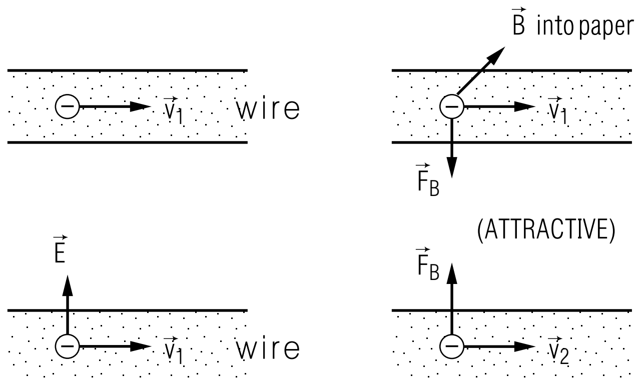
MISN-0-124

THE MAGNETIC FIELD OF A MOVING CHARGE



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

THE MAGNETIC FIELD OF A MOVING CHARGE

by

O. McHarris, Lansing Community College

Title: **The Magnetic Field of a Moving Charge**

Author: O. McHarris and P. Signell, Michigan State University

Version: 2/1/2000 Evaluation: Stage 0

Length: 1 hr; 20 pages

Input Skills:

1. Vocabulary: lattice (MISN-0-118); magnetic field, magnetic force (MISN-0-122).
2. Calculate the electrostatic field $\vec{E}$ due to a point charge q (MISN-0-115).
3. Calculate the Lorentz force on a charged particle (MISN-0-122).

Output Skills (Knowledge):

K1. Use the Lorentz force and the expression for the magnetic field of a moving charged particle to derive the rule that parallel currents attract and anti-parallel currents repel each other.

K2. Explain why the electric force between two moving charged particles is usually much larger than the magnetic force between the same two particles and why, on the other hand, the magnetic force is the important force between two current-carrying wires.

Output Skills (Problem Solving):

S1. Given a moving charged particle, calculate the electric and magnetic fields produced by that particle at any given point.

S2. Given a moving charged particle, calculate the Lorentz force on a second moving charged particle.

Post-Options:

1. "The Magnetic Field of a Current: The Ampere-Laplace Equation" (MISN-0-125).

THIS IS A DEVELOPMENTAL-STAGE PUBLICATION
OF PROJECT PHYSNET

The goal of our project is to assist a network of educators and scientists in transferring physics from one person to another. We support manuscript processing and distribution, along with communication and information systems. We also work with employers to identify basic scientific skills as well as physics topics that are needed in science and technology. A number of our publications are aimed at assisting users in acquiring such skills.

Our publications are designed: (i) to be updated quickly in response to field tests and new scientific developments; (ii) to be used in both classroom and professional settings; (iii) to show the prerequisite dependencies existing among the various chunks of physics knowledge and skill, as a guide both to mental organization and to use of the materials; and (iv) to be adapted quickly to specific user needs ranging from single-skill instruction to complete custom textbooks.

New authors, reviewers and field testers are welcome.

PROJECT STAFF

Andrew Schnepp	Webmaster
Eugene Kales	Graphics
Peter Signell	Project Director

ADVISORY COMMITTEE

D. Alan Bromley	Yale University
E. Leonard Jossem	The Ohio State University
A. A. Strassenburg	S. U. N. Y., Stony Brook

Views expressed in a module are those of the module author(s) and are not necessarily those of other project participants.

© 2001, Peter Signell for Project PHYSNET, Physics-Astronomy Bldg., Mich. State Univ., E. Lansing, MI 48824; (517) 355-3784. For our liberal use policies see:

http://www.physnet.org/home/modules/license.html.

THE MAGNETIC FIELD OF A MOVING CHARGE

by

O. McHarris, Lansing Community College

1. Introduction

A current loop placed in a magnetic field behaves much like a magnet placed in the same field because the loop of current produces a magnetic field just like the magnet does.[1] In fact, the magnetic field of a magnet is produced by tiny current loops within the magnetic material. Here we will study the magnetic field produced by that simplest of currents, a single moving charged particle. Elsewhere we will sum the fields produced by such particles to obtain the fields produced by the sets of moving charges called electric currents.

2. Magnetic Field of a Charge

2a. Three Inter-Particle Forces. If two particles interact it may be through some combination of gravitational, electric, and magnetic forces:[2]

$$\begin{aligned} \vec{F}_{g,12} &= -G\frac{m_1 m_2}{r_{12}^2}\,\hat{r}_{12} \\ \vec{F}_{e,12} &= k_e\frac{q_1 q_2}{r_{12}^2}\,\hat{r}_{12} \\ \vec{F}_{m,12} &= k_m\frac{q_1 q_2}{r_{12}^2}\,\vec{v}_1 \times (\vec{v}_2 \times \hat{r}_{12})\,. \end{aligned} \tag{1}$$

Here $\vec{F}_{12}$ is the force on particle #1 due to particle #2, G is the gravitational force constant, k_e is the electric force constant, k_m is the "magnetic force constant," q_1 and v_1 are the charge and velocity of particle #1, q_2 and v_2 are the charge and velocity of particle #2, and $\vec{r}_{12}$ is the position vector of particle #1 as seen from the position of particle #2 (see Fig. 1).

[1] See "Force on a Current in a Magnetic Field" (MISN-0-123) and "The Magnetic Field of a Current: The Ampere-Laplace Equation" (MISN-0-125).

[2] For the gravitational and electric interactions see "Newton's Law of Universal Gravitation," (MISN-0-101) and "Coulomb's Law" (MISN-0-114). The magnetic interaction is being introduced in this module.

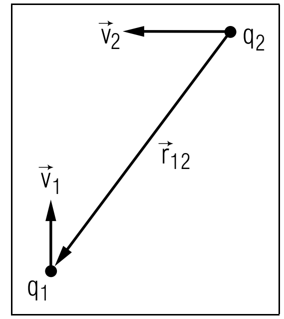


Figure 1. Vector relationships for two moving charged particles.

The magnetic and electric force constants are related by:

$$k_m \equiv \frac{k_e}{c^2} \equiv 10^{-7}\,\mathrm{N\,s^2\,C^{-2}}\,,$$

where $c \simeq 3 \times 10^8$ m/s is the speed of light in vacuum.

▷ Show that the magnetic force constant, k_m, is approximately $1/100,000,000,000,000,000$ the size of the electric force constant k_e.

▷ Show that there is no magnetic force between two particles unless *both* are moving, and that even if they *are* moving the vector products can still make the interaction zero.

2b. Forces from the Three Fields. The forces on a particle may be interpreted as being due to fields produced by other particles. For forces on a particle of mass m, charge q, and velocity v, due to gravitational, electric, and magnetic fields, we have:

$$\begin{aligned} \vec{F}_g &= m\,\vec{\mathcal{G}} \\ \vec{F}_e &= q\,\vec{E} \\ \vec{F}_m &= q\,\vec{v} \times \vec{B} \end{aligned} \tag{2}$$

where $\vec{\mathcal{G}}$ is the gravitational field, $\vec{E}$ is the electric field, and $\vec{B}$ is the magnetic field at the position of the particle. Note that there is no magnetic force on a particle unless it is moving, and even then the force will be zero if the particle's velocity is parallel or anti-parallel[3] to the magnetic field.

2c. The Three Fields from Particles. From Eqns. (1) and (2) we see that a particle of mass m, charge q, and velocity v produces these fields

[3] The commonly-used term "anti-parallel" means "opposite in direction."

at a space-point $\vec{r}$ (as seen *from* the position of the particle producing the fields, illustrated in Fig. 1):[4]

$$\begin{aligned} \vec{\mathcal{G}}(\vec{r}) &= -G\frac{m}{r^2}\,\hat{r} \\ \vec{E}(\vec{r}) &= k_e\frac{q}{r^2}\,\hat{r} \\ \vec{B}(\vec{r}) &= k_m\frac{q}{r^2}\,\vec{v}\times\hat{r}\,. \end{aligned} \tag{3}$$

Note that the magnetic field is only produced by a moving particle, and then only at space-points for which the relative-position vector is neither parallel nor anti-parallel to the particle's velocity.

2d. Comparison of F_e and F_m for Two Charges. For two moving charges, Eqs. (1) show that the magnitudes of F_m and F_e are related by:

$$F_m \le \frac{|v_1 v_2|}{c^2} F_e\,, \tag{4}$$

where the "$\le$" sign must be used because one or two sine functions from the relevant vector products might also multiply F_e. Since v_1 and v_2 are much smaller than c for ordinary electric currents, F_m is normally very much smaller than F_e. However, as we shall see later, F_e vanishes for the case of two current-carrying wires so for that case F_m is left as the only force between them.

2e. A Numerical Example. Given two moving charged particles, we can calculate the fields set up by one of them and then calculate the effect of those fields on the other particle. This means we can calculate the force that each exerts on the other. Take, for example, a charge $q_1 = -1.0\times10^{-6}$ C instantaneously at the origin of our coordinate system and another charge $q_2 = +1.0\times10^{-6}$ C at $\vec{r} = (4.0\,\text{cm})\,\hat{x} + (1.0\times10^{1}\,\text{cm})\,\hat{y}$ (see Fig. 2). Let $\vec{v}_1 = (3.5\times10^{-4}\,\text{m/s})\,\hat{y}$ and $\vec{v}_2 = (-3.5\times10^{-4}\,\text{m/s})\,\hat{x}$. For our example we calculate the force on q_2, taking q_1 as the source of the fields (see Figs. 2, 3, 4).[5] We now make life easier for ourselves by simplifying our notation:

$$\vec{r} \equiv \vec{r}_{21}\,.$$

[4]For a discussion of gravitational field-produced force see "The Gravitational Field" (MISN-0-108). For electric field-produced force see "Point Charge Field and Force," MISN-0-115). For both electric and magnetic field-produced forces, combined into one equation, see "Force on a Charged Particle in a Magnetic Field: The Lorentz Force" (MISN-0-122).

[5]Recall that one can only refer to a field *at a point in space*, and one can only refer to the force *on a particle at that point.*

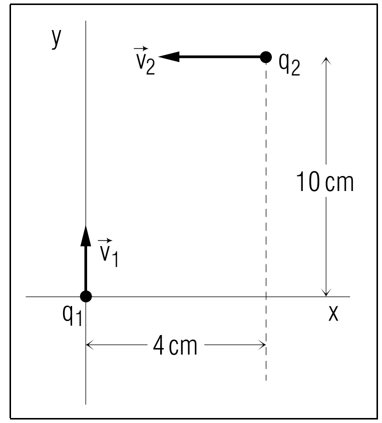


Figure 2. A specific example of two moving charged particles.

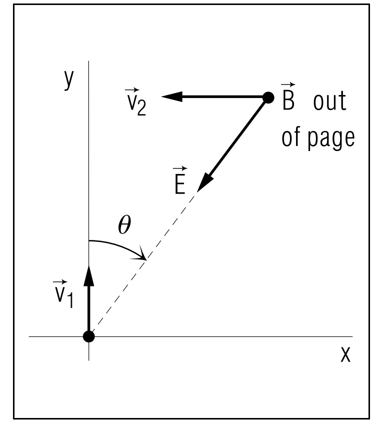


Figure 3. The electric and magnetic fields of q_1 at the position of q_2.

Then the electric and magnetic fields at $\vec{r}$, the position of q_2, due to q_1 are:

$$\vec{E}(\vec{r}) = k_e\frac{q_1}{r^2}\hat{r}\,, \tag{5}$$

$$\vec{B}(\vec{r}) = -k_m\frac{v_1 q_1 \sin\theta}{r^2}\hat{z}\,.\ \textit{Help: [S-9]} \tag{6}$$

Thus the electric and magnetic forces on charge q_2 are (see Fig. 4, *Help: [S-4]*):

$$\vec{F}_{e,21} = k_e\frac{q_1 q_2}{r^2}\,\hat{r} = -0.78\,\text{N}\,\hat{r}\,, \tag{7}$$

$$\vec{F}_{m,21} = q_2\vec{v}_2\times\vec{B}(\vec{r}) = -k_m\frac{v_1 v_2 q_1 q_2}{r^2}\sin\theta\hat{y} = 3.9\times10^{-25}\,\text{N}\,\hat{y}\,. \tag{8}$$

Note that $\vec{F}_m$ is negligible compared to $\vec{F}_e$. The total Lorentz force, $\vec{F}_m + F_e$, is, then, to quotable accuracy:

$$\vec{F}_{e+m,21} = -0.78\ \text{N}\,\hat{r}\,.$$

Note that Newton's third law is satisfied only when $\vec{v}_1$ and $\vec{v}_2$ are either parallel or anti-parallel (see the appendix).

When calculating magnetic interactions, calculate the force due to the fields on each particle in turn. Do not use the fields to obtain the force on one particle and then use Newton's third law to obtain the force on the other (see the Appendix).

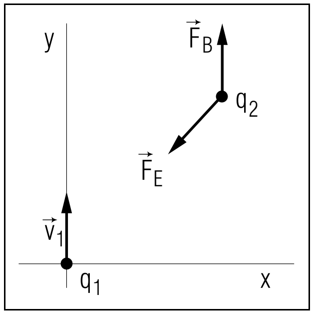


Figure 4. The electric and magnetic forces on q_2.

3. Forces between Currents In Wires

3a. F_E is Zero Between Current-Carrying Wires. Given the ability to calculate the electric and magnetic forces between two isolated charges, it is reasonable to ask: If those two charges were not isolated but were parts of two currents, what could we say about the forces between the two currents? The first thing to notice is that there is no electric force between current-carrying wires. The reason for this is that most materials are electrically neutral overall: the charge on the stationary positive ions in the metal's ionic "lattice" balances the negative charge on the mobile electrons in the current. Since both stationary and moving charges contribute to the total electric field, the total electric field is zero. In the magnetic interaction, on the other hand, only moving charges count; the moving electrons produce a magnetic field but the stationary lattice ions do not. Thus each wire exerts a magnetic force on the other wire but not an electric force.

3b. Example: Parallel Current-Carrying Wires Attract. Suppose the currents in two parallel wires are in the same direction: they are two streams of electrons moving in the same direction. Let us consider

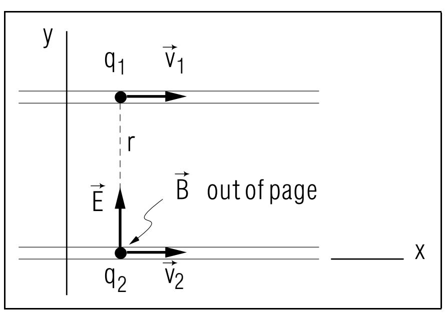


Figure 5. Two parallel current-carrying wires

the force between one electron in wire #1 and one electron in wire #2 and, for simplicity's sake, let those two electrons be at the ends of a perpendicular line between the two wires (see Fig. 5). Taking electron #1 as the source of a magnetic field acting on electron #2, we go through the calculation of the force on #2 with $q_1 = q_2 = -e$ for electrons:

$$\vec{B}(\vec{r}) = k_m \frac{v_1 e}{r^2} \hat{z}\,, \tag{9}$$

and

$$\vec{F}_{m,21} = q_2 \vec{v}_2 \times \vec{B}(\vec{r}) = k_m \frac{v_1 v_2 e^2}{r^2} \hat{y}\,. \text{ Help: [S-7]} \tag{10}$$

Thus $\vec{F}_{m,21}$ points in the positive y-direction. A similar analysis for q_1 shows that $F_{m,12}$ points in the negative y-direction so the force between the wires is attractive (see Fig. 6). *Help: [S-6]* You can easily modify the above argument to show that when the electrons move in opposite directions, so the electric currents in the two wires are in opposite directions, the wires repel each other.

The magnetic force between moving charges can sometimes have a dramatic effect. For example, when lightning strikes a hollow metal pipe, the attractive *magnetic* force between the electrons moving down the pipe can cause the pipe to collapse into a solid bar.

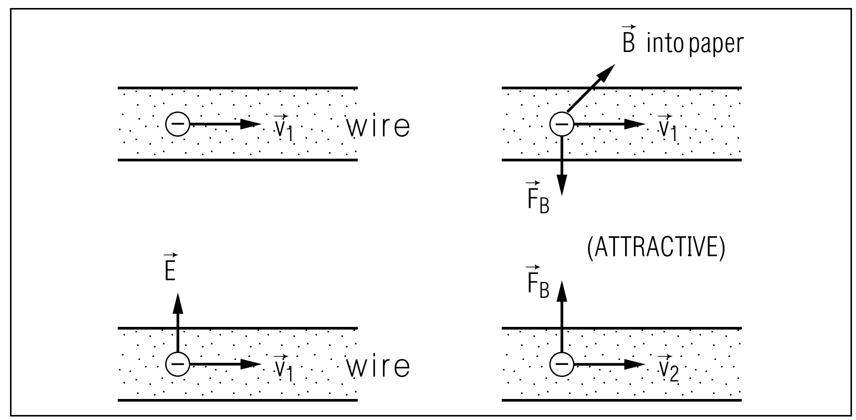


Figure 6. The magnetic interaction of two parallel current-carrying wires.

Acknowledgments

The Problem Supplement for this module was constructed by Kirby Morgan. We would like to thank C. Chatdorkmaiprai for suggesting an Appendix on the violation of Newton's third law. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

A. How Newton'S Third Law is Violated

(for those interested)

If we sum the magnetic forces of two charged particles on each other, we find this apparent violation of Newton's third law (after some manipulation):

$$\vec{F}_{m,21} + \vec{F}_{m,12} = k_e \frac{q_1 q_2}{r^2} \cdot \frac{v_1 v_2}{c^2} \cdot \hat{r} \times (\hat{v}_1 \times \hat{v}_2) \,.$$

The reason for the apparent violation is that the momentum of the electric and magnetic fields has been omitted. When that momentum is included (an advanced topic), Newton's third law is again obeyed.

▷ Under what circumstances is the right hand side non-zero?

PROBLEM SUPPLEMENT

Note: Problem 6 also occurs in this module's *Model Exam*.

1. A moving charge, $q = 0.50\,\text{C}$, is at the origin of a coordinate system at a certain instant of time. Calculate the value of the $\vec{E}$ and $\vec{B}$ fields it would set up at the point $x = 1.0\,\text{cm}$, $y = 0.0\,\text{cm}$, $z = 0.0\,\text{cm}$ for each of three different directions of motion:

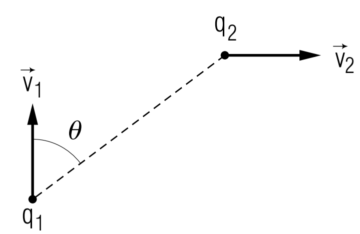
   a. $\vec{v} = 1.0 \times 10^3\,\text{m/s}\,\hat{x}$

   b. $\vec{v} = 1.0 \times 10^3\,\text{m/s}\,\hat{y}$

   c. $\vec{v} = (1.0 \times 10^3\,\text{m/s})[(\hat{x} + \hat{y})/\sqrt{2}]$

2. Two electrons separated by 0.10 mm move side by side along straight paths parallel to the x-axis with equal velocities of $1.0 \times 10^6\,\text{m/s}\hat{x}$. Find the electric and magnetic forces each exerts on the other. *Help: [S-1]*

3. Two charges move relative to each other as shown:



   Let $\vec{B}(\vec{r}_2)$ be the magnetic field at #2 due to #1

   Let $\vec{E}(\vec{r}_2)$ be the electric field at #2 due to #1

   Let $\vec{E}(\vec{r}_1)$ be the electric field at #1 due to #2

   a. Which of the following expressions is correct?

   (i) $\vec{B}(\vec{r}_2) = \dfrac{1}{c^2}\vec{v}_2 \times \vec{E}_2(\text{at } 1)$?

   (ii) $\vec{B}(\vec{r}_2) = \dfrac{1}{c^2}\vec{v}_2 \times \vec{E}_1(\text{at } 2)$?

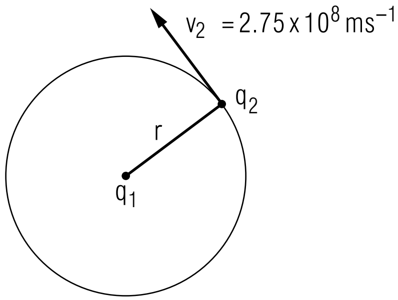
   (iii) $\vec{B}(\vec{r}_2) = \dfrac{1}{c^2}\vec{v}_1 \times \vec{E}_2(\text{at } 1)$?

(iv) $\vec{B}(\vec{r}_2) = \frac{1}{c^2}\vec{v}_1 \times \vec{E}_1(\text{at } 2)$?

b. Find F_m/F_e, the ratio of the magnitude of the magnetic force on particle #2 to the magnitude of the electric force on that particle.

c. If $F_e \neq 0$, when is F_m comparable to F_e?

4. An electric charge, $q_2 = -1.0\,\mathrm{C}$, moves in a circular orbit of radius $r = 0.25\,\mathrm{nm}$ around another charge $q_1 = 1.0\,\mathrm{C}$.



a. Calculate $\vec{B}$ at q_1. *Help: [S-2]*

b. What is the magnetic force F_m on the charge q_1?

5. Show that two parallel wires carrying currents in opposite directions will repel each other. *Help: [S-3]*

6. Given two charges with these specifications:

$$\begin{aligned}
Q_1 &= -1.0 \times 10^4\,\mathrm{C}\\
Q_2 &= -2.0 \times 10^4\,\mathrm{C}\\
\vec{r}_1 &= r_1\hat{x} = (3.0\,\mathrm{cm})\,\hat{x}\\
\vec{r}_2 &= r_2\hat{x} = (4.0\,\mathrm{cm})\,\hat{x}\\
\vec{v}_1 &= v_1\hat{y} = (3.0 \times 10^{-4}\,\mathrm{m/s})\,\hat{y}\\
\vec{v}_2 &= v_2\hat{y} = (4.0 \times 10^{-4}\,\mathrm{m/s})\,\hat{y}
\end{aligned}$$

a. Draw a diagram showing the positions and velocities of the charges.

b. Determine the symbolic answer for the total force of Q_2 on Q_1.

c. Determine the numerical value of the force of part (b).

Brief Answers:

1. $4.5 \times 10^{13}\,\mathrm{N/C}\,\hat{x}$

a. $\vec{B} = 0$

b. $\vec{B} = -0.50\,\mathrm{T}\,\hat{z}$

c. $\vec{B} = -0.35\,\mathrm{T}\,\hat{z}$

2. $\vec{F}_{e,21} = 2.3 \times 10^{-20}\,\mathrm{N}\,\hat{y}$ *Help: [S-1]*

$\vec{F}_{e,12} = -\vec{F}_{e,21}$ *Help: [S-1]*

$\vec{F}_{m,21} = -2.6 \times 10^{-25}\,\mathrm{N}\,\hat{y}$ *Help: [S-1]*

$\vec{F}_{m,12} = -\vec{F}_{m,21}$ *Help: [S-1]*

3. a. (iv)

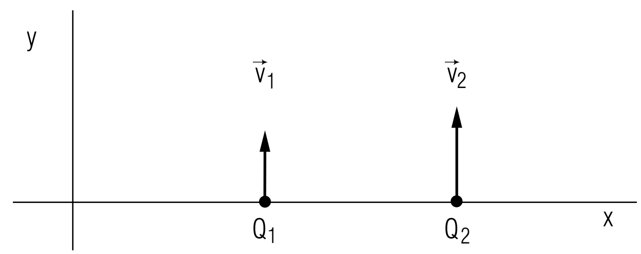
b. $\frac{F_m}{F_e} = \frac{v_1 v_2 \sin\theta}{c^2}$ *Help: [S-9]*

c. $\frac{F_m}{F_e} = 1$ if v_1 and v_2 are nearly c and $\theta \to 90°$

4. a. $\vec{B} = 4.4 \times 10^{20}\,\mathrm{T}$, into the page

b. $\vec{F}_m = q_1\vec{v}_1 \times \vec{B} = 0$ since $\vec{v}_1 = 0$

6. a.

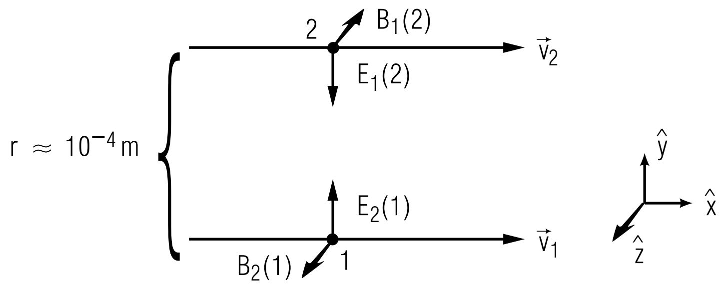


b. $\vec{F}_{12} = -k_e \frac{Q_1 Q_2}{(r_2 - r_1)^2}\left(\hat{x} - \frac{v_1 v_2}{c^2}\hat{x}\right)$

c. $\vec{F}_{12} = -1.8 \times 10^{22}\hat{x}\,\mathrm{N}$

SPECIAL ASSISTANCE SUPPLEMENT

S-1 *(from PS-problem 2)*



$\vec{F}_{e,21} = q_2\vec{E}(\vec{r}_2) = k_e\dfrac{q_2q_1}{r^2}\,\hat{y}$

$\vec{B}(\vec{r}_2) = k_m\dfrac{-v_1q_1}{r^2}\,\hat{z}$

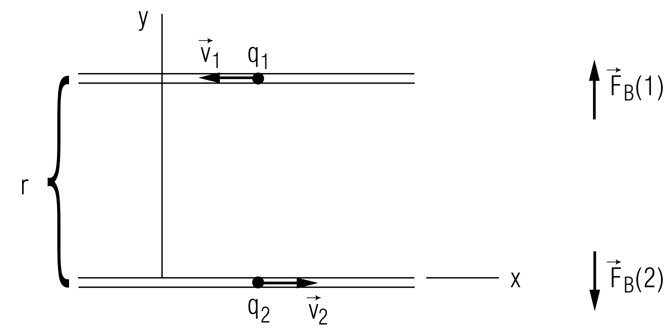
$\vec{F}_{m,21} = q_2\vec{v}_2 \times \vec{B}(\vec{r}_2) = -k_e\dfrac{v_1v_2q_1q_2}{r^2c^2}\,\hat{y}$

Signs ($\pm$) *Help: [S-9]*
Numerical Help: [S-8]
Value for the electron charge: see this book's Appendices

S-2 *(from PS-problem 4a)*

$\vec{B}(\vec{r}_1) = k_e\dfrac{q_2v_2}{r^2c^2}[\hat{v}_2 \times (-\hat{r})]$ *Numerical Help: [S-10]*

S-3 *(from PS-problem 5)*



$\vec{B}(\vec{r}_2) = k_m\dfrac{v_1q_1}{r^2}\,\hat{z}$

$\vec{F}_{m,21} = -k_m\dfrac{v_1v_2q_1q_2}{r^2}\,\hat{y}$

Likewise:

$\vec{F}_{m,12} = k_m\dfrac{v_1v_2q_1q_2}{r^2}\,\hat{y}$

S-4 *(from TX-3e)*

Note that $\vec{v}_2$ and $\vec{B}(\vec{r}_2)$ are mutually perpendicular so:

$$|\vec{v}_2 \times \vec{B}(\vec{r}_2)| = v_2\, B(\vec{r}_2)\,.$$

Also: $\sin\theta = \sin[\tan^{-1}(4\,\text{cm}/10\,\text{cm})] = 0.371$

S-6 *(from TX-3e)*

If the force on wire #1 is toward wire #2 and the force on wire #2 is toward wire #1, this is certainly the case of *attractive* forces.

S-7 *(from TX-3b)*

$q_2 = -e$ since it is an electron (e is a positive number, by definition).

S-8 *(from [S-1])*

$$8.99 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}\frac{(-1.602 \times 10^{-19}\,\mathrm{C})^2}{(0.0001\,\mathrm{m})^2} = 2.3 \times 10^{-20}\,\mathrm{N}$$

S-9 *(from TX-2e, PS-Problem 3b)*

If you are having trouble with signs ($\pm$) or with understanding where $\sin\theta$ came from, see the discussion of vector products and unit vectors in MISN-0-2, MISN-0-121, MISN-0-122, or MISN-0-123. Also be aware that the opposite direction to $+\hat{z}$ is $-\hat{z}$.

S-10 *(from [S-2])*

$$(8.99 \times 10^9\,\mathrm{N\,m^2\,C^{-2}})\frac{(1.0\,\mathrm{C})(2.75 \times 10^8\,\mathrm{m/s})}{(0.25 \times 10^{-9}\,\mathrm{m})^2(3.0 \times 10^8\,\mathrm{m/s})^2} = 4.4 \times 10^{20}\,\mathrm{T}$$

MODEL EXAM

$$k_e = 8.99 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}; \qquad k_m = 10^{-7}\,\mathrm{N\,s^2\,C^{-2}}$$

$$\text{elementary charge } \equiv e = 1.602 \times 10^{-19}\,\mathrm{C}$$

1. See Output Skills K1-K2 in this module's *ID Sheet*.

2. Given two charges with these specifications:

$$\begin{aligned} Q_1 &= -1.0 \times 10^4\,\mathrm{C} \\ Q_2 &= -2.0 \times 10^4\,\mathrm{C} \\ \vec{r}_1 &= r_1\hat{x} = (3\,\mathrm{cm})\,\hat{x} \\ \vec{r}_2 &= r_2\hat{x} = (4\,\mathrm{cm})\,\hat{x} \\ \vec{v}_1 &= v_1\hat{y} = (3 \times 10^{-4}\,\mathrm{m/s})\,\hat{y} \\ \vec{v}_2 &= v_2\hat{y} = (4 \times 10^{-4}\,\mathrm{m/s})\,\hat{y} \end{aligned}$$

   a. Draw a diagram showing the positions and velocities of the charges.
   b. Starting with the expression for $\vec{E}$ due to a point charge, derive the algebraic answer for the total force of Q_2 on Q_1.
   c. Determine the numerical value of the force of part (b).

Brief Answers:

1. See this module's *text*.

2. See Problem 6 in this module's *ID Sheet*.