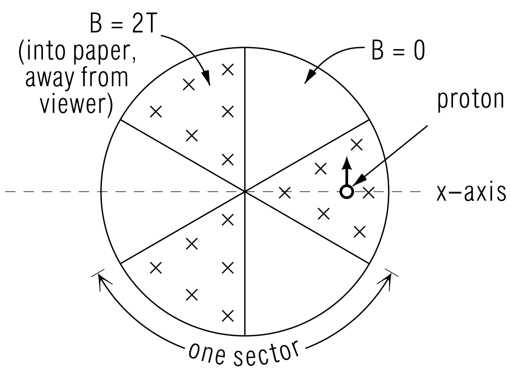
MISN-0-127

A CHARGED PARTICLE TRAJECTORY IN A MAGNETIC FIELD: CYCLOTRON ORBITS (A COMPUTER PROJECT)



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

A CHARGED PARTICLE TRAJECTORY IN A MAGNETIC FIELD:
CYCLOTRON ORBITS (A COMPUTER PROJECT)

by

J. S. Kovacs and P. Signell
Michigan State University

Title: **A Charged Particle Trajectory in a Magnetic Field: Cyclotron Orbits (A Computer Project)**

Author: J. Kovacs and P. Signell, Dept. of Physics, Mich. State Univ., E. Lansing, MI 48824.

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Input Skills:

1. Vocabulary: trajectory (MISN-0-72), magnetic field (MISN-0-122).
2. State the expression for the Lorentz force and state its significance for a moving charged particle in a magnetic field (MISN-0-122).

Output Skills (Project):

P1. Use a canned computer program to solve a design problem involving the trajectory of a charged particle in a magnetic field.

P2. Explain why the solution found is reasonable in terms of the forces involved.

External Resources (Required):

1. Canned computer program M127-P.BAS, 2 sheets of graph paper.

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Our publications are designed: (i) to be updated quickly in response to field tests and new scientific developments; (ii) to be used in both classroom and professional settings; (iii) to show the prerequisite dependencies existing among the various chunks of physics knowledge and skill, as a guide both to mental organization and to use of the materials; and (iv) to be adapted quickly to specific user needs ranging from single-skill instruction to complete custom textbooks.

New authors, reviewers and field testers are welcome.

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1. Introduction and Description

The trajectories of charged particles in zero and in uniform magnetic fields are obviously easy to obtain in terms of well-known functions, but such solutions are not known for more than a few other such simple fields. For the general case of an arbitrarily varying magnetic field, one must obtain or derive an approximate numerical technique to find the trajectory. Then either a hand calculator or a computer can be used to effect the solution, but use of the computer is by far the less tedious method. Computers are used both to determine charged particle trajectories in existing fields and to design magnetic fields in order to produce particular desired trajectories. The specific task in this module is that of using a Runge-Kutta algorithm in a computer program to find a unique orbit in a 3-sector cyclotron similar to the one at Michigan State University.[1]

2. The Problem

The general problem is to determine the trajectory of a charged particle in an arbitrary magnetic field. Your specific problem here is to find a unique orbit in a simplified version of a 3-sector cyclotron similar to the one at Michigan State University or its carbon copy at Princeton University. The charged particle to be tracked is a proton; the cyclotron's magnetic field is illustrated in Fig. 1. Given that the starting point for the proton is at a point along the positive x-axis and the initial velocity is a vector pointing along the positive y-axis, your job is to find the proper initial radius such that the trajectory of the proton closes on itself in one revolution. A solution and a non-solution (not meant to be realistic) are shown in Fig. 2.

[1]A cyclotron is a device that accelerates atomic nuclei in increments as the nuclei travel in roughly circular orbits inside the machine. In particular, a nucleus coasts around its orbit through approximately 180°, is accelerated through a very short distance, coasts through another 180°, is accelerated through another very short distance, and repeats the cycle. Every time the particle goes around it is accelerated to a higher speed and kinetic energy. After reaching a desired speed, the accelerations can be turned off and the nuclei will coast around their orbit (as in this module). The coasting segments obey the rules for charged particles in magnetic fields, while the acceleration segments obey the rules for charged particles in electric fields. The electric-field acceleration segments are not shown in the figures of this module.

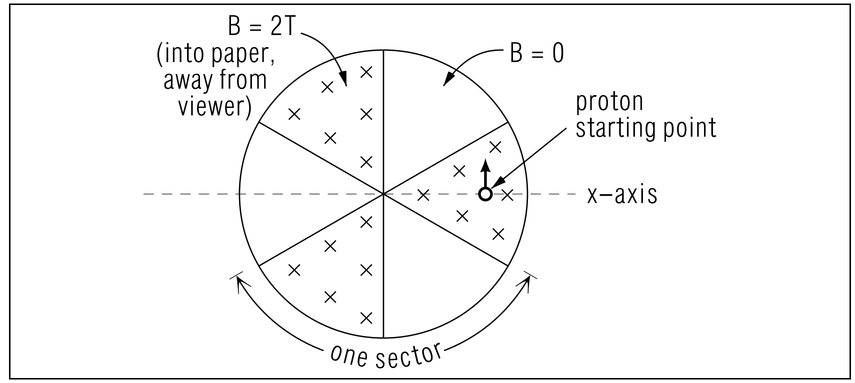

Figure 1. The magnetic field configuration in the MSU cyclotron.

How does one find the proper initial radius in order to get a single orbit? Call an initial trial radius r_0 and the radius resulting from a transit of one complete orbit r_1, as in Fig. 2. (The value of r_0 was read into the computer; the value of r_1 was taken from the computer output.) Plot the pair (r_0,r_1) as a point on a graph of r_0 vs r_1, as in Fig. 3. Then input another value of r_0 and get its corresponding value of r_1 from the output. Plot this pair as a point on the same graph. Your points on the graph will not define a smooth curve because the numerical method of solution cannot deal properly with the sharp edges of the magnetic field.

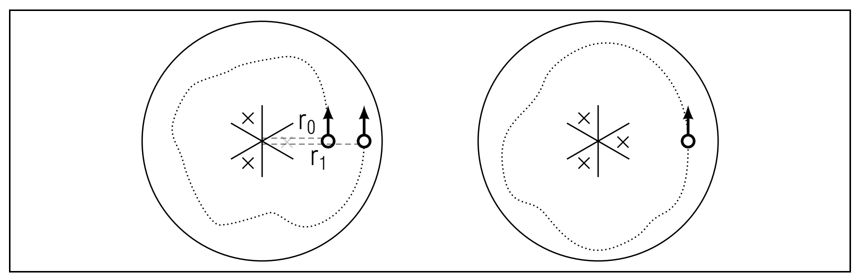

Figure 2. Non-Solution Solution

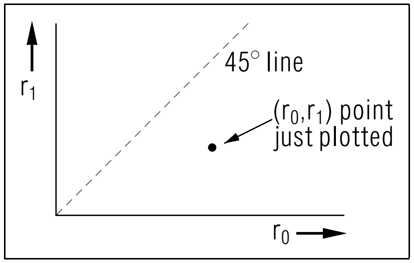

Figure 3. An example of successive pairs of radii.

However, the points will define a crude curve.

Through choosing judicious values for r_0, you should soon be able to draw a crude curve through the points and find where the curve crosses the 45° line. You should need only 3 or 4 points for this. The point where the crude curve crosses the 45° line is a crude solution value for r_0. Explain why this is so, on the graph.

Finally, put this solution value of r_0 into the computer program to obtain the cartesian coordinates of a closed orbit. Since the curve is crude, you should only expect your value of r_1 to agree with your value of r_0 to within about 5%. Plot the trajectory and the sector boundaries on a piece of graph paper and explain, on the graph paper, the various shapes you see in the orbit (straight lines, circular arcs, elliptical arcs, hyperbolic arcs, etc.). Then, on the graph paper, explain why those particular shapes occur for the case at hand.

The program M127-P.EXE assumes the cyclotron parameters in Fig. 4. as well as these parameters for the proton:

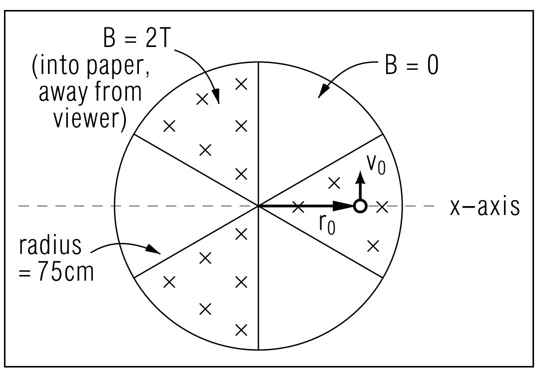

Figure 4. Cyclotron Parameters

$$q * B / m = 2/sec.$$
$$\vec{v}_0 = v_0\hat{y}$$
$$\vec{r}_0 = r_0\hat{x}.$$

3. Procedure

a. Obtain 2 sheets of linear (regular) graph paper from some source, such as a friend or a bookstore. There might or might not be some available in our Consulting Room or from our Exam Manager at the entrance to our Exam Room.

b. Go to our Consulting Room and get a copy of CBI Document #8-100, which tells you how to use a University microcomputer for this project. Use of the micro is free.

c. In order to run the program, follow the directions on #8-100. When you have reached the Microlab Class Menu - Page 2, select PHYCBI by hitting the 2 key. That will cause the program M127-P.EXE to be downloaded from a remote hard disk to the one in your machine. Your hard disk is called drive C (note the "C:\> " prompt that appears on the screen).

d. If all went well, lights will blink and the program will start running.

e. The first run of the program establishes the initial velocity, v_0. *Help: [S-1]* Initial values for the radial and tangential components of the velocity are assigned by the computer, using your CBI ID. An initial trial radius r_0 will be requested (*Help: [S-2]*) and a value of r_1 will be returned. *Help: [S-3]* The run will be quite slow, since these machines do not have a math chip. There is no need to see the whole orbit until you have a solution. Determine as many pairs of (r_0,r_1) values as you need to construct the graph of successive radii and determine a value of r_0 which yields a closed orbit (to within 5% or so). Then run again, this time asking for the whole orbit (we usually prefer to see every tenth point on this run).

f. At the " C:\WORK\PHYCBI> " prompt, type EXIT so the screen looks like this `C:\WORK\PHYCBI>EXIT` and then hit the RETURN key. That should take you back to the "classes" menu. Look at the menu and hit the key that takes you back to the "main menu."

g. Annotate your two graphs, showing the information asked for in the previous section of this module.

h. When finished annotating your two graphs, r_0 vs r_1 and x vs y (the crude solution orbit), bring them and the original of your computer output to the exam room and ask for your exam. Staple your annotated output to the exam answer sheet as the "exam answers" for this unit. Make sure your graphs are neat and well labeled. Axes should be labeled and units should be specified. The notations on the graphs should cover all aspects requested in the unit. You will receive zero credit for the unit if you do the computer and graphical parts but do not give the physics explanation.

Questions for Additional Understanding:

*1. What makes the 240°and 360°radii equal to the 0° and 120°radii for the solution-orbit? Is it related to the symmetry properties of the magnetic field geometry?

*2. How could you have solved this cyclotron problem exactly, in terms of common functions?

Acknowledgments

Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

SPECIAL ASSISTANCE SUPPLEMENT

S-1 *(from Procedure, part e)*

The speed v_0, called V0 in the program, is assigned by M127-P.BAS from your Physics Course ID number during your first run, and is kept at that value on succeeding runs as long as you do not leave the program.

S-2 *(from Procedure, part e)*

The maximum value r_0 can have is the radius of the cyclotron (See Fig. 4). Note that the program assumes values of r_0 that are in meters and v_0 in meters/sec.

S-3 *(from Procedure, part e)*

If the value of r_1 is greater than the dimensions of the cyclotron the prgram will print: "YOUR PARTICLE HAS EXCEEDED THE CYCLOTRON BOUNDARIES" and the program execution will terminate. If this occurs, start over again.

MODEL EXAM

Examinee:
On your computer output sheet(s):

(i) Mark page numbers in the upper right corners of all sheets.

(ii) Label all output, including all axes on all graphs.

On your Exam Answer Sheet(s), for each of the following parts of items (below this box), show:

(i) a reference to your annotated output; and

(ii) a blank area for grader comments.

When finished, staple together your sheets as usual, but include the original of your annotated output sheets just behind the Exam Answer Sheet.

1. a. Submit your hand-annotated output showing the (r_0, r_1) pairs. Be sure it shows that you homed in on rough agreement between the two numbers.
   b. Submit your graph showing the plotted (r_0, r_1) pairs.
   c. Submit your hand-annotated output showing equal-time points along the roughly-consistent trajectory.

2. Submit your graph showing the roughly-consistent trajectory. Be sure you have hand annotated the graph to point out the shapes and the physics reasons for those particular shapes.

INSTRUCTIONS TO GRADER

If the student has submitted copies rather than originals of the computer output, state that on the exam answer sheet and **immediately stop grading the exam and give it a grade of zero.**

Note that the award of points is set up in such a way that a student will get 50% or less on the exam if the student does only computer work and no physics.