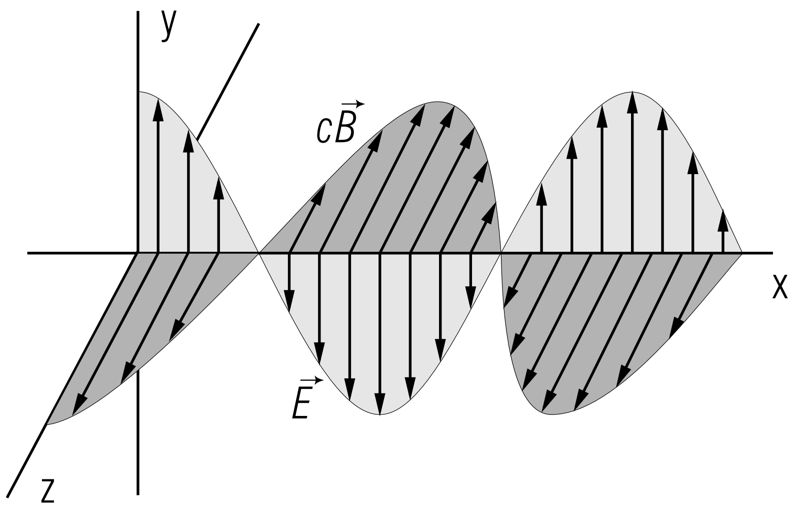
MISN-0-121

INTRODUCTION TO ELECTRICITY AND MAGNETISM



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

INTRODUCTION TO ELECTRICITY AND MAGNETISM
by
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Title: **Introduction to Electricity and Magnetism**

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Input Skills:

1. Vocabulary: force, Newton's laws, weight, acceleration (MISN-0-14), particle (MISN-0-7).

Output Skills (Knowledge):

K1. Vocabulary: electric charge, electric field, magnetic field.

K2. State how one can determine whether an electric force is present between two objects.

K3. State how one can tell whether an electric field is present.

K4. State how one can tell whether a magnetic field is present.

K5. Describe the retardation effect.

Post-Options:

1. "Coulomb's Law" (MISN-0-114).

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1. Introduction

1a. Electricity and Magnetism in General. The study of general electricity and magnetism involves the interaction of charged particles with each other through their electric and magnetic fields. "Charged particles" here means particles that are electrically charged, which means particles that can exert forces on each other which obey the equations governing electric and magnetic interactions. For two objects, both of which are less than the size of mountains, the electric force between the objects, if non-zero, is generally much larger than the gravitational force between the objects. In fact, the gravitational force between two ordinary objects is usually too small to be detectable, while the electric and magnetic forces between the two objects can be devastatingly strong, as in lightning and in other electrical discharges.

1b. Seeing Whether an Electric Force is Present. Suppose you have two objects that are obviously exerting a force on each other and you want to know whether the force is gravitational or electric. These are competing choices because electric and gravitational forces both have the same function of radius, $1/r^2$, where r is the distance between the objects. That means they cannot be distinguished from each other by their dependence on r.[1] The way to identify the force involved is simple: (1) measure each object's mass by weighing it or by measuring each object's acceleration under a known external force; (2) use the Law of Universal Gravitation[2] to calculate the gravitational force that does exist between the two objects; and (3) compare that gravitational force to the total $1/r^2$ force that actually exists between the two objects. If the total $1/r^2$ force doesn't equal the gravitational force, then an electric force must be present and the two objects must be charged electrically. Two other ways of saying it are that they must have electric charge or must be charges.

1c. The Nature of Electric Charge. When an object is electrically charged, it usually means that one of these things have happened: (1) some valence electrons have left the object, leaving it with a net positive charge;[3] or (2) some valence electrons from another object have attached themselves to the object at hand, giving it a net negative charge; or (3) some valence electrons have moved to one side of the object, making that side electrically negative and leaving the other side electrically positive.

[1] The magnetic force has other unique properties by which it can be identified.

[2] The Law: $F = Gm_1m_2/r^2$.

1d. The Equations Governing E & M. There are five equations that one must learn how to use in electricity and magnetism, a set of four called "Maxwell's Equations" plus one called "the Lorentz Force." Maxwell's equations can be stated in either differential or integral form: we shall use the integral form since it is simpler mathematically and it is the form more often used for applications. The Lorentz Force equation is algebraic. Once you have calculated the fields from Maxwell's Equations, and then the forces those fields exert on charged particles, you still need to use Newton's laws to determine the resulting motion of the charged particles.

2. Fields

2a. Fields vs. Particles. The primary quantities in electricity and magnetism are particles and fields, and these are very different concepts.

A particle has a specific location at any particular time. For example, we may write $\vec{r}_A(t)$ as the location of particle A. This means the position of particle A, at some particular time t, can be found by substituting that value of t into the expression $\vec{r}_A(t)$. The result is the radius vector from the origin to the particle's position at that instant. Thus at any instant there are three space numbers giving the particle's location (these might be the particle's three Cartesian coordinates, its x, y, and z values, or it might be a radius and two angles).

A field has a value at *every* point in space at every instant of time, in contrast to a particle which has only a single position in space at any one time. An example of a field is temperature, which has a value at each point in space at any one time. We write this functional dependence as $T(\vec{r}, t)$. If we replace the general symbols $\vec{r}$ and t by a particular point in space and a particular time t, and we get a number that is the temperature at that point at that time. Another example of a field is air pressure, where at any one time points in space have values for that physical quantity.

[3] Recall that electrons have negative charge and that, if all valence electrons (and no others) are present, an object will be electrically neutral.

▷ Write down two other quantities that obey the physics definition of a "field."

2b. The Electric Field. There is a non-zero value for the electric field $\vec{E}$ at a point in space at a particular time if a charged particle, being at that point at that time, would feel a force that is linearly proportional to the particle's charge in this way:

$$\vec{F} = q\vec{E}\,, \tag{1}$$

where q is the charge on the particle. The charge itself can be measured by letting it interact with a standard charge,[4] using the law for the interaction of two charged particles:

$$|\vec{F}| = k_e \frac{q_1 q_2}{r^2}\,, \tag{2}$$

where k_e is a universal constant with the value $9 \times 10^9\,\mathrm{N\,m^2/C^2}$. The symbol "C" denotes the unit of charge, the coulomb. Thus we might say that a particular particle has an electrical charge of $2.54 \times 10^{-9}\,\mathrm{C}$.

2c. The Magnetic Field. There is a non-zero value for the magnetic field $\vec{B}$ at a point in space at a particular time if a charged particle, being at that point at that time, would feel a force that is linearly proportional to the particle's charge *and velocity* in this way:[5]

$$\vec{F} = q\vec{v} \times \vec{B}\,. \tag{3}$$

Here q is the electric charge on the particle. Note that the force is at right angles to both the velocity vector v and the magnetic field vector, and that the magnetic field force is zero if the velocity is along the direction of the magnetic field B. Thus to test for the presence of a magnetic field B we must try several orientations for the charged particle's velocity.

A common way of testing for the presence of a significant magnetic field in some region of space is to bring a compass there and see if it deflects. A compass has internal electric charge moving in circles.[6] The force that makes the compass turn is just the force the external magnetic field force exerts on that internal circulating charge.

[4] As with all units, a more-or-less-arbitrary object is set up as the standard and all other objects are compared to this standard in order to determine their values.

[5] The "×" symbol in this equation indicates a vector product, also called a "cross product." See the Appendix.

[6] This circulating charge is internal to the electron and is called its "spin."

3. Producing the Fields

3a. The Electric Field. An electric field *is produced* by: (1) one or more charged particles; and/or (2) a time-changing magnetic field. Examples of the production by charged particles: (i) the electric field in a capacitor due to charged particles on its plates; and (ii) the electric field of the earth due to charged particles on its surface. An example of production by a time-changing magnetic field: the electric field in an inductor due to an alternating current that produces an alternating magnetic field.

3b. The Magnetic Field. A magnetic field is produced by: (1) an electric current (a stream of charged particles); and/or (2) a time-changing electric field. Examples of magnetic fields produced by electric currents: (i) magnets; (ii) the magnetic field of the earth (from internal currents inside the earth); and (iii) the magnetic field in a transformer. An example of a magnetic field produced by a time-changing electric field is an electromagnetic wave, such as a radio or TV wave, or an X-ray, or light. Here the electric field is oscillating sinusoidally and that produces a sinusoidally oscillating magnetic wave in step with the electric wave. In fact, the sinusoidally-varying magnetic wave also produces the electric wave so the two feed on each other. The result is that the wave can escape its charged-particle source and travel through space as an independent quantity.[7]

3c. The Equations Summarized. Here is what each of the four Maxwell's Equations implies:

- electric charges produce electric fields;
- there is no magnetic charge to produce magnetic fields, no magnetic analog to electric charge;
- time-changing magnetic fields produce electric fields; and
- electric currents and time-changing electric fields produce magnetic fields.

The Lorentz Force on a particle with charge q and velocity $\vec{v}$, moving in both an electric field E and a magnetic field B, is:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}\,. \tag{4}$$

[7] Of course that is precisely what radio and TV waves, X-rays, and light waves actually do.

4. Retardation

Finally, we mention the so-called "retardation" effect, wherein the force felt by one charged particle due to another is not instantaneous. For example, suppose two positive charges are near each other so each exerts a measurable repulsive force on the other. Because the charges are "like" charges, being of the same sign (positive), the forces on particle #1 points directly away from particle #2. Now suppose particle #1 is suddenly moved to a new location. The force on particle #2 must shift to point directly away from the new location of particle #1, but this shift only occurs after a period of time has elapsed. The length of this time delay is exactly the amount of time it would take a signal traveling at the speed of light to go from object #1 to object #2, informing it of particle #1's new location. This delay is called the "retardation" effect, but the times involved are so short that for many purposes the delay can be considered to be zero. That is what we will assume here, but the proper non-zero time must be used in some modern high-speed communication designs.

Acknowledgments

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Glossary

- **electric charge**: the basic property of a piece of matter that defines the strength of its interaction with other matter through electromagnetic forces. The SI unit of electric charge is the coulomb.
- **electric field**: a vector quantity at each space-point, being, at any particular space-point, the force per unit charge experienced by a test charge placed at that space-point. The electric field is written $\vec{E}$ and its SI unit is newtons/coulomb or, equivalently, volts/meter.
- **magnetic field**: a (psuedo-) vector quantity at each space-point, being, at any particular space-point, the force per unit charge per unit velocity normal to the field and to the force, experienced by a test charge with velocity $\vec{v}$ crossing that space-point. Technically, the magnetic field $\vec{B}$ is such that the force on a test charge q traveling with velocity

$\vec{v}$ obeys: $\vec{F} = q\vec{v} \times \vec{B}$. The SI unit of magnetic field is the tesla $\equiv$ newtons per ampere per meter.

A. The Vector Product

The vector product of, say, $\vec{A}$ and $\vec{B}$, is itself a vector and we will show separately how to find its magnitude and direction. The vector product is sometimes called the "cross product" because of the way it is indicated visually:

$$\vec{C} = \vec{A} \times \vec{B}.$$

The magnitude of $\vec{C}$ is defined as magnitude of $\vec{A}$ times the magnitude of $\vec{B}$ times the sine of the angle between them:

$$|\vec{C}| = |\vec{A} \times \vec{B}| = AB \sin\theta.$$

The direction of the vector $\vec{C}$ is perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$ and that narrows it to one of two directions. We will here describe two rules for obtaining the exact direction.

The "right-hand rule":

1. Extend the index finger of your right hand in the direction of the first vector ($\vec{A}$ in the example) with the rest of the fingers closed.

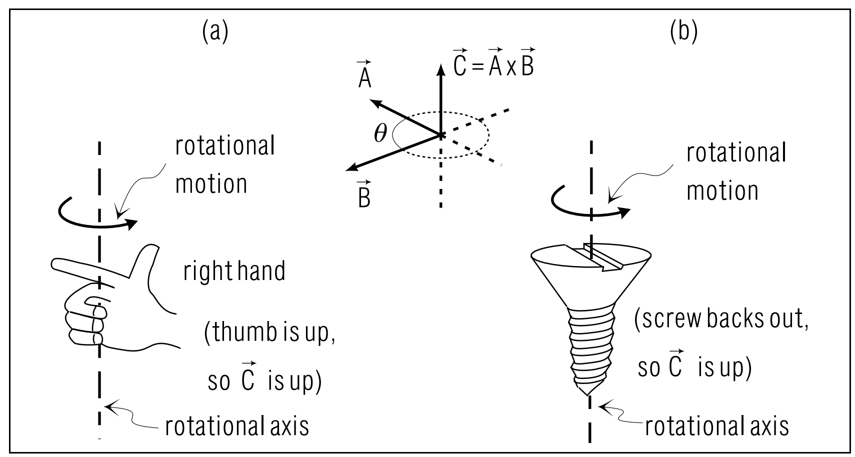

Figure 1. Two rules for finding the direction of $\vec{C} = \vec{A} \times \vec{B}$ by: (a) the "right hand" rule; (b) the "screw" rule.

2. Rotate the index finger and hand until the index finger aligns with the second vector ($\vec{B}$ in the above example). That is, the first of the two vectors denoted in the cross product is rotated toward the second through the smaller angle between them ($\vec{A}$ rotated toward $\vec{B}$ such that θ in Fig. 5 decreases) and the curled fingers of the right hand follow this rotation.

3. The direction of the extended thumb gives you the direction of $\vec{C}$ (see Fig. 5).

The "screw" rule:

1. Imagine placing vectors $\vec{A}$ and $\vec{B}$ tail-to-tail, as in Fig. 5.
2. Imagine a screw with right hand threads placed where the tails of the two vectors come together, with the axis of the screw perpendicular to the plane formed by the two vectors (see Fig. 1).
3. Imagine a screwdriver placed in the screw with the axis of the screwdriver being along the axis of the screw.
4. Imagine a spot on the screw, next to vector $\vec{A}$.
5. Turn the screwdriver through the shortest angle so the spot is now next to vector $\vec{B}$. The direction the screw went (in or out) is the direction of the product vector $\vec{C}$ (see Fig. 1).

▷ Show that the vector product of two vectors is zero if the two are parallel.

▷ Show that the vector product of a vector with itself is zero.

MODEL EXAM

1. See Output Skills K1-K5 in this module's *ID Sheet*.

Brief Answers:

1. See this module's *text*.