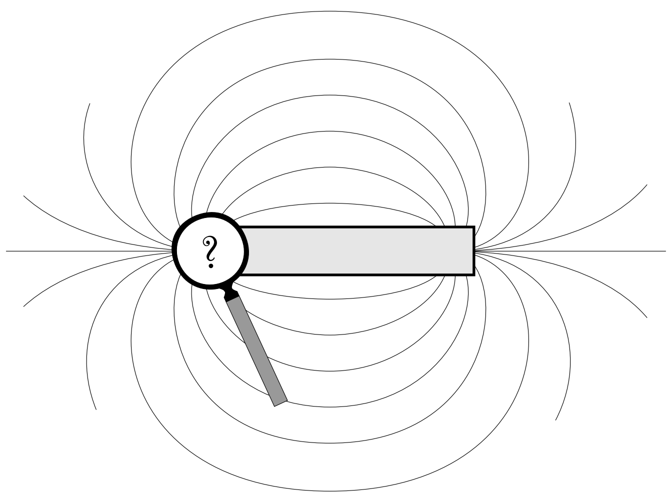
PHYSNET

MISN-0-140

A MAGNETIC MONOPOLE?



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

A MAGNETIC MONOPOLE?

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Input Skills:

1. Manipulate scalar and vector products of vectors (MISN-0-2).
2. Describe the meaning of the usually-assumed "Sourceless of the magnetic field flux" (MISN-0-139).
3. Write down Maxwell's Equations and define each symbol (MISN-0-146).
4. Define "Quantization of Angular Momentum" (MISN-0-251).

Output Skills (Knowledge):

K1. State the motivation for speculating on the existence of magnetic monopoles.

K2. Derive Dirac's reciprocal quantization rule.

K3. Calculate the numerical value of the fine structure constant and the Dirac magnetic charge unit.

K4. Give an argument why, if magnetic monopoles do exist, they have so far escaped detection.

External Resources (Required):

1. M. Alonso and E. J. Finn, *Physics*, Addison Wesley (1970). For access, see this module's *Local Guide*.

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1. Introduction

This topic, the magnetic monopole, should be viewed as an interesting speculation as no magnetic monopole has yet been found in nature. Nevertheless, the reciprocal electric and magnetic quantization condition of Dirac may be a key to the understanding of charge quantization. We shall present an elementary derivation of this rule due to Schwinger. We also recommend the derivation to you because the techniques and principle used are useful in other areas. It also serves to bring into better focus the nature of the quest for order and understanding that underlies the activity of physicists. Furthermore, the search for magnetic monopoles is still a very active research subject in high energy physics.

2. Readings

In AF[1] read Section 16.7. Note that $\epsilon_0 \equiv 1/(4\pi k_e)$ and $\mu_0 \equiv 4\pi k_m$. Answer question 7 on page 352. Read Section 20.11.

3. Possible Existence

3a. Motivation from Maxwell's Equations. The concept of magnetic monopole was probably first attractive to physicists inasmuch as the existence of such a charge would symmetrize the form of Maxwell's equation.

If there is no electric charge or current present, Maxwell's equations can be written as:

$$\oint_s \vec{E} \cdot \hat{N}\, ds = 0; \qquad \oint_s \vec{B} \cdot \hat{N}\, ds = 0$$

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt} \oint \vec{B} \cdot \hat{N}\, ds; \qquad \oint \vec{B} \cdot d\vec{\ell} = \frac{d}{dt} \oint \vec{E} \cdot \hat{N}\, ds$$

[1]Alonso and Finn, *Physics*, Addison-Wesley (1970). For access to this reference, see this module's *Local Guide*.

It is obvious that these equations are unchanged under $\vec{E} \to \vec{B}$ and $\vec{B} \to -\vec{E}$. That is, there exists a symmetry between the electric and magnetic quantities. Since we know electric charges are present, but their counterparts, magnetic monopoles, have not been found, we have the Maxwell equation given on page 472 of AF. It is obvious that in the absence of magnetic monopoles the intrinsic symmetry between electric and magnetic quantities is lost. This is the conventional view.

3b. Another Approach. There is another possibility: (1) magnetic monopoles exist but so far have not been detected, and (2) the electromagnetic field equations are unchanged under $\vec{E} \to \vec{B}$ and $\vec{B} \to -\vec{E}$ plus q_e (electric charge) $\to q_m$ (magnetic charge). In order to take this possibility seriously one must be able to answer:

(i) why magnetic monopole has so far escaped detection;

(ii) how the existence of a magnetic monopole could provide further understanding of nature.

4. The Reciprocal Quantization Rule

4a. Introduction. For questions raised by the idea of magnetic monopoles, a partial answer was given by the reciprocal quantization rule of Dirac. This rule is such that the unit of magnetic charge, deduced from the known unit of electric charge, is quite large. A strong attractive force between opposite magnetic charged particles tend to bind them together to produce magnetic neutral matter. It should also be very difficult to separate opposite magnetic charges in what is normally magnetically neutral matter. Furthermore, if magnetic charge exists, it would lead to a quantization of electric charge in which only integral multiples of a fundamental unit could occur. Of course this is precisely what is observed.

4b. The Forces Involved. Consider a particle of mass m carrying electric charge e and magnetic charge g_1 which moves with velocity v in the presence of a stationary body that possesses charges e_2 and g_2.

The force acting on the particle is given by

$$\vec{F} = \vec{F}_e + \vec{F}_m\,, \tag{1}$$

where $\vec{F}_e$ is the force acting on an electric charge by the fields produced by the stationary body and F_m is the force acting on a magnetic charge by the fields produced by the stationary body.

4b. Symmetry of the Equations. As given in page 472 of AF:

$$\vec{F}_e = e_1(\vec{E} + \frac{1}{c}\vec{v} \times \vec{B})\,. \tag{2}$$

Under the symmetry consideration $\vec{E} \to \vec{B}$, $\vec{B} \to -\vec{E}$, $q_e \to q_m$ we have:

$$\vec{F}_m = g_1(\vec{B} - \frac{1}{c}\vec{v} \times \vec{E})\,, \tag{3}$$

where $\vec{E}$, $\vec{B}$ are the field strengths at the position of the moving particle produced by the stationary charge.

Let the stationary body with charges e_2, g_2 be located at the origin of our coordinate system. The moving particle has a position vector $\vec{r} = r\hat{u}_r$, where $\hat{u}_r$ is a unit vector along the direction from the stationary charge to the moving particle.

From Coulomb's Law, we have:

$$\vec{E} = e_2\frac{\vec{r}}{r^3}\ . \tag{4}$$

Under the symmetry consideration, $\vec{E} \to \vec{B}$, $\vec{B} \to -\vec{E}$, $q_e \to q_m$ we have:

$$\vec{E} = g_2\frac{\vec{r}}{r^3}\ . \tag{5}$$

Therefore combining Eqs. (1) - (5),

$$\vec{F} = e_1\left(e_2\frac{\vec{r}}{r^3} + \frac{1}{c}\vec{v} \times g_2\frac{\vec{r}}{r^3}\right) + g_1\left(g_2\frac{\vec{r}}{r^3} - \frac{1}{c}\vec{v} \times e_1\frac{\vec{r}}{r^3}\right). \tag{6}$$

Together with Newton's second law,

$$\vec{F} = m\frac{d\vec{v}}{dt}\,, \tag{7}$$

we have:

$$m\frac{d\vec{v}}{dt} = (e_1e_2 + g_1g_2)\frac{\vec{r}}{r^3} + (e_1g_2 - e_2g_1)\frac{1}{c}\vec{v} \times \frac{\vec{r}}{r^3} \tag{8}$$

$$\vec{r} \times m\frac{d\vec{v}}{dt} = (e_1g_2 - e_2g_1)\frac{1}{c}\frac{\vec{r} \times (\vec{v} \times \vec{r})}{r^3}, \tag{9}$$

where we have used: $\vec{r} \times \vec{r} = 0$.

4c. Completing the Derivation. Using vector identities, one can show that:

$$\frac{\vec{r} \times (\vec{v} \times \vec{r})}{r^3} = \frac{d}{dt}\left(\frac{\vec{r}}{r}\right).$$

The proof is outlined in Appendix A.

Furthermore,

$$\frac{d\vec{r}}{dt} \times m\vec{v} = \left(\frac{d}{dt}\vec{r}\right) \times m\vec{v} + \vec{r} \times m\frac{d\vec{v}}{dt} = \vec{v} \times m\vec{v} + \vec{r} \times m\frac{d\vec{v}}{dt} = \vec{r} \times m\frac{d\vec{v}}{dt}\ .$$

Therefore recalling Eq. (9):

$$\frac{d\vec{r}}{dt} \times m\vec{v} = (e_1g_2 - e_2g_1)\frac{1}{c}\frac{d}{dt}\left(\frac{\vec{r}}{r}\right), \tag{10}$$

and we recognize that angular momentum would be conserved if $\vec{J}$ were given by:

$$\vec{J} = \vec{v} \times m\vec{v} - (e_1g_2 - e_2g_1)\frac{1}{c}\left(\frac{\vec{r}}{r}\right).$$

Now the component of $\vec{J}$ along the direction of $\vec{r}$ is:

$$J_r = (e_1g_2 - e_2g_1)\frac{1}{c}\ .$$

From angular momentum quantization, we know that the LHS (left-hand-side) of the above equation must be:

$$n\hbar \equiv n\frac{h}{2\pi}\,,$$

where n is an integer and h is Planck's constant ($\hbar = 1.05 \times 10^{-34}\,\mathrm{J\,s}$).

Therefore:

$$(e_1g_2 - e_2g_1)/\hbar c = n\,,$$

where n is an integer. This is the reciprocal quantization rule of Dirac. [In Dirac's original paper n = half integer are also allowed. We shall neglect that subtlety here.]

4d. Charge Quantization. In the present experimental situation, with only electric charge known, the consideration of a hypothetical magnetic charge g gives the electric quantization condition:

$$\frac{(e_1 - e_2)g}{\hbar c} = n \qquad \text{for any charge } e_1,\ e_2\,.$$

Choose $e_2 = 0$ and we have the quantization condition:

$$\frac{e_1 g}{\hbar c} = n \qquad \text{for any } \mathrm{e}_1\,.$$

Therefore, we have:

$$\frac{eg}{\hbar c} = n \qquad \text{for any e}\,.$$

That is the quantization condition for electric charge.

▷ From the observed unit of electric charge, show that:

$$\frac{e^2}{\hbar c} \approx \frac{1}{137}\,.$$

This number is known as the "fine structure constant."

Choosing $n = 1$, we find the unit of magnetic charge g_0.

▷ Show:

$$\frac{g_0^2}{\hbar c} \approx \frac{1}{137} = 137e^+\,.$$

It is clear that the forces between magnetic charges are super-strong.

Acknowledgments

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A. Vector Identities

1. $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$

   Proof: Take the same one component on both the left and right sides of the equation and prove the two sides equal for that case. Then by cycle the components, $(x, y, z) \to (y, z, x)$ and then $(y, z, x) \to (z, x, y)$ to immediately prove the equality for the other two components.

2. $(d/dt)\,(1/r) = -(1/r^3)\,(\vec{r} \cdot \vec{v})$.

   Proof: $(d/dt)\,(1/r) = (d/dr)\,(1/r)\,(dr/dt) = (-1/r^2)\,v_r = -(1/r^2)\,(\hat{r} \cdot \vec{v}) = -(1/r^3)\,(\vec{r} \cdot \vec{v})$.

3. $\dfrac{\vec{r} \times (\vec{v} \times \vec{r})}{r^3} = \dfrac{d}{dt}\left(\dfrac{\vec{r}}{r}\right)$.

   Proof: In the equation in #3, use the equation in #1 on the left side and and the equation in #2 on the right side, and the equality will emerge.

Note: In the original paper of Dirac, $n =$ half integer was also allowed. In that case, the unit of magnetic charge g is obtained by choosing $n = 1/2$. That leads to $g_0 = (137/2)\,e$ and this is known as the Dirac magnetic charge.

B. Additional Readings

1. J. Schwinger, *Science* **165**, 757 (1969).
2. K. Ford, "Magnetic Monopole," *Scientific American*, Dec, 1963, page 122. We strongly recommend that you read this well-written article.
3. L. Alvarez, P. H. Eberhard, R. R. Ross, and R. D. Watt, *Science* **167**, 701 (1970).

LOCAL GUIDE

The readings for this unit are on reserve for you in the Physics-Astronomy Library, Room 230 in the Physics-Astronomy Building. Ask for them as "The readings for CBI Unit 140." Do **not** ask for them by book title.

MODEL EXAM

$$k_e = 8.99 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$$
$$\epsilon_0 \equiv \frac{1}{4\pi k_e} = 8.85 \times 10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$$
$$k_m = 10^{-7}\,\mathrm{T\,m/A}$$
$$\mu_0 \equiv 4\pi k_m = 4\pi \times 10^{-7}\,\mathrm{T\,m/A}$$
$$c = 3.00 \times 10^8\,\mathrm{m/s}$$
$$h = 6.63 \times 10^{-34}\,\mathrm{J\,s}$$
$$e = 1.60 \times 10^{-19}\,\mathrm{C}$$

1. Write down in a few clearly stated sentences the motivation for postulating the existence of magnetic monopoles.

2. Starting from the assumption of symmetry between $\vec{E}$ and $\vec{B}$ fields in Maxwell's equations and the Lorentz Force equation, derive the reciprocal quantization condition of Dirac.

3. Evaluate the numerical values of the fine structure constant and the Dirac magnetic charge unit.

4. Give an argument why even if magnetic monopoles do exist they have escaped detection.

Brief Answers:

1-4. See this module's *text*.