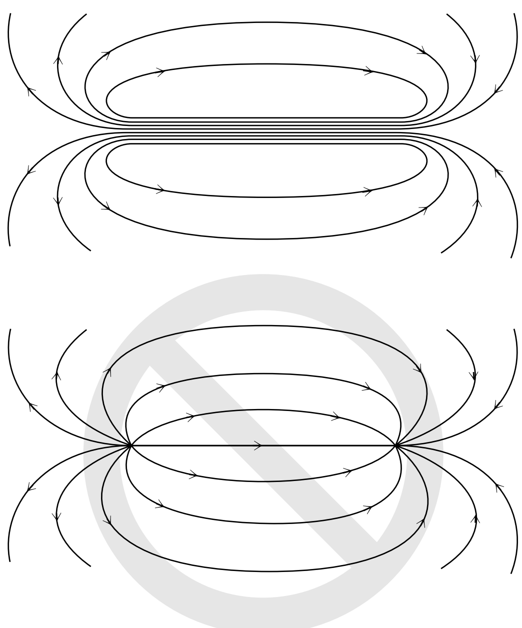
PHYSNET

MISN-0-139

SOURCELESSNESS OF THE MAGNETIC FIELD



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

SOURCELESSNESS OF THE MAGNETIC FIELD

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Title: **Sourcelessness of the Magnetic Field**

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Version: 2/1/2000 Evaluation: Stage 0

Length: 1 hr; 16 pages

Input Skills:

1. Some in "Gauss's Law Applied to Charge Distributions with Cylindrical and Planar Symmetry" (MISN-0-133).
2. Some in "Electrostatic Capacitance," (MISN-0-135).

Output Skills (Knowledge):

K1. Explain how this Gauss's Law for magnetic fields is a result of the physical property that magnetic "lines of force" are continuous and closed—that there are no isolated magnetic poles (or magnetic charges.)

Output Skills (Problem Solving):

S1. Evaluate the flux of $\vec{B}$ through surfaces (rectangular, cylindrical, spherical) directly and through the use of Gauss's Law for magnetic fields.

External Resources (Required):

1. M. Alonso and E. J. Finn, *Physics*, Addison-Wesley, Reading (1970); for access see this module's *Local Guide*.

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OF PROJECT PHYSNET

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Our publications are designed: (i) to be updated quickly in response to field tests and new scientific developments; (ii) to be used in both classroom and professional settings; (iii) to show the prerequisite dependencies existing among the various chunks of physics knowledge and skill, as a guide both to mental organization and to use of the materials; and (iv) to be adapted quickly to specific user needs ranging from single-skill instruction to complete custom textbooks.

New authors, reviewers and field testers are welcome.

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1. Introduction

Gauss's Law applied to Magnetic fields differs from Guass's Law applied to electric fields in that the expression of the law for magnetic fields has the net flux of $\vec{B}$ across any closed surface always equaling zero: $\oint \vec{B} \cdot \hat{N}\, ds = 0$. For electric fields the right side equals zero only if the net charge enclosed by the closed surface is zero, otherwise it's proportional to the net charge enclosed. This then indicates that "magnetic charges," the analog of electric charges which are the sources of the electric fields, do not exist.

2. Assigned Readings and Problems

1. In AF[1] read section 19.14. Note that $\epsilon_0 \equiv 1/(4\pi k_e)$ and $\mu_0 \equiv 4\pi k_m$.

   Study again Section 19.2 and Figure 19.1 on page 419 where the flux of a vector field through a surface is defined.

2. Work problems 19.26* and 19.27*. In Problem 19.26, evaluate the flux across surface $(aefd)$ two ways: first directly, then secondly, knowing the flux over all other pieces of the closed surface using Gauss's Law for $\vec{B}$.

3. Comments

Referring to Figure 19.1 of AF, note that while the direction of vector $\vec{V}$ (or $\vec{B}$, for our purposes) at a point is unique (determined by the physics of the situation in the case of $\vec{B}$), the direction of the unit vector, $\hat{N}$ normal to the element of surface on the integration surface is somewhat ambiguous. It is normal to the surface, but is it "up" or "down"? To remove this ambiguity, the convention is adopted that $\hat{N}$ is normal to the surface in such a direction that it points out of the volume enclosed by the surface if the surface is closed. If the surface is not closed, $\hat{N}$ points away from the concavity enveloped by the surface. If the surface is an isolated plane surface the unit normal needs to be specified another way.

[1]M. Alonso and E. J. Finn, *Physics*, Addison-Wesley, Reading (1970); for access see this module's *Local Guide*.

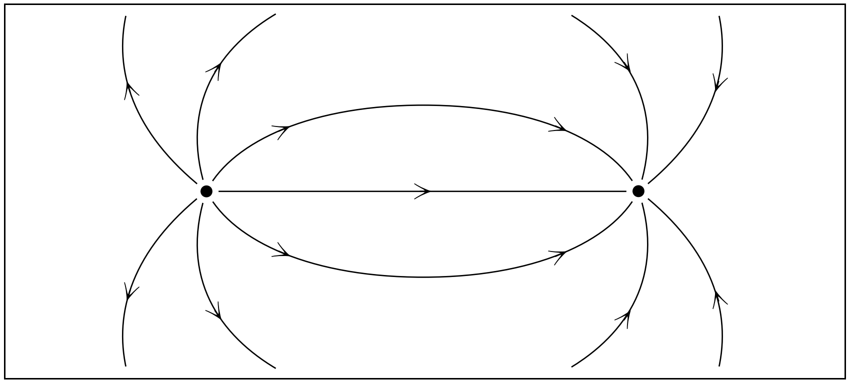


Figure 1. .

Compare the Gauss's Law for magnetic fields, $\oint \vec{B} \cdot \hat{N}\, ds = 0$, over *all* closed surfaces, with that for electric fields:

$$\oint_S \vec{E} \cdot \hat{N}\, ds = 4\pi k_e q_0\,,$$

where q_0 is the charge enclosed by the surface S. For example, consider two unlike charges and the electric field lines in their vicinity, as shown in Fig. 1.

If we surround the positive charge with some imaginary closed surface (excluding the negative charge) then notice that the lines of $\vec{E}$ point out of the surface, everywhere on the surface, so that $\vec{E} \cdot \hat{N}$ at every point on the surface is always a positive number. Thus when this is multiplied by the element of surface area, ΔS, and summed over the whole surface, this adds up to a positive number. This positive number is $(4\pi k_e)$ times the charge enclosed.

Similarly, if we surround only the negative charge then $\vec{E}$ everywhere points into the surface while $\hat{N}$ points outward so that $\vec{E} \cdot \hat{N}$ is always negative.

Now if our imaginary closed surface surrounds some volume between the two charges, with the charges not enclosed, then over parts of the surface $\vec{E} \cdot \hat{N}$ is positive while on other parts it is negative. Multiplying $\vec{E} \cdot \hat{N}$ by ΔS and summing over the whole closed surface we will be adding plus and minus contributions. Gauss's Law, in fact, tells us that these

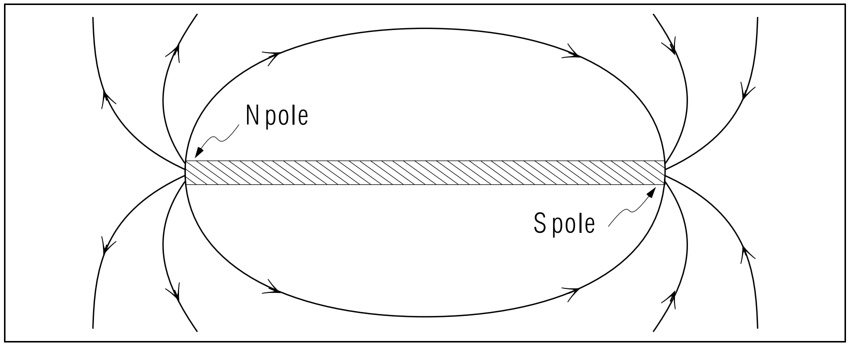


Figure 2. .

contributions add to zero exactly if no charge is enclosed. Now in this latter case notice that the lines of $\vec{E}$ are continuous, entering the closed surface on one side and leaving it on another, whereas in the cases where a charge is enclosed the lines of $\vec{E}$ emerge from or terminate on the charge. The charges are the sources of $\vec{E}$, and the surface integral of tells us the "strength" of the source.

In the case of magnetic fields there are no sources, no magnetic "charges," no isolated magnetic poles. Hence, over any closed surface,

$$\oint_S \vec{B} \cdot \hat{N}\, ds = 0.$$

The lines of $\vec{B}$ are continuous, and the negative and positive contributions of $\vec{B} \cdot \hat{N}$ always cancel over any closed surface.

We might ask: what about a bar magnet?

All of the lines of $\vec{B}$ appear to emerge from the N-pole and terminate on the S-pole (see Fig. 2). So, we might ask, would not the flux of $\vec{B}$ over a closed surface surrounding only the N-pole have only positive contributions making a net zero flux impossible? The answer is that while the lines do emerge from the N-pole outside of the magnet, these lines enter the S-pole of the magnet, are continuous through the inside of the magnet, directed toward the N-pole inside the magnet, connecting continuously with the lines that leave the N-pole, as shown in Fig. 3.

The net effect is that when we take into account the contribution to the flux through the closed surface from the $\vec{B}$ inside the magnet, the total flux is zero over the closed surface.

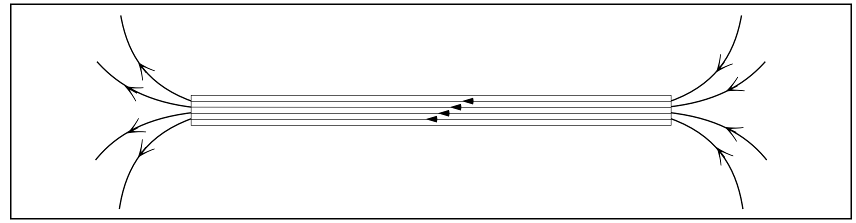


Figure 3. .

4. Answers to Assigned Problems

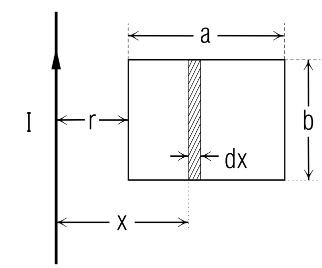
19.26 a. $-2400\,\mathrm{T\,cm}$. Be sure you understand why the sign is negative.

b. zero.

c. $+2400\,\mathrm{T\,cm} = (B\cos\theta) \times 1500$ where $\cos\theta = 4/5$, θ being the angle between $\hat{N}$ and $\vec{B}$

19.27 $(\mu_0 I b)/(2\pi)\ell n(1 + a/r)$ = flux of $\vec{B}$ through coil. This flux is + or −, depending upon how we defined $\hat{N}$ direction of coil. To get this answer we need to integrate $\vec{B} \cdot \hat{N}\, ds$ over the area of the coil. However, $\vec{B}$ is not constant so we cannot just multiply $|\vec{B}|$ by the coil's area.

In the sketch below, $|\vec{B}|$ at distance x from the wire is $(\mu_0 I)/(2\pi x)$ so the flux element through the shaded region is $d\phi = (b\mu_0 I)/(2\pi x)\, dx$.



Acknowledgments

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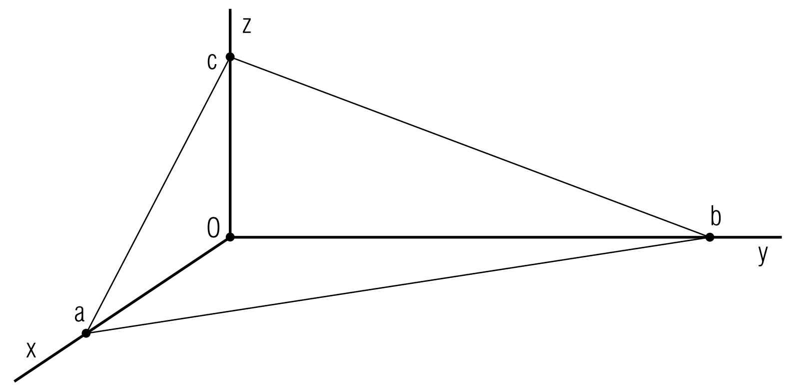
LOCAL GUIDE

The readings for this unit are on reserve for you in the Physics-Astronomy Library, Room 230 in the Physics-Astronomy Building. Ask for them as "The readings for CBI Unit 139." Do **not** ask for them by book title.

PROBLEM SUPPLEMENT

1.



The pyramid above is a closed surface formed by the four plane triangular surfaces aOc, aOb, cOb and abc. In this region of space there exists a uniform magnetic field $\vec{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$ where B_x is the x-component of $\vec{B}$, etc.

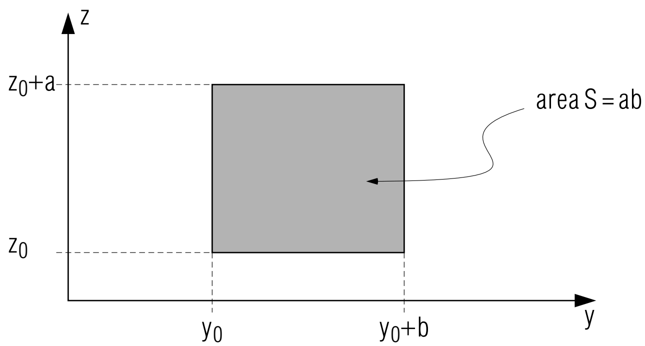
a. What is the flux of $\vec{B}$ through surface aOc? [H]

b. What is the flux of $\vec{B}$ through surface aOb? [D]

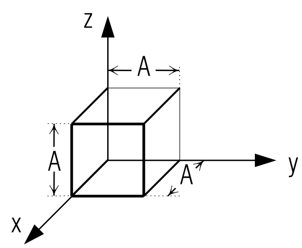
c. What is the flux of $\vec{B}$ through surface cOb? [B]

d. For surface abc, it could be a tedious chore to find the unit normal to the surface. Use, instead, Gauss's Law and the results of parts (a), (b), and (c) to find the flux of $\vec{B}$ through the surface. [G]

2. Consider the field $\vec{B} = B_1 y\hat{x} + B_1 x\hat{y}$ where B_1 is a constant and x and y are the cartesian coordinates of a point relative to a fixed coordinate system. Evaluate the magnitude of the flux of $\vec{B}$ through the rectangular surface of area $S = ab$ shown below, oriented in the y-z plane: [C]



3. Consider the field $\vec{B} = B_0 x\hat{x}$ where B_0 is constant and x is the distance along the x-axis. Evaluate the flux of this field through the sides of a closed cube, A meters on a side whose edges are parallel to the coordinate axes and one corner of which is at the origin of the coordinates: [F]



Use the result to prove that no physical $\vec{B}$ field can have the functional dependence on the coordinates as that written above. [A]

4. Explain how Gauss's Law for magnetic fields is a consequence of the physical properties that:

   a. there are no point sources of magnetic fields as there are for electric fields; and

   b. the lines of $\vec{B}$ are continuous and close on themselves.

   In your explanation, illustrate with sketches what Gauss's Law gives you if you have a source of the field enclosed by the Gaussian (integration) surface and contrast this with a sketch for the case of continuous lines of the vector field. [E]

Brief Answers:

A. For a physically realizable field, the flux of $\vec{B}$ over any closed surface must be zero. A field whose $\vec{B} = B_0 x\hat{x}$ is thus not physically realizable.

B. $-(1/2)bcB_x$.

C. $(1/2)SB_1(b + 2y_0)$.

D. $-(1/2)abB_z$.

E. See text and comments section of this study guide.

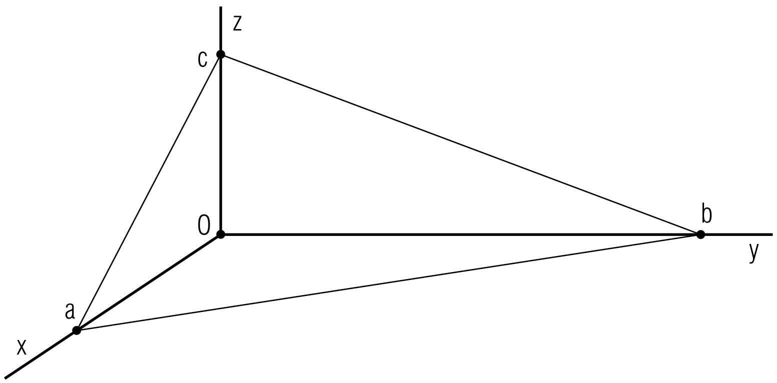
F. B_0A^3.

G. $(1/2)(bcB_x + acB_y + abB_z)$.

H. $-(1/2)acB_y$.

MODEL EXAM

1. See Output Skill K1 in this module's *ID Sheet*.

2. 

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   a. What is the flux of $\vec{B}$ through surface aOc?
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   d. For surface abc, it could be a tedious chore to find the unit normal to the surface. Use, instead, Gauss's Law and the results of parts (a), (b), and (c) to find the flux of $\vec{B}$ through the surface.

Brief Answers:

1. See this module's *text*.

2. See this module's *Problem Supplement*, problem 1.