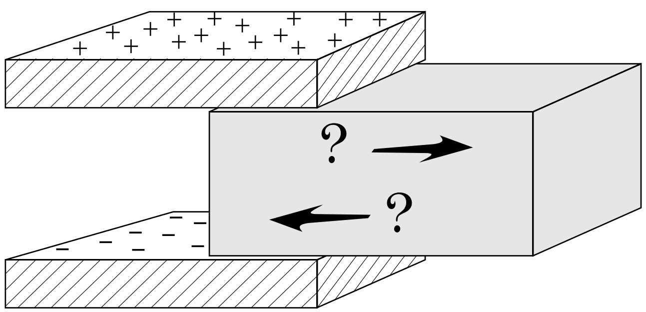
PHYSNET

MISN-0-137

ELECTROSTATIC FIELD ENERGY



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

ELECTROSTATIC FIELD ENERGY

by

O. McHarris and P. Signell

Title: **Electrostatic Field Energy**

Author: O. McHarris and P. Signell, Michigan State University

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Input Skills:

1. State the change in potential energy of a system that is initially and finally stationary, given the work done on that system (MISN-0-20).
2. Calculate the work done on a given charge moved through a given potential difference (MISN-0-117).
3. Calculate the work capable of being done on a given charge by a given seat of $\mathcal{EMF}$ (MISN-0-119).
4. . Given the dimensions, calculate the capacitance of a metal sphere and of a parallel plate capacitor, both in air and with a given dielectric as part of the system (MISN-0-135).

Output Skills (Knowledge):

K1. Derive the expression for the energy stored in the electric field of a capacitor.

K2. State the general expression for the stored-energy-per-unit-volume associated with an electric field; derive that expression for the case of a parallel-plate capacitor.

Output Skills (Problem Solving):

S1. Calculate the potential energy of capacitor-type charge configurations given any two of the three quantities: capacitance, charge, and potential difference.

S2. Calculate the forces on the plates of a capacitor or on a slab of dielectric between the plates of a capacitor, given the electrical charge state of the capacitor.

External Resources (Required):

1. M. Alonso and E. J. Finn, *Physics*, Addison-Wesley, Reading (1970). For access, see this module's *Local Guide*.

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New authors, reviewers and field testers are welcome.

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1. Introduction

In this unit we are primarily concerned with using the energy stored in charge configurations which produce electric fields. The expression for that stored energy has been derived for the particular case of the electric field of capacitors. One of the advantages of knowing the potential energy of any system is that one can use it to calculate the mechanical forces operating on the system,[1] and this is true in the case of capacitors also. One must be certain, however, to include all energy changes associated with slight movement of the object for which the force is being calculated.

2. Readings and Problems to Solve

In AF[2] study Section 19.12 (p. 437) and do Problems 19.10(c), 19.15, 19.16 (b,c), and 19.17. Note that AF use the term "relative permittivity" and the symbol ϵ_r to label exactly the same quantity for which other authors use the term "dielectric constant" and the symbol K or κ: $\epsilon_r \equiv K \equiv \kappa$.

Brief Answers:

19.10		energy (series)= 1.3×10^{-4} joules
		energy (parallel)= 1.3×10^{-4} joules
19.16	b)	4.5×10^{-9} joules
	c)	3.4×10^{-9} joules. The "lost" energy has heated up the connecting wire.

[1]See "Work, Power, Kinetic Energy, Work-Energy Principle" (MISN-0-20).

[2]M. Alonso and E. J. Finn, *Physics*, Addison-Wesley, Reading (1970). For access, see this module's *Local Guide*.

19.17 d) $\vec{E}$ takes account of the charges on both plates. $\vec{F}$ must take account only of the charges on one plate attracting the charges on the second plate. Multiplying Q times $\vec{E}$ would give $2\vec{F}$.
For the case in which V is kept constant, $d(\text{energy}) = (V^2\,A)/(8\pi k_e\,x^2)\,dx$ and $F = (V^2\,A)/(8\pi k_e\,x^2)$.

3. Notes

The energy of a capacitor has been shown (see above) to be:

$$U = \frac{CV^2}{2} = \frac{Q^2}{2C},$$

and we also know[3] that C depends on the physical dimensions and dielectric constant of the capacitor. Thus we may ask how a capacitor's energy changes as one of its dimensions changes, and in answering that we obtain the expression for the force on the object that moves to change the dimension. For example, Problem 19.17 in AF asks you to calculate the force of attraction between the two plates of a parallel-plate capacitor; it asks you to do this by calculating the change in energy of the capacitor when the plate separation is increased by a small amount and by then equating the increase in potential energy to the work done on the capacitor.

Since $C = Q/V$, a change in C requires a change in charge, potential difference, or both. One must examine the physical set-up for any given case in order to determine which of Q and V changes. If the capacitor is first charged, and then the charging device is disconnected before we change C, Q will be unable to change when C changes: there is no way for the charge carriers to leave or enter the capacitor plates. Since Q is then a constant, the chain rule for derivatives gives:

$$dU = -\frac{Q^2}{2C^2}\,dC.$$

The proportionality constant $(Q^2/2C^2)$ is positive, so our equation tells us that if, for example, U increases by dU, C decreases by $|dC|$. Thus the

[3]See "Electrostatic Capacitance" (MISN-0-135).

forces would initially act in such a way[4] as to increase C if the capacitor parts could move.

A second typical case is where we wish to know the forces in a capacitor while a constant voltage source, such as a battery, is kept connected to it. As we change C in this case, only Q will change as the battery adjusts the charges on the plates to keep $Q = CV$. In analogy with the constant-Q situation, one is tempted to say: write the capacitor's energy in terms of the constant V and then differentiate. However, this would give only part of the energy change. Since the battery is still connected to the capacitor, we must also include the work it does in moving the charge carriers as the capacitance changes.[5]

$$dW = V\,dQ.$$

Thus in this case the change in energy of the whole system, as C changes, is:

$$d(\text{energy}) = dU - dW = -(V^2/2)\,dC.$$

Notice that for a given V, Q, C and dC, and with the battery maintaining a constant voltage, the initial small change in energy with C is the same as in the case where the capacitor was charged but unconnected to the battery.[6] This makes sense. Consider, for example, the force of attraction between the two plates of a capacitor. The force exists because of the Coulombic forces between the charges on the two plates and in the dielectric and, for a given capacitor with a given charge, the magnitude and especially the direction of the force will not depend on whether the capacitor is connected to a battery.

Acknowledgments

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[4]See "Potential Energy and Motion: Potential Curves, Turning Points" (MISN-0-22).

[5]See "Resistive D.C. Circuits" (MISN-0-119).

[6]However, a real change in the capacitor, resulting in a real change in C, would require different energies in the two cases. In the present unit we have only calculated forces within a fixed capacitor.

LOCAL GUIDE

The readings for this unit are on reserve for you in the Physics-Astronomy Library, Room 230 in the Physics-Astronomy Building. Ask for them as "The readings for CBI Unit 137." Do **not** ask for them by book title.

MODEL EXAM

1. See Output Skills K1-K2 in this module's *ID Sheet*. One or more of these skills may be on the actual exam.

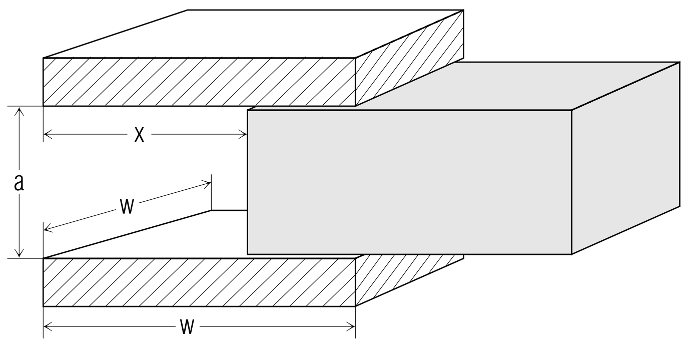
2. A dielectric slab is partially inserted between two plates of a parallel-plate capacitor, as shown below. Assuming that the potential applied to the capacitor is constant, calculate:

   a. the capacitance of the system.

   b. the energy of the capacitor as shown.

   c. the force exerted on the slab.

   d. whether the slab is ejected or attracted in.



Brief Answers:

1. See this module's *ID Sheet*.

2. This system is equivalent to two connected capacitors, one of length x and one of length $(w - x)$.
   Are they connected in series or parallel?

   a. $C = \dfrac{(Kw^2 - xw(K-1)}{4\pi k_e a}$.

   b. $E = v^2 \dfrac{Kw^2 - xw(K-1)}{8\pi k_e a}$.

   c. Find F by calculating the change in energy with change in x of the entire system.
   $F = \dfrac{v^2 w(K-1)}{8\pi k_e a}$.

   d. Potential energy increases as x increases. Therefore the system, if left to itself, tends to decrease x in order to decrease the potential energy. Thus the slab is attracted inward.