
GAUSS'S LAW APPLIED TO CYLINDRICAL AND PLANARCHARGE DISTRIBUTIONS
by
Peter Signell, Michigan State University

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## Input Skills:

1. Vocabulary: cylindrical symmetry, planar symmetry (MISN-0153); Gaussian surface, volume charge density (MISN-0-132).
2. State Gauss's law and apply it in cases of spherical symmetry (MISN-0-132).

## Output Skills (Knowledge):

K1. Vocabulary: coaxial cable, cylinder of charge, line of charge, sheet of charge, linear charge density.
K2. Justify the Gaussian-Surface shapes that are appropriate for cylindrical and planar charge distributions.
K3. State Gauss's Law in equation form and define each symbol. For cylindrical and planar charge distributions, define needed parameters and then, justifying each step as you go, solve Gauss's Law for the symbolic electric field at a space-point.

## Output Skills (Problem Solving):

S1. Given a specific charge distribution with cylindrical or planar symmetry, use Gauss's law to determine the electric field produced by the charge distribution.

## Post-Options:

1. "Electric Fields and Potentials Across Charge Layers and In Capacitors" (MISN-0-134).
2. "Electrostatic Capacitance" (MISN-0-135).

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# GAUSS'S LAW APPLIED TO CYLINDRICAL AND PLANAR CHARGE DISTRIBUTIONS 

by<br>Peter Signell, Michigan State University

## 1. Introduction

1a. Overview. In this module Gauss's law is used to find the electric field in the neighborhood of charge distributions that have cylindrical and planar symmetry. For each of these two symmetries, useful Gaussian surfaces are easily constructed. Once an appropriate Gaussian surface is constructed, the electric field is easily found from Gauss's law: ${ }^{1}$

$$
\begin{equation*}
\oint \vec{E} \cdot \hat{n} d S=4 \pi k_{e} q_{S} \tag{1}
\end{equation*}
$$

where $q_{S}$ is the net charge enclosed by the Gaussian surface $S$ and $k_{e}$ is the electrostatic force constant.
1b. Usefulness. It is very useful to know the electric field in the neighborhood of cylindrical and planar charge distributions, for these geometries are the ones used in coaxial cables and capacitors. Knowing the electric fields helps one determine how these devices will react in electronic circuits. In addition, the same general ideas are used in determining the magnetic fields produced in solenoids, transformers, coaxial cables, chokes, and transmission lines.

## 2. Cylindrical Symmetry: Line Charge

2a. Approximating a Real Line by an Infinite One. When dealing with a line of charge, we will treat it as though its ends had been extended to infinity. This approximation makes the resulting electric field especially simple and easy to solve for. The solutions we get for the infinitely long line will be applicable to the finite-line case for electric field points that are much closer to the middle part of the line than to its ends. For practical applications the "infinite line" is almost always a good approximation to the actual finite line.

[^0]

Figure 1. A cylindrical Gaussian surface is used to apply Gauss's law to a line of charge.

2b. The Gaussian Surface. For an infinitely long line with uniform linear charge density ${ }^{2}$ along it, the preferred Gaussian surface is cylindrical (see Fig. 1). This follows from taking the two rules for constructing Gaussian surfaces and combining them with knowledge of the electric field's directions and equi-magnitude surfaces. ${ }^{3}$ The axis of the cylindrical surface is along the line of charge, while the surface's radius is that of the point at which you wish to know the electric field. The length of the cylindrical surface is immaterial.

2c. The Electric Field. Applying Gauss's law, Eq. (1), to the case of a straight line of charge with uniform linear charge density (charge per unit length) $\lambda$, we will show that the magnitude of the electric field at a distance $r$ from the line is:

$$
\begin{equation*}
E=2 k_{e} \frac{\lambda}{r} \tag{2}
\end{equation*}
$$

Proof: If the length of the cylindrical Gaussian surface is $L$, then the charge enclosed by the surface is:

$$
\begin{equation*}
q_{S}=\lambda L \tag{3}
\end{equation*}
$$

The component of the electric field normal to either flat end of the closed cylindrical surface is zero, but the component normal to the cylindrical

[^1]part of the surface is just the field itself:
$\oint \vec{E} \cdot \hat{n} d S=\int_{\text {cyl. }} \vec{E} \cdot \hat{n} d S+\int_{\text {ends }} \vec{E} \cdot \hat{n} d S=E \int_{\text {cyl. }} d S+0=E(2 \pi r L)$.
Using Gauss's law, Eq. (1), to combine Eqs. (3) and (4), we obtain the solution, Eq. (2). Of course in a real problem our solution would be valid only in the region where the distance to the line of charge is much smaller than the distance to the line's nearest end. As an amusing "aside," notice that Eq. (2) says that the sound of a long "line" of traffic will only die off as $r^{-1}$ rather than the $r^{-2}$ one obtains for a point source.

## 3. Other Cylindrical Distributions

3a. Electric Field of a Cylindrical Surface. A cylindrical surface with finite radius, constant surface charge density, and infinite extent, has an electric field whose preferred Gaussian surfaces are identical to those for an infinite charged line (see Fig. 2). This is because both the line and the cylindrical surface have the same geometrical symmetry and hence the same electric field directions and equi-magnitude surfaces. ${ }^{4}$ For a charged surface of radius $R$ and surface charge density $\sigma$, the amount of charge
${ }^{4}$ See "Electric Field from Symmetric Charge Distributions," (MISN-0-153), the section on infinitely long cylindrical charge distributions.


Figure 2. The Gaussian surface for a cylindrical surface charge distribution (on a cylindrical surface of radius $R$ ).
enclosed by a Gaussian surface of radius $r$ and length $L$ is:

$$
\begin{align*}
q_{S} & =2 \pi R L \sigma & & \text { for } r>R  \tag{5}\\
& =0 & & \text { for } r<R
\end{align*}
$$

Exactly as in the case of the line of charge, the integral of the normal component of the electric field over the Gaussian surface is:

$$
\begin{equation*}
\oint \vec{E} \cdot \hat{n} d S=(2 \pi r L) E . \tag{6}
\end{equation*}
$$

Then using Eqs. (5) and (6) in Gauss's law, Eq. (1), we find:

$$
\begin{align*}
E & =4 \pi k_{e} \frac{\sigma R}{r} & & \text { for } r>R  \tag{7}\\
& =0 & & \text { for } r<R
\end{align*}
$$

3b. Linear vs. Surface Charge Density. We may describe the charge distribution on a cylindrical surface as either a surface charge density or a linear charge density.

The surface charge density $\sigma$ is the charge per unit area on the cylindrical surface:

$$
\begin{equation*}
\sigma=\frac{q}{2 \pi r \ell} \tag{8}
\end{equation*}
$$

where $2 \pi r \ell$ is the surface area of a cylinder of radius $r$ and length $\ell$.
The linear charge density $\lambda$ is the (total) charge per unit length along the cylindrical surface:

$$
\begin{equation*}
\lambda=\frac{q}{\ell}=2 \pi r \sigma \tag{9}
\end{equation*}
$$

$\triangleright$ Show that if $q=1.0 \times 10^{-6} \mathrm{C}, r=1.0 \mathrm{~cm}$, and $\ell=1.0 \mathrm{~m}$, then $\sigma=$ $1.6 \times 10^{-5} \mathrm{C} / \mathrm{m}^{2}$ and $\lambda=1.0 \times 10^{-6} \mathrm{C} / \mathrm{m}$.

3c. The Coaxial Cable. A coaxial cable, such as that used to transport TV signals or the signal from a pickup to a stereo amplifier, consists of two metallic conductors with cylindrical symmetry, sharing a common cylinder axis and separated by some kind of insulator. The center cylinder is usually a solid copper wire while the outer one is usually a sheath of braided wire (see Fig. 3). This construction makes the cable mechanically flexible. As far as the electrical properties are concerned, there might as well be two concentric cylindrical surfaces ${ }^{5}$ as shown in Fig. 4. The magnitude of the linear charge density on the two cylinders is the same, so the

[^2]

Figure 3. A cross-sectional view of a typical coaxial cable.


Figure 4. A crosssectional view of the charge surfaces in a coaxial cable. The radius of the inner cylinder is exaggerated for the purpose of illustration.
magnitude of the surface charge density is higher on the inner cylinder. For ordinary uses the charges on the two cylinders are of opposite sign.
3d. Electric Field of the Coaxial Cable. Gauss's law shows that the electric field of a charged coaxial cable is zero except between the conducting cylindrical surfaces, where it is equal to the field produced by the inner cylinder. Applying Gauss's law to the coaxial cable's various regions, as shown in Fig. 4, the electric field is readily found to be:

$$
\begin{align*}
\vec{E}_{I} & =0 \\
\vec{E}_{I I} & =2 k_{e} \frac{\lambda}{r} \hat{r}  \tag{10}\\
\vec{E}_{I I I} & =0 \quad \text { Help }:[S-1]
\end{align*}
$$

In region II of Fig. 4, $\lambda$ is negative, so that particular electric field is directed radially inward.


Figure 5. (a) A cross-sectional view of a Gaussian surface (dashed lines) for an infinite plane of charge; and (b) a threedimensional view of the Gaussian surface.

## 4. A Single Sheet of Charge

4a. Approximation: An Infinite Sheet. We will restrict ourselves to the case of a uniform planar charge distribution of infinite extent: in other words, a flat sheet with a uniform surface charge density that extends to infinity. These restrictions make the resulting electric field especially simple and easy to determine. The solutions we get will be valid for any application in which the sheet of charge can be approximated by an infinite sheet with the same surface charge density. This approximation will be a good one when the distances from relevant electric field points to the edges of the physical sheet are all much larger than the distance to the nearest point on the sheet of charge. Thus the edges will "look" an almost infinite distance away (in comparison). This will be the case for important charge-storing components in electronic circuits.

4b. The Gaussian Surface. For a uniform planar charge distribution of infinite extent, all parts of the preferred Gaussian surface can be shown to be either parallel or perpendicular to the plane of the charge. This requirement would be satisfied, for example, by a box-like surface that is cut by the plane into two equal boxes (see Fig. 5). The "parallel or perpendicular" requirement follows from taking the two rules for constructing Gaussian surfaces and combining them with our knowledge of the electric field's directions and equi-magnitude surfaces. Any surface that satisfies the "parallel or perpendicular" requirement is acceptable,
but rectangular and cylindrical boxes are the easiest to use in computing the surface areas and volumes that enter into Gauss's law.
4c. The Electric Field. Applying Gauss's law to a flat infinite sheet with uniform surface charge density $\sigma$, we find that the magnitude of the electric field is everywhere the same:

$$
\begin{equation*}
E=2 \pi k_{e} \sigma \tag{11}
\end{equation*}
$$

The direction of the field is normal to the sheet of charge, directed away from the sheet for a positive charge density and toward the sheet for a negative charge density. If the area of one end of the box-like Gaussian surface is $A$, then the charge enclosed by the surface is:

$$
\begin{equation*}
q_{S}=\sigma A \tag{12}
\end{equation*}
$$

The component of the electric field normal to the side of the Gaussian surface is zero on the four sides that cut through the plane and equal to the electric field on the other two sides:

$$
\begin{equation*}
\oint \vec{E} \cdot \hat{n} d S=2 E A \tag{13}
\end{equation*}
$$

Using Gauss's law to combine Eqs. (12) and (13) we obtain Eq. (11), the solution. Of course in a real problem the constancy of the electric field is restricted to regions where the distance to the sheet of charge is much smaller than the distance to the sheet's nearest edge.

## 5. Two Parallel Sheets of Charge

5a. Unequal Surface Charge Densities. Gauss's law can be easily applied to the case of two infinite parallel sheets having uniform surface charge densities $\sigma$ and $\sigma^{\prime}$, respectively. To obtain the electric field at some particular point, apply Gauss's law to each of the sheets separately and then add the fields from the two sheets vectorially. Note that the two Gaussian surfaces have one side in common. You should obtain the answers: Help: [S-3]

$$
\begin{array}{ll}
E=2 \pi k_{e}\left(\sigma+\sigma^{\prime}\right), & \text { (outside the planes) } \\
E=2 \pi k_{e}\left(\sigma-\sigma^{\prime}\right), & \text { (between the planes) } \tag{15}
\end{array}
$$

Note that these equations become particularly simple when $\sigma$ and $\sigma^{\prime}$ are equal. Note also that this problem has no symmetry plane so a single preferred Gaussian surface could not be drawn: the two rules for constructing such a surface could not be satisfied with only our usual prior knowledge of the field.

5b. Equal Surface Charge Densities. For two infinite parallel sheets of charge with identical surface charge densities, $\sigma$, we can apply Gauss's law using a single Gaussian surface. There is a plane of symmetry that is parallel to the two sheets and half way between them, so the Gaussian surface must be symmetrical with respect to that plane of symmetry. In practical terms, the symmetry plane must cut the boxlike surface into two identical box-like surfaces. Note how this symmetry of the Gaussian surface with respect to the problem's symmetry plane ensures that the two rules for constructing the preferred surface can be satisfied. Help: [S-2] If one end of the surface has area $A$, the charge enclosed by the surface is:

$$
\begin{equation*}
q_{S}=2 \sigma A \tag{16}
\end{equation*}
$$

Then Gauss's law produces:

$$
\begin{equation*}
E=4 \pi k_{e} \sigma, \quad \text { (outside the sheets) } \tag{17}
\end{equation*}
$$

and

$$
E=0, \quad \text { (between the sheets). }
$$

These two equations agree with Eqs. (14) and (15).

## Acknowledgments

I would like to thank Professor J. Linnemann for a valuable suggestion. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant \#SED 74-20088 to Michigan State University.

## Glossary

- coaxial cable: an electrical cable consisting of two metallic concentric cylinders separated by some kind of insulator. The electric field is zero both inside the inner cylinder and outside the outer cylinder.
- cylinder of charge: a charge distribution with cylindrical symmetry. For a cylinder of charge with constant density extending to infinity, the associated electric field falls off as $(1 / r)$ outside the surface ( $r$ is the radius from the axis of the cylinder).
- line of charge: a charge distribution along a straight line. This is a special case of cylinder of charge.
- linear charge density: the charge per unit length along a line.
- sheet of charge: a charge distribution in a plane. For a plane of charge with constant density extending to infinity in all directions, the associated electric field is everywhere constant and normal to the plane.


## PROBLEM SUPPLEMENT

Note: Problems 7 and 8 also occur in this module's Model Exam.

1. Two parallel lines of charge, shown coming out of the page in the sketch, are a distance $2.0 d$ apart. If both are positively charged, find the electric field as a function of $y$ at points along the perpendicular bisector of the line connecting the two [find $\vec{E}(y)$ for $x=0$ ].

2. An infinitely long cylinder of charge has a radius $R$ and a volume charge density $\rho$. Find the electric field in the regions $r<R$ and $r>R$ and show that both lead to the same result at $r=R$. Help: [S-9]

3. Find the electric field inside and outside a hollow cylinder of charge whose radius is $R$, and whose linear charge density is $\lambda$.

4. A cylinder of charge has volume charge density $\rho$ and radius $R_{1}$. Outside it and concentric with it is a cylindrical surface of charge with radius $R_{2}$. The charge on this outer surface has a linear charge density $\lambda$. Find the electric field in each of the three regions defined by the cylinder surfaces at $R_{1}$ and $R_{2}$. Show that the result in Problem 3 can be obtained from the results in this Problem
 by letting $R_{1}=0, R_{2}=R$.
5. Find the electric field in the four regions defined by the three infinite planes (sheets) of charge in the sketch.

6. An infinite slab of charge of thickness $d$ has a uniform charge density $\rho$, as shown below in cross section in the sketch. Use Gauss's law to find the electric field inside and outside the slab. Help: [S-4]

7. Use Gauss's law to find the electric field outside and inside two large parallel plates with equal surface charge densities $\sigma$ on the plates and with volume charge density $\rho$ between the plates. The distance between the plates is $d$. Help: [S-11]

8. Use Gauss's law to find the electric field in each of the three regions defined by two coaxial cylindrical surfaces, each with linear charge density $\lambda$, and with a uniform volume charge density $\rho$ inside the inner cylindrical surface. The radii of the two cylindrical surfaces are $R_{1}$ and $R_{2}$ (see diagram below).


## Brief Answers:

1. $E_{x}=0$
$E_{y}=4 k_{e} \frac{\lambda y}{\left(d^{2}+y^{2}\right)}$ Help: [S-10]
2. $\vec{E}(r)=2 \pi k_{e} \rho r \hat{r}$ for $r<R$ Help: $[S-6]$
$\vec{E}(r)=2 \pi k_{e} \frac{\rho R^{2}}{r} \hat{r}$ for $r>R$
At the surface: $\vec{E}(R)=2 \pi k_{e} \rho R \hat{r}$ for $r=R$
3. Region $1\left(r<R_{1}\right): \vec{E}(r)=2 \pi k_{e} \rho r \hat{r}$ Help: [S-7]

Region $2\left(R_{1}<r<R_{2}\right): \vec{E}(r)=2 \pi k_{e} \frac{\rho R_{1}^{2}}{r} \hat{r}$
Region $3\left(R_{2}<r\right): \vec{E}(r)=2 k_{e} \frac{\rho \pi R_{1}^{2}+\lambda}{r} \hat{r}$
4. $\lim _{R_{1} \rightarrow 0} 2 \pi k_{e}\left(\frac{\rho R_{1}^{2}}{r}\right)=0 ; \vec{E}=0($ for $r<R)$
$\lim _{R_{1} \rightarrow 0}\left(2 \pi k_{e} \frac{\rho R_{1}^{2}}{r}+2 k_{e} \frac{\lambda}{r}\right)=2 k_{e} \frac{\lambda}{r} ; \vec{E}(r)=2 k_{e} \frac{\lambda}{r} \hat{r}($ for $r>R)$
5. In the figure below, note that each row of four arrows shows the field that would be produced by just one plane of charge alone (see the annotations down the right side of the figure).


$$
\vec{E}_{I}=-2 \pi k_{e} \sigma \hat{x} ; \text { Help: [S-8] }
$$

$$
\vec{E}_{I I}=2 \pi k_{e} \sigma \hat{x} ; \quad \vec{E}_{I I I}=-2 \pi k_{e} \sigma \hat{x} ; \quad \vec{E}_{I V}=2 \pi k_{e} \sigma \hat{x}
$$

6. "Inside": $\vec{E}=4 \pi k_{e} \rho y \hat{y}$;
"Outside": $\vec{E}=2 \pi k_{e} \rho d \hat{y}$
7. Outside: $\begin{aligned} \vec{E}_{\text {right }} & =+2 \pi k_{e}(2 \sigma+d \rho) \hat{x} \\ & \vec{E}_{\text {left }}\end{aligned}$

Inside: $\vec{E}=4 \pi k_{e} \rho x \hat{x}$, where $x$ is measured from the symmetry plane between the plates.
8. "Inside" region: $\vec{E}(r)=2 \pi k_{e} \rho r \hat{r}$
"Between" region: $\vec{E}(r)=2 k_{e} \frac{\pi R_{1}^{2} \rho+\lambda}{r} \hat{r}$
"Outside" region: $\vec{E}(r)=2 k_{e} \frac{\pi R_{1}^{2} \rho+2 \lambda}{r} \hat{r}$

## SPECIAL ASSISTANCE SUPPLEMENT

## S-1 (from TX-3d)

$\oint_{S_{I}} \vec{E}_{I} \cdot \hat{n} d S=4 \pi k_{e} q_{S_{I}}=0$
$\vec{E}_{I} \cdot \hat{n}=0$ on cyl. surf.
$\vec{E}_{I}=0$
$\oint_{S_{I I}} \vec{E}_{I I} \cdot \hat{n} d S=4 \pi k_{e} q_{S_{I I}}$
$\begin{aligned} & =4 \pi k_{e} \lambda \ell \\ \vec{E}_{I I} & \\ E_{I I} & \\ = & 4 \pi k_{e} \lambda \ell\end{aligned}$
$\vec{E}_{I I}=2 k_{e} \frac{\lambda}{r} \hat{r}$
$\oint_{S_{S I I}} \vec{E}_{I I I} \cdot \hat{n} d S=4 \pi k_{e} q_{S_{I I I}}=0$
$\vec{E}_{I I I I} \cdot \hat{n}=0$ on cyl. surf.
$\vec{E}_{I I I}=0$


## S-2 (from TX-5b)

$\oint_{S} \vec{E} \cdot \hat{n} d S=4 \pi k_{e} q_{S}=4 \pi k_{e} 2 \sigma A$
on ends: $\vec{E} \cdot \hat{n}(2 A)=4 \pi k_{e} 2 \sigma A$
outside the planes: $\vec{E}=4 \pi k_{e} \sigma \hat{n}$

$\oint_{S_{1}} \vec{E} \cdot \hat{n} d S=4 \pi k_{e} q_{S}=0$
on ends: $\vec{E} \cdot \hat{n}(2 A)=0$
between the planes: $\vec{E}=0$

## S-3 (from TX-5a)

Multiply all



## S-4 (from PS-Problem 6)

Since the slab has planar symmetry, the field direction is everywhere normal to the slab and the equi-magnitude surfaces are parallel to the slab faces (see MISN-0-153). When you construct Gaussian surfaces, take advantage of the reflection symmetry about the $x-z$ plane by choosing the two faces that are parallel to the slab to be equidistant from the slab as well. That way you can write the surface integral of $\vec{E}$ as:

$$
\oint_{S} \vec{E} \cdot d \vec{S}=2 E A
$$

since the field is uniform over the two Gaussian surface areas, and the field must be the same on each surface by symmetry.

## S-5 (from [S-10])

Draw a sketch of the situation and on it mark the given quantities. Then figure out and mark on the sketch the $\theta$ and $r$ we use here:
$E_{y}($ total $)=E_{y}($ from $\# 1)+E_{y}($ from $\# 2)$
$E_{y}($ from $\# 1)=E($ from $\# 1) \cos \theta$
$E_{y}($ from $\# 2)=E($ from $\# 2) \cos \theta$
$E($ from $\# 1)=E($ from $\# 2)=2 k_{e} \lambda /(r)$
where $r=\left(d^{2}+y^{2}\right)^{1 / 2}, \lambda$ is the charge per unit length along each wire, and $\cos \theta=y / r$.
$E_{y}($ total $)=4 k_{e} \lambda y /\left(r^{2}\right)=4 k_{e} \lambda y /\left(d^{2}+y^{2}\right)$

## S-6 (from PS-Problem 2)

"Charge volume density" is "charge per unit volume" and in the MKS system is measured in $\mathrm{C} / \mathrm{m}^{3}$.
Then: total charge $=$ charge volume density $\times$ volume,
where all three quantities refer to "inside the surface."

## S-7 (from PS-Problem 4)

Completely solve Problems 1 and 2 first.
"Linear charge density" is "charge per unit length" and in the MKS system is measured in $\mathrm{C} / \mathrm{m}$.
It is the amount of charge per unit length down the entire cylindrical surface.

## S-8 (from PS-Problem 5)

Use Gauss's Law separately on each plane (as though the other did not exist) to get: $\vec{E}_{I}$ (due to $\# 1$ ), $\vec{E}_{I}$ (due to $\# 2$ ), and $\vec{E}_{I}$ (due to $\# 3$ ). Then add them to get $\vec{E}$ in region I, here labeled $\vec{E}_{I}$.

## S-9 (from PS-problem 2)

The concept that has caused students trouble in the past (in Problem 2) is directly and clearly handled in this module's text. Read and understand it there.

## S-10 (from PS-problem 1)

First, try your very best to solve this problem without Special Assistance. Go back and work through the text again, this time paying special attention to sections relevant to this problem. Remember that you will not have the Special Assistance available at exam time so you need to learn how to work without it.
If you try and try and truly fail, then try the Special Assistance in [S-5].

## S-11 (from PS-problem 7)

Calculating the enclosed charge involves techniques you used in the two previous problems.

## MODEL EXAM

1. See Output Skill K1 in this module's $I D$ Sheet.
2. Use Gauss's law to find the electric field outside and inside two large parallel plates with equal surface charge densities $\sigma$ on the plates and with volume charge density $\rho$ between the plates. The distance between the plates is $d$.

3. Use Gauss's law to find the electric field in each of the three regions defined by two coaxial cylindrical surfaces, each with linear charge density $\lambda$, and with a uniform volume charge density $\rho$ inside the inner cylindrical surface. The radii of the two cylindrical surfaces are $R_{1}$ and $R_{2}$ (see diagram below).


## Brief Answers:

1. See this module's text.
2. See this module's Problem Supplement, Problem 7.
3. See this module's Problem Supplement, Problem 8.

[^0]:    ${ }^{1}$ See "Gauss's Law and Spherically Symmetric Charge Distributions" (MISN-0-132) for an introduction to Gauss's law and the rules for using it.

[^1]:    ${ }^{2}$ The term "linear charge density" means the charge is being described as a certain amount of charge per unit length along the wire. This is in contrast to "volume charge density" where the charge is described as a certain amount of charge per unit volume within the wire. For uniform cross-sectional distributions, the linear charge density equals the cross-sectional area times the volume charge density.
    ${ }^{3}$ For the two rules for constructing Gaussian surfaces, see Ref. 1. For derivation of the electric field directions and equi-magnitude surfaces see "Electric Fields from Symmetric Charge Distributions" (MISN-0-153).

[^2]:    ${ }^{5}$ Electrostatic charges reside on the surfaces of metallic conductors: see "Electrostatic Properties of Conductors" (MISN-0-136).

